CONTENTS

➤➤➤ NUMBER SYSTEM ◀◀◀

CLASSIFICATION OF NUMBERS

(I) Natural numbers:

Set of all non-fractional number from 1 to $+ \infty$, N = {1,2,3,4,....}.

(II) Whole numbers :

Set of numbers from 0 to $+ \infty$, W = {0,1,2,3,4,.....}.

(III) Integers :

Set of all-non fractional numbers from ∞ to $+ \infty$, I or Z = (...., -3,-2,-1,0,1,2,3,....}.

(IV) Rational numbers :

These are real numbers which can be expressed in the form of p/q, where **p** and **q** are integers and $q \neq 0$.

e.g. 2/3, 37/15, -17/19.

❖   All natural numbers, whole numbers and integers are rational.

❖   Rational numbers include all Integers (without any decimal part to it), terminating fractions (fractions in which the decimal parts terminating e.g. 0.75, - 0.02 etc.) and also non-terminating but recurring decimals e.g. 0.666...., -2.333....., etc.

Fractions :

(a) Common fraction : Fractions whose denominator is not 10.

(b) Decimal fraction : Fractions whose denominator is 10 or any power of 10.

(c) Proper fraction : Numerator < Denominator i.e. $\dfrac{3}{5}$.

(d) Improper fraction : Numerator > Denominator i.e. $\dfrac{5}{3}$.

(e) Mixed fraction : Consists of integral as well as fractional part i.e. $3\dfrac{2}{7}$.

(f) Compound fraction : Fraction whose numerator and denominator themselves are fractions. i.e. $\dfrac{2/3}{5/7}$.

❖ Improper fraction can be written in the form of mixed fractions.

(v) Irrational Numbers :

All real number which are not rational are irrational numbers. These are non-recurring as well as non-terminating type of decimal numbers e.g. $\sqrt{2}, \sqrt[3]{4}, 2+\sqrt{3}, \sqrt{2+\sqrt{3}}, \sqrt[4]{\sqrt[7]{3}}$ etc.

(vi) Real numbers : Number which can represent actual physical quantities in a meaningful way are known as **real numbers.** These can be represented on the number line. Number line in geometrical straight line with arbitrarily defined zero (origin).

(vii) Prime number : All natural numbers that have one and itself only as their factors are called prime numbers i.e. prime numbers are exactly divisible by 1 and themselves. e.g. 2,3,5,7,11,13,17,19,23....etc. If P is the set of prime number then **P** = {2,3,5,7....}.

(viii) Composite numbers : All natural number, which are not prime are composite numbers. If C is the set of composite number then C = {4,6,8,9,10,12,.....}.

❖ 1 is neither prime nor composite number.

(ix) Co-prime numbers : If the H.C.F. of the given numbers (not necessarily prime) is 1 then they are known as co-prime numbers. e.g. 4, 9, are co-prime as H.C.F. of (4, 9) = 1.

❖ Any two consecutive numbers will always be co-prime.

(x) Even Numbers : All integers which are divisible by 2 are called even numbers. Even numbers are denoted by the expression 2n, where n is any integer. So, if E is a set even numbers, then E = {...., -4, -2, 0, 2, 4,....}.

(xi) Odd Numbers: All integers which are not divisible by 2 are called odd numbers. Odd numbers are denoted by the general expression 2n - 1 where n is any integer. If O is a set of odd numbers, then O = {...., -5, -3, -1, 1, 3, 5,....}.

(xii) Imaginary Numbers: All the numbers whose square is negative are called imaginary numbers. e.g. 3i, 4i, i, ... where i = $\sqrt{-1}$.

(xiii) Complex Numbers : The combined form of real and imaginary numbers is known as complex numbers. It is denoted by Z = A + iB where A is real part and B is imaginary part of Z and A, B ∈ R.

❖ The set of complex number is the super set of all the sets of numbers.

IDENTIFICATION PRIME NUMBER

Step 1 : Find approximate square root of given number.

Step 2 : Divide the given number by prime numbers less than approximate square root of number. If given number is not divisible by any of this prime number then the number is prime otherwise not.

Ex.1 571, is it a prime ?

Sol. Approximate square root of 571 = 24.

Prime number < 24 are 2, 3, 5, 7, 11, 13, 17, 19, & 23. But 571 is not divisible by any of these prime numbers so 571 is a prime number.

Ex.2 Is 1 prime or composite number ?

Sol. 1 is neither prime nor composite number.

REPRESENTATIO FO RATIONAL NUMBER OF A REAL NUMBER LINE

(i) 3/7 Divide a unit into 7 equal parts.

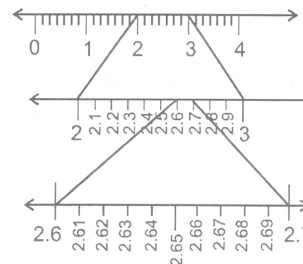(ii) $\dfrac{13}{7}$

(iii) $-\dfrac{4}{9}$

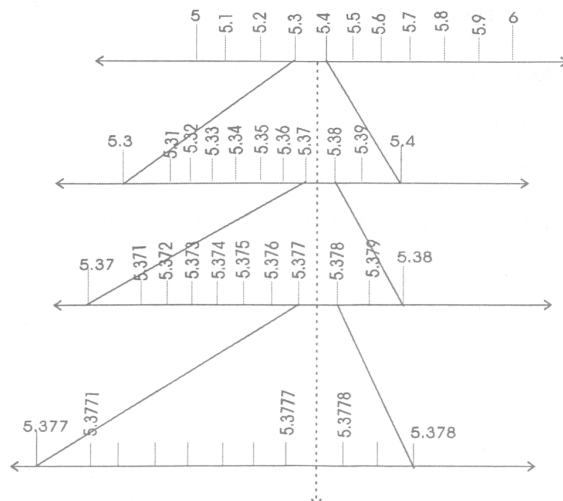**(a) Decimal Number (Terminating) :**

(i) 2.5

(ii) 2.65 (process of magnification)

Ex.3 Visualize the representation of $5.3\overline{7}$ on the number line upto 5 decimal place. i.e. 5.37777.

(b) Find Rational Numbers Between Two Integral Numbers :

Ex.4 Find 4 rational numbers between 2 and 3.

Sol. **Steps :**

(i) Write 2 and 3 multipling in N^r and D^r with (4+1).

(ii) i.e. $2 \dfrac{2 \times (4+1)}{(4+1)} = \dfrac{1}{5}$ & $3 = \dfrac{3 \times (4+1)}{(4+1)} = \dfrac{15}{5}$

(iii) So, the four required numbers are $\dfrac{11}{5}, \dfrac{12}{5}, \dfrac{13}{5}, \dfrac{14}{5}$.

Ex.5 Find three rational no's between a and b (a < b).

Sol.   a < b

$\Rightarrow$ a + a < b + a

$\Rightarrow$ 2a < a + b

$\Rightarrow$ $a < \dfrac{a+b}{2}$

Again,   a < b

$\Rightarrow$ a + b < b + b.

$\Rightarrow$ a + b < 2b

$\Rightarrow$ $\dfrac{a+b}{2} < b.$

$\therefore$ $a < \dfrac{a+b}{2} < b.$

i.e. $\dfrac{a+b}{2}$ lies between a and b.

Hence 1st rational number between a and b is $\dfrac{a+b}{2}$.

For next rational number

$$\dfrac{a + \dfrac{a+b}{2}}{2} = \dfrac{\dfrac{2a+a+b}{2}}{2} = \dfrac{3a+b}{4} \quad \therefore \quad a < \dfrac{3a+b}{4} < \dfrac{a+b}{2} < b.$$

Next, $\dfrac{\dfrac{a+b}{2} + b}{2} = \dfrac{a+b+2b}{2 \times 2} = \dfrac{a+3b}{4}$

$\therefore$ $a < \dfrac{3a+b}{4} < \dfrac{a+b}{2} < \dfrac{a+3b}{4} < b$, and continues like this.

Ex.6 Find 3 rational numbers between $\dfrac{1}{3}$ & $\dfrac{1}{2}$.

Sol. 1st Method $\dfrac{\dfrac{1}{3} + \dfrac{1}{2}}{2} = \dfrac{\dfrac{2+3}{6}}{2} = \dfrac{5}{12}$ $\therefore$ $\dfrac{1}{3}, \dfrac{5}{12}, \dfrac{1}{2}$

$= \dfrac{\dfrac{1}{3} + \dfrac{5}{12}}{2} = \dfrac{\dfrac{4+5}{12}}{2} = \dfrac{9}{24}$ $\therefore$ $\dfrac{1}{3}, \dfrac{9}{24}, \dfrac{5}{12}, \dfrac{1}{2}$

$= \dfrac{\dfrac{5}{12} + \dfrac{1}{2}}{2} = \dfrac{\dfrac{5}{12} + \dfrac{6}{12}}{2} = \dfrac{11}{24}$ $\therefore$ $\dfrac{1}{3}, \dfrac{9}{24}, \dfrac{5}{12}, \dfrac{11}{24}, \dfrac{1}{2}$.

Verify : $\dfrac{8}{24} < \dfrac{9}{24} < \dfrac{10}{24} < \dfrac{11}{24} < \dfrac{12}{24} \left(as \dfrac{8}{24} = \dfrac{1}{3} \& \dfrac{1}{2} \right)$

2nd Method : Find n rational numbers between a and b (a < b).

(i) Find d = $\dfrac{b-a}{n+1}$.

(ii) 1st rational number will be a + d.
2nd rational number will be a + 2d.
3rd rational number will be a + 3d and so on....
nth rational number is a + nd.

Ex.7 Find 5 rational number between $\dfrac{3}{5}$ and $\dfrac{4}{5}$

Here, $a = \dfrac{3}{5}, b = \dfrac{4}{5} \quad d = \dfrac{b-a}{n+1} = \dfrac{\dfrac{4}{5} - \dfrac{3}{5}}{5+1} = \dfrac{1}{5} \times \dfrac{1}{6} = \dfrac{1}{30}$.

$1st = a + b = \dfrac{3}{5} + \dfrac{1}{30} = \dfrac{19}{20}$, $2nd = a + 2d = \dfrac{3}{5} + \dfrac{2}{30}$,

$3rd = a + 3d = \dfrac{3}{5} + \dfrac{3}{30} = \dfrac{21}{30}$, $4th = a + 4d = \dfrac{3}{5} + \dfrac{4}{30} = \dfrac{22}{30}$,

$5th = a + 5d = \dfrac{3}{5} + \dfrac{5}{30} = \dfrac{23}{30}$.

RATIONAL NUMBER IN DECIMAL REPRESENTATION

(a) Terminating Decimal :

In this a finite number of digit occurs after decimal i.e. $\dfrac{1}{2}$ = 0.5, 0.6875, 0.15 etc.

(b) Non-Terminating and Repeating (Recurring Decimal) :

In this a set of digits or a digit is repeated continuously.

Ex.8 $\dfrac{2}{3} = 0.6666 ------- = 0.\overline{6}$.

Ex.9 $\dfrac{5}{11} = 0.454545 ------ = 0.\overline{45}$.

PROPERTIES OF RATIONAL NUMBER

If a,b,c are three rational numbers.

(i) Commutative property of addition. a + b = b + a (ii) Associative property of addition (a+b)+c = a+(b+c)

(iii) Additive inverse a + (-a) = 0 0 is identity element, -a is called additive inverse of a.

(iv) Commutative property of multiplications a.b. = b.a. (v) Associative property of multiplication (a.b).c = a.(b.c)

(vi) Multiplicative inverse $(a) \times \left(\dfrac{1}{a} \right) = 1$

1 is called multiplicative identity and $\dfrac{1}{a}$ is called multiplicative inverse of a or reciprocal of a.

(vii) Distributive property a.(b+c) = a.b + a.c

∴   L.H.S. ≠ R.H.S.

Hence in contradicts our assumption that $2 + \sqrt{3}$ rational.

∴ $2 + \sqrt{3}$ is irrational.

Ex.9   Prove that $\sqrt{3} - \sqrt{2}$ is an irrational number

Sol.   Let $\sqrt{3} - \sqrt{2} = r$ where r be a rational number

Squaring both sides

$\Rightarrow$ $\left(\sqrt{3} - \sqrt{2}\right)^2 = r^2$

$\Rightarrow$ $3 + 2 - 2\sqrt{6} = r^2$

$\Rightarrow$ $5 - 2\sqrt{6} = r^2$

Here, $5 - 2\sqrt{6}$ is an irrational number but r^2 is a rational number

∴   L.H.S. ≠ R.H.S.

Hence it contradicts our assumption that $\sqrt{3} - \sqrt{2}$ is a rational number.

(b) Irrational Number in Decimal Form :

$\sqrt{2}$ = 1.414213 ...... i.e. it is not-recurring as well as non-terminating.

$\sqrt{3}$ = 1.732050807 ...... i.e. it is non-recurring as well as non-terminating.

