विध्न विचारत भीरु जन, नहीं आरम्भे काम, विपति देख छोड़े तुरंत मध्यम मन कर श्याम।
पुरुष सिंह संकल्प कर, सहते विपति अनेक, 'बना' न छोड़े ध्येय को, रघुबर राखे टेक।।

रचितः मानव धर्म प्रणेता

सद्‌गुरु श्री रणछोड़दासजी महाराज

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Question bank on Quadratic Eqaution, Determinant & Matrices

***Select the correct alternative* : (Only one is correct)**

Q.1$_{4/qe}$ If $a^2 + b^2 + c^2 = 1$ then $ab + bc + ca$ lies in the interval :

(A) $\left[\frac{1}{2}, 2\right]$ (B) $[-1, 2]$ (C*) $\left[-\frac{1}{2}, 1\right]$ (D) $\left[-1, \frac{1}{2}\right]$

[Hint: $\sum (a-b)^2 \ge 0 \Rightarrow 2\sum a^2 - 2\sum ab \ge 0 \Rightarrow \sum ab \le \sum a^2 \Rightarrow ab + bc + ca \le 1$
Also note that $(a + b + c)^2 \ge 0$]

Q.2$_{1/det}$ The value of the determinant $\begin{vmatrix} a^2 & a & 1 \\ \cos(nx) & \cos(n+1)x & \cos(n+2)x \\ \sin(nx) & \sin(n+1)x & \sin(n+2)x \end{vmatrix}$ is independent of :

(A*) n (B) a (C) x (D) a , n and x

[Hint: Directly open by R_1 to get a form of $\sin(A - B)$ etc.]

Q.3$_{2/mat}$ A is an involutary matrix given by $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$ then the inverse of $\frac{A}{2}$ will be

(A*) 2A (B) $\frac{A^{-1}}{2}$ (C) $\frac{A}{2}$ (D) A^2

[Hint: A is involutary $\Rightarrow$ $A^2 = I$ $\Rightarrow$ $A = A^{-1}$

also $(KA)^{-1} = \frac{1}{k}(A)^{-1}$; hence $\left(\frac{1}{2}A\right)^{-1} = 2(A)^{-1}$ $\Rightarrow$ 2A]

Q.4$_{5/qe}$ If $P(x) = ax^2 + bx + c$ & $Q(x) = -ax^2 + dx + c$, where $ac \ne 0$, then $P(x) . Q(x) = 0$ has

(A) exactly one real root (B*) atleast two real roots
(C) exactly three real roots (D) all four are real roots .

[Hint : D_1 : $b^2 - 4ac$ & D_2 : $d^2 + 4ac$. Hence atleast one of either D_1 or D_2 is zero]

Q.5$_{2/det}$ If a, b, c are all different from zero & $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = 0$, then the value of $a^{-1} + b^{-1} + c^{-1}$ is

(A) abc (B) $a^{-1} b^{-1} c^{-1}$ (C) $-a - b - c$ (D*) -1

[Hint: $C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$ & then open by R_1 to get $ab + abc + ac + bc = 0$; divided by abc]

Q.6$_{3/mat}$ If A and B are symmetric matrices, then ABA is

(A*) symmetric matrix (B) skew symmetric
(C) diagonal matrix (D) scalar matrix

[Sol. We have $(ABA)' = A'B'A' = ABA \Rightarrow ABA$ is symmetric]

Q.7$_{11/qe}$ Let $a > 0, b > 0$ & $c > 0$. Then both the roots of the equation $ax^2 + bx + c = 0$

(A) are real & negative (B*) have negative real parts
(C) are rational numbers (D) none

[Hint: $\alpha, \beta = -\dfrac{b}{2a} \pm \dfrac{\sqrt{b^2 - 4ac}}{2a}$; consider the examples $x^2 + x + 1 = 0$ and $x^2 + 3x + 2 = 0$]

Q.8$_{4/det}$ If α, β & γ are real numbers, then $D = \begin{vmatrix} 1 & \cos(\beta-\alpha) & \cos(\gamma-\alpha) \\ \cos(\alpha-\beta) & 1 & \cos(\gamma-\beta) \\ \cos(\alpha-\gamma) & \cos(\beta-\gamma) & 1 \end{vmatrix} =$

(A) -1 (B) $\cos\alpha \cos\beta \cos\gamma$
(C) $\cos\alpha + \cos\beta + \cos\gamma$ (D*) zero

[Hint: Write 1 as $\sin^2\alpha + \cos^2\alpha$ etc. to get

$$\begin{vmatrix} \sin^2\alpha + \cos^2\alpha & \cos\beta\cos\alpha + \sin\beta\sin\alpha & \cos\gamma\cos\alpha + \sin\gamma\sin\alpha \\ \cos\alpha\cos\beta + \sin\alpha\sin\beta & \cos^2\beta + \sin^2\beta & \cos\gamma\cos\beta + \sin\gamma\sin\beta \\ \cos\alpha\cos\gamma + \sin\alpha\sin\gamma & \cos\beta\cos\gamma + \sin\beta\sin\gamma & \sin^2\gamma + \cos^2\gamma \end{vmatrix}$$

can be factorized into 2 determinant

$$\begin{vmatrix} \cos\alpha & \sin\alpha & x \\ \cos\beta & \sin\beta & x \\ \cos\gamma & \sin\gamma & x \end{vmatrix} \begin{vmatrix} \cos\alpha & \cos\beta & \cos\gamma \\ \sin\alpha & \sin\beta & \sin\gamma \\ x & x & x \end{vmatrix} = 0 \quad]$$

Alternatively: $\alpha - \beta = x$; $\beta - \gamma = y$; $\gamma - \alpha = z \Rightarrow x + y + z = 0$ Now expand]

Q.9$_{14/qe}$ The real values of 'a' for which the quadratic equation, $2x^2 - (a^3 + 8a - 1)x + a^2 - 4a = 0$ possesses roots of opposite signs is given by :

(A) $a > 5$ (B*) $0 < a < 4$ (C) $a > 0$ (D) $a > 7$

[Hint: $f(0) < 0$]

Q.10$_{4/mat}$ If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, A^{-1} is given by

(A) $-A$ (B*) A^T (C) $-A^T$ (D) A

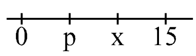
[Sol. For Adj A interchange the diagonal elements and change the sign of off diagonal elements.

