विध्न विचारत भीरु जन, नहीं आरम्भे काम, विपति देख छोड़े तुरंत मध्यम मन कर श्याम।
पुरुष सिंह संकल्प कर, सहते विपति अनेक, 'बना' न छोड़े ध्येय को, रघुबर राखे टेक।।

रचितः मानव धर्म प्रणेता
सद्गुरु श्री रणछोड़दासजी महाराज

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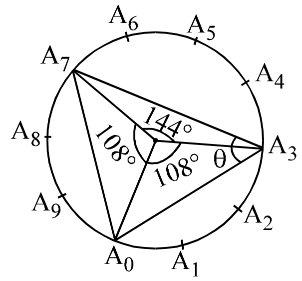
Select the correct alternative : (Only one is correct)

Q.1$_{13/ph-1}$ A regular decagon $A_0, A_1, A_2......A_9$ is given in the xy plane. Measure of the $\angle A_0A_3A_7$ in degrees is
(A) 108° (B) 96°
(C) 72° (D*) 54°



[Hint: angle substended by each sides is 36° at the centre as shown

$$\theta = \frac{108}{2} = 54°\ ]$$

Q.2$_{4/qe}$ If $a^2 + b^2 + c^2 = 1$ then $ab + bc + ca$ lies in the interval :

(A) $\left[\frac{1}{2}, 2\right]$ (B) $[-1, 2]$ (C*) $\left[-\frac{1}{2}, 1\right]$ (D) $\left[-1, \frac{1}{2}\right]$

[Hint: $\sum (a-b)^2 \geq 0 \Rightarrow 2\sum a^2 - 2\sum ab \geq 0 \Rightarrow \sum ab \leq \sum a^2 \Rightarrow ab + bc + ca \leq 1$
Also note that $(a + b + c)^2 \geq 0$]

Q.3$_{13/s\&p}$ If the roots of the cubic $x^3 - px^2 + qx - r = 0$ are in G.P. then
(A*) $q^3 = p^3r$ (B) $p^3 = q^3r$ (C) $pq = r$ (D) $pr = q$

[Hint: Let $\frac{\alpha}{\delta}, \alpha, \alpha\delta$ are the roots of the given cubic

$$\therefore \quad \alpha^3 = r \ ; \quad \alpha\left[\frac{1}{\delta}+1+\delta\right] = p \quad ; \quad \frac{\alpha^2}{\delta} + \alpha^2\delta + \alpha^2 = q \text{ (Taken two at a time)}$$

hence $\alpha^2\left(\frac{1}{\delta} + \delta + 1\right) = q$; $\therefore$ $\alpha = \frac{q}{p}$, also $\alpha^3 = r$; $\therefore$ $\frac{q^3}{p^3} = r \Rightarrow q^3 = p^3r$]

Q.4$_{13/ph-3}$ In a triangle ABC, $a : b : c = 4 : 5 : 6$. Then $3A + B =$
(A) $4C$ (B) 2π (C) $\pi - C$ (D*) π

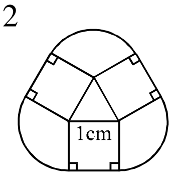
[Hint : $\cos A = \frac{25+36-16}{2\,.\,5\,.\,6} = \frac{3}{4} \Rightarrow \cos 3A = 4\cos^3 A - 3\cos B = -\frac{9}{16}$

Also $\cos B = \frac{36+16-25}{2\,.\,6\,.\,4} = \frac{9}{16} \Rightarrow \cos 3A = -\cos B = \cos(A - B) \Rightarrow 3A + B = \pi$]

Q.5$_{16/ph-1}$ An equilateral triangle has sides 1 cm long. An ant walks around the triangle, maintaining a distance of 1 cm from the triangle at all time. Distance travelled by the ant in one round is

(A) $3+3\sqrt{3}$ (B) $3+6\sqrt{3}$ (C*) $3 + 2\pi$ (D) $3 + \frac{3\pi}{2}$

[Hint: The Ant must trace 3 sides of length 1 cm and the 3 arcs around each corner of length $\frac{2\pi}{3}$ for a total distance of $(3 + 2\pi)$. ref. figure.



]

Q.6$_{5/qe}$ If $P(x) = ax^2 + bx + c$ & $Q(x) = -ax^2 + dx + c$, where $ac \neq 0$, then $P(x) \cdot Q(x) = 0$ has

(A) exactly one real root (B*) atleast two real roots

(C) exactly three real roots (D) all four are real roots .

[Hint : $D_1 : b^2 - 4ac$ & $D_2 : d^2 + 4ac$. Hence atleast one of either D_1 or D_2 is zero]

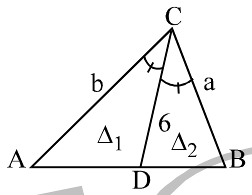
Q.7$_{23/log}$ The set of all real numbers x for which $\log_{2004}\left(\log_{2003}\left(\log_{2002}\left(\log_{2001} x\right)\right)\right)$ is defined as $\{x \mid x > c\}$. The value of c is

(A) 0 (B*) $(2001)^{2002}$ (C) $(2003)^{2004}$ (D) $(2001)^{2002^{2003}}$

Q.8$_{19/ph-3}$ In a triangle ABC, CD is the bisector of the angle C. If $\cos\frac{C}{2}$ has the value $\frac{1}{3}$ and $l(CD) = 6$, then $\left(\frac{1}{a} + \frac{1}{b}\right)$ has the value equal to

(A*) $\frac{1}{9}$ (B) $\frac{1}{12}$ (C) $\frac{1}{6}$ (D) none

[Hint : $\Delta = \Delta_1 + \Delta_2 = \frac{1}{2}\, ab \sin C = ab \sin\frac{C}{2}\cos\frac{C}{2}$



$= \frac{1}{2}\, 6b \sin\frac{C}{2} + \frac{1}{2}\, 6a \sin\frac{C}{2} \Rightarrow \frac{1}{a} + \frac{1}{b} = \frac{1}{9}$]

Q.9$_{14/qe}$ The real values of 'a' for which the quadratic equation, $2x^2 - (a^3 + 8a - 1)x + a^2 - 4a = 0$ possesses roots of opposite signs is given by :

(A) $a > 5$ (B*) $0 < a < 4$ (C) $a > 0$ (D) $a > 7$

[Hint: $f(0) < 0$]

Q.10$_{30/s\&p}$ The arithmetic mean of the nine numbers in the given set {9, 99, 999, ... 999999999} is a 9 digit number N, all whose digits are distinct. The number N does not contain the digit

(A*) 0 (B) 2 (C) 5 (D) 9

[Hint: $N = \frac{1}{9}\{9, 99, 999, ... 999999999\} = 1 + 11 + 111 + ...+111111111$

$= 1\,2\,3\,4\,5\,6\,7\,8\,9 \Rightarrow$ (A)]

Q.11$_{12/ph-2}$ If $x = \frac{n\pi}{2}$, satisfies the equation $\sin\frac{x}{2} - \cos\frac{x}{2} = 1 - \sin x$ & the inequality $\left|\frac{x}{2} - \frac{\pi}{2}\right| \leq \frac{3\pi}{4}$, then:

(A) n = –1, 0, 3, 5 (B*) n = 1, 2, 4, 5

(C) n = 0, 2, 4 (D) n = –1, 1, 3, 5

[Sol. $\left|\frac{x}{2} - \frac{\pi}{2}\right| \leq \frac{3\pi}{4}$ possible x are

$-\frac{3\pi}{4} \leq \frac{x}{2} - \frac{\pi}{2} \leq \frac{3\pi}{4}$

$-\frac{\pi}{4} \leq \frac{x}{2} \leq \frac{5\pi}{4}$

$-\frac{\pi}{2} \leq x \leq \frac{5\pi}{4}$

$-\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \frac{5\pi}{2}$

$\sin\frac{x}{2} - \cos\frac{x}{2} = (\sin\frac{x}{2} - \cos\frac{x}{2})^2$

factors $\sin\frac{x}{2} - \cos\frac{x}{2} = 0$

or $\sin\frac{x}{2} - \cos\frac{x}{2} = 1$

only circled angle satisfy one of the above equation when n = 1, 2, 4, 5]

Q.12$_{21/ph-3}$ With usual notations, in a triangle ABC, a cos(B – C) + b cos(C – A) + c cos(A – B) is equal to

(A*) $\frac{abc}{R^2}$ (B) $\frac{abc}{4R^2}$ (C) $\frac{4abc}{R^2}$ (D) $\frac{abc}{2R^2}$

[Sol. Here a(cosB cosC + sinB sinC) + ...

using $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$

$a\,(\cos B \cos C + \frac{bc}{4R^2}) + \ldots\ldots$

$= \frac{3abc}{4R^2} + a\cos B\cos C + b\cos C\cos A + c\cos A\cos B = \frac{3abc}{4R^2} + c\cos C + c\cos A\cos B$

$= \frac{3abc}{4R^2} + c\,[\cos A\cos B - \cos(A+B)] = \frac{3abc}{4R^2} + c\sin A\sin B = \frac{3abc}{4R^2} + \frac{abc}{4R^2} = \frac{abc}{R^2}$ Ans.]