Ex.10   Insert an irrational number between 2 and 3.

Sol. $\sqrt{2 \times 3} = \sqrt{6}$

Ex.11   Find two irrational number between 2 and 2.5.

Sol.   1st **Method :** $\sqrt{2 \times 2.5} = \sqrt{5}$

Since there is no rational number whose square is 5. So $\sqrt{5}$ is irrational..

Also $\sqrt{2 \times \sqrt{5}}$ is a irrational number.

2nd Method : 2.101001000100001.... is between 2 and 5 and it is non-recurring as well as non-terminating.

Also, 2.201001000100001......... and so on.

Ex.12   Find two irrational number between $\sqrt{2}$ and $\sqrt{3}$.

Sol.   **1st Method :** $\sqrt{\sqrt{2} \times \sqrt{3}} = \sqrt{\sqrt{6}} = \sqrt[4]{6}$ Irrational number between $\sqrt{2}$ and $\sqrt[4]{6}$

$\sqrt{\sqrt{2} \times \sqrt[4]{6}} = \sqrt[4]{2} \times \sqrt[8]{6}$

2nd Method : As $\sqrt{2}$ = 1.414213562 ...... and $\sqrt{3}$ = 1.732050808......

As , $\sqrt{3} > \sqrt{2}$ and $\sqrt{2}$ has 4 in the 1st place of decimal while $\sqrt{3}$ has 7 is the 1st place of decimal.

∴   1.501001000100001......., 1.601001000100001...... etc. are in between $\sqrt{2}$ and $\sqrt{3}$

Ex.13 Find two irrational number between 0.12 and 0.13.

Sol. 0.1201001000100001......., 0.12101001000100001 ....etc.

Ex.14 Find two irrational number between 0.3030030003..... and 0.3010010001 ....

Sol. 0.302020020002...... 0.302030030003.... etc.

Ex.15 Find two rational number between 0.2323323332..... and 0.252552555255552.......

Sol. 1st place is same 2.

2nd place is 3 & 5.
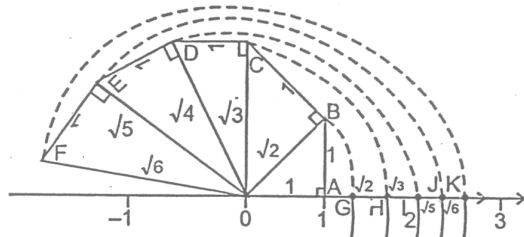
3rd place is 2 in both.

4th place is 3 & 5.

Let a number = 0.25, it falls between the two irrational number.

Also a number = 0.2525 an so on.

(c) Irrational Number on a Number Line :

Ex.16 Plot $\sqrt{2}, \sqrt{3}, \sqrt{5}\sqrt{6}$ on a number line.

Sol.

Another Method for :

(i) Plot $\sqrt{2}, \sqrt{3}$

So, OC = $\sqrt{2}$ and OD = $\sqrt{3}$
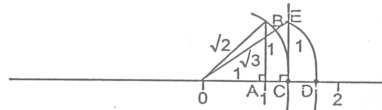
(ii) Plot $\sqrt{5}, \sqrt{6}, \sqrt{7}\sqrt{8}$

OC = $\sqrt{5}$

OD = $\sqrt{6}$ OH = $\sqrt{7}$ ....
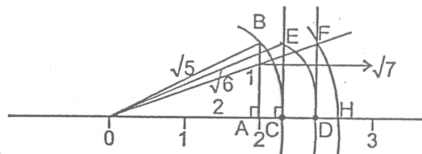
(d) Properties of Irrational Number :

(i) Negative of an irrational number is an irrational number e.g. $-\sqrt{3} - \sqrt[4]{5}$ are irrational.

(ii) Sum and difference of a rational and an irrational number is always an irrational number.

(iii) Sum and difference of two irrational numbers is either rational or irrational number.

(iv) Product of a non-zero rational number with an irrational number is either rational or irrationals

(v) Product of an irrational with a irrational is not always irrational.

Ex.17 Two number's are 2 and $\sqrt{3}$, then

Sum = 2 + $\sqrt{3}$, is an irrational number.

Difference = 2 - $\sqrt{3}$, is an irrational number.

Also $\sqrt{3} - 2$ is an irrational number.

Ex.18 Two number's are 4 and $\sqrt[3]{3}$, then

Sum = 4 + $\sqrt[3]{3}$, is an irrational number.

Difference = 4 - $\sqrt[3]{3}$, is an irrational number.

Ex.19 Two irrational numbers are $\sqrt{3}, -\sqrt{3}$, then

Sum = $\sqrt{3} + \left(-\sqrt{3}\right) = 0$ which is rational.

Difference = $\sqrt{3} - \left(-\sqrt{3}\right) = 2\sqrt{3}$, which is irrational.

Ex.20 Two irrational numbers are 2 + $\sqrt{3}$ and 2 - $\sqrt{3}$, then

Sum = $\left(2 + \sqrt{3}\right) + \left(2 - \sqrt{3}\right) = 4$, a rational number

Two irrational numbers are $\sqrt{3} + 3m\sqrt{3} - 3$

Difference = $\sqrt{3} + 3 - \sqrt{3} + 3 = 6$, a rational number

Ex.21 Two irrational numbers are $\sqrt{3} - \sqrt{2}, \sqrt{3} + \sqrt{2}$, then

Sum = $\sqrt{3} - \sqrt{2} + \sqrt{3} + \sqrt{2} = 2\sqrt{3}$, an irrational

Ex.22 2 is a rational number and $\sqrt{3}$ is an irrational.

$2 \times \sqrt{3} = 2\sqrt{3}$, an irrational.

Ex.23 0 a rational and $\sqrt{3}$ an irrational.

$0 \times \sqrt{3} = 0$, a rational.

Ex.24 $\dfrac{4}{3} \times \sqrt{3} = \dfrac{4}{3}\sqrt{3} = \dfrac{4}{\sqrt{3}}$ is an irrational.

Ex.25 $\sqrt{3} \times \sqrt{3} = \sqrt{3 \times 3} = \sqrt{9} = 3$ a rational number.

Ex.26 $2\sqrt{3} \times 3\sqrt{2} = 2 \times 3\sqrt{3 \times 2} = 6\sqrt{6}$ and irrational number.

Ex.27 $\sqrt[3]{3} \times \sqrt[3]{3^2} = \sqrt[3]{3 \times 3^2} = \sqrt[3]{3^3} = 3$ a rational number.

Ex.28 $\left(2 + \sqrt{3}\right)\left(2 - \sqrt{3}\right) = (2)^2 - \left(\sqrt{3}\right)^2 = 4 - 3 = 1$ a rational number.

Ex.29 $(2+\sqrt{3})(2+\sqrt{3}) = (2+\sqrt{3})^2$

$$= (2)^2 + (\sqrt{3})^2 + 2(2) \times (\sqrt{3})$$

$$= 4 + 3 + 4\sqrt{3}$$

$$= 7 + 4\sqrt{3} \text{ an irrational number}$$

NOTE :

(i) $\sqrt{-2} \ne -\sqrt{2}$, it is not a irrational number.

(ii) $\sqrt{-2} \times \sqrt{-3} \ne (\sqrt{-2 \times -3} = \sqrt{6})$

Instead $\sqrt{-2}, \sqrt{-3}$ are called Imaginary numbers.

$\sqrt{-2} = i\sqrt{2}$, where i (= iota) = $\sqrt{-1}$

$\therefore$ (A) $i^2 = -1$

     (B) $i^3 = i^2 \times i = (-1) \times i = -i$

     (C) $i^4 = i^2 \times i^2 = (-1) \times (-1) = 1$

(iii) Numbers of the type (a + ib) are called complex numbers where (a, b) $\in$ R.e.g. 2 + 3i, -2 + 4i, -3i, 11 - 4i, are complex numbers.

GEOMETRICAL REPRESENTATION OF REAL NUMBERS

To represent any real number of number line we follows the following steps :

STEP I : Obtain the positive real number x (say).

STEP II : Draw a line and mark a point A on it.

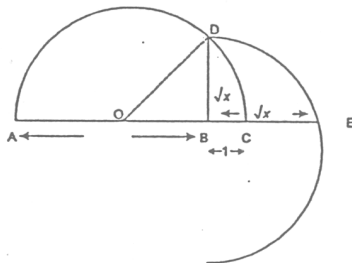STEP III : Mark a point B on the line such that AB = x units.

STEP IV : From point B mark a distance of 1 unit and mark the new point as C.

STEP V : Find the mid - point of AC and mark the point as O.

STEP VI : Draw a circle with centre O and radius OC.

STEP VII : Draw a line perpendicular to AC passing through B and intersecting the semi circle at D.

Length BD is equal to $\sqrt{x}$.

4. Examine whether the following numbers are rational or irrational :

(i) $\left(2-\sqrt{3}\right)^2$ (ii) $\left(\sqrt{2}+\sqrt{3}\right)^2$ (iii) $\left(3+\sqrt{2}\right)\left(3-\sqrt{2}\right)$ (iv) $\dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

5. Represent $\sqrt{8.3}$ on the number line.

6. Represent $(2+\sqrt{3})$ on the number line.

7. Prove that $\left(\sqrt{2}+\sqrt{5}\right)$ is an irrational number.

8. Prove that $\sqrt{7}$ is not a rational number.

9. Prove that $(2+\sqrt{2})$ is an irrational number.

10. Multiply $\sqrt{27a^3b^2c^4}\times\sqrt[3]{128a^7b^9c^2}\times\sqrt[6]{729ab^{12}c^2}$.

11. Express the following in the form of p/q.

(i) $0.\overline{3}$ (ii) $0.\overline{37}$ (iii) $0.\overline{54}$ (iv) $0.\overline{05}$ (v) $1.\overline{3}$ (vi) $0.\overline{621}$

12. Simplify : $0.\overline{4}+.0\overline{18}$

➤➤➤ NUMBER SYSTEM ◄◄◄

SURDS

Any irrational number of the form $\sqrt[n]{a}$ is given a special name surd. Where 'a' is called radicand, it should always be a rational number. Also the symbol $\sqrt[n]{}$ is called the radical sign and the index n is called order of the surd.

$\sqrt[n]{a}$ is read as 'nth **root a**' and can also be written as $a^{\frac{1}{n}}$.

(a) Some Identical Surds :

(i) $\sqrt[3]{4}$ is a surd as radicand is a rational number.

Similar examples $\sqrt[3]{5},\sqrt[4]{12},\sqrt[5]{7},\sqrt{12},\ldots\ldots$

(i) $2\sqrt{3}$ is a surd (as surd + rational number will give a surd)

Similar examples $\sqrt{3}+1,\sqrt[3]{3}+1,\ldots$

(iii) $\sqrt{7 - 4\sqrt{3}}$ is a surd as $7 - 4\sqrt{3}$ is a perfect square of $\left(2 - \sqrt{3}\right)$

Similar examples $\sqrt{7 + 4\sqrt{3}}, \sqrt{9 - 4\sqrt{5}}, \sqrt{9 + 4\sqrt{5}}, \ldots\ldots$

(i) $\sqrt[3]{\sqrt{3}}$ is a surd as $\sqrt[3]{\sqrt{3}} = \left(3^{\frac{1}{2}}\right)^{\frac{1}{3}} = 3^{\frac{1}{6}} = \sqrt[6]{3}$

Similar examples $\sqrt[3]{\sqrt[3]{5}}, \sqrt[4]{\sqrt[5]{6}}, \ldots\ldots\ldots$

(b) Some Expression are not Surds :

(i) $\sqrt[3]{8}$ because $\sqrt[3]{8} = \sqrt[3]{2^3} = 2$, which is a rational number.

(ii) $\sqrt{2 + \sqrt{3}}$ because $2 + \sqrt{3}$ is not a perfect square.

(iii) $\sqrt[3]{1 + \sqrt{3}}$ because radicand is an irrational number.

LAWS OF SURDS

(i) $\left(\sqrt[n]{a}\right)^n = \sqrt[n]{a^n} = a$

e.g. (A) $\sqrt[3]{8} = \sqrt[3]{2^3} = 2$ (B) $\sqrt[4]{81} = \sqrt[4]{3^4} = 3$

(ii) $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$ [Here order should be same]

e.g. (A) $\sqrt[3]{2} \times \sqrt[3]{6} = \sqrt[3]{2 \times 6} = \sqrt[3]{12}$

but, $\sqrt[3]{3} \times \sqrt[4]{6} \neq \sqrt{3 \times 6}$ [Because order is not same]

1st make their order same and then you can multiply.