We have $A^{-1} = \dfrac{1}{\cos^2\theta + \sin^2\theta}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = A^T \Rightarrow$ A is orthogonal matrix]

Q.11$_{20/qe}$ The minimum value of the expression $|x - p| + |x - 15| + |x - p - 15|$ for 'x' in the range $p \le x \le 15$ where $0 < p < 15$, is

(A) 10 (B*) 15 (C) 30 (D) 0

[Sol. $|x - p| = x - p$ (since $x \ge p$)
$|x - 15| = 15 - x$ (since $x \le 15$)
$|x - (p + 15)| = (p + 15) - x$ (as $15 + p > x$)
$\therefore$ expression reduces to
$E = x - p + 15 - x + p + 15 - x$



$E = 30 - x$

$\therefore$ E_{min} occurs when $x = 15$

$\therefore$ $E_{min} = 15$ Ans.]

Q.12$_{6/det}$ If the system of equations $ax + y + z = 0$, $x + by + z = 0$ & $x + y + cz = 0$ $(a, b, c \neq 1)$ has a non-trivial solution, then the value of $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$ is :

(A) -1 (B) 0 (C*) 1 (D) none of these

[Hint: $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix}$ Use $R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_1$ & open by C_1 to get

$(1 - a)[(1 - b)c + (1 - c)] + (1 - b)(1 - c) = 0$ divide by $(1 - a)(1 - b)(1 - c)$ to get the result]

Q.13$_{22/qe}$ If a, b, c are real numbers satisfying the condition $a + b + c = 0$ then the roots of the quadratic equation $3ax^2 + 5bx + 7c = 0$ are :

(A) positive (B) negative (C*) real & distinct (D) imaginary

[Hint: $D = 25b^2 - 84ac$

$= 25(a + c)^2 - 84ac$ using $b = -(a + c)$

$= 21[(a+c)^2 - 4ac] + 4(a+c)^2 > 0$]

Q.14$_{6/mat}$ Consider the matrices $A = \begin{bmatrix} 4 & 6 & -1 \\ 3 & 0 & 2 \\ 1 & -2 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 4 \\ 0 & 1 \\ -1 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$. Out of the given matrix products

(i) $(AB)^TC$ (ii) $C^TC(AB)^T$ (iii) C^TAB and (iv) A^TABB^TC

(A) exactly one is defined (B) exactly two are defined

(C*) exactly three are defined (D) all four are defined

[Hint: (i), (iii) and (iv) are correct]

Q.15$_{24/qe}$ If the difference of the roots of the equation, $x^2 + ax + b = 0$ is equal to the difference of the roots of the equation $x^2 + bx + a = 0$ then :

(A) $a + b = 4$ (B*) $a + b = -4$ (C) $a - b = 4$ (D) $a - b = -4$

Q.16$_{7/det}$ The value of a for which the system of equations ; $a^3x + (a+1)^3 y + (a+2)^3 z = 0$, $ax + (a+1)y + (a+2)z = 0$ & $x + y + z = 0$ has a non-zero solution is :

(A) 1 (B) 0 (C*) -1 (D) none of these

[Hint: Use $c_2 \to c_2 - c_1$ & $c_3 \to c_3 - c_2$ & then open by R_3.]

Q.17$_{29/qe}$ Suppose a, b, and c are positive numbers such that $a + b + c = 1$. Then the maximum value of $ab + bc + ca$ is

(A*) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$

[Sol. $a^2 + b^2 + c^2 = 1 - 2\sum ab$ ....(1)

also $(a - b)^2 \geq 0$ etc.

hence $a^2 + b^2 + c^2 \geq ab + bc + ca$

$1 - 2\sum ab \geq \sum ab$

$1 \geq 3\sum ab$

$\therefore \quad \sum ab \leq \frac{1}{3}$ Ans.]

Q.18$_{10/mat}$ If $A = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$, then A^n (where $n \in N$) equals

(A*) $\begin{pmatrix} 1 & na \\ 0 & 1 \end{pmatrix}$ (B) $\begin{pmatrix} 1 & n^2a \\ 0 & 1 \end{pmatrix}$ (C) $\begin{pmatrix} 1 & na \\ 0 & 0 \end{pmatrix}$ (D) $\begin{pmatrix} n & na \\ 0 & n \end{pmatrix}$

[Sol. We have $A^2 = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2a \\ 0 & 1 \end{pmatrix}$

$$A^3 = A^2A = \begin{pmatrix} 1 & 2a \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3a \\ 0 & 1 \end{pmatrix}$$

In general by induction, $A^n = \begin{pmatrix} 1 & na \\ 0 & 1 \end{pmatrix}, \forall n \in N$]

Q.19$_{30/qe}$ The roots of $(x-1)(x-3) + K(x-2)(x-4) = 0$, $K > 0$ are :

(A*) real (B) real & equal (C) imaginary (D) one real & one imaginary

[Hint : check f(1) , f(2) , f(3) & f(4) and interpret
note that one root lie between 1 and 2 and the other between 3 and 4]

Q.20$_{10/det}$ Let $f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 1+\cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 1+4\sin 2x \end{vmatrix}$, then the maximum value of $f(x) =$

(A) 2 (B) 4 (C*) 6 (D) 8

[Hint : Use $R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$ and expand to get $f(x) = 2 + 4\sin 2x$]

Q.21$_{35/qe}$ Largest integral value of *m* for which the quadratic expression
$y = x^2 + (2m+6)x + 4m + 12$ is always positive, $\forall x \in R$, is

(A) – 1 (B) – 2 (C*) 0 (D) 2

[Hint: $D < 0 \Rightarrow -3 < m < 1 \Rightarrow m = 0$]

Q.22$_{12/mat}$ If $A = \begin{bmatrix} 3 & 4 \\ 1 & -6 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 5 \\ 6 & 1 \end{bmatrix}$ then X such that $A + 2X = B$ equals

(A) $\begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 3 & 5 \\ -1 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 5 & 2 \\ -1 & 0 \end{bmatrix}$ (D*) none of these

[Sol: $X = \frac{1}{2}(B-A) = \frac{1}{2}\begin{bmatrix} -5 & 1 \\ 5 & 7 \end{bmatrix}$]

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Q.23$_{11/det}$ If $px^4 + qx^3 + rx^2 + sx + t \equiv \begin{vmatrix} x^2+3x & x-1 & x+3 \\ x+1 & 2-x & x-3 \\ x-3 & x+4 & 3x \end{vmatrix}$ then $t =$

(A) 33 (B) 0 (C*) 21 (D) none

[Hint: Put $x = 0$ & then evaluate]

Q.24$_{36/qe}$ Let $P(x) = kx^3 + 2k^2x^2 + k^3$. Find the sum of all real numbers k for which $x - 2$ is a factor of $P(x)$.