Q.13$_{20/ph-1}$ If in a Δ ABC, $\sin^3A + \sin^3B + \sin^3C = 3\sin A \cdot \sin B \cdot \sin C$ then

(A) Δ ABC may be a scalene triangle (B) Δ ABC is a right triangle
(C) Δ ABC is an obtuse angled triangle (D*) Δ ABC is an equilateral triangle

[Hint: Use : $a^3+b^3+c^3-3abc = (1/2)(a+b+c)\,[(a-b)^2+(b-c)^2+(c-a)^2$
Hint: either $\sin A + \sin B + \sin C = 0$ (which is not possible)
or $\sin A = \sin B = \sin C \Rightarrow$ equilateral]

Q.14$_{22/qe}$ If a, b, c are real numbers satisfying the condition a + b + c = 0 then the roots of the quadratic equation $3ax^2 + 5bx + 7c = 0$ are :

(A) positive (B) negative (C*) real & distinct (D) imaginary

[Hint: $D = 25b^2 - 84\,ac$
$= 25(a+c)^2 - 84ac$ using $b = -(a+c)$
$= 21[(a+c)^2 - 4ac] + 4(a+c)^2 > 0$]

Q.15$_{40/ph-1}$ If $\sin^3x \cdot \cos 3x + \cos^3x \cdot \sin 3x = \frac{3}{8}$, then the value of sin 4x is

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C*) $\frac{1}{2}$ (D) $\frac{3}{8}$

[Sol. $4\sin^3x \cdot \cos 3x + 4\cos^3x \cdot \sin 3x = 3/2$

$(3\sin x - \sin 3x)\cos 3x + (3\cos x + \cos 3x)\sin 3x = \frac{3}{2}$

$3[\sin x\cos 3x + \cos x\sin 3x] = \frac{3}{2} \Rightarrow \sin 4x = \frac{1}{2}$

$4x = n\pi + (-1)^n\left(\frac{\pi}{6}\right) \Rightarrow x = \frac{n\pi}{4} + (-1)^n\frac{\pi}{6}$]

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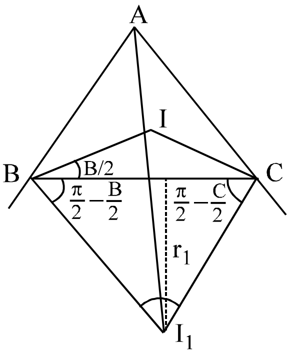
Q.16$_{25/ph-3}$ With usual notations in a triangle ABC, $(I\,I_1)\cdot(I\,I_2)\cdot(I\,I_3)$ has the value equal to
(A) R^2r (B) $2R^2r$ (C) $4R^2r$ (D*) $16R^2r$

[Hint: $BICI_1$ is a cyclic quadrilateral with $I\,I_1$ as the diameter



also $\angle BI_1C = \frac{\pi}{2} - \frac{A}{2}$

applying sine law in BCI_1

$$\frac{a}{\cos\frac{A}{2}} = I\,I_1$$

$$\therefore \quad I\,I_1 = \frac{2R\cdot 2\sin\frac{A}{2}\cdot\cos\frac{A}{2}}{\cos\frac{A}{2}} = 4R\sin\frac{A}{2}$$

$$\therefore \quad \prod I\,I_1 = 64R^3\sin\frac{A}{2}\,\sin\frac{B}{2}\,\sin\frac{C}{2} = 16R^2\,r \quad]$$

Q.17$_{29/s\&p}$ Consider the pattern shown below

Row 1	1				
Row 2	3	5			
Row 3	7	9	11		
Row 4	13	15	17	19	etc

The number at the end of row 80, is
(A*) 6479 (B) 6319 (C) 6481 (D) 6531

[Sol. 1^{st} term is given by
$n^2 - n + 1$
$n = 80$ $\quad 80^2 - 80 + 1$
$80\cdot 79 + 1 = 6321$
number at the end $= 6321 + 2\times 79 = 6321 + 158 = 6479$ Ans.]

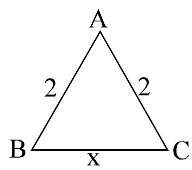
Q.18$_{25/log}$ For all positive integers n let $f(n) = \log_{2002}n^2$. Let $N = f(11) + f(13) + f(14)$ which of the following relations is true?
(A) $0 < N < 1$ (B) $N = 1$ (C) $1 < N < 2$ (D*) $N = 2$

[Hint: $2002 = 2\cdot 7\cdot 11\cdot 13$; $N = \log_N 11^2 + \log_N 13^2 + \log_N 14^2 = \log_{2002}(11^2\cdot 13^2\cdot 14)$
$N = \log_{2002}(11^2\cdot 13^2\cdot 2^2\cdot 7^2) = 2 \Rightarrow$ (D)]

Q.19$_{30/qe}$ The roots of $(x-1)(x-3) + K(x-2)(x-4) = 0$, $K > 0$ are :
(A*) real (B) real & equal (C) imaginary (D) one real & one imaginary

[Hint : check f(1) , f(2) , f(3) & f(4) and interpret
note that one root lie between 1 and 2 and the other between 3 and 4]

Q.20$_{26/ph-3}$ An isosceles triangle has sides of length 2, 2, and x. The value of x for which the area of the triangle is maximum, is
(A) 1 (B) $\sqrt{2}$
(C) 2 (D*) $2\sqrt{2}$



[Hint: $\frac{1}{2}\times 2\times 2\sin A$ which is maximum if $A = 90°$ $\Rightarrow$ $x = 2\sqrt{2}$]

Q.21$_{41/ph-1}$ If $x \sec\alpha + y \tan\alpha = x \sec\beta + y \tan\beta = a$, then $\sec\alpha \cdot \sec\beta =$

(A) $\frac{a^2+y^2}{x^2+y^2}$ (B*) $\frac{a^2+y^2}{x^2-y^2}$ (C) $\frac{x^2+y^2}{a^2+y^2}$ (D) $\frac{x^2-y^2}{a^2-y^2}$

[Sol. α and β satisfy the equation

$x \sec\theta + y \tan\theta = a$

or $(x \sec\theta - a^2) = y^2 \tan^2\theta = y^2(x \sec^2\theta - 1)$

$\sec^2\theta\,(x^2 - y^2) - 2ax \sec\theta + a^2 + y^2 = 0$

This is a quadratic in $\sec\theta$, whose roots are $\sec\beta$ and $\sec\beta$

$\sec\alpha \cdot \sec\beta = \frac{a^2+y^2}{x^2-y^2}$]

Q.22$_{35/qe}$ Largest integral value of *m* for which the quadratic expression

$y = x^2 + (2m + 6)x + 4m + 12$ is always positive, $\forall\, x \in R$, is

(A) – 1 (B) – 2 (C*) 0 (D) 2

[Hint: $D < 0$ $\Rightarrow$ $-3 < m < 1$ $\Rightarrow$ $m = 0$]

Q.23$_{16/ph-2}$ The general solution of the trigonometric equation

$\tan x + \tan 2x + \tan 3x = \tan x \cdot \tan 2x \cdot \tan 3x$ is

(A) $x = n\pi$ (B) $n\pi \pm \frac{\pi}{3}$ (C) $x = 2n\pi$ (D*) $x = \frac{n\pi}{3}$

where $n \in I$

[Hint: $\tan x - \tan 2x - \tan 3x = \tan 3x - \tan 2x - \tan x$

$\Rightarrow$ $\tan x + \tan 2x = 0$

$\therefore$ $\tan 2x = \tan(-x)$

$2x = n\pi - x$

$x = \frac{n\pi}{3}$, $n \in I$]

Q.24$_{30/ph-3}$ With usual notation in a ΔABC $\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\left(\frac{1}{r_2}+\frac{1}{r_3}\right)\left(\frac{1}{r_3}+\frac{1}{r_1}\right) = \frac{K R^3}{a^2 b^2 c^2}$ where K has the value equal to :

(A) 1 (B) 16 (C*) 64 (D) 128

[Hint : 1st term $= \frac{1}{\Delta}(s - a + s - b) = \frac{c}{\Delta}$ $\Rightarrow$ $L H S = \frac{abc}{\Delta^3}$. Use $\Delta = \frac{abc}{4R}$ to get the result]

Q.25$_{35/s\&p}$ If the sum of the roots of the quadratic equation, $ax^2 + bx + c = 0$ is equal to sum of the squares of their reciprocals, then $\frac{a}{c}, \frac{b}{a}, \frac{c}{b}$ are in :

(A) A.P. (B) G.P. (C*) H.P. (D) none

[Sol. Let the roots of the equation are $ax^2 + bx + c = 0$ are α and β

$$\alpha+\beta = \frac{1}{\alpha^2}+\frac{1}{\beta^2} = \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha^2\beta^2}$$

$$(\alpha+\beta)(\alpha\beta)^2 = (\alpha+\beta)^2 - 2\alpha\beta$$

$$\left(-\frac{b}{a}\right)\left(\frac{c^2}{a^2}\right)=\frac{b^2}{a^2}-\frac{2c}{a}$$

$$-bc^2 = ab^2 - 2a^2c$$

$$\frac{ab^2+bc^2=2a^2c}{abc}$$

⇒ (Dividing by abc)

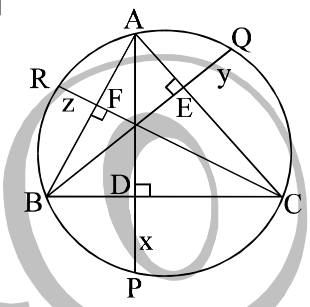
$$\frac{b}{c}+\frac{c}{a}=\frac{2a}{b} \quad \text{or} \quad \frac{a}{b}=\frac{\frac{b}{c}+\frac{c}{a}}{2} \quad \Rightarrow \quad \frac{b}{c},\frac{a}{b},\frac{c}{a} \text{ are in A.P.} \quad \Rightarrow \text{ result} \quad]$$