(iii) $\sqrt[n]{a} + \sqrt[n]{b} = \sqrt[n]{\dfrac{a}{b}}$

(iv) $\sqrt[n]{\sqrt[m]{a}} = \sqrt[nm]{a} = \sqrt[m]{\sqrt[n]{a}}$ e.g. $= \sqrt{\sqrt{\sqrt{2}}} = \sqrt[8]{8}$

(v) $\sqrt[n]{a} = \sqrt[n \times p]{a^p}$ [Important for changing order of surds]

or, $\sqrt[n]{a^m} = \sqrt[n \times p]{a^{m \times p}}$

e.g. $\sqrt[3]{6^2}$ make its order 6, then $\sqrt[3]{6^2} = \sqrt[3 \times 2]{6^{2 \times 2}} = \sqrt[6]{6^4}$.

e.g. $\sqrt[3]{6}$ make its order 15, then $\sqrt[3]{6} = \sqrt[3 \times 5]{6^{1 \times 5}} = \sqrt[15]{6^5}$.

OPERATION OF SURDS

(a) Addition and Subtraction of Surds :

Addition and subtraction of surds are possible only when order and radicand are same i.e. only for surds.

Ex.1 Simplify

(i) $\sqrt{6} - \sqrt{216} + \sqrt{96} = 15\sqrt{6} - \sqrt{6^2 \times 6} + \sqrt{16 \times 6}$ [Bring surd in simples form]

$\qquad = 15\sqrt{6} - 6\sqrt{6} + 4\sqrt{6}$

$\qquad = (15 - 6 + 4)\sqrt{6}$

$\qquad = 13\sqrt{6}$ **Ans.**

(ii) $5\sqrt[3]{250} + 7\sqrt[3]{16} - 14\sqrt[3]{54}$ $\quad = 5\sqrt[3]{125 \times 2} + 7\sqrt[3]{8 \times 2} - 14\sqrt[3]{27 \times 2}$

$\qquad\qquad = 5 \times 5\sqrt[3]{2} + 7 \times 2\sqrt[3]{2} - 14 \times 3 \times \sqrt[3]{2}$

$\qquad\qquad = (25 + 14 - 42)\sqrt[3]{2}$

$\qquad\qquad = -3\sqrt[3]{2}$ **Ans.**

(ii) $5\sqrt[3]{250} + 7\sqrt[3]{16} - 14\sqrt[3]{54}$ $\quad = 5\sqrt[3]{125 \times 2} + 7\sqrt[3]{8 \times 2} - 14\sqrt[3]{27 \times 2}$

$\qquad\qquad = 5 \times 5\sqrt[3]{2} + 7 \times 2\sqrt[3]{2} - 14 \times 3 \times \sqrt[3]{2}$

$\qquad\qquad = (25 + 14 - 42)\sqrt[3]{2}$

$\qquad\qquad = -3\sqrt[3]{2}$ **Ans.**

(iii) $4\sqrt{3} + 3\sqrt{48} - \dfrac{5}{2}\sqrt{\dfrac{1}{3}} = 4\sqrt{3} + 3\sqrt{16 \times 3} - \dfrac{5}{2}\sqrt{\dfrac{1 \times 3}{3 \times 3}}$

$\qquad\qquad = 4\sqrt{3} + 3 \times 4\sqrt{3} - \dfrac{5}{2} \times \dfrac{1}{3}\sqrt{3}$

$\qquad\qquad = 4\sqrt{3} + 12\sqrt{3} - \dfrac{5}{6}\sqrt{3}$

$\qquad\qquad = \left(4 + 12 - \dfrac{5}{6}\right)\sqrt{3}$

$\qquad\qquad = \dfrac{91}{6}\sqrt{3}$ **Ans.**

(b) Multiplication and Division of Surds :

Ex.2 (i) $\sqrt[3]{4} \times \sqrt[3]{22} = \sqrt[3]{4 \times 22} = \sqrt[3]{2^3 \times 11} = 2\sqrt[3]{11}$

(i) $\sqrt[3]{2} \times \sqrt[4]{3} = \sqrt[12]{2^4} \times \sqrt[12]{3^3} = \sqrt[12]{2^4 \times 3^3} = \sqrt[12]{16 \times 27} = \sqrt[12]{432}$

Ex.3 Simplify $\sqrt{8a^5 b} \times \sqrt[3]{4a^2 b^2}$

Hint : $\sqrt[6]{8^3 a^{15} b^3} \times \sqrt[6]{4^2 a^4 b^4} = \sqrt[6]{2^{13} a^{19} b^7} = \sqrt[6]{2ab}$. **Ans.**

Ex.4   Divide $\sqrt{24} \div \sqrt[3]{200} = \dfrac{\sqrt{24}}{\sqrt[3]{200}} = \dfrac{\sqrt[6]{(24)^3}}{\sqrt[6]{(200)^2}} = \sqrt[6]{\dfrac{216}{625}}$ **Ans..**

(c) Comparison of Surds :

It is clear that if $x > y > 0$ and $n > 1$ is a positive integer then $\sqrt[n]{x} > \sqrt[n]{y}$.

Ex.5 $\sqrt[3]{16} > \sqrt[3]{12}, \sqrt[5]{35} > \sqrt[5]{25}$ and so on.

Ex.6   Which is greater is each of the following :

(i) $\sqrt[3]{16}$ and $\sqrt[5]{8}$ (ii) $\sqrt{\dfrac{1}{2}}$ and $\sqrt[3]{\dfrac{1}{3}}$

L.C.M. of 3 and 5 15.   L.C.M. of 2 and 3 is 6.

$\sqrt[3]{6} = \sqrt[3 \times 5]{6^5} = \sqrt[15]{7776}$ $\sqrt[6]{\left(\dfrac{1}{2}\right)^3}$ and $\sqrt[3]{\left(\dfrac{1}{3}\right)^2}$

$\sqrt[5]{8} = \sqrt[3 \times 5]{8^5} = \sqrt[15]{512}$ $\sqrt[6]{\dfrac{1}{8}}$ and $\sqrt[6]{\dfrac{1}{9}}$ $\left[\text{As} \, 8 < 9 \therefore \dfrac{1}{8} > \dfrac{1}{9}\right]$

$\therefore$ $\sqrt[75]{7776} > \sqrt[15]{512}$ so, $\sqrt[6]{\dfrac{1}{8}} > \sqrt[6]{\dfrac{1}{9}}$

$\Rightarrow$ $\sqrt[3]{6} > \sqrt[5]{8}$ $\Rightarrow$ $\sqrt{\dfrac{1}{2}} > \sqrt[3]{\dfrac{1}{3}}$

Ex.7   Arrange $\sqrt{2}, \sqrt[3]{3}$ and $\sqrt[4]{5}$ is ascending order.

Sol.   L.C.M. of 2, 3, 4 is 12.

$\therefore$ $\sqrt{2} = \sqrt[2 \times 6]{2^6} = \sqrt[12]{64}$

     $\sqrt[3]{3} = \sqrt[3 \times 4]{3^4} = \sqrt[12]{81}$

     $\sqrt[4]{5} = \sqrt[4 \times 3]{5^3} = \sqrt[12]{125}$

As, $64 < 81 < 125$.

$\therefore$ $\sqrt[12]{64} < \sqrt[12]{81} < \sqrt[12]{125}$

$\Rightarrow$ $\sqrt{2} < \sqrt[3]{3} < \sqrt[4]{5}$

Ex.8   Which is greater $\sqrt{7} - \sqrt{3}$ or $\sqrt{5} - 1$?

Sol. $\sqrt{7} - \sqrt{3} = \dfrac{(\sqrt{7} - \sqrt{3})(\sqrt{7} + \sqrt{3})}{(\sqrt{7} + \sqrt{3})} = \dfrac{7 - 3}{\sqrt{7} + \sqrt{3}} = \dfrac{4}{\sqrt{7} + \sqrt{3}}$

And, $\sqrt{5} - 1 = \dfrac{(\sqrt{5} - 1)(\sqrt{5} + 1)}{(\sqrt{5} + 1)} = \dfrac{5 - 1}{\sqrt{5} + 1} = \dfrac{4}{\sqrt{5} + 1}$

Now, we know that $\sqrt{7} > \sqrt{5}$ and $\sqrt{3} > 1$, add

So, $\sqrt{7} + \sqrt{3} > \sqrt{5} + 1$

$$\Rightarrow \quad \frac{1}{\sqrt{7}+\sqrt{3}} < \frac{1}{\sqrt{5}+1} \quad \Rightarrow \quad \frac{4}{\sqrt{7}+\sqrt{3}} < \frac{4}{\sqrt{5}+1} \quad \Rightarrow \quad \sqrt{7}-\sqrt{3} < \sqrt{5}-1$$

So, $\sqrt{5}-1 > \sqrt{7}-\sqrt{3}$

RATIONALIZATION OF SURDS

Rationalizing factor product of two surds is a rational number then each of them is called the rationalizing factor (R.F.) of the other. The process of converting a surd to a rational number by using an appropriate multiplier is known as **rationalization.**

Some examples :

(i) R.F. of $\sqrt{a}$ is $\sqrt{a}$ $\left(\because \sqrt{a} \times \sqrt{a} = a\right)$.

(ii) R.F. of $\sqrt[3]{a}$ is $\sqrt[3]{a^2}$ $\left(\because \sqrt[3]{a} \times \sqrt[3]{a^2} = \sqrt[3]{a^3} = a\right)$.

(iii) R.F. of $\sqrt{a}+\sqrt{b}$ is $\sqrt{a}-\sqrt{b}$ & vice versa $\left[\because \left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right) = a-b\right]$.

(iv) R.F. of $a+\sqrt{b}$ is $a-\sqrt{b}$ & vice versa $\left[\because \left(a+\sqrt{b}\right)\left(a-\sqrt{b}\right) = a^2-b\right]$

(v) R.F. of $\sqrt[3]{a}+\sqrt[3]{b}$ is $\left(\sqrt[3]{a^2}-\sqrt[3]{ab}+\sqrt[3]{b^2}\right)\left[\because \left(\sqrt[3]{a}+\sqrt[3]{b}\right)\left(\sqrt[3]{a^2}-\sqrt[3]{ab}+\sqrt[3]{b^2}\right)\right]$

$\left[\because \left(\sqrt[3]{a}\right)^3 + \left(\sqrt[3]{b}\right)^3 = a+b\right]$ which is rational.

(vi) R.F. of $\left(\sqrt{a}+\sqrt{b}+\sqrt{c}\right)$ is $\left(\sqrt{a}+\sqrt{b}-\sqrt{c}\right)$ nd $\left(a+b-c+2\sqrt{ab}\right)$.

Ex.9 Find the R.G. (rationalizing factor) of the following :

(i) $\sqrt{10}$ (ii) $\sqrt{12}$ (iii) $\sqrt{162}$ (iv) $\sqrt[3]{4}$ (v) $\sqrt[3]{16}$ (vi) $\sqrt[4]{162}$ (vii) $2+\sqrt{3}$

(viii) $7-4\sqrt{3}$ (ix) $3\sqrt{3}+2\sqrt{2}$ (x) $\sqrt[3]{3}+\sqrt[3]{2}$ (xi) $1+\sqrt{2}+\sqrt{3}$

(i) $\sqrt{10}$

Sol. $[\because \sqrt{10} \times \sqrt{10} = \sqrt{10 \times 10} = 10]$ as 10 is rational number.

$\therefore$ R.F. of $\sqrt{10}$ is $\sqrt{10}$ **Ans.**

(ii). $\sqrt{12}$ **Sol.**   First write it's simplest from i.e. $2\sqrt{3}$.

Now find R.F. (i.e. R.F. of $\sqrt{3}$ is $\sqrt{3}$)

$\therefore$ R.F. of $\sqrt{12}$ is $\sqrt{3}$ **Ans.**

(iii) $\sqrt{162}$ **Sol.**   Simplest from of $\sqrt{162}$ is $9\sqrt{2}$. R.F. of $\sqrt{2}$ is $\sqrt{2}$.

$\therefore$ R.F. of $\sqrt{162}$ is $\sqrt{2}$ **Ans.**

(iv) $\sqrt[3]{4}$ **Sol.** $\sqrt[3]{4} \times \sqrt[3]{4^2} = \sqrt[3]{4^3} = 4$ $\therefore$ **R.F.** of $\sqrt[3]{4}$ is $\sqrt[3]{4^2}$ **Ans.**

(v). $\sqrt[3]{16}$

Sol.   Simplest from of $\sqrt[3]{16}$ is $2\sqrt[3]{2}$ Now R.F. of $\sqrt[3]{2}$ is $\sqrt[3]{2^2}$

$\therefore$ R.F. of $\sqrt[3]{16}$ is $\sqrt[3]{2^2}$ **Ans.**

(vi) $\sqrt[4]{162}$

Sol. Simplest form of $\sqrt[4]{162}$ is $3\sqrt[4]{2}$

Now R.F. of $\sqrt[4]{2}$ is $\sqrt[4]{2^3}$

R.F. of $(\sqrt[4]{162})$ is $\sqrt[4]{2^3}$ **Ans.**

(vii) $2+\sqrt{3}$

Sol. As $\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)=(2)^2-\left(\sqrt{3}\right)^2=4-3=1$, , which is rational.