(A) 4 (B) 8 (C) – 4 (D*) – 8

[Hint: put $x = 2$, $P(2) = 0$, $k^3 + 8k^2 + 8k = 0 \Rightarrow k_1 + k_2 + k_3 = -8$]

Q.25$_{15/mat}$ If A and B are invertible matrices, which one of the following statements is not correct

(A) Adj. $A = |A|\, A^{-1}$ (B) $\det(A^{-1}) = |\det(A)|^{-1}$

(C*) $(A + B)^{-1} = B^{-1} + A^{-1}$ (D) $(AB)^{-1} = B^{-1}A^{-1}$

[Sol. $AA^{-1} = I \Rightarrow |AA^{-1}| = |I| = 1$

hence $|A|\,|A^{-1}| = 1 \Rightarrow |A^{-1}| = \dfrac{1}{|A|} \Rightarrow$ (B) is correct]

Q.26$_{39/qe}$ The sum of all the value of m for which the roots x_1 and x_2 of the quadratic equation $x^2 - 2mx + m = 0$ satisfy the condition $x_1^3 + x_2^3 = x_1^2 + x_2^2$, is

(A) $\dfrac{3}{4}$ (B) 1 (C) $\dfrac{9}{4}$ (D*) $\dfrac{5}{4}$

[Hint: $x_1 + x_2 = 2m$; $x_1 x_2 = m$

$(x_1 + x_2)^3 - 3x_1x_2(x_1 + x_2) = (x_1 + x_2)^2 - 2x_1x_2$

$8m^3 - 3m(2m) = 4m^2 - 2m$

$8m^3 - 10m^2 + 2m = 0$

$2m(4m^2 - 5m + 1) = 0 \Rightarrow m = 0$

$(m - 1)(4m - 1) = 0 \Rightarrow m = 1$ or $m = 1/4$]

Q.27$_{13/det}$ If $D = \begin{vmatrix} a^2+1 & ab & ac \\ ba & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$ then $D =$

(A*) $1 + a^2 + b^2 + c^2$ (B) $a^2 + b^2 + c^2$ (C) $(a + b + c)^2$ (D) none

[Hint: Multiply R_1 by a, R_2 by b & R_3 by c & divide the determinant by abc. Now take a, b & c common from c_1, c_2 & c_3. Now use $C_1 \to C_1 + C_2 + C_3$ to get]

$(a^2 + b^2 + c^2 + 1)\begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$. Now use $c_1 \to c_1 - c_2$ & $c_2 \to c_2 - c_3$ to get the value as 1.]

Q.28$_{19/mat}$ If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ satisfies the equation $x^2 - (a + d)x + k = 0$, then

(A) $k = bc$ (B) $k = ad$ (C) $k = a^2 + b^2 + c^2 + d^2$ (D*) $ad - bc$

[Sol. We have $A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix}$; $(a + d)A = \begin{bmatrix} a(a+d) & b(a+d) \\ c(a+d) & d(a+d) \end{bmatrix}$; $\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$

$\therefore \quad A^2 -(a + d)A = \begin{bmatrix} bc-ad & 0 \\ 0 & bc-da \end{bmatrix} = (bc - ad)\, I$

As $A^2 - (a + d)A + kI = 0$, we get $(bc - ad)I + kI = 0 \Rightarrow k = ad - bc$]

Q.29$_{41/qe}$ Let r_1, r_2 and r_3 be the solutions of the equation $x^3 - 2x^2 + 4x + 5074 = 0$ then the value of $(r_1 + 2)(r_2 + 2)(r_3 + 2)$ is

(A) 5050 (B) 5066 (C*) – 5050 (D) – 5066

[Sol. $x^3 - 2x^2 + 4x + 5074 = (x - r_1)(x - r_2)(x - r_3)$

put $x = -2$

$-8 - 8 - 8 + 5074 = -(2 + r_1)(2 + r_2)(2 + r_3)$

$\therefore \quad 5050 = -(2 + r_1)(2 + r_2)(2 + r_3)$

$(2 + r_1)(2 + r_2)(2 + r_3) = -5050$ **Ans.**]

Q.30$_{14/det}$ If $a, b, c > 0$ & $x, y, z \in R$, then the determinant $\begin{vmatrix} (a^x + a^{-x})^2 & (a^x - a^{-x})^2 & 1 \\ (b^y + b^{-y})^2 & (b^y - b^{-y})^2 & 1 \\ (c^z + c^{-z})^2 & (c^z - c^{-z})^2 & 1 \end{vmatrix} =$

(A) $a^x b^y c^z$ (B) $a^{-x} b^{-y} c^{-z}$ (C) $a^{2x} b^{2y} c^{2z}$ (D*) zero

[Use : $C_1 \to C_1 - C_2$ and take 4 common $\Rightarrow$ two identical columns]

Q.31$_{47/qe}$ The sum of the roots of the equation $(x + 1) = 2\log_2(2^x + 3) - 2\log_4(1980 - 2^{-x})$ is

(A) 3954 (B*) $\log_2 11$ (C) $\log_2 3954$ (D) indeterminate

Q.32$_{24/mat}$ Identify the incorrect statement in respect of two square matrices A and B conformable for sum and product.

(A) $t_r(A + B) = t_r(A) + t_r(B)$ (B) $t_r(\alpha A) = \alpha\, t_r(A), \alpha \in R$

(C) $t_r(A^T) = t_r(A)$ (D*) $t_r(AB) \neq t_r(BA)$

Q.33$_{49/qe}$ If $a + b + c = 0$ & $a^2 + b^2 + c^2 = 1$ then the value of $a^4 + b^4 + c^4$ is

(A) 3/ 2 (B) 3/4 (C*) 1/2 (D) 1/4

Q.34$_{17/det}$ The determinant $\begin{vmatrix} \cos(\theta+\phi) & -\sin(\theta+\phi) & \cos 2\phi \\ \sin\theta & \cos\theta & \sin\phi \\ -\cos\theta & \sin\theta & \cos\phi \end{vmatrix}$ is :

(A) 0 (B*) independent of θ

(C) independent of ϕ (D) independent of θ & ϕ both

[Hint: Directly open by R_1 to get

$\cos^2(\theta + \phi) + \sin^2(\theta + \phi) + \cos 2\phi$

$= 1 + \cos 2\phi$. Which is independent of θ]

Q.35$_{50/qe}$ The equation whose roots are $\sec^2\alpha$ & $\text{cosec}^2\alpha$ can be :

(A) $2x^2 - x - 1 = 0$ (B) $x^2 - 3x + 3 = 0$ (C*) $x^2 - 9x + 9 = 0$ (D) none

[Hint : Note that $\sec^2\alpha + \text{cosec}^2\alpha = \sec^2\alpha \cdot \text{cosec}^2\alpha \geq 4$]

Q.36$_{25/mat}$ If A and B are non singular Matrices of same order then Adj. (AB) is

(A) Adj. (A) (Adj. B) (B*) (Adj. B) (Adj. A)

(C) Adj. A + Adj. B (D) none of these

[Hint: A adj A = |A| I ....(1)

(AB) (adj AB) = |AB| I

Also (AB)(adj B . adj A) = A (B adj B) adj A (associativity)

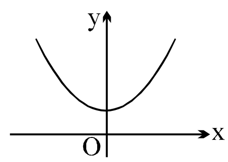
= A |B| I_n AdjA

= |B| A adj A

(AB) (adj.B · adj A) = |B| |A| I_n or |AB| I_n ....(2)

from (1) and (2) adj (AB) = (adj B) · (adj A)]

Q.37$_{51/qe}$ The graph of a quadratic polynomial $y = ax^2 + bx + c$ $(a, b, c \in R)$ with vertex on y-axis is as shown in the figure. Then which one of the following statement is INCORRECT?