Q.26$_{36/log}$ The set of values of x satisfying the inequality $\frac{1}{\log_4 \frac{x+1}{x+2}} \le \frac{1}{\log_4 (x+3)}$ is :

(A) $(-3, -2)$ (B) $(-3, -2) \cup (-1, \infty)$ (C*) $(-1, \infty)$ (D) none

[Hint : Domain is $(-3, 2) \cup (-1, \infty)$; for $x > -1$,
LHS is negative & RHS is positive
and for $-3 < x < -2$ it is the other way ⇒ $x > -1$ is the final answer]

Q.27$_{56/ph-1}$ As shown in the figure AD is the altitude on BC and AD produced meets the circumcircle of ΔABC at P where DP = x. Similarly EQ = y and FR = z. If a, b, c respectively denotes the sides BC, CA and AB then $\frac{a}{2x}+\frac{b}{2y}+\frac{c}{2z}$ has the value equal to

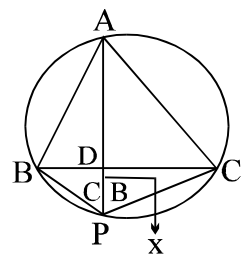


(A*) tanA + tanB + tanC (B) cotA + cotB + cotC
(C) cosA + cosB + cosC (D) cosecA + cosecB + cosecC

[Hint: BD = x tanC in ΔPDB
and DC = x tanB for ΔPDC
∴ BD + DC = a = x (tanB + tanC)

$$\frac{a}{x} = \tan B + \tan C$$

⇒ result]



Q.28$_{31/ph-3}$ In a Δ ABC, the value of $\frac{a\cos A + b\cos B + c\cos C}{a+b+c}$ is equal to :

(A*) $\frac{r}{R}$ (B) $\frac{R}{2r}$ (C) $\frac{R}{r}$ (D) $\frac{2r}{R}$

[Hint : LHS $\frac{R\left[\sin 2A + \sin 2B + \sin 2C\right]}{2R\left[\sin A + \sin B + \sin C\right]} = \frac{4\sin A \sin B \sin C}{2 \cdot 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}}$

$$= 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} = \frac{r}{R}$$]

Q.29$_{52/s\&p}$ If the sum of the first 11 terms of an arithmetical progression equals that of the first 19 terms, then the sum of its first 30 terms, is

(A*) equal to 0 (B) equal to – 1 (C) equal to 1 (D) non unique

[Sol. $\frac{11}{2}[2a + 10d] = \frac{19}{2}[2a + 18d]$

$11 \cdot 2(a + 5d) = 19 \cdot 2(a + 9d)$

$11a + 55d = 19a + 171d$

$8a + 116d = 0 \Rightarrow 2a + 29d = 0$

$\Rightarrow S_{30} = 0 \Rightarrow$ (A)]

Q.30$_{39/qe}$ The sum of all the value of m for which the roots x_1 and x_2 of the quadratic equation $x^2 - 2mx + m = 0$ satisfy the condition $x_1^3 + x_2^3 = x_1^2 + x_2^2$, is

(A) $\frac{3}{4}$ (B) 1 (C) $\frac{9}{4}$ (D*) $\frac{5}{4}$

[Hint: $x_1 + x_2 = 2m$; $x_1 x_2 = m$

$(x_1 + x_2)^3 - 3x_1x_2(x_1 + x_2) = (x_1 + x_2)^2 - 2x_1x_2$

$8m^3 - 3m(2m) = 4m^2 - 2m$

$8m^3 - 10m^2 + 2m = 0$

$2m(4m^2 - 5m + 1) = 0 \Rightarrow m = 0$

$(m - 1)(4m - 1) = 0 \Rightarrow m = 1$ or $m = 1/4$]

Q.31$_{27/s\&p}$ In an A.P. with first term 'a' and the common difference d (a, d ≠ 0), the ratio ' ρ ' of the sum of the first n terms to sum of n terms succeeding them does not depend on n. Then the ratio $\frac{a}{d}$ and the ratio ' ρ ', respectively are

(A) $\frac{1}{2}, \frac{1}{4}$ (B) $2, \frac{1}{3}$ (C*) $\frac{1}{2}, \frac{1}{3}$ (D) $\frac{1}{2}, 2$

[Hint : $\dfrac{\frac{n}{2}[2a+(n-1)d]}{\frac{2n}{2}[2a+(2n-1)d]-\frac{n}{2}[2a+\overline{n-1}\ d]} = \dfrac{(2a-d)+nd}{2[(2a-d)+2nd]-[2a-d+nd]}$

if 2a = d then ratio $= \frac{nd}{4nd - nd} = \frac{1}{3}$

$\therefore \quad \frac{a}{d} = \frac{1}{2}$; ratio $= \frac{1}{3} \Rightarrow$ C]

Q.32$_{42/ph-3}$ AD, BE and CF are the perpendiculars from the angular points of a Δ ABC upon the opposite sides. The perimeters of the Δ DEF and Δ ABC are in the ratio :

(A) $\frac{2r}{R}$ (B) $\frac{r}{2R}$ (C*) $\frac{r}{R}$ (D) $\frac{r}{3R}$

where r is the in radius and R is the circum radius of the Δ ABC

[Hint : Note that Δ DEF is a pedal triangle whose sides are $R \sin 2A$, $R \sin 2B$ and $R \sin 2C$.

$$\Rightarrow \text{ratio} = \frac{R\,\Sigma \sin 2A}{a+b+c} = \frac{4R\,\Pi \sin A}{2R\,\Sigma \sin A} = \frac{2 \,.\, 8\,\Pi \sin\frac{A}{2}\,\Pi \cos\frac{A}{2}}{4\,\Pi \cos\frac{A}{2}} = 4\,\Pi \sin\frac{A}{2} = \frac{r}{R}$$]

Q.33$_{78/ph-1}$ If $\cos 25° + \sin 25° = p$, then $\cos 50°$ is

(A) $\sqrt{2-p^2}$ (B) $-\,p\sqrt{2-p^2}$ (C*) $p\sqrt{2-p^2}$ (D) $-\,p\sqrt{2-p^2}$

[Hint: $\cos 50° = \cos^2 25° - \sin^2 25° = p(\cos 25° - \sin 25°)$]

Q.34$_{41/qe}$ Let r_1, r_2 and r_3 be the solutions of the equation $x^3 - 2x^2 + 4x + 5074 = 0$ then the value of $(r_1 + 2)(r_2 + 2)(r_3 + 2)$ is

(A) 5050 (B) 5066 (C*) – 5050 (D) – 5066

[Sol. $x^3 - 2x^2 + 4x + 5074 = (x - r_1)(x - r_2)(x - r_3)$

put $x = -2$

$-8 - 8 - 8 + 5074 = -(2 + r_1)(2 + r_2)(2 + r_3)$

$\therefore \quad 5050 = -(2 + r_1)(2 + r_2)(2 + r_3)$

$(2 + r_1)(2 + r_2)(2 + r_3) = -5050$ **Ans.**]

Q.35$_{58/s\&p}$ If p , q, r in H.P. and p & r be different having same sign then the roots of the equation $px^2 + qx + r = 0$ are

(A) real & equal (B) real & distinct (C) irrational (D*) imaginary

[Sol. $D = q^2 - 4pr$ also $q = \dfrac{2pr}{p+r}$

$$= \left(\frac{2pr}{p+r}\right)^2 - 4pr = -4pr\left[1 - \frac{pr}{(p+r)^2}\right] \Rightarrow -4pr\left[\frac{p^2 + r^2 + pr}{(p+r)^2}\right]$$

$\therefore D < 0$ roots are imaginary]

Q.36$_{43/ph-3}$ In a ΔABC if $b + c = 3a$ then $\cot\dfrac{B}{2} \cdot \cot\dfrac{C}{2}$ has the value equal to :

(A) 4 (B) 3 (C*) 2 (D) 1

[Hint : $$\cot\frac{B}{2} \cdot \cot\frac{C}{2} = \frac{s(s-b)}{\Delta} \,.\, \frac{s(s-c)}{\Delta} \,.\, \frac{(s-a)}{s-a} = \frac{s}{s-a} = \frac{2s}{2s-2a}$$

but given that $a + b + c = 4a \Rightarrow 2s = 4a$ Hence $\cot\dfrac{B}{2} \cdot \cot\dfrac{C}{2} = \dfrac{4a}{2a} = 2$]

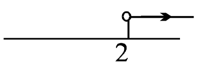
Q.37$_{37/log}$ Indicate the correct choice : If $\log_{0.3}(x-1) < \log_{0.09}(x-1)$, then x lies in the interval ;

(A*) $(2, \infty)$ (B) $(1, 2)$ (C) $(-2, -1)$ (D) none of these

[Hint: $\log_{0.3}(x-1) < 1/2 \log_{0.03}(x-1)$ or $\log_{0.3}(x-1) < \log_{0.09}(x-1)^{1/2}$

$x - 1 > \sqrt{x-1} \Rightarrow x - 1 > 0$ or $x > 1$

$\Rightarrow (x-1)^2 > x - 1 \Rightarrow (x-2)(x-1) > 0$



]