∴ R.F. of $(2+\sqrt{3})$ is $(2-\sqrt{3})$ **Ans.**

(viii) $7-4\sqrt{3}$

Sol. As $\left(7-4\sqrt{3}\right)\left(7+4\sqrt{3}\right)=(7)^2-\left(4-\sqrt{3}\right)^2$ = 49 - 48 = 1, which is rational

∴ R.F. of $(7-4\sqrt{3})$ is $\left(7+4\sqrt{3}\right)$ **Ans.**

(ix). $3\sqrt{3}+2\sqrt{2}$

Sol. As $\left(3\sqrt{3}+2\sqrt{2}\right)\left(3\sqrt{3}-2\sqrt{2}\right)=\left(3\sqrt{3}\right)^2-\left(2\sqrt{2}\right)^2=27-8=19$, which is rational.

∴ R.F. of $(3\sqrt{3}+2\sqrt{2})$ is $(3\sqrt{3}-2\sqrt{2})$ **Ans.**

(x) $\sqrt[3]{3}+\sqrt[3]{2}$

Sol. As $\left(\sqrt[3]{3}+\sqrt[3]{2}\right)\left(\sqrt[3]{3^2}-\sqrt[3]{3}\times\sqrt[3]{2}+\sqrt[3]{2^2}\right)=\left(\sqrt[3]{3^3}+\sqrt[3]{2^3}\right)$ = 3 + 2 = 5, which is rational.

∴ R.F. of $(\sqrt[3]{3}+\sqrt[3]{2})$ is $\left(\sqrt[3]{3^2}-\sqrt[3]{3}\times\sqrt[3]{2}+\sqrt[3]{2^2}\right)$ **Ans.**

(xi) $1+\sqrt{2}+\sqrt{3}$

Sol. $\left(1+\sqrt{2}+\sqrt{3}\right)\left(1+\sqrt{2}-\sqrt{3}\right)=\left(1+\sqrt{2}\right)^2-\left(\sqrt{3}\right)^2$

$$=1)^2+\left(\sqrt{2}\right)^2+2(1)\left(\sqrt{2}\right)-3$$

$$=1+2+2\sqrt{2}-3$$

$$=3+2\sqrt{2}-3$$

$$=2\sqrt{2}$$

$$2\sqrt{2}\times\sqrt{2}=2\times2=4$$

∴ R.F. of $1+\sqrt{2}+\sqrt{3}$ is $\left(1+\sqrt{2}-\sqrt{3}\right)$ and $\sqrt{2}$. **Ans.**

NOTE : R.F. of $\sqrt{a}+\sqrt{b}$ or $\sqrt{a}-\sqrt{b}$ type surds are also called conjugate surds & vice versa.

Ex.10 (i) $2 - \sqrt{3}$ is conjugate of $2 + \sqrt{3}$

(ii) $\sqrt{5} + 1$ is conjugate of $\sqrt{5} - 1$

NOTE : Sometimes conjugate surds and reciprocals are same.

Ex.11 (i) $2 + \sqrt{3}$, it's conjugate is $2 - \sqrt{3}$, its reciprocal is $2 - \sqrt{3}$ & vice versa.

(ii) $5 - 2\sqrt{6}$, it's conjugate is $5 + 2\sqrt{6}$, its reciprocal is $5 - 2\sqrt{6}$ & vice versa.

(iii) $6 - \sqrt{35}, 6 + \sqrt{35}$

(iv) $7 - 4\sqrt{3}, 7 + 4\sqrt{3}$

(v) $8 + 3\sqrt{7}, 8 - 3\sqrt{7}$ .... and so on.

Ex.12 Express the following surd with a rational denominator.

Sol. $\dfrac{8}{\sqrt{15} + 1 - \sqrt{5} - \sqrt{3}} = \dfrac{8}{\left[\left(\sqrt{15} + 1\right) - \left(\sqrt{15} + \sqrt{3}\right)\right]} \times \left[\dfrac{\left(\sqrt{15} + 1\right) + \left(\sqrt{5} + \sqrt{3}\right)}{\left(\sqrt{15} + 1\right) + \left(\sqrt{5} + \sqrt{3}\right)}\right]$

$= \dfrac{8\left(\sqrt{15} + 1 + \sqrt{5} + \sqrt{3}\right)}{\left(\sqrt{15} + 1\right)^2 - \left(\sqrt{5} + \sqrt{3}\right)^2}$

$= \dfrac{8\left(\sqrt{15} + 1 + \sqrt{5} + \sqrt{3}\right)}{15 + 1 + 2\sqrt{15} - \left(5 + 3 + 2\sqrt{15}\right)}$

$= \dfrac{8\left(\sqrt{15} + 1 + \sqrt{5} + \sqrt{3}\right)}{8}$

$= (\sqrt{15} + 1 + \sqrt{5} + \sqrt{3})$　　　　**Ans.**

Ex.13 Rationalize the denominator of $\dfrac{a^2}{\sqrt{a^2 + b^2} + b}$

Sol. $\dfrac{a^2}{\sqrt{a^2 + b^2} + b} = \dfrac{a^2}{\sqrt{a^2 + b^2} + b} \times \dfrac{\sqrt{a^2 + b^2} - b}{a^2 + b^2 - b}$

$= \dfrac{a^2\left(\sqrt{a^2 + b^2} - b\right)}{\left(\sqrt{a^2 + b^2}\right)^2 - (b)^2}$

$= \dfrac{a^2\left(\sqrt{a^2 + b^2} - b\right)}{a^2 + b^2 - b^2} = \left(\sqrt{a^2 + b^2} - b\right)$　　　　**Ans.**

Ex.14   If $\dfrac{3+2\sqrt{2}}{3-\sqrt{2}} = a + b\sqrt{2}$, where a and b are rational then find the values of a and b.

Sol.   L.H.S. $\dfrac{3+2\sqrt{2}}{3-\sqrt{2}} = \dfrac{(3+2\sqrt{2})(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})}$

$$= \dfrac{9 + 3\sqrt{2} + 6\sqrt{2} + 4}{9-2}$$

$$= \dfrac{13 + 9\sqrt{2}}{7}$$

$$= \dfrac{13}{7} + \dfrac{9}{7}\sqrt{2}$$

$\therefore$ $\dfrac{13}{7} + \dfrac{9}{7}\sqrt{2} = a + b\sqrt{2}$

Equating the rational and irrational parts

We get $a = \dfrac{13}{7}, b = \dfrac{9}{7}$ **Ans.**

Ex.15   If $\sqrt{3} = 1.732$, find the value of $\dfrac{1}{\sqrt{3}-1}$

Sol. $\dfrac{1}{\sqrt{3}-1} = \dfrac{1}{\sqrt{3}-1} \times \dfrac{\sqrt{3}+1}{\sqrt{3}+1}$

$$= \dfrac{\sqrt{3}+1}{3-1}$$

$$= \dfrac{\sqrt{3}+1}{2}$$

$$= \dfrac{1.732+1}{2}$$

$$= \dfrac{2.732}{2}$$

$$= 1.366 \qquad\qquad\qquad \textbf{Ans.}$$

Ex.16   If $\sqrt{5}$ = 2.236 and $\sqrt{2}$ = 1.414, then

Evaluate : $\dfrac{3}{\sqrt{5}+\sqrt{2}} + \dfrac{4}{\sqrt{5}-\sqrt{2}}$

Sol. $\dfrac{3}{\sqrt{5}+\sqrt{2}} + \dfrac{4}{\sqrt{5}-\sqrt{2}} = \dfrac{3\sqrt{5}-\sqrt{2} + 4(\sqrt{5}+\sqrt{2})}{(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})}$

$$= \dfrac{3\sqrt{5} - 3\sqrt{2} + 4\sqrt{5} + 4\sqrt{2}}{5-2}$$

$$= \dfrac{7\sqrt{5} + \sqrt{2}}{5-2}$$

$$= \frac{7\sqrt{5} + \sqrt{2}}{3}$$

$$= \frac{7 \times 2.236 + 1.414}{3}$$

$$= \frac{15.652 + 1.414}{3}$$

$$= \frac{17.066}{3}$$

= 5.689 (approximate)

Ex.17 If $= \dfrac{1}{2 + \sqrt{3}}$ find the value of $x^3 - x^2 - 11x + 3$.

Sol. As, $x = \dfrac{1}{2 + \sqrt{3}} = 2 - \sqrt{3}$

$\Rightarrow$ $x - 2 = -\sqrt{3}$

$\Rightarrow$ $(x - 2)^2 = \left(-\sqrt{3}\right)^2$ [By squaring both sides]

$\Rightarrow$ $x^2 + 4 - 4x = 3$

$\Rightarrow$ $x^2 - 4x + 1 = 0$

Now, $x^3 - x^2 - 11x + 3 = x^3 - 4x^2 + x + 3x^2 - 12x + 3$

$\qquad\qquad\qquad\qquad = x(x^2 - 4x + 1) + 3(x^2 - 4x + 1)$

$\qquad\qquad\qquad\qquad = x(0) + 3(0)$

$\qquad\qquad\qquad\qquad = 0 + 0 = 0$ **Ans.**

Ex.18 If $x = 3 - \sqrt{8}$, find the value of $x^3 + \dfrac{1}{x^3}$.

Sol. $x = 3 - \sqrt{8}$

$\therefore$ $\dfrac{1}{x} = \dfrac{1}{3 - \sqrt{8}}$

$\Rightarrow$ $\dfrac{1}{x} = 3 + \sqrt{8}$

Now, $x + \dfrac{1}{x} = 3 - \sqrt{8} + 3 + \sqrt{8} = 6$

$\Rightarrow$ $x^3 + \dfrac{1}{x^3} = \left(x + \dfrac{1}{x}\right)^3 - 3x\dfrac{1}{x}\left(x + \dfrac{1}{x}\right)$

$\Rightarrow$ $x^3 + \dfrac{1}{x^3} = (6)^3 - 3(6)$

$\Rightarrow$ $x^3 + \dfrac{1}{x^3} = 216 - 18$

$\Rightarrow$ $x^3 + \dfrac{1}{x^3} = 198$ **Ans.**

Ex.19 If $x = 1 + 2^{1/3} + 2^{2/3}$, show that $x^3 - 3x^2 - 3x - 1 = 0$

Sol. $x = 1 + 2^{1/3} + 2^{2/3}$

$\Rightarrow$ $x - 1 (2^{1/3} + 2^{2/3})$

$\Rightarrow$ $(x - 1)^3 = (2^{1/3} + 2^{2/3})^3$

$\Rightarrow$ $(x - 1)^3 = (2^{1/3}) + (2^{2/3})^3 + 3.2^{1/3}.2^{2/3}(2^{1/3} + 2^{1/3})$

$\Rightarrow$ $(x - 1)^3 = 2 + 2^2 + 3.2^1 (x - 1)$

$\Rightarrow$ $(x - 1)^3 = 6 + 6 (x - 1)$

$\Rightarrow$ $x^3 - 3x^2 + 3x - 1 = 6x$

$\Rightarrow$ $x^3 - 3x^2 - 3x - 1 = 0$ **Ans.**

Ex.20 Solve : $\sqrt{x+3} + \sqrt{x-2} = 5$.

Sol. $\sqrt{x+3} = 5 - \sqrt{x-2}$

$\Rightarrow$ $\left(\sqrt{x+3}\right)^2 = \left(5 - \sqrt{x-2}\right)^2$ [By squaring both sides]

$\Rightarrow$ $x + 3 = 25 + (x - 2) - 10\sqrt{x-2}$

$\Rightarrow$ $x + 3 = 25 + x - 2 - 10\sqrt{x-2}$

$\Rightarrow$ $3 - 23 = -10\sqrt{x-2}$

$\Rightarrow$ $-20 = -10\sqrt{x-2}$

$\Rightarrow$ $2 = \sqrt{x-2}$

$\Rightarrow$ $x - 2 = 4$ [By squaring both sides]

$\Rightarrow$ $x = 6$ **Ans.**

Ex.21 If $x = 1 + \sqrt{2} + \sqrt{3}$, prove that $x^4 - 4x^3 - 4x^2 + 16 - 8 = 0$.

Hint : $x = 1 + \sqrt{2} + \sqrt{3}$

$\Rightarrow$ $x - 1 = \sqrt{2} + \sqrt{3}$

$\Rightarrow$ $(x-1)^2 = \left(\sqrt{2} + \sqrt{3}\right)^2$ [By squaring both sides]

$\Rightarrow$ $x^2 + 1 - 2x = 2 + 3 + 2\sqrt{6}$

$\Rightarrow$ $x^2 - 2x - 4 = 2\sqrt{6}$

$\Rightarrow$ $(x^2 - 2x - 4)^2 = (2\sqrt{6})^2$

$\Rightarrow$ $x^4 + 4x^2 + 16 - 4x^3 + 16x - 8x^2 = 24$

$\Rightarrow$ $x^4 - 4x^3 - 4x^2 + 16x + 16 - 24 = 0$

$\Rightarrow$ $x^4 - 4x^3 - 4x^2 + 16x - 8 = 0$ **Ans.**

EXPONENTS OF REAL NUMBER

(a) Positive Integral Power :

For any real number a and a positive integer 'n' we define a^n as :

$a^n = a \times a \times a \times \ldots\ldots\ldots\ldots\ldots \times a$ (n times)

a^n is called then n^{th} power of a. The real number 'a' is called the base and 'n' is called the exponent of the nth power of a.

e.g. $2^3 = 2 \times 2 \times 2 = 8$

NOTE : For any non-zero real number 'a' we define $a^0 = 1$.

e.g. thus, $3^0 = 1$, 5^0, $\left(\dfrac{3}{4}\right)^0 = 1$ and so on.