(A) Product of the roots of the corresponding quadratic equation is positive.

(B) Discriminant of the quadratic equation is negative.

(C*) Nothing definite can be said about the sum of the roots, whether positive, negative or zero.

(D) Both roots of the quadratic equation are purely imaginary.

[Sol. Roots are purely imaginary

i.e. $i\beta$ and $-i\beta$

$\therefore$ sum of roots = 0

incorrect (C)

product of roots $= -i^2\beta^2 = \beta^2 \Rightarrow$ product > 0 ; $\frac{c}{a} > 0 \Rightarrow c = +$ ve]

Q.38$_{18/det}$ If $\begin{vmatrix} a+1 & a+2 & a+p \\ a+2 & a+3 & a+q \\ a+3 & a+4 & a+r \end{vmatrix} = 0$, then p, q, r are in :

(A*) AP (B) GP (C) HP (D) none

[Hint: Use $R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_2$ & then

$c_1 \to c_1 - c_2$ to get $\begin{vmatrix} -1 & a+2 & a+p \\ 1 & 1 & q-p \\ 1 & 1 & r-q \end{vmatrix}$ open by c_1 to get $p + r = 2q$]

Q.39$_{55/qe}$ The number of solution of the equation $e^{2x} + e^x + e^{-2x} + e^{-x} = 3(e^{-2x} + e^x)$ is

(A) 0 (B) 2 (C*) 1 (D) more than 2

[Hint: $x = \ln 2$]

Q.40$_{28/mat}$ Let $A = \begin{bmatrix} x+\lambda & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda \end{bmatrix}$, then A^{-1} exists if

(A) $x \neq 0$ (B) $\lambda \neq 0$

(C*) $3x + \lambda \neq 0, \lambda \neq 0$ (D) $x \neq 0, \lambda \neq 0$

[Sol. We have $|A| = \begin{vmatrix} x+\lambda & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda \end{vmatrix} = \begin{vmatrix} 3x+\lambda & x & x \\ 3x+\lambda & x+\lambda & x \\ 3x+\lambda & x & x+\lambda \end{vmatrix} = (3x+\lambda)\begin{vmatrix} 1 & x & x \\ 1 & x+\lambda & x \\ 1 & x & x+\lambda \end{vmatrix}$

$= (3x + \lambda)\begin{vmatrix} 1 & x & x \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix} = \lambda^2(3x + \lambda)$ [Take $3x + \lambda$ common and use $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$]

Thus, A^{-1} will exist if $\lambda \neq 0$ and $3x + \lambda \neq 0$]

Q.41$_{56/qe}$ Let a, b, c be the three roots of the equation $x^3 + x^2 - 333x - 1002 = 0$ then the value of $a^3 + b^3 + c^3$.

(A*) 2006 (B) 2005 (C) 2003 (D) 2002

[Sol. Let t be the root of the given cubic where t can take values a, b, c

hence $t^3 + t^2 - 333t - 1002 = 0$ or $t^3 = 1002 + 333t - t^2$

$\therefore \quad \sum t^3 = \sum 1002 + 333\sum t - \sum t^2 = 3006 + 333\sum t - \left[\left(\sum t\right)^2 - 2\sum t_1 t_2\right]$

but $\sum t = -1$; $\sum t_1 t_2 = -333$

$\therefore \quad a^3 + b^3 + c^3 = 3006 - 333 - [1 + 666] = 3006 - 333 - 667 = 3006 - 1000 = 2006$ Ans.]

Q.42$_{19/det}$ For positive numbers x, y & z the numerical value of the determinant $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$ is

(A*) 0 (B) 1 (C) 3 (D) none

[Hint: $D = \begin{vmatrix} 1 & \dfrac{\log y}{\log x} & \dfrac{\log z}{\log x} \\ \dfrac{\log x}{\log y} & 1 & \dfrac{\log z}{\log y} \\ \dfrac{\log x}{\log z} & \dfrac{\log y}{\log z} & 1 \end{vmatrix}$

Multiply R_1 by log x & R_2 by log y to get 2 identical rows. Hence D = 0]

Q.43$_{29/mat}$ If $K \in R_0$ then det. {adj (KI_n)} is equal to

(A) K^{n-1} (B*) $K^{n(n-1)}$ (C) K^n (D) K

[Sol. $(KI_n)\,\text{adj}(KI_n) = |\,KI_n\,|\,I_n$ [Using $A\,(\text{adj}\,A) = |A|\,I$]

$\text{adj}\,(KI_n) = K^{n-1}\,I_n$

$|\,\text{adj}\,(KI_n)\,| = K^{n(n-1)}$]

Q.44$_{61/qe}$ The number of real roots of the equation $\sqrt{x^2+1} - \sqrt{2x^2+5} = 1$ is

(A) 4 (B) 2 (C) 1 (D*) 0

[Hint: $x^2 + 1 < 2x^2 + 5,\ \forall\ x \in R \Rightarrow$ no roots]

Q.45$_{23/det}$ The determinant $\begin{vmatrix} b_1+c_1 & c_1+a_1 & a_1+b_1 \\ b_2+c_2 & c_2+a_2 & a_2+b_2 \\ b_3+c_3 & c_3+a_3 & a_3+b_3 \end{vmatrix}$ =

(A) $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ (B*) 2$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ (C) 3$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ (D) 4$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

[Hint: $C_1 \to C_1 + C_2 + C_3$]

Q.46$_{30/mat}$ Which of the following is an orthogonal matrix

(A*) $\begin{bmatrix} 6/7 & 2/7 & -3/7 \\ 2/7 & 3/7 & 6/7 \\ 3/7 & -6/7 & 2/7 \end{bmatrix}$ (B) $\begin{bmatrix} 6/7 & 2/7 & 3/7 \\ 2/7 & -3/7 & 6/7 \\ 3/7 & 6/7 & -2/7 \end{bmatrix}$