Q.38$_{18/ph-2}$ Number of roots of the equation, $\sin(\cos x) = \cos(\sin x)$ in $[0, 2\pi]$ is
(A*) 0 (B) 1 (C) 2 (D) 4

[Sol. $\sin(\cos x) = \sin\left(\frac{\pi}{2} - \sin x\right)$

$\cos x + \sin x = \frac{\pi}{2}$

no solution as $-\sqrt{2} \le \cos x + \sin x \le \sqrt{2}$]

Q.39$_{47/qe}$ The sum of the roots of the equation $(x + 1) = 2\log_2(2^x + 3) - 2\log_4(1980 - 2^{-x})$ is
(A) 3954 (B*) $\log_2 11$ (C) $\log_2 3954$ (D) indeterminate

Q.40$_{91/ph-1}$ Let n be a positive integer such that $\sin\frac{\pi}{2n} + \cos\frac{\pi}{2n} = \frac{\sqrt{n}}{2}$. Then
(A) $6 \le n \le 8$ (B) $4 \le n \le 8$ (C) $4 \le n < 8$ (D*) $4 < n < 8$

[Hint: Given $\sin\frac{\pi}{2n} + \cos\frac{\pi}{2n} = \frac{\sqrt{n}}{2}$ $\Rightarrow$ $\sqrt{2}\sin\left(\frac{\pi}{4} + \frac{\pi}{2n}\right) = \frac{\sqrt{n}}{2}$

As $n \in N$, $\frac{1}{\sqrt{2}} < \sin\left(\frac{\pi}{4} + \frac{\pi}{2n}\right) < 1$; hence $\frac{1}{\sqrt{2}} < \frac{\sqrt{n}}{2\sqrt{2}} < 1$

$2 < \sqrt{n} < 2\sqrt{2}$ $\Rightarrow$ $4 < n < 8$ Ans.]

Q.41$_{45/ph-3}$ Let f, g, h be the lengths of the perpendiculars from the circumcentre of the Δ ABC on the sides a, b and c respectively . If $\frac{a}{f} + \frac{b}{g} + \frac{c}{h} = \lambda \frac{a\,b\,c}{f\,g\,h}$ then the value of λ is :
(A*) 1/4 (B) 1/2 (C) 1 (D) 2

[Hint : $\tan A = \frac{a}{2f}$ $\Rightarrow$ $\frac{1}{2}\Sigma\tan A = \frac{1}{2}\Pi\tan A$

$= \frac{1}{4}\left(\frac{a}{f}\cdot\frac{b}{g}\cdot\frac{c}{h}\right)$ $\Rightarrow$ A]

Q.42$_{50/qe}$ The equation whose roots are $\sec^2\alpha$ & $\text{cosec}^2\alpha$ can be :
(A) $2x^2 - x - 1 = 0$ (B) $x^2 - 3x + 3 = 0$ (C*) $x^2 - 9x + 9 = 0$ (D) none
[Hint : Note that $\sec^2\alpha + \text{cosec}^2\alpha = \sec^2\alpha \,.\, \text{cosec}^2\alpha \ge 4$]

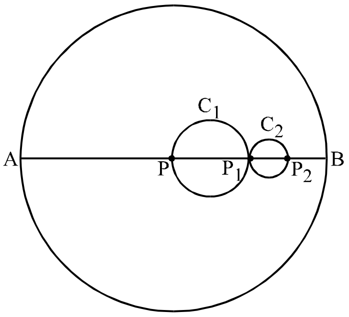
Q.43$_{92/ph-1}$ Minimum vertical distance between the graphs of $y = 2 + \sin x$ and $y = \cos x$ is
(A) 2 (B) 1 (C) $\sqrt{2}$ (D*) $2 - \sqrt{2}$

[Hint: $d_{min} = \min(2 + \sin x - \cos x) = 2 + \sqrt{2}\sin\left(x - \frac{\pi}{4}\right) = 2 - \sqrt{2}$]

Q.44$_{53/s\&p}$ Let C be a circle with centre P_0 and AB be a diameter of C. Suppose P_1 is the mid point of the line segment P_0B, P_2 is the mid point of the line segment P_1B and so on. Let $C_1, C_2, C_3, ...$ be circles with diameters P_0P_1, P_1P_2, P_2P_3 ...... respectively. Suppose the circles $C_1, C_2, C_3, ...$ are all shaded. The ratio of the area of the unshaded portion of C to that of the original circle C is

(A) 8 : 9 (B) 9 : 10 (C) 10 : 11 (D*) 11 : 12

[Sol. area of circle $C_1 = \frac{\pi}{4}\left(\frac{r}{2}\right)^2$ (area of circle = $\frac{\pi d^2}{4}$)



area of circle $C_2 = \frac{\pi}{4}\left(\frac{r}{4}\right)^2$

area of circle $C_3 = \frac{\pi}{4}\left(\frac{r}{8}\right)^2$ and so on

$\therefore$ shaded area $= \frac{\pi}{4}\left[\frac{r^2}{4}+\frac{r^2}{16}+\frac{r^2}{64}+.......\right] = \frac{\pi r^2}{4}\cdot\frac{\frac{1}{4}}{1-\frac{1}{4}} = \frac{\pi r^2}{12}$

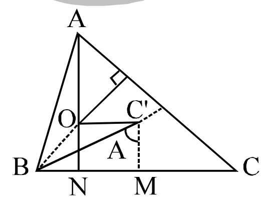
Hence ratio $= \frac{\pi r^2 - \frac{\pi r^2}{12}}{\pi r^2} = \frac{11}{12}$]

Q.45$_{47/ph-3}$ If the orthocentre and circumcentre of a triangle ABC be at equal distances from the side BC and lie on the same side of BC then tanB tanC has the value equal to :

(A*) 3 (B) $\frac{1}{3}$ (C) – 3 (D) $-\frac{1}{3}$

[Hint : R cos A = 2 R cosB cosC (C′ M = ON = distance of orthocentre from the side)

$\therefore$ $\frac{\cos(B+C)}{\cos B\cos C} = -2$ (ON = 2R cosB cosC)

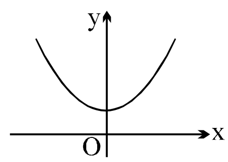


$\frac{\cos B\cos C - \sin B\sin C}{\cos B\cos C} = -2$ (C'B = R)

1 – tan B tanC = – 2

$\therefore$ tan B tan C = – 3]

Q.46$_{51/qe}$ The graph of a quadratic polynomial $y = ax^2 + bx + c$ (a, b, c $\in$ R) with vertex on y-axis is as shown in the figure. Then which one of the following statement is INCORRECT?



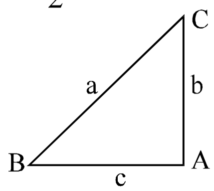
(A) Product of the roots of the corresponding quadratic equation is positive.
(B) Discriminant of the quadratic equation is negative.
(C*) Nothing definite can be said about the sum of the roots, whether positive, negative or zero.
(D) Both roots of the quadratic equation are purely imaginary.

[Sol. Roots are purely imaginary
i.e. $i\beta$ and $-i\beta$
$\therefore$ sum of roots = 0
incorrect (C)

product of roots $= -i^2\beta^2 = \beta^2 \Rightarrow$ product > 0 ; $\frac{c}{a} > 0 \Rightarrow c = +$ ve]

Q.47$_{75/ph-3}$ If in a triangle ABC $\frac{2\cos A}{a}+\frac{\cos B}{b}+\frac{2\cos C}{c}=\frac{a}{bc}+\frac{b}{ca}$ then the value of the angle A is:

(A) $\frac{\pi}{8}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D*) $\frac{\pi}{2}$



[Sol. $\frac{2(b^2+c^2-a^2)}{2abc}+\frac{c^2+a^2-b^2}{2abc}+\frac{2(a^2+b^2-c^2)}{2abc}=\frac{a}{bc}+\frac{b}{ca}$

$(b^2 + c^2 - a^2) + \frac{(c^2+a^2-b^2)}{2} + a^2 + b^2 - c^2 = a^2 + b^2$

$2b^2 - 2a^2 + c^2 + a^2 - b^2 = 0$

$b^2 - a^2 + c^2 = 0$

$b^2 + c^2 = a^2$]

Q.48$_{94/ph-1}$ If $\sin(\theta+\alpha) = a$ & $\sin(\theta+\beta) = b$ $(0 < \alpha, \beta, \theta < \pi/2)$ then $\cos 2(\alpha-\beta) - 4ab\cos(\alpha-\beta) =$

(A) $1 - a^2 - b^2$ (B*) $1 - 2a^2 - 2b^2$ (C) $2 + a^2 + b^2$ (D) $2 - a^2 - b^2$

[Sol. $2\cos^2(\alpha-\beta) - 4ab\cos(\alpha-\beta) - 1$

$2\cos^2\left(\overline{\alpha+\theta}-(\beta+\theta)\right) - 4ab\cos\left(\overline{\alpha+\theta}-(\beta+\theta)\right) - 1$

$2\left[\sqrt{1-a^2}.\sqrt{1-b^2}+ab\right]^2 - 4ab\left[\sqrt{1-a^2}.\sqrt{1-b^2}+ab\right] - 1$

On solving we get

$= 2[1 - b^2 - a^2 + a^2b^2] - 2a^2b^2 - 1$

$= 1 - 2a^2 - 2b^2$ Ans]

Q.49$_{51/s\&p}$ Concentric circles of radii 1, 2, 3......100 cms are drawn. The interior of the smallest circle is coloured red and the angular regions are coloured alternately green and red, so that no two adjacent regions are of the same colour. The total area of the green regions in sq. cm is equal to

(A) 1000π (B*) 5050π (C) 4950π (D) 5151π

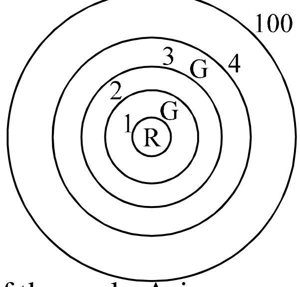
[Hint: $\pi\left[\left(r_2^2-r_1^2\right)+\left(r_4^2-r_3^2\right)+......+\left(r_{100}^2-r_{99}^2\right)\right]$

$\therefore\ r_2 - r_1 = r_4 - r_3 = ... = r_{100} - r_{99} = 1$

$= \pi\,[r_1 + r_2 + r_3 + r_4 + ... + r_{100}]$

$= \pi\,[1 + 2 + 3 + ... + 100]$

$= 5050\pi$ sq. cm.]