(b) Negative Integral Power :

For any non-zero real number 'a' and a positive integer 'n' we define $a^{-n} = \dfrac{1}{a^n}$

Thus we have defined a^n find all integral values of n, positive, zero or negative. a^n is called the nth power of a.

RATIONAL EXPONENTS OR A REAL NUMBER

(a) Principal of nth Root of a Positive Real Numbers :

If 'a' is a positive real number and 'n' is a positive integer, then the principal nth root of a is the unique positive real number x such that x^n = a.

The principal nth root of a positive real number a is denoted by $a^{1/n}$ or $\sqrt[n]{a}$.

(b) Principal of nth Root of a Negative Real Numbers :

If 'a' is a negative real number and 'n' is an odd positive integer, then the principle nth root of a is define as - $|a|^{1/n}$ i.e. the principal nth root of -a is negative of the principal nth root of $|a|$.

Remark :

It 'a' is negative real number and 'n' is an even positive integer, then the principle nth root of a is not defined, because an even power of real number is always positive. Therefore $(-9)^{1/2}$ is a meaningless quantity, if we confine ourselves to the set of real number, only.

(c) Rational Power (Exponents) :

For any positive real number 'a' and a rational number $\dfrac{p}{q} \neq$ where $q \neq 0$, we define $a^{p/q} = (a^p)^{1/q}$ i.e. $a^{p/q}$ is the principle qth root of a^p.

LAWS OF RATIONAL EXPONETNS

The following laws hold the rational exponents

(i) $a^m \times a^n = a^{m+1}$
(ii) $a^m \div a^n = a^{m-n}$

(ii) $(a^m)^n = a^{mn}$
(iv) $a^{-n} = \dfrac{1}{a^n}$

(v) $a^{m/n} = (a^m)^{1/n} = (a^{1/n})^m$ i.e. $a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$
(vi) $(ab)^m = a^m b^m$

(vii) $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$
(viii) $a^{bn} = a^{b+b+b\ldots n \, tmes}$

Where a,b are positive real number and m,n are relational numbers.

ILLUSTRATIONS :

Ex.22 Evaluate each of the following:

(i) $5^2 \times 5^4$
(ii) $5^8 \div 5^3$
(iii) $\left(3^2\right)^2$
(iv) $\left(\dfrac{11}{12}\right)^3$
(v) $\left(\dfrac{3}{4}\right)^{-3}$

Sol. Using the laws of indices, we have

(i) $5^2 . 5^4 = 5^{2+4} = 5^6 = 15625$ $\because a^m \times a^n = a^{m+n}$

(ii) $5^8 \div 5^3 = \dfrac{5^8}{5^3} = 5^{8-3} = 5^5 = 3125$ $\because a^m + a^n = a^{m-n}$

(iii) $\left(3^2\right)^3 = 3^{2\times3} = 3^6 = 729$ $\because (a^m)^n = a^{m\times n}$

(iv) $\left(\dfrac{11}{12}\right)^3 = \dfrac{11^3}{12^3} = \dfrac{1331}{1728}$ $\because \left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$

(v) $\left(\dfrac{3}{4}\right)^{-3} = \dfrac{1}{\left(\dfrac{3}{4}\right)^3} = \dfrac{1}{\dfrac{3^3}{4^3}} = \dfrac{1}{\dfrac{27}{64}} = \dfrac{64}{27}$ $\because a^{-n} = \dfrac{1}{a^n}$

Ex.23 **Evaluate each of the following :**

(i) $\left(\dfrac{2}{11}\right)^4 \times \left(\dfrac{11}{3}\right)^2 \times \left(\dfrac{3}{2}\right)^3$ (ii) $\left(\dfrac{1}{2}\right)^5 \times \left(\dfrac{-2}{3}\right)^4 \times \left(\dfrac{3}{5}\right)^{-1}$

(iii) $2^{55} \times 2^{60} - 2^{97} \times 2^{18}$ (iv) $\left(\dfrac{2}{3}\right)^3 \times \left(\dfrac{2}{5}\right)^{-3} \times \left(\dfrac{3}{5}\right)^2$

Sol. We have.

(i) $\left(\dfrac{2}{11}\right)^4 \times \left(\dfrac{11}{3}\right)^2 \times \left(\dfrac{3}{2}\right)^3 = \dfrac{2^4}{11^4} \times \dfrac{11^2}{3^2} \times \dfrac{3^3}{2^3}$

$$= \dfrac{2 \times 3}{11^2}$$

$$= \dfrac{6}{121} \qquad \textbf{Ans.}$$

(ii) We have,

$$\left(\dfrac{1}{2}\right)^5 \times \left(\dfrac{-2}{3}\right)^4 \times \left(\dfrac{3}{5}\right)^{-1} = \left(\dfrac{1}{2}\right)^5 \times \left(\dfrac{-2}{3}\right)^4 \times \left(\dfrac{\dfrac{1}{3}}{5}\right)$$

$$= \dfrac{1^5}{2^5} \times \dfrac{(-2)^2}{3^4} \times \dfrac{5}{3}$$

$$= \dfrac{1 \times 16 \times 5}{32 \times 81 \times 3}$$

$$= \dfrac{5}{2 \times 81 \times 3}$$

$$= \dfrac{5}{486} \qquad \textbf{Ans.}$$

(iii) We have,

$$2^{55} \times 2^{60} - 2^{97} \times 2^{18} = 2^{55+60} - 2^{97+18}$$

$$= 2^{15} - 2^{115}$$

$$= 0 \qquad \textbf{Ans.}$$

(iv) We have,

$$\left(\frac{2}{3}\right)^3 \times \left(\frac{2}{5}\right)^{-3} \times \left(\frac{3}{5}\right)^{-2} = \frac{2^3}{3^3} \times \frac{1}{(2/5)} \times \frac{3^2}{5^2}$$

$$= \frac{2^3}{3^3} \times \frac{1}{2^3/5^3} \times \frac{3^2}{5^2}$$

$$= \frac{2^3 \times 5^3 \times 3^2}{3^3 \times 2^3 \times 5^2}$$

$$= \frac{5}{3} \qquad \textbf{Ans.}$$

Ex.24 Simplify :

(i) $\dfrac{(25)^{3/2} \times (243)^{3/5}}{(16)^{5/4} \times (8)^{4/3}}$ \qquad (ii) $\dfrac{16 \times 2^{n+1} - 4 \times 2^n}{16 \times 2^{n+2} - 2 \times 2^{n+2}}$

Sol. We have,

(i) $\dfrac{(25)^{3/2} \times (243)^{3/5}}{(16)^{5/4} \times (8)^{4/3}} = \dfrac{\left(5^2\right)^{3/2} \times \left(3^5\right)^{3/5}}{\left(2^4\right)^{5/4} \times \left(2^3\right)^{4/3}}$

$$= \frac{5^{2 \times 3/2} \times 3^{5 \times 3/5}}{2^{4 \times 5/4} \times 2^{3 \times 4/3}}$$

$$= \frac{5^3 \times 3^3}{2^5 \times 2^4}$$

$$= \frac{125 \times 27}{32 \times 16}$$

$$= \frac{3375}{512} \qquad \textbf{Ans.}$$

(ii) $\dfrac{16 \times 2^{n+1} - 4 \times 2^n}{16 \times 2^{n+2} - 2 \times 2^{n+2}} = \dfrac{2^4 \times 2^{n+1} - 2^2 \times 2^n}{2^4 \times 2^{n+2} - 2 \times 2^{n+2}}$

$$= \frac{2^{n+5} - 2^{n+2}}{2^{n+6} - 2^{n+3}}$$

$$= \frac{2^{n+5} - 2^{n+2}}{2.2^{n+5} - 2.2^{n+2}}$$

$$= \frac{2^{n+5} - 2^{n+2}}{2\left(2^{n+5} - 2^{n+2}\right)} = \frac{1}{2} \qquad \textbf{Ans.}$$

Ex.25 Simplify $\left(\dfrac{81}{16}\right)^{-3/4} \times \left[\left(\dfrac{25}{9}\right)^{-3/2} \div \left(\dfrac{5}{2}\right)^{-3}\right]$

Sol. We have

$$\left(\dfrac{81}{16}\right)^{-3/4} \times \left[\left(\dfrac{25}{9}\right)^{-3/2} \div \left(\dfrac{5}{2}\right)^{-3}\right] = \left(\dfrac{3^4}{2^4}\right)^{-3/4} \times \left[\left(\dfrac{5^2}{3^2}\right)^{-3/2} \div \left(\dfrac{5}{2}\right)^{-3}\right]$$

$$= \left[\left(\dfrac{3}{2}\right)^4\right]^{-3/4} \times \left[\left[\left(\dfrac{5}{3}\right)^2\right]^{-3/2} \div \left[\left(\dfrac{5}{2}\right)^{-3}\right]\right]$$

$$= \left(\dfrac{3}{2}\right)^{4 \times -3/4} \times \left[\left(\dfrac{5}{3}\right)^{2 \times -3/2} \div \left(\dfrac{5}{2}\right)^{-3}\right]$$

$$= \left(\dfrac{3}{2}\right)^{-3} \times \left[\left(\dfrac{5}{3}\right)^{-3} \div \left(\dfrac{5}{2}\right)^{-3}\right]$$

$$= \left(\dfrac{2}{3}\right)^{3} \times \left[\left(\dfrac{5}{3}\right)^{-3} \times \left(\dfrac{5}{2}\right)^{-3}\right]$$

$$= \dfrac{2^3}{3^3} \times \left[\dfrac{3^3}{5^3} \div \dfrac{2^3}{5^3}\right]$$

$$= \dfrac{2^3}{3^3} \times \left[\dfrac{3^3}{5^3} \times \dfrac{5^3}{2^3}\right]$$

$$= 1 \textbf{ Ans.}$$

OBJECTIVE DPP - 3.1

1. If $x = 3 + \sqrt{8}$ and $y = 3 - \sqrt{8}$ then $\dfrac{1}{x^2} + \dfrac{1}{y^2} =$

(A) -34   (B) 34   (C) $12\sqrt{8}$ (D) $-12\sqrt{8}$

2. If $\dfrac{3 + \sqrt{7}}{3 - \sqrt{7}} = a + b\sqrt{7}$ then $(a,b) =$

(A) (8, -3)   (B) (-8, -3)   (C) (-8, 3)   (D) (8, 3)

3. $\dfrac{\sqrt{5} - 2}{\sqrt{5} + 2} - \dfrac{\sqrt{5} + 2}{\sqrt{5} - 2} =$

(A) $8\sqrt{5}$ (B) $-8\sqrt{5}$ (C) $6\sqrt{5}$ (D) $-6\sqrt{5}$

4. If $x = \dfrac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}}$ and $y = 1$, the value of $\dfrac{x - y}{x - 3y}$ is :

(A) $\dfrac{5}{\sqrt{5} - 4}$ (B) $\dfrac{5}{\sqrt{6} + 4}$ (C) $\dfrac{\sqrt{6} - 4}{5}$ (D) $\dfrac{\sqrt{6} + 4}{5}$

5. Which one is greatest in the following :

(A) $\sqrt{2}$ (B) $\sqrt[3]{3}$ (C) $\sqrt[3]{4}$ (D) $\sqrt[3]{2}$

6. The value of $\sqrt[5]{(32)^{-3}}$ is :

(A) 1/8   (B) 1/16   (C) 1/32   (D) None

7. If $x = \dfrac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$ and $y = \dfrac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}}$ the value of $x^2 + xy + y^2$ is :

(A) 99   (B) 100   (C) 1   (D) 0

8. Simplify : $\dfrac{2}{\sqrt{5} + \sqrt{3}} + \dfrac{1}{\sqrt{3} + \sqrt{2}} - \dfrac{3}{\sqrt{5} + \sqrt{2}}$

(A) 1   (B) 0   (C) 10   (D) 100

9. Which of the following is smallest ?

(A) $\sqrt[4]{5}$ (B) $\sqrt[5]{4}$ (C) $\sqrt{4}$ (D) $\sqrt{3}$

10　　The product of $\sqrt{3}$ and $\sqrt[3]{5}$ is :

(A) $\sqrt[6]{375}$　　　　　(B) $\sqrt[6]{675}$　　　　　(C) $\sqrt[6]{575}$　　　　　(D) $\sqrt[6]{475}$

11.　　The exponential from of $\sqrt{\sqrt{2} \times \sqrt{2} \times \sqrt{2}}$ is :

(A) $2^{1/16}$　　　　　(B) $8^{3/4}$　　　　　(C) $2^{3/4}$　　　　　(D) $8^{1/2}$

12.　　The value of x, if $5^{x-3} \cdot 3^{2x-8} = 225$, is :

(A) 1　　　　　(B) 2　　　　　(C) 3　　　　　(D) 5

13.　　If $2^{5x} \div 2^x = \sqrt[5]{2^{20}}$ then x =

(A) 0　　　　　(B) - 1　　　　　(C) $\dfrac{1}{2}$　　　　　(D) 1

14.　　$\sqrt[3]{(729)^{2.5}} =$

(A) $\dfrac{1}{81}$　　　　　(B) 81　　　　　(C) 243　　　　　(D) 729

15.　　$\sqrt[4]{\sqrt[3]{x^2}} =$

(A) x　　　　　(B) $x^{\frac{1}{2}}$　　　　　(C) $x^{\frac{1}{3}}$　　　　　(D) $x^{\frac{1}{6}}$

SUBJECTIVE DPP - 3.2

1.　　Arrange the following surds in ascending order of magnitude :

(i) $4\sqrt{10}, 3\sqrt{6}, \sqrt{3}$　　　　　　　(ii) $3\sqrt{4}, 4\sqrt{5}, \sqrt{3}$

2.　　Whish is greater :

$\sqrt{17} - \sqrt{12}$ or $\sqrt{11} - \sqrt{6}$.