(C) $\begin{bmatrix} -6/7 & -2/7 & -3/7 \\ 2/7 & 3/7 & 6/7 \\ -3/7 & 6/7 & 2/7 \end{bmatrix}$ (D) $\begin{bmatrix} 6/7 & -2/7 & 3/7 \\ 2/7 & 2/7 & -3/7 \\ -6/7 & 2/7 & 3/7 \end{bmatrix}$

[Hint: Matrix $\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$ is orthogonal if

$$\sum a_i^2 = \sum b_i^2 = \sum c_i^2 = 1 \ ; \ \sum a_i b_i = \sum b_i c_i = \sum c_i a_i = 0 \ \Rightarrow \ (A) \ ]$$

Q.47$_{64/qe}$ Number of integral values of x satisfying the inequality $\left(\frac{3}{4}\right)^{6x+10-x^2} < \frac{27}{64}$ is

(A) 6 (B*) 7 (C) 8 (D) infinite

[Hint : $6x + 10 - x^2 > 3$

$\therefore$ $x^2 - 6x - 7 < 0$

$(x + 1)(x - 7) < 0$

$-1 \quad +7$ $\Rightarrow$ 0, 1, 2, 3, 4, 5, 6 $\Rightarrow$ B]

Q.48$_{24/det}$ The determinant $\begin{vmatrix} 1+a+x & a+y & a+z \\ b+x & 1+b+y & b+z \\ c+x & c+y & 1+c+z \end{vmatrix}$ =

(A*) $(1 + a + b + c)(1 + x + y + z) - 3(ax + by + cz)$

(B) $a(x + y) + b(y + z) + c(z + x) - (xy + yz + zx)$

(C) $x(a + b) + y(b + c) + z(c + a) - (ab + bc + ca)$

(D) none of these

[Hint : $1 + a + b + c = k$ and use $R_1 + R_2 + R_3$ we get

$$D = \begin{vmatrix} k+3x & k+3y & k+3z \\ b+x & 1+b+y & b+z \\ c+x & c+y & 1+c+z \end{vmatrix}$$

$$= k \begin{vmatrix} 1 & 1 & 1 \\ b+x & 1+b+y & b+z \\ c+x & c+y & 1+c+z \end{vmatrix} + 3 \begin{vmatrix} x & y & z \\ b+x & 1+b+y & b+z \\ c+x & c+y & 1+c+z \end{vmatrix}$$

now proceed]

Q.49$_{32/mat}$ Which of the following statements is incorrect for a square matrix A. ($|A| \neq 0$)

(A) If A is a diagonal matrix, A^{-1} will also be a diagonal matrix

(B) If A is a symmetric matrix, A^{-1} will also be a symmetric matrix

(C*) If $A^{-1} = A \Rightarrow$ A is an idempotent matrix

(D) If $A^{-1} = A \Rightarrow$ A is an involutary matrix

[Hint: $A^2 = I \rightarrow$ Involutary Matrix

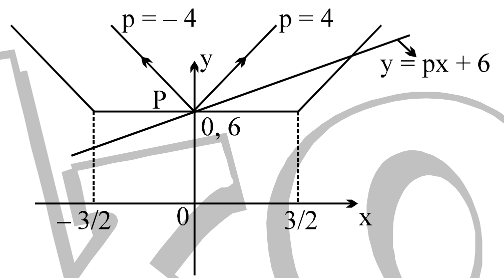
$A^2 = A \rightarrow$ Idempotent Matrix]

Q.50$_{67/qe}$ The set of real value(s) of p for which the equation, $|2x+3| + |2x-3| = px+6$ has more than two solutions is :

(A) (0, 4] (B) (− 4, 4) (C) R − {4, - 4, 0} (D*) {0}

[Hint : Draw graphs of :

$$y = \begin{cases} 4x & \text{if} \quad x \geq \frac{3}{2} \\ 6 & \text{if} \quad -\frac{3}{2} < x < \frac{3}{2} \\ -4x & \text{if} \quad x \leq -\frac{3}{2} \end{cases}$$



and $y = px + 6$

From the graph it is obvious that if,

$p = 0$ we have infinite solutions ranging from $\left[-\frac{3}{2}, \frac{3}{2}\right]$

if $0 < p < 4$ or $-4 < p < 0$,

two solutions , if $p = 4$ or -4 we have

$x = 0$ is the only solution]

Q.51$_{25/det}$ The determinant $\begin{vmatrix} {}^xC_1 & {}^xC_2 & {}^xC_3 \\ {}^yC_1 & {}^yC_2 & {}^yC_3 \\ {}^zC_1 & {}^zC_2 & {}^zC_3 \end{vmatrix} =$

(A) $\frac{1}{3}$ xyz (x+y) (y+z) (z+x) (B) $\frac{1}{4}$ xyz (x+y−z) (y+z−x)

(C*) $\frac{1}{12}$ xyz (x−y) (y−z) (z−x) (D) none

[Hint: $\begin{vmatrix} x & \frac{x(x-1)}{2} & \frac{x(x-1)(x-2)}{6} \\ y & \frac{y(y-1)}{2} & \frac{y(y-1)(y-2)}{6} \\ z & \frac{z(z-1)}{2} & \frac{z(z-1)(z-2)}{6} \end{vmatrix} = \frac{xyz}{12}\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \Rightarrow$ (C)

$R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$]

Q.52$_{33/mat}$ Which of the following is a nilpotent matrix

(A) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ (C*) $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

[Hint: A is nilpotent if $A^m = 0$ and $A^{m-1} \neq 0$ and the order of nilpotency is m. General form of a null matrix is $\begin{bmatrix} 0 & 0 \\ a & 0 \end{bmatrix}$]

Q.53$_{70/qe}$ The absolute term in the quadratic expression $\sum_{k=1}^{n}\left(x-\frac{1}{k+1}\right)\left(x-\frac{1}{k}\right)$ as $n \to \infty$ is

(A*) 1 (B) –1 (C) 0 (D) 1/2

[Sol. $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{1}{2}\sum_{k=1}^{n}\left(\frac{1}{k} - \frac{1}{k+1}\right) = \frac{1}{2}\left(1 - \frac{1}{n+1}\right)$

$\therefore$ absolute term $= \lim_{n\to\infty} \frac{1}{2}\left(1 - \frac{1}{n+1}\right) = \frac{1}{2}$]

Q.54$_{26/det}$ If a, b, c are all different and $\begin{vmatrix} a & a^3 & a^4-1 \\ b & b^3 & b^4-1 \\ c & c^3 & c^4-1 \end{vmatrix} = 0$, then :

(A*) abc (ab + bc + ca) = a + b + c (B) (a + b + c) (ab + bc + ca) = abc
(C) abc (a + b + c) = ab + bc + ca (D) none of these

[Hint : Split the determinant into 2 & then evaluate $R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$]

Q.55$_{35/mat}$ Give the correct order of initials **T** or **F** for following statements. Use **T** if statement is true and **F** if it is false.