Q.50$_{49/ph-3}$ In a ΔABC if $b = a\left(\sqrt{3}-1\right)$ and $\angle C = 30^0$ then the measure of the angle A is

(A) 15^0 (B) 45^0 (C) 75^0 (D*) 105^0

[Hint: use $\tan\frac{A-B}{2}=\frac{a-b}{a+b}\cot\frac{C}{2}$ to get A – B and $A + B = 150^0$ (given)]

Q.51$_{55/qe}$ The number of solution of the equation $e^{2x} + e^x + e^{-2x} + e^{-x} = 3(e^{-2x} + e^x)$ is

(A) 0 (B) 2 (C*) 1 (D) more than 2

[Hint: $x = \ln 2$]

Q.52$_{71/ph-3}$ If in a triangle $\sin A : \sin C = \sin(A-B) : \sin(B-C)$ then $a^2 : b^2 : c^2$

(A*) are in A.P. (B) are in G.P.
(C) are in H.P. (D) none of these

[Sol. $\frac{\sin A}{\sin C} = \frac{\sin(A-B)}{\sin(B-C)}$

$\frac{\sin(B+C)}{\sin(A+B)} = \frac{\sin(A-B)}{\sin(B-C)}$

$\sin^2 B - \sin^2 C = \sin^2 A - \sin^2 B$

$\therefore$ $2b^2 = a^2 + c^2 \Rightarrow a^2, b^2, c^2$ are in A.P.]

Q.53$_{46/s\&p}$ The number of natural numbers less than 400 that are not divisible by 17 or 23 is

(A) 382 (B) 359 (C*) 360 (D) 376

[Hint: divisible by 17

17, 34, 51, ... 391 (23)

divisible by 23

23, 46, 69,.........391 (17)

divisible by both

391 (1)

divisible by 17 or 23 = 23 + 17 – 1 = 39

not divisible by 17 or 23 and < 400

= 399 – 39 = 360]

Q.54$_{98/ph-1}$ The minimum value of the expression $\frac{9x^2\sin^2 x + 4}{x \sin x}$ for $x \in (0, \pi)$ is

(A) $\frac{16}{3}$ (B) 6 (C*) 12 (D) $\frac{8}{3}$

[Sol. $E = 9x\sin x + \frac{4}{x \sin x}$ [note that $x \sin x > 0$ in $(0, \pi)$]

$E = \left(3\sqrt{x\sin x} - \frac{2}{\sqrt{x \sin x}}\right)^2 + 12$

$\therefore$ $E_{min} = 12$ which occurs when $3x\sin x = 2 \Rightarrow x \sin x = 2/3$]

note that $x \sin x$ is continuous at $x = 0$ and attains the value $\pi/2$ which is greater than $2/3$ at $x = \pi/2$, hence it must take the 2/3 in $(0, \pi/2)$]

Q.55$_{50/ph-3}$ In a ΔABC, $a = a_1 = 2$, $b = a_2$, $c = a_3$ such that $a_{p+1} = \frac{5^p}{3^{2-p}} a_p \left(2^{2-p} - \frac{4p-2}{5^p} a_p\right)$

where p = 1,2 then

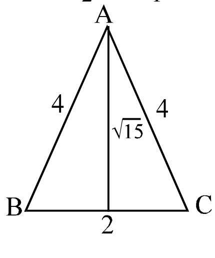
(A) $r_1 = r_2$ (B) $r_3 = 2r_1$ (C) $r_2 = 2r_1$ (D*) $r_2 = 3r_1$

[Hint: put $p = 1$, we get $a_2 = 4 \Rightarrow b = 4$

put $p = 2$, we get $a_3 = 4 \Rightarrow c = 4$

Hence the ΔABC is isosceles

now $\Delta = \sqrt{15}$



$\therefore$ $r_1 = \frac{\Delta}{s-a} = \frac{\sqrt{15}}{3}$ and $r_2 = \frac{\Delta}{s-b} = \frac{\sqrt{15}}{1} = r_3$

hence $r_2 = r_3 = 3r_1$]

Q.56$_{43/s\&p}$ The sum $\sum_{n=1}^{\infty} \frac{1}{n^2+2n}$ equals

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C*) $\frac{3}{4}$ (D) 1

[Sol. $T_n = \frac{1}{n(n+2)} = \frac{1}{2}\left[\frac{1}{n} - \frac{1}{n+2}\right]$

$T_1 = \frac{1}{2}\left[1 - \frac{1}{3}\right]$

$T_2 = \frac{1}{2}\left[\frac{1}{2} - \frac{1}{4}\right]$

$T_3 = \frac{1}{2}\left[\frac{1}{3} - \frac{1}{5}\right]$

$T_4 = \frac{1}{2}\left[\frac{1}{4} - \frac{1}{6}\right]$

add

$S = \frac{1}{2}\left[1 + \frac{1}{2}\right] = \frac{3}{4} \Rightarrow$ (C)]

Q.57$_{67/ph-3}$ The product of the arithmetic mean of the lengths of the sides of a triangle and harmonic mean of the lengths of the altitudes of the triangle is equal to :

(A) Δ (B*) 2Δ (C) 3Δ (D) 4Δ

[where Δ is the area of the triangle ABC]

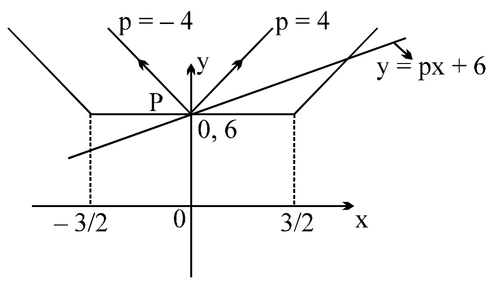
[Hint: $ah_1 = bh_2 = ch_3 = 2\Delta \Rightarrow \frac{1}{h_1} + \frac{1}{h_2} + \frac{1}{h_3} = \frac{a+b+c}{2\Delta} \Rightarrow \frac{a+b+c}{3} \cdot \frac{3}{\frac{1}{h_1}+\frac{1}{h_2}+\frac{1}{h_3}} = 2\Delta$]

Q.58$_{67/qe}$ The set of real value(s) of p for which the equation, $|2x+3| + |2x-3| = px + 6$ has more than two solutions is :

(A) (0, 4] (B) (– 4, 4) (C) R – {4, - 4, 0} (D*) {0}

[Hint : Draw graphs of :

$$y = \begin{cases} 4x & \text{if } x \ge \frac{3}{2} \\ 6 & \text{if } -\frac{3}{2} < x < \frac{3}{2} \\ -4x & \text{if } x \le -\frac{3}{2} \end{cases}$$



and $y = px + 6$

From the graph it is obvious that if,

$p = 0$ we have infinite solutions ranging from $\left[-\frac{3}{2}, \frac{3}{2}\right]$

if $0 < p < 4$ or $-4 < p < 0$,

two solutions , if $p = 4$ or -4 we have

$x = 0$ is the only solution]

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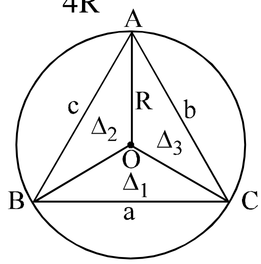
Q.59$_{51/ph-3}$ If 'O' is the circumcentre of the Δ ABC and R_1, R_2 and R_3 are the radii of the circumcircles of triangles OBC, OCA and OAB respectively then $\frac{a}{R_1} + \frac{b}{R_2} + \frac{c}{R_3}$ has the value equal to:

(A) $\frac{abc}{2R^3}$ (B) $\frac{R^3}{abc}$ (C*) $\frac{4\Delta}{R^2}$ (D) $\frac{\Delta}{4R^2}$



[Hint : Using $R = \frac{abc}{4\Delta}$ $\Rightarrow$ $\frac{a}{R} = \frac{4\Delta}{bc}$

$$\frac{a}{R_1} + \frac{b}{R_2} + \frac{c}{R_3} = \frac{4}{R^2}(\Delta_1 + \Delta_2 + \Delta_3) = \frac{4\Delta}{R^2} \Rightarrow C \text{]}$$