3.　　Simplify : $\dfrac{8}{\sqrt{15} + 1 - \sqrt{5} - \sqrt{3}}$.

4.　　If p and q are rational number and $p - \sqrt{q} = \dfrac{4 + \sqrt{2}}{3 + \sqrt{2}}$ find p and q.

5.　　Find the simplest R.F. of :

(i) $\sqrt[3]{32}$　　　　　(ii) $\sqrt[3]{36}$　　　　(iii) $2^{3/5}$

6.　　Retionalise the denominator :

(i) $\dfrac{3}{\sqrt{5}}$　　　　　　(ii) $\dfrac{\sqrt{2} + \sqrt{5}}{\sqrt{3}}$

7. Retionalise the denominator and simplify :

(i) $\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ (ii) $\dfrac{1+\sqrt{2}}{3+2\sqrt{2}}$ (iii) $\dfrac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$

8. Simplify :

(i) $\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ (ii) $\dfrac{7+3\sqrt{5}}{3+\sqrt{5}}-\dfrac{7-3\sqrt{5}}{3-\sqrt{5}}$

9. Find the value of a and b

(i) $\dfrac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}}=a-b\sqrt{77}$ (ii) $\dfrac{5+\sqrt{6}}{5-\sqrt{6}}=a+b\sqrt{6}$

10. If $x=\dfrac{\sqrt{3}+1}{2}$ find the value of $4x^3 + 2x^2 - 8x + 7$.

11. If $x=\dfrac{5-\sqrt{21}}{2}$ show that $\left(x^3+\dfrac{1}{x^3}\right)-5\left(x^2+\dfrac{1}{x^2}\right)+\left(x+\dfrac{1}{x}\right)=0$.

12. Show that a = x + 1/x, where x = $\dfrac{\sqrt{a+2}+\sqrt{a-2}}{\sqrt{a+2}-\sqrt{a-2}}$.

13. Prove that : $\dfrac{1}{3-\sqrt{8}}-\dfrac{1}{\sqrt{8}-\sqrt{7}}+\dfrac{1}{\sqrt{7}-\sqrt{6}}-\dfrac{1}{\sqrt{6}-\sqrt{5}}+\dfrac{1}{\sqrt{5}-2}=5$.

14. If $x=\dfrac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$ and $y=\dfrac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$ find the value of $3x^2 + 4xy - 3y^2$.

15. Evaluate:

$$\dfrac{\sqrt{\sqrt{5}+2}+\sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}+1}}-\sqrt{3-2\sqrt{2}}.$$

16. If x= 3, b = 4 then find the values of :

(i) $a^b + b^a$ (ii) $a^a + b^b$ (iii) $a^b - b^a$

17. Simplify :

$\left(\sqrt{x}\right)^{-2/3}\sqrt{y^4}\div\sqrt{xy^{-1/2}}$.

18. Simplify :

(i) $\left[16^{-1/5}\right]^{5/2}$ (ii) $[0.001]^{\frac{1}{3}}$

19. If $\dfrac{9^n\times3^2\times\left[3-^{-n/2}\right]^{-2}-(27)^n}{3^{3m}\times2^3}=\dfrac{1}{27}$, then prove than m - n = 1.

20. Find the value of x, if $5^{x-3(2x-3)}=625$.

ANSWER KEY

(Objective DPP # 1.1)

Qus.	1	2	3	4	5	6
Ans.	A	B	C	B	D	C

(Subjective DPP # 1.2)

1. (i) Non-terminating and repeating   (ii) Non-terminating and non-repeating

(iii) Non-terminating and repeating   (iv) Terminating

2. $\dfrac{-7}{6}, \dfrac{-4}{3}, \dfrac{-3}{2}, \dfrac{-5}{3}, \dfrac{-11}{6}$

3. -4, -3, -2, -1

4. $\dfrac{-5}{24}$

5. $\dfrac{22}{7}, \dfrac{23}{7}, \dfrac{24}{7}, \dfrac{25}{7}, \dfrac{26}{7}, \dfrac{27}{7}$

6. $\dfrac{9}{24}, \dfrac{10}{24}, \dfrac{11}{24}$

7. $\dfrac{-5}{14}, \dfrac{-4}{14}, \dfrac{-3}{14}$

(Objective DPP # 2.1)

Qus.	1	2	3	4	5	6	7	8
Ans.	A	B	A	A	C	D	C	C

(Subjective DPP # 2.2)

3. 0.1101010010000100001

4. (i) irrational   (ii) irrational   (iii) rational   (iv) irrational

10. $36A^4B^6C^3\sqrt[6]{108}$

11. (i) 1/3   (ii) $\dfrac{37}{99}$ (iii) $\dfrac{6}{11}$ (iv) $\dfrac{5}{99}$ (v) $\dfrac{4}{3}$ (vi) $\dfrac{4}{3}$ (v) $\dfrac{23}{37}$

12. $\dfrac{19}{30}$

(Objective DPP # 3.1)

Qus.	1	2	3	4	5	6	7	8	9	10
Ans.	B	D	B	D	C	A	A	B	B	B
Qus.	11	12	13	14	15					
Ans.	C	D	D	C	D					

(Subjective DPP # 3.2)

1. (i) $4\sqrt{10} > 3\sqrt{6} > \sqrt{3}$ (ii) $4\sqrt{5} > 3\sqrt{4} > \sqrt{3}$ **2.** $\sqrt{11} - \sqrt{6}$

3. $\sqrt{16} + 1 + \sqrt{5} + \sqrt{5}$ **4.** $P = \dfrac{10}{7}, Q = \dfrac{2}{49}$ **5.** (i) $\sqrt[3]{2}$ (ii) $\sqrt[3]{6}$ (iii) $2^{2/5}$

6. (i) $\dfrac{3}{2}\sqrt{5}$ (ii) $\dfrac{\sqrt{6} + \sqrt{15}}{3}$ **7.** (i) $5 - 2\sqrt{6}$ (ii) $7 + 5\sqrt{2}$ (iii) $\dfrac{9 + 4\sqrt{6}}{15}$

8. (i) 8 (ii) $\sqrt{5}$ **9.** (i) a = 9/2, b = 1/2 (ii) $a = \dfrac{31}{19}, b = \dfrac{10}{19}$

10. 10 **14.** $\dfrac{12 + 56\sqrt{10}}{3}$

15. 1 **16.** (i) 145 (ii) 283 (iii) 17

17. $\dfrac{y^{9/4}}{x^{5/6}}$ **18.** (i) 1/4 (ii) 0.1 **20.** 1

►►► POLYNOMIALS ◄◄◄

POLYNOMIALS

An algebraic expression (f(x) of the form $f(x) = a_0 + a_1x + a_2x^2 + ... + a^nx^n$, where $a_0, a_1, a_2 ...., a_n$ are real numbers and all the index of 'x' are non-negative integers is called a polynomials in x.

(a) Degree of the Polynomial :

Highest Index of x in algebraic expression is called the degree of the polynomial, here $a_0, a_1x , a_2x^2 ..... a_nx^n$, are called the terms o the polynomial and $z_0, a_1, a_2......, a_n$ are called various coefficients of the polynomial f(x).

NOTE : A polynomial in x is said to be in standard form when the terms are written either in increasing order or decreasing order of the indices of x in various terms.

(b) Different Types of Polynomials :

Generally, we divide the polynomials in the following categories.

(i) Based on degrees :

There are four types of polynomials based on degrees. These are listed below :

(A) **Linear Polynomials :** A polynomials of degree one is called a linear polynomial. The general formula of linear polynomial is ax + b, where a and b are any real constant and a ≠ 0.

(B) **Quadratic Polynomials :** A polynomial of degree two is called a quadratic polynomial. The general form of a quadratic polynomial is $ax^2 + b + c$, where a ≠ 0.

(C) **Cubic Polynomials :** A polynomial of degree three is called a cubic polynomial. The general form of a cubic polynomial is $ax^3 + bx^2 + cx + d$, where a ≠ 0.

(D) **Biquadratic (or quadric) Polynomials :** A polynomial of degree four is called a biquadratic (quadratic) polynomial. The general form of a biquadratic polynomial is $ax^4 + bx^3 + cx^2 + dx + e$, where a ≠ 0.

NOTE : A polynomial of degree five or more than five does not have any particular name. Such a polynomial usually called a polynomial of degree five or six or ....etc.

(ii) Based on number of terms

There are three types of polynomials based on number of terms. These are as follows :

(A) Monomial : A polynomial is said to be monomial if it has only one term. e.g. x, $9x^2$, $5x^3$ all are monomials.

(B) Binomial : A polynomial is said to be binomial if it contains two terms e.g. $2x^2 + 3x$, $\sqrt{3} x + 5x^3$, $-8x^3 + 3$, all are binomials.

(C) Trinomials : A polynomial is said to be a trinomial it if contains three terms. e.g. $3x^3 - 8 + \dfrac{5}{2}$,

$\sqrt{7}x^{10} 8x^4 - 3x^2, 5 - 7x + 8x^9$, are all trinomials.

NOTE : A polynomial having four or more than four terms does not have particular Name. These are simply called polynomials.

(iii) Zero degree polynomial : Any non-zero number (constant) is regarded as polynomial of degree zero or zero degree polynomial. i.e. $f(x) = a$, where $a \neq 0$ is a zero degree polynomial, since we can write $f(x) = a$ as $f(x) = ax^0$.

(iv) Zero polynomial : A polynomial whose all coefficients are zeros is called as zero polynomial i.e. $f(x) = 0$, we cannot determine the degree of zero polynomial.

ALGEBRAIC IDENTITY

An identity is an equality which is true for all values of the variables

Some important identities are

(i) $(a + b)^2 = a^2 + 2ab + b^2$

(ii) $(a - b)^2 = a^2 - 2ab + b^2$

(iii) $a^2 - b^2 = (a + b)(a - b)$

(iv) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

(v) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

(vi) $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

(vii) $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

(viii) $a^4 + a^2b^2 + b^4 = (a^2 + ab + b^2)(a^2 - ab + b^2)$

(ix) $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$

Special case : if $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$.