Statement-1 : If A is an invertible 3×3 matrix and B is a 3×4 matrix, then $A^{-1}B$ is defined
Statement-2 : It is never true that A + B, A – B, and AB are all defined.
Statement-3 : Every matrix none of whose entries are zero is invertible.
Statement-4 : Every invertible matrix is square and has no two rows the same.

(A) TFFF (B) TTFF (C*) TFFT (D) TTTF

Q.56$_{72/qe}$ Number of values of the parameter $\alpha \in [0, 2\pi]$ for which the quadratic function,

$(\sin\alpha)x^2 + 2\cos\alpha\, x + \frac{1}{2}(\cos\alpha + \sin\alpha)$ is the square of a linear function is

(A*) 2 (B) 3 (C) 4 (D) 1

[Hint : Let $f(x) = \left(\sqrt{\sin\alpha}\ x + b\right)^2$ now compare the coefficient and eliminate b. divide by $\cos^2\alpha$ to get

$(\tan\alpha - 1)(\tan\alpha + 2) = 0 \qquad \Rightarrow \alpha = \dfrac{\pi}{4}$ or $\pi - \tan^{-1}(2)$]

Q.57$_{28/det}$ If ω is one of the imaginary cube roots of unity, then the value of the determinant $\begin{vmatrix} 1 & \omega^3 & \omega^2 \\ \omega^3 & 1 & \omega \\ \omega^2 & \omega & 1 \end{vmatrix} =$

(A) 1 (B) 2 (C*) 3 (D) none

[Hint: Put $\omega^3 = 1$ $\begin{vmatrix} 1 & 1 & \omega^2 \\ 1 & 1 & \omega \\ \omega^2 & \omega & 1 \end{vmatrix}$ and open by R_1 to get $(1 - \omega^2) + (1 - \omega) = 3$]

Q.58$_{37/mat}$ Identify the correct statement :

(A) If system of n simultaneous linear equations has a unique solution, then coefficient matrix is singular

(B*) If system of n simultaneous linear equations has a unique solution, then coefficient matrix is non singular

(C) If A^{-1} exists, $(\text{adj}A)^{-1}$ may or may not exist

(D) $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 0 \end{bmatrix}$, then $F(x) . F(y) = F(x - y)$

[Hint: (A) It should be non singular

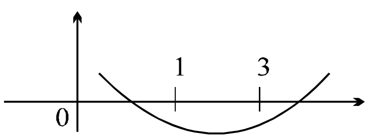
(C) since $A^{-1} = \dfrac{\text{adj}\,A}{|A|}$, hence adj A must be a non singular matrix. Its inverse must exist.

(D) It should be $F(x + y)$]

Q.59$_{75/qe}$ The set of values of 'a' for which the inequality, $(x - 3a)(x - a - 3) < 0$ is satisfied for all $x \in [1, 3]$ is:

(A) (1/3, 3) (B*) (0, 1/3) (C) (–2, 0) (D) (–2, 3)

[Hint : Equation is $x^2 - (4a + 3)x + 3a(a + 4)$



$f(1) < 0$ and $f(3) < 0$

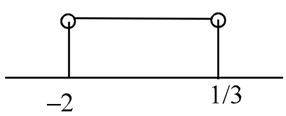
$(1-3a)(1 - a - 3) < 0 \Rightarrow 1 - a - 3 - 3a + 3a^2 + 9a < 0$

$\Rightarrow \quad 3a^2 + 5a - 2 < 0$

$3a^2 + 6a - a - 2 < 0$

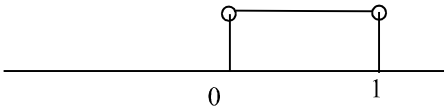
$3a(a + 2) - (a + 2) < 0$

$(a + 2)(3a - 1) < 0 \Rightarrow$



again $(3-3a)(-a) < 0 \Rightarrow (a - 1)a < 0$

$\Rightarrow \quad 0 < a < 1$



Hence $a \in \left(0, \frac{1}{3}\right)$]

Q.60$_{29/det}$ If the determinant $\begin{vmatrix} a+p & 1+x & u+f \\ b+q & m+y & v+g \\ c+r & n+z & w+h \end{vmatrix}$ splits into exactly K determinants of order 3, each element of which contains only one term, then the value of K, is

(A) 6 (B*) 8 (C) 9 (D) 12

[Hint : Divide c_1 by x, c_2 by y and c_3 by z and use $c_1 \to c_1 + c_2 + c_3$]

Q.61$_{38/mat}$ A and B are two given matrices such that the order of A is 3×4 , if $A'B$ and BA' are both defined then

(A) order of B' is 3×4 (B*) order of $B'A$ is 4×4

(C) order of $B'A$ is 3×3 (D) $B'A$ is undefined

[Hint: $A = 3 \times 4$; $A' = 4 \times 3$

As $A'B$ is defined $\Rightarrow$ let order of $B = 3 \times n$

now $BA' = (3 \times n) \times (4 \times 3) \Rightarrow n = 4$

$\therefore$ order of B is 3×4

$\therefore$ order of $B' = 4 \times 3$

order of $B'A = (4 \times 3) \times (3 \times 4) = 4 \times 4$ Ans]

Q.62$_{76/qe}$ If α, β & γ are the roots of the equation, $x^3 - x - 1 = 0$ then, $\frac{1+\alpha}{1-\alpha} + \frac{1+\beta}{1-\beta} + \frac{1+\gamma}{1-\gamma}$ has the value equal to :

(A) zero (B) -1 (C*) -7 (D) 1

[Hint: Let α, β, γ be the roots of 2nd $\Rightarrow \Sigma\alpha = A$; $\Sigma\alpha\beta = B$; $\Sigma\alpha\beta\gamma = C$

Let $y = \frac{1+x}{1-x} \Rightarrow x = \frac{y-1}{y+1} \Rightarrow \left(\frac{y-1}{y+1}\right)^3 - \frac{y-1}{y+1} - 1 = 0$

$\Rightarrow \quad y^3 + 7y^2 - y + 1 = 0 \Rightarrow \sum \frac{1+\alpha}{1-\alpha} = -7$]

Q.63$_{30/det}$ If the system of equations $x + 2y + 3z = 4$, $x + py + 2z = 3$, $x + 4y + \mu z = 3$ has an infinite number of solutions , then :

(A) $p = 2$, $\mu = 3$ (B) $p = 2$, $\mu = 4$ (C) $3p = 2\mu$ (D*) none of these

[Hint : For 2nd and 3rd equation , $1 = p/4 = 2/\mu = 1 \Rightarrow p = 4$; $\mu = 1$]

Q.64$_{39/mat}$ If $A = \begin{bmatrix} \cos^2\alpha & \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha & \sin^2\alpha \end{bmatrix}$; $B = \begin{bmatrix} \cos^2\beta & \sin\beta\cos\beta \\ \sin\beta\cos\beta & \sin^2\beta \end{bmatrix}$ are such that, AB is a null matrix, then which of the following should necessarily be an odd integral multiple of $\frac{\pi}{2}$.