Q.60$_{36/s\&p}$ If for an A.P. $a_1, a_2, a_3, \ldots, a_n, \ldots$
$a_1 + a_3 + a_5 = -12$ and $a_1 a_2 a_3 = 8$
then the value of $a_2 + a_4 + a_6$ equals

(A) – 12 (B) – 16 (C) – 18 (D*) – 21

[Hint: Let the 1st 5 terms of the A.P. are
$a - 2d, a - d, a, a + d, a + 2d$
now $a_1 + a_3 + a_5 = -12$
$\therefore$ $3a = -12$ $\Rightarrow$ $a = -4$
also $a_1 . a_2 . a_3 = 8$
$(a - 2d)(a - d)a = 8$
$-4(-4 - 2d)(-4 - d) = 8 \Rightarrow d = -3$
Hence the A.P. is 2 , – 1 , – 4, – 7, – 10, – 13 ,......
Hence $a_2 + a_4 + a_6 = -21$]

Q.61$_{64/ph-3}$ If in a Δ ABC, $\cos A \cdot \cos B + \sin A \sin B \sin 2C = 1$ then, the statement which is incorrect, is

(A) Δ ABC is isosceles but not right angled (B) Δ ABC is acute angled

(C*) Δ ABC is right angled (D) least angle of the triangle is $\frac{\pi}{4}$

[Hint : $\sin 2C = \frac{1 - \cos A \cos B}{\sin A \sin B} \le 1$. Now proceed $A = B = \frac{3\pi}{8}$ and $C = \frac{\pi}{4}$
$1 \le \cos(A - B)$ $\Rightarrow$ $\cos(A - B) = 1$ $\Rightarrow$ $A = B$]

Q.62$_{70/qe}$ The absolute term in the quadratic expression $\sum_{k=1}^{n}\left(x - \frac{1}{k+1}\right)\left(x - \frac{1}{k}\right)$ as $n \to \infty$ is

(A*) 1 (B) –1 (C) 0 (D) 1/2

[Sol. $$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{k(k+1)} = \frac{1}{2}\sum_{k=1}^{n}\left(\frac{1}{k} - \frac{1}{k+1}\right) = \frac{1}{2}\left(1 - \frac{1}{n+1}\right)$$

$\therefore$ absolute term $= \lim_{n\to\infty}\frac{1}{2}\left(1 - \frac{1}{n+1}\right) = \frac{1}{2}$]

Q.63$_{19/ph-2}$ The number of roots of the equation, $\sin x + 2\sin 2x = 3 + \sin 3x$ is :

(A*) 0 (B) 1 (C) 2 (D) infinite

[Hint : $(\sin x - \sin 3x) + 2\sin 2x - 3 = 0$

$\Rightarrow$ $2\sin x \cos 2x - 2\sin 2x + 3 = 0$

$2\sin x\cos 2x - 2\sin 2x + (\sin^2 x + \cos^2 x) + (\sin^2 2x + \cos^2 2x) + 1 = 0$

$\Rightarrow$ $(1 - \sin 2x)^2 + (\sin x + \cos 2x)^2 + \cos^2 x = 0$ $\Rightarrow$ No root]

Q.64$_{54/ph-3}$ In a triangle the expression $\dfrac{a^2b^2c^2(\sin 2A + \sin 2B + \sin 2C)}{\Delta^3}$ simplifies to

(A) 8 (B) 16 (C*) 32 (D) 64

[Hint: $\left(\dfrac{abc}{\Delta}\right)^2 \cdot \dfrac{4\prod \sin A}{\Delta} = 16R^2 \cdot \dfrac{2}{R^2} = 32$]

Q.65$_{99/qe}$ Number of real values of x satisfying the equation

$$\sqrt{x^2 - 6x + 9} + \sqrt{x^2 - 6x + 6} = 1 \text{ is}$$

(A*) 0 (B) 1 (C) 2 (D) more than 2

[Hint: $(x^2 - 6x + 9) - (x^2 - 6x + 6) = 3$

$\sqrt{x^2 - 6x + 9} - \sqrt{x^2 - 6x + 6} = 3$

adding $\sqrt{x^2 - 6x + 9} = 2$

$x^2 - 6x + 9 = 4$

$x^2 - 6x + 5 = 0$

$(x - 5)(x - 1) = 0$ $\Rightarrow$ $x = 1$ or 5

but none satisfies.]

Q.66$_{100/qe}$ If the roots of the equation $x^3 - px^2 - r = 0$ are $\tan\alpha$, $\tan\beta$ and $\tan\gamma$ then the value of $\sec^2\alpha \cdot \sec^2\beta \cdot \sec^2\gamma$ is

(A) $p^2 + r^2 + 2rp + 1$ (B*) $p^2 + r^2 - 2rp + 1$ (C) $p^2 - r^2 - 2rp + 1$ (D) None

[Sol. $\sum \tan\alpha = p$; $\sum \tan\alpha \cdot \tan\beta = 0$; $\prod \tan\alpha = r$

now $\sec^2\alpha \cdot \sec^2\beta \cdot \sec^2\gamma = (1 + \tan^2\alpha)(1 + \tan^2\beta)(1 + \tan^2\gamma)$

$= 1 + \sum \tan^2\alpha + \sum \tan^2\alpha \cdot \tan^2\beta + \tan^2\alpha \cdot \tan^2\beta \cdot \tan^2\gamma$

now $\sum \tan^2\alpha = \left(\sum \tan\alpha\right)^2 - 2\sum \tan\alpha \cdot \tan\beta = p^2$

$\sum \tan^2\alpha \cdot \tan^2\beta = \left(\sum \tan\alpha \cdot \tan\beta\right)^2 - 2\tan\alpha \cdot \tan\beta \cdot \tan\gamma \left(\sum \tan\alpha\right)$

$= 0 - 2rp$

$\prod \tan^2\alpha = r^2$

$\therefore$ $\prod \sec^2\alpha = 1 + p^2 - 2rp + r^2 = 1 + (p - r)^2$]

Q.67$_{56/ph-3}$ If r_1, r_2, r_3 be the radii of excircles of the triangle ABC, then $\dfrac{\sum r_1}{\sqrt{\sum r_1 r_2}}$ is equal to :

(A) $\sum \cot\dfrac{A}{2}$ (B) $\sum \cot\dfrac{A}{2}\cot\dfrac{B}{2}$ (C*) $\sum \tan\dfrac{A}{2}$ (D) $\prod \tan\dfrac{A}{2}$

[Hint : $\dfrac{s\sum \tan\dfrac{A}{2}}{\sqrt{s^2}} = \sum \tan\dfrac{A}{2} \Rightarrow$ C]

Q.68$_{25/s\&p}$ There is a certain sequence of positive real numbers. Beginning from the third term, each term of the sequence is the sum of all the previous terms. The seventh term is equal to 1000 and the first term is equal to 1. The second term of this sequence is equal to

(A) 246 (B*) $\dfrac{123}{2}$ (C) $\dfrac{123}{4}$ (D) 124

[Sol. sequence is $t_1 + t_2 + t_3 + t_4 + ...$

$t_3 = t_1 + t_2$; $t_7 = 1000$

$t_1 = 1$

but $t_7 = t_1 + t_2 + t_3 + t_4 + t_5 + t_6$

$1000 = 2(t_1 + t_2 + t_3 + t_4 + t_5)$

$= 4(t_1 + t_2 + t_3 + t_4)$

$= 8(t_1 + t_2 + t_3)$

$1000 = 16(t_1 + t_2)$

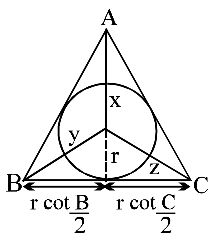
$t_1 + t_2 = \dfrac{1000}{16} \Rightarrow \quad t_2 = \dfrac{1000}{16} - 1 = \dfrac{125}{2} - 1 = \dfrac{123}{2}$ Ans]

Q.69$_{57/ph-3}$ If x, y and z are the distances of incentre from the vertices of the triangle ABC respectively then $\dfrac{a\,b\,c}{x\,y\,z}$ is equal to

(A) $\prod \tan\dfrac{A}{2}$ (B*) $\sum \cot\dfrac{A}{2}$ (C) $\sum \tan\dfrac{A}{2}$ (D) $\sum \sin\dfrac{A}{2}$

[Sol. $x = r\operatorname{cosec}\dfrac{A}{2}$

$a = r\left(\cot\dfrac{B}{2} + \cot\dfrac{C}{2}\right)$



$\dfrac{a}{x} = \left(\cot\dfrac{B}{2} + \cot\dfrac{C}{2}\right).\sin\dfrac{A}{2} = \dfrac{\sin\dfrac{A}{2}.\cos\dfrac{A}{2}}{\sin\dfrac{B}{2}.\sin\dfrac{C}{2}}$

$\therefore \quad \dfrac{abc}{xyz} = \dfrac{\cos\dfrac{A}{2}.\cos\dfrac{B}{2}.\cos\dfrac{C}{2}}{\sin\dfrac{A}{2}\sin\dfrac{B}{2}.\sin\dfrac{C}{2}} = \cot\dfrac{A}{2}.\cot\dfrac{B}{2}.\cot\dfrac{C}{2}$

In a triangle $\prod \cot\dfrac{A}{2} = \sum \cot\dfrac{A}{2}$]

Q.70$_{102/qe}$ If the equation $a(x-1)^2 + b(x^2 - 3x + 2) + x - a^2 = 0$ is satisfied for all $x \in R$ then the number of ordered pairs of (a, b) can be

(A) 0 (B*) 1 (C) 2 (D) infinite

[Sol. equation is an identity $\Rightarrow$ coefficient of $x^2 = 0 =$ coefficient of x = constant term

$\therefore$ $a + b = 0$ ....(1)

$-2a - 3b + 1 = 0$ ....(2)

and $a + 2b - a^2 = 0$ ....(3)

from (1) and (2) $a = -1$ and $b = 1$

which also satisfies (3) $\Rightarrow$ $(a, b) = (-1, 1)$ $\Rightarrow$ (B)]

Q.71$_{60/ph-3}$ In a ΔABC, a semicircle is inscribed, whose diameter lies on the side c. Then the radius of the semicircle is

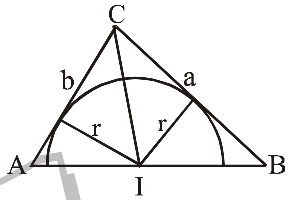
(A*) $\frac{2\Delta}{a+b}$ (B) $\frac{2\Delta}{a+b-c}$ (C) $\frac{2\Delta}{s}$ (D) $\frac{c}{2}$

Where Δ is the area of the triangle ABC.