(a) Value Form :

(i) $a^2 + b^2 = (a + b)^2 - 2ab$,	if $a + b$ and ab are given
(ii) $a^2 + b^2 = (a - b)^2 + 2ab$	if $a - b$ and ab are given
(iii) $a + b = \sqrt{(a-b)^2 + 4ab}$	if $a - b$ and ab are given
(iv) $a - b = \sqrt{(a+b)^2 - 4ab}$	if $a + b$ and ab are given
(v) $a^2 + \dfrac{1}{a^2} = \left(a + \dfrac{1}{a}\right)^2 - 2$	if $a + \dfrac{1}{a}$ is given
(vi) $a^2 + \dfrac{1}{a^2} = \left(a + \dfrac{1}{a}\right)^2 + 2$	if $a - \dfrac{1}{a}$ is given
(vii) $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$	if $(a + b)$ and ab are given
(viii) $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$	if $(a - b)$ and ab are given

(ix) $x^3 + \dfrac{1}{a^3} = \left(a + \dfrac{1}{a}\right)^3 - 3\left(a + \dfrac{1}{a}\right)$ if $a + \dfrac{1}{a}$ is given

(x) $a^3 - \dfrac{1}{a^3} = \left(a - \dfrac{1}{a}\right)^3 + 3\left(a - 1\dfrac{1}{a}\right),$ if $\left(a - \dfrac{1}{a}\right)$ is given

(xi) $a^4 + b^4 = (a^2 + b^2)^2 - 2a^2b^2 = [(a + b)^2 - 2ab]^2 - 2a^2b^2$, if $(a + b)$ and ab are given

(xii) $a^4 - b^4 = (a^2 + b^2)(a^2 - b^2) = [(a + b)^2 - 2ab](a + b)(a - b)$

(xiii) $a^5 + b^5 = (a^3 + b^3)(a^2 + b^2) - a^2b^2(a + b)$

ILLUSTRATION

Ex.1   Find the value of :

(i) $36x^2 + 49y^2 + 84xy$, when $x = 3$, $y = 6$

(ii) $25x^2 + 16y^2 - 40xy$, when $x = 6$, $y = 7$

Sol.   (i) $36x^2 + 49y^2 + 84xy$ $= (6x)^2 + (7y)^2 + 2 \times (6x) \times (7y)$

$= (6x + 7y)^2$

$= (6 \times 3 + 7 \times 6)^2$ [When $x = 3$, $y = 6$]

$= (18 + 42)^2$

$= (60)^2$

$= 3600.$ **Ans.**

(ii) $25x^2 + 16y^2 - 40xy$ $= (5x)^2 + (4y)^2 - 2 \times (5x) \times (4y)$

$= (5x - 4y)^2$

$= (5 \times 6 - 4 \times 7)^2$ [When $x = 6$, $y = 7$]

$= (30 - 28)^2$

$= 2^2$

$= 4$

     Ans.

Ex.2   If $x^2 + \dfrac{1}{x^2} = 23$, find the value of $\left(x + \dfrac{1}{x}\right)$.

Sol. $x^2 + \dfrac{1}{x^2} = 23$ ....(i)

$\Rightarrow$ $x^2 + \dfrac{1}{x^2} + 2 = 25$ [Adding 2 on both sides of (i)]

$\Rightarrow$ $(x^2) + \left(\dfrac{1}{x}\right)^2 + 2 \cdot x \cdot \dfrac{1}{x} = 25$

$\Rightarrow$ $\left(x + \dfrac{1}{x}\right)^2 = (5)^2$

$\Rightarrow$ $x + \dfrac{1}{x} = 5$ **Ans.**

Ex.3 Prove that $a^2 + b^2 + c^2 - ab - bc - ca = \dfrac{1}{2}\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right]$.

Sol. Here, L.H.S. $= a^2 + b^2 + c^2 - ab - bc - ca$

$= \dfrac{1}{2}\ [2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca]$

$= \dfrac{1}{2}\ [(a^2 - 2ab + b^2) + (b^2 - 2bc + c^2) + (c^2 - 2ca + a^2)]$

$= \dfrac{1}{2}\ [(a - b)^2 + (b - c)^2 + (c - a)^2]$

$= $ RHS   Hence Proved.

Ex.4 **Evaluate :**

(i) $(107)^2$ (ii) $(94)^2$ (iii) $(0.99)^2$

Sol. (i) $(107)^2 = (100 + 7)^2$

$= (100)^2 + (7)^2 + 2 \times 100 \times 7$

$= 10000 + 49 + 1400$

$= 11449$ **Ans.**

(ii) $(94)^2 = (100 - 6)^2$

$= (100)^2 + (6)^2 - 2 \times 100 \times 6$

$= 10000 + 36 - 1200$

$= 8836$ **Ans.**

(iii) $(0.99)^2 = (1 - 0.01)^2$

$= (1)^2 + (0.01)^2 - 2 \times 1 \times 0.01$

$= \ + 0.0001 - 0.02$

$= 0.9801$ **Ans.**

NOTE : We may extend the formula for squaring a binomial to the squaring of a trinomial as given below.

$(a + b + c)^2 = [a + (b + c)]^2$

$= a^2 + (b + c)^2 + 2 \times a \times (b + c)$ [Using the identity for the square of binomial]

$= a^2 + b^2 + c^2 + 2bc + 2\ (b + c)$ [Using $(b + c)^2 = b^2 + c^2 + 2bc$]

$= a^2 + b^2 + c^2 + 2bc + 2ab + 2ac$ [Using the distributive law]

$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

$\therefore$ $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

Ex.5 Simplify : $(3x + 4)^3 - (3x - 4)^3$.

Sol. We have,

$(3x + 4)^3 - (3x - 4)^3 = [(3x)^3 + (4)^3 + 3 \times 3x \times 4 \times (3x + 4)] - [(3x)^3 - (4)^3 - 3 \times 3x \times 4 \times (3x - 4)]$

$= [27^3 + 64 + 36x\ (3x + 4)] - [27^3 - 64 - 36x\ (3x - 4)]$

$= [27x^3 + 64 + 108x^2 + 144x] - [27x^3 - 64 - 108x^2 + 144x]$

$= 27x^3 + 64 + 108x^2 + 144x - 27x^3 + 64 + 108x^2 - 144x$

$= 128 + 216x^2$

$\therefore$ $(3x + 4)^3 - (3x - 4)^3 = 128 + 216x^2$ **Ans.**

Ex.6 Evaluate :

(i) $(1005)^3$ (ii) $(997)^3$

Sol. (i) $(1005)^3 = (1000 + 5)^3$

$= (1000)^3 + (5)^3 + 3 \times 1000 \times 5 \times (1000 + 5)$

$= 1000000000 + 125 + 15000 + (1000 + 5)$

$= 1000000000 + 125 + 15000000 + 75000$

$= 1015075125.$ **Ans.**

(ii) $(997)^3 = (1000 - 3)^3$

$= (1000)^3 - (3)^3 - 3 \times 1000 \times 3 \times (1000 - 3)$

$= 1000000000 - 27 - 9000 \times (1000 - 3)$

$= 1000000000 - 27 - 900000 + 27000$

$= 991026973$ **Ans.**

Ex.7 If $x - \dfrac{1}{x} = 5$, find the value of $x^3 - \dfrac{1}{x^3}$

Sol. We have, $x - \dfrac{1}{x} = 5$ ...(i)

$\Rightarrow$ $\left(x - \dfrac{1}{x} \right)^3 = (5)^3$ [Cubing both sides of (i)]

$\Rightarrow$ $x^3 - \dfrac{1}{x^3} - 3x.\dfrac{1}{x}\left(x - \dfrac{1}{x} \right) = 125$

$\Rightarrow$ $x^3 - \dfrac{1}{x^3} - 3\left(x - \dfrac{1}{x} \right) = 125$

$\Rightarrow$ $x^3 - \dfrac{1}{x^3} - 3 \times 5 = 125$ [Substituting $\left(x - \dfrac{1}{x} \right) = 5$]

$\Rightarrow$ $x^3 - \dfrac{1}{x^3} - 15 = 125$

$\Rightarrow$ $x^3 - \dfrac{1}{x^3} = (125 + 15) = 140$ **Ans.**

Ex.8 Find the following products of the following expression :

(i) $(4x + 3y) (16x^2 - 12xy + 9y^2)$ (ii) $(5x - 2y) (25x^2 + 10xy + 4y^2)$

Sol. (i) $(4x + 3y) (16x^2 - 12xy + 9y^2)$

$= (4x + 3y) [(4x)^2 - (4x) \times (3y) + (3y)^2]$

$= (x + b) (x^2 - ab + b^2)$ [Where a = 4x, b = 3y]

$= a^3 + b^3$

$= (4x)^3 + (3y)^3 = 64x^3 + 27y^3$ **Ans.**

(ii) $(5x - 2y) (25x^2 + 10xy + 4y^2)$

$= (5x - 2y) [(5x^2 + (5x) \times (2y) + (2y)^2]$

$= (a - b) (a^2 + ab + b^2)$ [Where a = 5x, b = 2y]

$= a^3 - b^3$

$= (5x)^3 - (2y)^3$

$= 125x^3 - 8y^3$ **Ans.**

Ex.9   Simplify : $\dfrac{\left(a^2-b^2\right)^3+\left(v^2-c^2\right)^3+\left(c^2-a^2\right)^3}{(a-b)^3+(b-c)^3+(c-a)^3}$.

Sol.   Here $\left(a^2-b^2\right)+\left(b^2-c^2\right)^3+\left(c^2-a^2\right)=0$

$\therefore$ $\left(a^2-b^2\right)^3+\left(b^2-c^2\right)^3+\left(c^2-a^2\right)^3=3\left(a^2-b^2\right)\left(b^2-c^2\right)\left(c^2-a^2\right)$

Also, $(a-b)+(b-c)+(c-a)=0$

$\therefore$ $(a-b)^3+(b-c)^3+(c-a)^3=3(a-b)(b-c)(c-a)$

$\therefore$ Given expression $=\dfrac{3(a-b)(a+b)(b-c)(b+c)(c-a)(c+a)}{3(a-b)(b-c)(c-a)}$

$=\dfrac{3(a-b)(a+b)(b-c)(b+c)(c-a)(c+a)}{3(a-b)(b-c)(c-a)}$

$=(a+b)(b+c)(c+a)$ **Ans.**

Ex.10   Prove that : $(x-y)^3+(y-z)^3+(z-x)^3=3(x-y)(y-z)(z-x)$.

Sol.   Let $(x-y)=a,(y-z)=b$ and $(z-x)=c$.

Then, $a+b+c=(x-y)+(y-z)+(z-x)=0$

$\therefore a^3+b^3+c^3=3abc$

Or $(x-y)^3+(y-z)^3+(z-x)^3=3(x-y)(y-z)(z-x)$ **Ans.**

Ex.11   Find the value of $(28)^3-(78)^3+(50)^3$.

Sol.   Let $a=28,b=-78,c=50$

Then, $a+b+c=28-78+50=0$

$\therefore$ $a^3+b^3+c^3=3abc$.

So, $(28)^3+(-78)^3+(50)^3=3\times28\times(-78)\times50$

Ex.12   If $a+b+c=9$ and $ab+bc+ac=26$, find the value of $a^3+b^3+c^3-3abc$.

Sol.   We have $a+b+c=9$ ...(i)

$\Rightarrow$ $(a+b+c)^2=81$ [On squaring both sides of (i)]

$\Rightarrow$ $a^2+b^2+c^2+2(ab+bc+ac)=81$

$\Rightarrow$ $a^2+b^2+c^2+2\times26=81$ [$\because ab+bc+ac=26$]

$\Rightarrow$ $a^2+b^2+c^2=(81-52)$

$\Rightarrow$ $a^2+b^2+^2=29$.

Now, we have

$a^3+b^3+c^3-3abc$ $=(a+b+c)(a^2+b^2+c^2-ab-bc-ac)$

$=(a+b+c)[(a^2+b^2+c^2)-(ab+bc+ac)]$

$=9\times[(29-26)]$

$=(9\times3)$

$=27$ **Ans.**

(b) A Special Product :

We have $(x + a) (x + b)$ $= x (x + b) + a (x + b)$

$\qquad\qquad\qquad\qquad = x^2 + xb + ax + ab$

$\qquad\qquad\qquad\qquad = x^2 + bx + ax + ab \qquad [\because xb = bx]$

$\qquad\qquad\qquad\qquad = x^2 + ax + bx + ab$

$\qquad\qquad\qquad\qquad = x^2 + (a + b) x + ab$

Thus, we have the following identity

$\qquad (x + a) (x + b) = x^2 + (a + b)x + ab.$

Ex.13 Find the following products :