(A) α (B) β (C*) $\alpha - \beta$ (D) $\alpha + \beta$

[Hint: $AB = \begin{pmatrix} \cos^2\alpha & \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha & \sin^2\alpha \end{pmatrix} \begin{pmatrix} \cos^2\beta & \sin\beta\cos\beta \\ \sin\beta\cos\beta & \sin^2\beta \end{pmatrix}$

$= \begin{pmatrix} \cos^2\alpha\cos^2\beta + \sin\alpha\cos\alpha\sin\beta\cos\beta & \cos^2\alpha\sin\beta\cos\beta + \sin\alpha\cos\alpha\sin^2\beta \\ \cos^2\beta\sin\alpha\cos\alpha + \sin^2\alpha\sin\beta\cos\beta & \sin\alpha\cos\alpha\sin\beta\cos\beta + \sin^2\alpha\sin^2\beta \end{pmatrix}$

$= \begin{pmatrix} \cos\alpha\cos\beta\cos(\alpha-\beta) & \cos\alpha\sin\beta\cos(\alpha-\beta) \\ \sin\alpha\cos\beta\cos(\alpha-\beta) & \sin\alpha\sin\beta\cos(\alpha-\beta) \end{pmatrix}$

$\Rightarrow$ $\alpha - \beta$ must be an odd integral multiple of $\pi/2 \Rightarrow$ (C)]

Q.65$_{80/qe}$ For every $x \in R$, the polynomial $x^8 - x^5 + x^2 - x + 1$ is :

(A*) positive
(B) never positive
(C) positive as well as negative
(D) negative

[Hint : for $x \ge 1$ $E = x^5(x^3 - 1) + (x - 1) + 1 > 0$

for $1 < x < 0$, $E = (1 - x) + x^2(1 - x^3) + x^8 > 0$

For $x < 0$, all terms are positive $\Rightarrow$ > 0 Hence A]

Q.66$_{32/det}$ Let $D_1 = \begin{vmatrix} a & b & a+b \\ c & d & c+d \\ a & b & a-b \end{vmatrix}$ and $D_2 = \begin{vmatrix} a & c & a+c \\ b & d & b+d \\ a & c & a+b+c \end{vmatrix}$ then the value of $\frac{D_1}{D_2}$ where $b \ne 0$ and $ad \ne bc$, is

(A*) -2
(B) 0
(C) $-2b$
(D) 2b

[Sol. Using: $C_3 \to C_3 - (C_1 + C_2)$, $D_1 = \begin{vmatrix} a & b & a+b \\ c & d & c+d \\ a & b & a-b \end{vmatrix}$ and $D_2 = \begin{vmatrix} a & c & a+c \\ b & d & b+d \\ a & c & a+b+c \end{vmatrix}$

$\therefore$ $\frac{D_1}{D_2} = \frac{-2b(ad - bc)}{b(ad - bc)} = -2$ Ans.]

Q.67$_{40/mat}$ For a given matrix $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ which of the following statement holds good?

(A) $A = A^{-1}$ $\forall \theta \in R$
(B) A is symmetric, for $\theta = (2n + 1)\frac{\pi}{2}$, $n \in I$
(C*) A is an orthogonal matrix for $\theta \in R$
(D) A is a skew symmetric, for $\theta = n\pi$; $n \in I$

[Hint: Obv. A is orthogonal as $a_{11}^2 + a_{12}^2 = 1 = a_{21}^2 + a_{22}^2 = a_{11}^2 + a_{22}^2$

for skew symmetric matrix $a_{ii} = 0 \Rightarrow \theta = (2n + 1)\frac{\pi}{2}$

for symmetric matrix , $A = A^T \Rightarrow \sin\theta = 0 \Rightarrow \theta = n\pi$

Also $\text{adj}A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ and $|A| = 1$ hence $A = A^{-1}$ is possible if $\sin\theta = 0$]

Q.68$_{82/qe}$ If the roots of the equation, $x^3 + Px^2 + Qx - 19 = 0$ are each one more than the roots of the equaton, $x^3 - Ax^2 + Bx - C = 0$ where A, B, C, P & Q are constants then the value of $A + B + C$ =

(A*) 18 (B) 19 (C) 20 (D) none

[Hint : Let roots of $x^3 - Ax^2 + Bx - C = 0$ are x_1, x_2, x_3

$\Rightarrow \quad \Sigma x_1 = A \; ; \quad \Sigma x_1 x_2 = B \; ; \quad \Sigma x_1 x_2 x_3 = C$

Now $x_1 + 1$, $x_2 + 1$, $x_3 + 1$ are the roots of $x^3 + Px^2 + Qx - 19 = 0$

$(x_1 + 1)(x_2 + 1)(x_3 + 1) = 19 \qquad 1 + (x_1 + x_2 + x_3) + (\Sigma x_1 x_2) + x_1 x_2 x_3 = 19$

$1 + A + B + C = 19 \quad \Rightarrow \quad A + B + C = 18$]

Q.69$_{33/det}$ If $a^2 + b^2 + c^2 = -2$ and $f(x) = \begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$ then f (x) is a polynomial of degree

(A) 0 (B) 1 (C*) 2 (D) 3

[Sol. $C_1 \to C_1 + C_2 + C_3$

$$f(x) = \begin{vmatrix} 1+2x+x(a^2+b^2+c^2) & (1+b^2)x & (1+c^2)x \\ 1+2x+x(a^2+b^2+c^2) & 1+b^2x & (1+c^2)x \\ 1+2x+x(a^2+b^2+c^2) & (1+b^2)x & 1+c^2x \end{vmatrix} \qquad (\text{as } a^2 + b^2 + c^2 = -2)$$

$$\begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 1 & 1+b^2x & (1+c^2)x \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix}$$