[Hint: $\frac{1}{2} r a + \frac{1}{2} r b = \frac{1}{2} a b \sin C$

$r(a+b) = 2\Delta$

$r = \frac{2\Delta}{a+b}$]



Q.72$_{107/qe}$ Number of quadratic equations with real roots which remain unchanged even after squaring their roots, is

(A) 1 (B) 2 (C*) 3 (D) 4

[Hint: $\alpha\beta = \alpha^2\beta^2$ ....(1)

and $\alpha^2 + \beta^2 = \alpha + \beta$ ....(2)

Hence $\alpha\beta(1-\alpha\beta) = 0 \Rightarrow \alpha = 0$ or $\beta = 0$ or $\alpha\beta = 1$

if $\alpha = 0$ then from (2) $\beta = 0$ or $\beta = 1 \Rightarrow$ roots are (0,0) or (0,1)

if $\beta = 0$ then $\alpha = 0$ or $\beta = 1$

if $\beta = \frac{1}{\alpha}$ then $\alpha^2 + \frac{1}{\alpha^2} = \alpha + \frac{1}{\alpha} \Rightarrow \left[\alpha + \frac{1}{\alpha}\right]^2 - 2 = \alpha + \frac{1}{\alpha}$

hence $t^2 - t - 2 = 0 \Rightarrow (t-2)(t+1) = 0 \Rightarrow t = 2$ or $t = -1$

if $t = 2 \Rightarrow \alpha = 1$ & $\beta = 1$, if $t = -1$ roots are imaginary (ω or ω^2)]

Q.73$_{14/s\&p}$ Along a road lies an odd number of stones placed at intervals of 10 m. These stones have to be assembled around the middle stone. A person can carry only one stone at a time. A man carried out the job starting with the stone in the middle, carrying stones in succession, thereby covering a distance of 4.8 km. Then the number of stones is

(A) 15 (B) 29 (C*) 31 (D) 35

[Hint: Let there be 2n + 1 stones ; i.e. n stones on each side of the middle stone. The man will run 20 m, to pick pick up the first stone and return, 40 m. for the second stone and so on. So he runs $(n/2)(2 \times 20 + (n-1)20) = 10n(n+1)$ meters to pick up the stones on one side, and hence $20n(n+1)$ m , to pick up all the stones.

$\therefore$ $20n(n+1) = 4800$, or n = 15.

$\therefore$ there are 2n + 1 = 31 stones]

Q.74$_{61/ph-3}$ If in a Δ ABC, $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$ then the triangle is

(A) right angled (B) isosceles (C*) equilateral (D) obtuse

[Hint : $\frac{\cos A}{2R\sin A} = \frac{\cos B}{2R\sin B} = \frac{\cos C}{2R\sin C} \Rightarrow \tan A = \tan B = \tan C \Rightarrow A = B = C \Rightarrow$ (C)]

Q.75$_{118/qe}$ The quadratic equation $ax^2 + bx + c = 0$ has imaginary roots if :

(A) $a < -1,\ 0 < c < 1,\ b > 0$ (B) $a < -1,\ -1 < c < 0,\ 0 < b < 1$

(C) $a < -1,\ c < 0,\ b > 1$ (D*) none

Q.76$_{138/qe}$ The equations $x^3 + 5x^2 + px + q = 0$ and $x^3 + 7x^2 + px + r = 0$ have two roots in common. If the third root of each equation is represented by x_1 and x_2 respectively, then the ordered pair (x_1, x_2) is :

(A*) $(-5, -7)$ (B) $(1, -1)$ (C) $(-1, 1)$ (D) $(5, 7)$

[Hint : The common roots must be roots of the equation $2x^2 + (r - q) = 0$

$\Rightarrow$ sum is zero. Hence third root of first is -5 and third root of 2nd is -7]

Q.77$_{63/ph-3}$ If $\cos A + \cos B + 2\cos C = 2$ then the sides of the Δ ABC are in

(A*) A.P. (B) G.P (C) H.P. (D) none

[Hint : $\cos A + \cos B = 2(1-\cos C) = 4\sin^2\frac{C}{2}$ or $2\cos\frac{A+B}{2}\cos\frac{A-B}{2} = 4\sin^2\frac{C}{2}$

or $\cos\frac{A-B}{2} = 2\sin\frac{C}{2}$ or $2\cos\frac{C}{2}\cos\frac{A-B}{2} = 4\sin\frac{C}{2}\cos\frac{C}{2} = 2\sin C$

$2\sin\frac{A+B}{2}\cos\frac{A-B}{2} = 2\sin C$ or $\sin A + \sin B = 2\sin C \Rightarrow$ a, c, b are in A.P.]

<u>Select the correct alternatives</u> : (More than one are correct)

Q.78$_{501(x)/ph-2}$ If $\sin\theta = \sin\alpha$ then $\sin\frac{\theta}{3}$ equal to

(A*) $\sin\frac{\alpha}{3}$ (B*) $\sin\left(\frac{\pi}{3} - \frac{\alpha}{3}\right)$ (C) $\sin\left(\frac{\pi}{3} + \frac{\alpha}{3}\right)$ (D*) $-\sin\left(\frac{\pi}{3} + \frac{\alpha}{3}\right)$

[Sol. $\sin\theta = \sin\alpha$

$\theta = n\pi + (-1)^n\alpha$

$n = 0$ $\theta = \alpha$ $\sin\theta/3 = \sin\alpha/3$ $\Rightarrow$ (A)

$n = 1$ $\theta = \pi - \alpha$ $\sin\theta/3 = \sin(\pi/3 - \alpha/3)$ $\Rightarrow$ (B)

$n = -1$ $\theta = -\pi - \alpha$ $\sin\theta/3 = \sin(-\pi/3 - \alpha/3) = -\sin(\pi/3 + \alpha/3)$ $\Rightarrow$ (D)]

Q.79$_{501/qe}$ $\cos\alpha$ is a root of the equation $25x^2 + 5x - 12 = 0,\ -1 < x < 0$, then the value of $\sin 2\alpha$ is :

(A*) 24/25 (B) $-12/25$ (C*) $-24/25$ (D) 20/25

Q.80$_{507/s\&p}$ If $\sin(x - y)$, $\sin x$ and $\sin(x + y)$ are in H.P., then $\sin x . \sec\frac{y}{2}$ =

(A) 2 (B*) $\sqrt{2}$ (C*) $-\sqrt{2}$ (D) -2

[Hint : $\sin x = \dfrac{2\sin(x-y)\sin(x+y)}{\sin(x-y)+\sin(x+y)} = \dfrac{2(\sin^2 x - \sin^2 y)}{2\sin x\cos y}$

$\sin^2 x\cos y = \sin^2 x - \sin^2 y$ $\qquad \sin^2 x(1-\cos y) = \sin^2 y$

$\sin^2 x + 2\sin^2\frac{y}{2} = 4\sin^2\frac{y}{2}\cos^2\frac{y}{2}$ $\qquad \sin^2 x\sec^2\frac{y}{2} = 2$ $\qquad \sin x\sec\frac{y}{2} = \pm\sqrt{2}$]

Q.81$_{510/ph-1}$ Which of the following functions have the maximum value unity ?

(A*) $\sin^2 x - \cos^2 x$ $\qquad$ (B*) $\dfrac{\sin 2x - \cos 2x}{\sqrt{2}}$

(C*) $-\dfrac{\sin 2x - \cos 2x}{\sqrt{2}}$ $\qquad$ (D*) $\sqrt{\dfrac{6}{5}}\left(\dfrac{1}{\sqrt{2}}\sin x + \dfrac{1}{\sqrt{3}}\cos x\right)$

Q.82$_{503/ph-3}$ In a triangle, the lengths of the two larger sides are 10 and 9 respectively . If the angles are in A.P., then the length of the third side can be :

(A*) $5-\sqrt{6}$ $\qquad$ (B) $3\sqrt{3}$ $\qquad$ (C) 5 $\qquad$ (D*) $6 \pm \sqrt{5}$

[Hint : $a > b > c$; $B = 60°$; use $\cos B$ as cosine rule to get two values of c]

Q.83$_{504/log}$ If $\left(a^{\log_b x}\right)^2 - 5\, x^{\log_b a} + 6 = 0$, where $a > 0$, $b > 0$ & $ab \neq 1$, then the value of x can be equal to

(A) $2^{\log_b a}$ $\qquad$ (B*) $3^{\log_a b}$ $\qquad$ (C*) $b^{\log_a 2}$ $\qquad$ (D) $a^{\log_b 3}$

[Hint: $\quad a^{\log_b x} = x^{\log_b a} = y$

$y^2 - 5y + 6 = 0 \Rightarrow y = 2, 3$

when $\quad y = 2 = a^{\log_b x} \Rightarrow \quad \log_b x = \log_a 2 \quad \Rightarrow x = b^{\log_a 2}$

when $\quad y = 3 = a^{\log_b x} \Rightarrow x = 3^{\log_a b}$]

Q.84$_{510/s\&p}$ If the roots of the equation, $x^3 + px^2 + qx - 1 = 0$ form an increasing G.P. where p and q are real, then

(A*) p+q = 0 $\qquad$ (B*) $p \in (-\infty, -3)$

(C*) one of the roots is one $\qquad$ (D*) one root is smaller than 1 & one root is greater than 1.