(i) $(x + 2) (x + 3)$ $\qquad\qquad\qquad$ (ii) $(x + 7) (x - 2)$

(ii) $(y - 4) (y + 3)$ $\qquad\qquad\qquad$ (iv) $(y - 7) (y + 3)$

(v) $(2x - 3) (2x - 5)$ $\qquad\qquad\quad$ (vi) $(3x + 4) (3x - 5)$

Sol. Using the identity : $(x + a) (x + b) = x^2 + (a + b) x + ab$, we have

(i) $(x + 2) (x + 3) = x^2 + (2 + 3) x + 2 \times 3$

$\qquad = x^2 + 5x + 6.$ $\qquad\qquad$ **Ans.**

(ii) $(x + 7) (x - 2) = (x + 7) (x + (-2))$

$\qquad = x^2 + 7x + (-2)x + 7 \times (-2)$

$\qquad = x^2 + 5x - 14.$ $\qquad\qquad$ **Ans.**

(iii) $(y - 4) (y - 3) = \{y + (-4)\} \{y + (-3)\}$

$\qquad = y^2 + \{(-4) + (-3)\}y + (-4) \times (-3)$

$\qquad = y^2 - 7y + 12$ $\qquad\qquad$ **Ans.**

(iv) $(y - 7) (y + 3) = \{y + (-7)\} (y + 3)$

$\qquad = y^2 + \{(-7) + 3\} + (-7) \times 3$

$\qquad = y^2 - 4y - 21.$ $\qquad\qquad$ **Ans.**

(v) $(2x - 3) (2x + 5) = (y - 3) (y + 5)$, where $y = 2x$

$\qquad = \{y + (-3)\} (y + 5)$

$\qquad = y^2 + \{(-3) + 5\} y + (-3) \times 5$

$\qquad = y^2 + 2y - 15$

$\qquad = (2x)^2 + 2 \times 2x - 15$

$\qquad = 4x^2 + 4x - 15.$ $\qquad\quad$ **Ans.**

(vi) $(3x + 4) (3x - 5) = (y + 4) (y - 5)$, where $y = 3x$

$\qquad = (y + 4) \{y + (-5)\}$

$\qquad = y^2 + \{4 + (-5)\} + 4 \times (-5)$

$\qquad = y^2 - y - 20$

$\qquad = (3x)^2 - 3x - 20$

$\qquad = 9x^2 - 3x - 20.$ $\qquad\quad$ **Ans.**

Ex.14 Evaluate : (i) 35×37 $\qquad$ (ii) 103×96

Sol. (i) $35 \times 37 = (40 - 5) (40 - 3)$

$\qquad = (40 + (-5)) (40 + (-3))$

$\qquad = 40^2 + (-5 - 3) \times 40 + (-5 \times -3)$

$\qquad = 1600 - 320 + 15$

$\qquad = 1615 - 320$

$\qquad = 1295$ $\qquad\qquad$ **Ans.**

(ii)   103 × 96

$= (100 + 3) [100 + (-4)]$

$= 100^2 + (3 + (-4)) \times 100 + (3 \times -4)$

$= 10000 - 100 - 12$

$= 9888$ **Ans.**

FACTORS OF A POLYNOMIAL

If a polynomial $f(x)$ can be written as a product of two or more other polynomial $f_1(x)$, $f_2(x)$, $f_3(x)$,..... then each of the polynomials $f_1(x)$, $f_2(x)$,..... is called a factor of polynomial $f(x)$. The method of finding the factors of a polynomials is called factorisations.

(a) Factorisation by Making a Trinomial a Perfect Square :

Ex.15 $81a^2b^2c^2 + 64a^6b^2 - 144a^4b^2c$

Sol. $81a^2b^2bc^2 + 64a^6b^2 - 144a^4b^2c$

$= [9abc]^2 - 2 [9abc][8a^3b] + [8a^3b]^2$

$= [9abc - 8a^3b]^2 = a^2b^2[9c - 8a^2]^2$ **Ans.**

Ex.16 $\left(3a - \dfrac{1}{b}\right)^2 - \left(3a - \dfrac{1}{b}\right) + 9 + \left(c + \dfrac{1}{b} - 2a\right)\left(3a - \dfrac{1}{b} - 3\right)$

Sol. $\left(3a - \dfrac{1}{b}\right)^2 - 6\left(3a - \dfrac{1}{b}\right) + 9 + \left(c + \dfrac{1}{b} - 2a\right)\left(3a - \dfrac{1}{b} - 3\right)$

$= \left(3a - \dfrac{1}{b}\right)^2 - 2.3\left(3a - \dfrac{1}{b}\right) + (3)^2 + \left(c + \dfrac{1}{b} - 2a\right)\left(3a - \dfrac{1}{b} - 3\right)$

$= \left(3a - \dfrac{1}{b} - 3\right)^2 + \left(c + \dfrac{1}{b} - 2a\right)\left(3a - \dfrac{1}{b} - 3\right)$

$= \left(3a - \dfrac{1}{b} - 3\right)\left[3a - \dfrac{1}{b} + 3 + \dfrac{1}{b} - 2a\right]$

$= \left(3a - \dfrac{1}{b} - 3\right)[a + c - 3]$ **Ans.**

(b) Factorisation by Using the Formula for the Difference of Two Squares :

$a^2 - b^2 = (+ b)(a - b)$

Ex.17   Factorise   : $4(2a + 3b - 4c)^2 - (a - 4b + 5c).2$

Sol. $= 4(2a + 3b - 4c)^2 - (a - 4b + 5c)^2$

$= [2(2a + 3b - 4c)]^2 - (a - 4b + 5c)^2$

$= [4a + 6b - 8c + a - 4b + 5c] [4a + 6b - 8c - a + 4b - 5c]$

$= [5a + 2b - 3c] [3a + 10b - 13c]$ **Ans.**

Ex.18 Factorise : $4x^2 + \dfrac{1}{4x^2} + 2 - 9y^2$.

Sol. $4x^2 + \dfrac{1}{4x^2} + 2 - 9y^2$

$= (2x)^2 + 2.(2x).\left(\dfrac{1}{2x}\right) + \left(\dfrac{1}{2x}\right)^2 - (3y)^2$

$= \left(2x + \dfrac{1}{2x}\right)^2 - (3y)^2$

$= \left(2x + \dfrac{1}{2x} + 3y\right)\left(2x + \dfrac{1}{2x} - 3y\right)$ **Ans.**

Ex.19 Factorise : $x^4 + \dfrac{1}{a^4} - 3$.

Sol. $(a^2)^2 + \left(\dfrac{1}{a^2}\right)^2 - 2.(a^2)\left(\dfrac{1}{a^2}\right) - 1$

$= \left(a^2 - \dfrac{1}{a^2}\right)^2 - (1)^2$

$= \left(a^2 - \dfrac{1}{a^2} + 1\right)\left(a^2 - \dfrac{1}{a^2} - 1\right)$ **Ans.**

Ex.20 Factorise : $x^4 + x^2y^2 + y^4$.

Sol. $x^2 + x^2y^2 + y^4 = (x^2)^2 + 2.x^2.y^2 + (y^2)^2 - x^2y^2$

$= x^2 + y^2)^2 - (xy)^2$

$= (x^2 + y^2 + xy(\,(x^2 + y^2 - xy)$ **Ans.**

(c) Factorisation by Using Formula of $a^3 + b^3$ and $a^3 - b^3$:

Ex.21 Factorise : $64a^{13}b + 343ab^{13}$.

Sol. $64a^{13}b + 343ab^{13} = ab[64a^{12} + 343b^{12}]$

$= ab[(4a^4)^3 + (7b^4)^3]$

$= ab[4a^4 + 7b^4]\,[(4a^4)^2 - (4a^4)\,(7b^4) + (7b^4)^2]$

$= ab[4a^4 + 7b^4][16a^8 - 28a^4b^4 + 49b^8]$ **Ans.**

Ex.22 Factorise : $p^3q^2x^4 + 3p^2qx^3 + 3px^2 + \dfrac{x}{q} - q^2r^3x$

Sol. In above question, If we take common then it may become in the form of $^3 + b^3$.

$\therefore$ $p^3q^2x^4 + 3p^2qx^3 + 3px^2 + \dfrac{x}{q} - q^2r^3x$

$= \dfrac{x}{q}$ [p^{3}q^{3}x^3 + 3p^{2}q^{2}x^2 + 3pqx + 1 - q^{3}r^3]

$= \dfrac{x}{q}$ [(pqx)^3 + 3(pqx)^2 .1 + 3pqx . (1)^2 + (1)^3 - q^{3}r^3]

Let pqx = A & 1 = B

$= \dfrac{x}{q}$ [A^3 + 3A^{2}B + 3AB^2 + B^3 - q^{3}r^3]

$= \dfrac{x}{q}$ [(pqx + 1)^3 - (qr)^3] $= \dfrac{x}{q}$ [pqx + 1 - qr][(pqx + 1)^2 + (pqx + 1) qr + (qr)^2]

$= \dfrac{x}{q}$ [pqx + 1 - qr][p^{2}q^{2}x^2 + 1 + 2pqx + pq^{2}xr + qr + q^{2}r^2]   **Ans.**

Ex.23 Factories : x^3 - 6x^2 + 32

Sol : x^3 + 32 - 6x^2

= x^3 + 8 + 24 - 6x^2

= [(x)^3 + (2)^3] + 6[4 - x^2]

= (x + 2)[x^2 - 2x + 4] + 6[2 + x][2 - x]

= (x + 2) [x^2 -2x + 4 + 6(2 - x)]

= (x + 2)[x^2 -2x + 4 + 12 - 6x]

= (x + 2) [x^2 - 8x + 16]

= (x + 2) (x - 4)^2 **Ans.**

OBJECTIVE DPP - 4.1

1.   The product of $(x + a)(x + b)$ is :

(A) $x^2 + (a + b)x + ab$ (B) $x^2 - (a - b)x + ab$ (C) $a^2 + (a - b)x + ab$ (D) $x^2 + (a - b)x - ab$.

2.   The value of 150×98 is :

(A) 10047   (B) 14800   (C) 14700   (D) 10470

3   The expansion of $(x + y - z)^2$ is :

(A) $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$ (B) $x^2 + y^2 - z^2 - 2xy + yz + 2zx$

(C) $x^2 + y^2 + z^2 + 2xy - 2yz - 2zx$ (D) $x^2 + y^2 - z^2 + 2zy - 2yz - 2zx$

4.   The value of $(x + 2y + 2z)^2 + (x - 2y - 2z)^2$ is:

(A) $2x^2 + 8y^2 + 8z^2$ (B) $2x^2 + 8y^2 + 8z^2 + 8xyz$

(C) $2x^2 + 8y^2 + 8z^2 - 8yz$ (D) $2x^2 + 8y^2 + 8z^2 + 16yz$

5.   The value of $25x^2 + 16y^2 + 40xy$ at $x = 1$ and $y = -1$ is :

(A) 81   (B) -49   (C) 1   (D) None of these

6.   On simplifying $(a + b)^3 + (a - b)^3 + 6a(a^2 - b^2)$ we get :

(A) $8a^2$ (B) $8a^2b$ (C) $8a^3b$ (D) $8a^3$

7.   Find the value of $\dfrac{a^3 + b^3 + c^3 - 3abc}{ab + bc + ca - a^2 - b^2 - c^2}$, when a = -5, 5 = -6 , c = 10.

(A) 1   (B) -1   (C) 2   (D) -2

8.   If $(x + y + z) = 1$, $xy + yz + zx = -1$ $xyz = -1$ then value of $x^3 + y^3 + z^3$ is :

(A) -1   (B) 1   (C) 2   (D) -2

9.   In method of factorisation of an algebraic expression. Which of the following statement is false ?

(A) Taking out a common factor from two or more terms.

(B) Taking out a common factor from a group of terms.

(C) By using remainder theorem.

(D) By using standard identities.

10.   Factors of $(a + b)^3 - (a - b)^3$ is :

(A) $2ab(3a^2 + b^2)$ (B) $ab(3a^2 + b^2)$ (C) $2b(3a^2 + b^2)$ (D) $3a^2 + b^2$

11.   Degree of zero polynomial is :

(A) 0   (B) 1   (C) Both 0 & 1   (D) Not defined

1. If $a^4 + \dfrac{1}{a^4} = 119$, then find the value of $-\dfrac{1}{a^3}$.

2. If x = 152, y = -91 find the value of $9x^2 + 30xy + 25y^2$.

3. Evaluate :

(i) $(5x + 4y)^2$ (ii) $(4x - 5y)^2$ (iii) $\left(2x - \dfrac{1}{x}\right)^2$

4. If x + y = 3 and xy = - 18, find the value of $x^3 + y^3$.

5. If $x^2 + \dfrac{1}{x^2} = 51$ find the value of $x^3 - \dfrac{1}{x^3}$..

6. Evaluate :

(i) $25^3 - 75^3 + 50^3$ (ii) $\left(\dfrac{1}{2}\right)^3 + \left(\dfrac{1}{3}\right)^3 - \left(\dfrac{5}{6}\right)^3$ (iii) $(0.2)^3 - (0.3)^3 + (0.1)^3$

7. Find the product of :

(i) (x + 4) (x + 7)   (ii) $(x + \dfrac{1}{5})(x + 5)$ (iii) $(P^2 + 16) (P^2 - \dfrac{1}{4})$

8. Evaluate :

(i) 102 × 106   (ii) 994 × 1006   (iii) 34 × 36

9. Factorise : $4x^4 + (7a)^4$.

10. Factorise : $x^{12} = 1$.

11. Evaluate $\dfrac{(a-b)^2}{(b-c)(c-a)} + \dfrac{(b-c)^2}{(a-b)(c-a)} + \dfrac{(c-a)^2}{(a-b)(b-c)}$.

12. Write the following polynomials in standard forms :

(i) $x^6 - 3a^4 + \sqrt{2}\,x + 5x^2 + 7x^5 + 4$

(ii) $m^7 + 8m^5 + 4m^6 + 6m - 3m^2 - 11$

13. Factorise : (x + 1) (x + 2) (x + 3) (x + 4) - 3.

14. Factorise : $64a^3 - 27b^3 - 144a^2b + 108ab^2$.

15. Factorise : $x^4 + 2x^3y - 2xy^3 - y^4$.