[Sol. roots are a/r , a, ar : where a >0 , r > 1]

Now $\quad a/r + a + ar = -p \qquad$ ....(1)

$a.a/r + a.\ ar + ar.\ a/r = q \qquad$ ....(2)

$a/r.\ a.\ ar = 1 \qquad$ ....(3)

$a^3 = 1 \Rightarrow a = 1 \qquad \Rightarrow$ [C]

from (1) putting a = 1 we get

$1/r + 1 + r = -p \qquad$ ....(4)

$\left(\sqrt{r} - \dfrac{1}{\sqrt{r}}\right)^2 + 3 = -p$

$p + 3 < 0 \qquad \Rightarrow$ (B)

from (2) putting a = 1 we get

$1/r + r + 1 = q \qquad$ ....(5)

from (4) and (5) we have $-p = q \Rightarrow p+q = 0 \Rightarrow$ [A]

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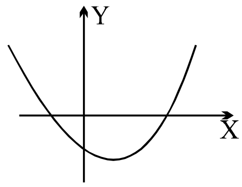
Now as , $r > 1$

$a/r = 1/r < 1$

and $ar = r > 1 \Rightarrow$ [D]]

Q.85$_{502/qe}$ The graph of the quadratic polynomial ;

$y = ax^2 + bx + c$ is as shown in the figure . Then :



(A*) $b^2 - 4ac > 0$ (B*) $b < 0$

(C*) $a > 0$ (D*) $c < 0$

Q.86$_{504/log}$ If $\left(a^{\log_b x}\right)^2 - 5\, x^{\log_b a} + 6 = 0$, where $a > 0$, $b > 0$ & $ab \neq 1$, then the value of x can be equal to

(A) $2^{\log_b a}$ (B*) $3^{\log_a b}$ (C*) $b^{\log_a 2}$ (D) $a^{\log_b 3}$

[Hint: $a^{\log_b x} = x^{\log_b a} = y$

$y^2 - 5y + 6 = 0 \Rightarrow y = 2 , 3$

when $y = 2 = a^{\log_b x} \Rightarrow \log_b x = \log_a 2 \Rightarrow x = b^{\log_a 2}$

when $y = 3 = a^{\log_b x} \Rightarrow x = 3^{\log_a b}$]

Q.87$_{513/qe}$ Let Δ^2 be the discriminant and α, β be the roots of the equation $ax^2 + bx + c = 0$. Then $2a\alpha + \Delta$ and $2a\beta - \Delta$ can be the roots of the equation :

(A*) $x^2 + 2bx + b^2 = 0$ (B) $x^2 - 2bx + b^2 = 0$

(C*) $x^2 + 2bx - 3b^2 + 16ac = 0$ (D) $x^2 - 2bx - 3b^2 + 16ac = 0$

[Hint : $\alpha, \beta = \dfrac{-b \pm \sqrt{\Delta^2}}{2a} \Rightarrow \alpha = \dfrac{-b + \Delta}{2a}$ and $\beta = \dfrac{-b - \Delta}{2a}$

$\Rightarrow$ $2a\alpha - \Delta = -b$ and $2a\beta + \Delta = -b$ ....(1)

or $2a\alpha + \Delta = -b$ and $2a\beta - \Delta = -b$ ....(2)

From (1) $2a\alpha + \Delta = 2\Delta - b$ and $2a\beta - \Delta = -2\Delta - b$

sum = $-2b$; Product = $b^2 - 4\Delta^2 = b^2 - 4(b^2 - 4ac) = 16ac - 3b^2$ ....(3)

From (2) sum = $-2b$; Product = b^2 ....(4)

Hence QE is $x^2 + 2bx + b^2 = 0$

or $x^2 + 2bx + 16ac - 3b^2 = 0$ $\Rightarrow$ A and C]

Q.88$_{514/log}$ Which of the following statement(s) is/are true?

(A*) $\log_{10} 2$ lies between $\dfrac{1}{3}$ & $\dfrac{1}{4}$ (B*) $\log_{\text{cosec}(5\pi/6)} \cos \dfrac{5\pi}{3} = -1$

(C) $e^{\ln(\ln 3)}$ is smaller than 1

(D*) $\log_{10} 1 + \dfrac{1}{2} \log_{10} 3 + \log_{10}\left(2 + \sqrt{3}\right) = \log\left(1 + \sqrt{3} + \left(2 + \sqrt{3}\right)\right)$

[Hint: (A) $\log_{10} 2 = \dfrac{1}{\log_2 10}$

$\log_2 8 < \log_2 10 < \log_2 16$

$3 < \log_2 10 < 4 \Rightarrow$ T

(B) L.H.S. = $\log_2 1/2 = -1 \Rightarrow$ T

(C) $e^{\ln(\ln 3)} = \ln 3 = \log_{2.7} 3 > 1 \Rightarrow$ F

(D) L.H.S. = $\log_{10}(3 + \sqrt{3}) = \log_{10} 3 + 2\sqrt{3}$ = R.H.S. $\Rightarrow$ T]

Q.89$_{502/ph-2}$ Select the statement(s) which are true in respect of a triangle ABC, all symbols have their usual meaning.

(A*) The inradius, circumradius and one of the exradii of an equilateral triangle are in the ratio of $1 : 2 : 3$.

(B) $abc = \frac{1}{4}$ Rrs

(C*) If r = 3 then the value of $\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{3}$

(D*) If the diameter of any escribed cricle is equal to the perimeter then the triangle must be a right triangle .

[Hint: (A) Let a = b = c = 1

$\therefore \quad r = \frac{\Delta}{s} = \frac{\sqrt{3}/4}{3/2} = \frac{\sqrt{3}}{4}.\frac{2}{3} = \frac{\sqrt{3}}{6}$

$R = \frac{abc}{\Delta} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

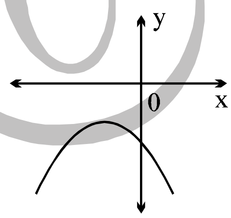
$r_1 = s\tan\frac{A}{2} = \frac{3}{2}\tan 30^0 = \frac{\sqrt{3}}{2} \qquad \Rightarrow r : R : r_1 = \sqrt{3} : 2\sqrt{3} : 3\sqrt{3}$

(B) obviously wrong

(C) note that $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$

(D) $2r_1 = 2s \quad \Rightarrow \quad s\tan\frac{A}{2} = s \Rightarrow \frac{A}{2} \quad 45^0 \Rightarrow A = 90^0$]

Q.90$_{515/qe}$ The graph of a quadratic polynomial $y = ax^2 + bx + c$ (a, b, c $\in$ R, a $\neq$ 0) is as shown. Then the incorrect statement(s) is/are



(A*) c > 0 (B) b < 0

(C*) product of the roots is negative (D*) sum of the roots is positive

Q.91$_{512/s\&p}$ The points $A(x_1, y_1)$; $B(x_2, y_2)$ and $C(x_3, y_3)$ lie on the parabola $y = 3x^2$. If x_1, x_2, x_3 are in A.P. and y_1, y_2, y_3 are in G.P. then the common ratio of the G.P. is

(A*) $3 + 2\sqrt{2}$ (B) $3 + \sqrt{2}$ (C) $3 - \sqrt{2}$ (D*) $3 - 2\sqrt{2}$

[Sol. Let $x_1 = t - d \quad ; \quad y_1 = 3(t - d)^2$

$x_2 = t \quad ; \quad y_2 = 3t^2$

$x_3 = t + d \quad ; \quad y_3 = 3(t + d)^2$

since y_1, y_2 and y_3 are in G.P.

however $9t^4 = 9(t - d)^2(t + d)^2$

$t^2 = (t - d)(t + d) \quad$ or $\quad -(t - d)(t + d)$

$t^2 = t^2 - d^2$ rejected as a $\neq$ 0

$\therefore \quad t^2 = d^2 - t^2$

$2t^2 = d^2 \quad \Rightarrow \quad d = \sqrt{2}\,t$ or $-\sqrt{2}\,t$

$$r = \frac{t^2}{(t-d)^2} = \frac{t^2}{(t-\sqrt{2}\,t)^2} = \frac{1}{(\sqrt{2}-1)^2} = \frac{1}{3-2\sqrt{2}} = 3 + 2\sqrt{2}$$

if $d = -\sqrt{2}\,t$ then $r = 3 - 2\sqrt{2}$]