$R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$

$$\begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 0 & 1-x & 0 \\ 0 & 1-x & 1-x \end{vmatrix}$$

$f(x) = (1 - x)^2 = 1 - 2x + x^2 \quad \Rightarrow \quad$ (C)]

Q.70$_{41/mat}$ Matrix $A = \begin{bmatrix} x & 3 & 2 \\ 1 & y & 4 \\ 2 & 2 & z \end{bmatrix}$, if $xyz = 60$ and $8x + 4y + 3z = 20$, then $A(\text{adj } A)$ is equal to

(A) $\begin{bmatrix} 64 & 0 & 0 \\ 0 & 64 & 0 \\ 0 & 0 & 64 \end{bmatrix}$ (B) $\begin{bmatrix} 88 & 0 & 0 \\ 0 & 88 & 0 \\ 0 & 0 & 88 \end{bmatrix}$ (C*) $\begin{bmatrix} 68 & 0 & 0 \\ 0 & 68 & 0 \\ 0 & 0 & 68 \end{bmatrix}$ (D) $\begin{bmatrix} 34 & 0 & 0 \\ 0 & 34 & 0 \\ 0 & 0 & 34 \end{bmatrix}$

[Sol. $A.\,\text{adj } A = |A|\, I$

$|A| = xyz - 8x - 3(z - 8) + 2(2 - 2y)$

$|A| = xyz - (8x + 3z + 4y) + 28 \quad \Rightarrow 60 - 20 + 28 = 68 \quad \Rightarrow \quad$ (C)]

Q.71$_{84/qe}$ If the roots of the cubic, $x^3 + ax^2 + bx + c = 0$ are three consecutive positive integers. Then the value of $\dfrac{a^2}{b+1}$ is equal to

(A*) 3 (B) 2 (C) 1 (D) 1/3

[Sol. $n,\ n+1,\ n+2$ **[11th test (2-10-2005)]**

sum $= 3(n+1) = -a$

$\therefore \quad a^2 = 9(n+1)^2$

sum of the roots taken 2 at atime $= +b$

$\therefore \quad n(n+1) + (n+1)(n+2) + (n+2)n + 1 = b + 1$ (adding 1 both sides)

$n^2 + n + n^2 + 3n + 2 + n^2 + 2n + 1 = b + 1$

$\therefore \quad b + 1 = 3n^2 + 6n + 3 = 3(n+1)^2$

$b + 1 = 3(n+1)^2 = \dfrac{a^2}{3}$; $\quad \therefore \quad \dfrac{a^2}{b+1} = 3 \quad \Rightarrow \quad$ (A)]

Q.72$_{34/det}$ The values of θ, λ for which the following equations

$\sin\theta x - \cos\theta y + (\lambda+1)z = 0;\ \cos\theta x + \sin\theta y - \lambda z = 0;\ \lambda x + (\lambda+1)y + \cos\theta\, z = 0$

have non trivial solution, is

(A) $\theta = n\pi,\ \lambda \in R - \{0\}$

(B) $\theta = 2n\pi,\ \lambda$ is any rational number

(C) $\theta = (2n+1)\pi,\ \lambda \in R^+,\ n \in I$

(D*) $\theta = (2n+1)\dfrac{\pi}{2},\ \lambda \in R,\ n \in I$

[Hint: for non trivial solution $\begin{vmatrix} \sin\theta & -\cos\theta & \lambda+1 \\ \cos\theta & \sin\theta & -\lambda \\ \lambda & \lambda+1 & \cos\theta \end{vmatrix} = 0$; this gives $2\cos\theta(\lambda^2 + \lambda + 1) = 0$]

Q.73$_{42/mat}$ If A is matrix such that $A^2 + A + 2I = \mathbf{O}$, then which of the following is INCORRECT ?

(A) A is non-singular (B) $A \neq \mathbf{O}$ (C*) A is symmetric (D) $A^{-1} = -\dfrac{1}{2}(A+I)$

(Where I is unit matrix of order 2 and $\mathbf{O}$ is null matrix of order 2)

[Sol. We have $A(A+I) = -2I$

$\Rightarrow \quad |A(A+I)| = |-2I| \quad \Rightarrow \quad |A||A+I| = 2 \neq 0$

Thus , $|A| \neq 0 \quad \Rightarrow \quad$ A is non singular $\quad \Rightarrow \quad$ A is correct

Also , $A\left(-\dfrac{1}{2}(A+I)\right) = I \quad \Rightarrow \quad A^{-1} = -\dfrac{1}{2}(A+I) \quad \Rightarrow \quad$ D is correct

Also $A = 0$ does not satisfy the given eqaution $\Rightarrow \quad A \neq 0$

again $\left.\begin{array}{l} A^2 + A + 2I = 0 \\ (A^T)^2 + A^T + 2I = 0 \end{array}\right]$ subtract again will $A^T = B$

$(A^2 - B^2) + (A - B) = 0$

$(A - B)(A + B + I) = 0$

$\not\Rightarrow \quad A - B = 0 \quad$ or $\quad A + B + I = 0$]

Q.74$_{89/qe}$ Number of possible ordered pair(s) (a, b) for each of which the equality,

$a(\cos x - 1) + b^2 = \cos(ax + b^2) - 1$ holds true for all $x \in R$ are :

(A) 0 (B) 1 (C*) 2 (D) infinite

[Hint : Put $x = 0 \quad b^2 = \cos b^2 - 1$ or $\cos b^2 = 1 + b^2 \quad \Rightarrow \quad b = 0$

Now the equation becomes $\ -2a\sin^2\dfrac{x}{2} = \cos ax - 1 = -2\sin^2\dfrac{ax}{2}$

or $\ a\sin^2\dfrac{x}{2} = \sin^2\dfrac{ax}{2}$ must be true $\forall\ x \in R \quad \Rightarrow \quad a = 0$ or $a = 1$]

Q.75$_{35/det}$ The system of equations :

$2x\cos^2\theta + y\sin 2\theta - 2\sin\theta = 0$

$x\sin 2\theta + 2y\sin^2\theta = -2\cos\theta$

$x\sin\theta - y\cos\theta = 0$, for all values of θ, can

(A) have a unique non - trivial solution (B*) not have a solution

(C) have infinite solutions (D) have a trivial solution

[Hint: slope of (1) and (2) is cot $\theta \Rightarrow$ (1) and (2) are parallel and slope of (3) is $\tan\theta$

$\Rightarrow$ no solution.

Using $R_2 \to R_2 - (2\cos\theta)R_3$ and $R_1 \to R_1 + (2\sin\theta)R_3$, the value of determinat is 4 .]

Q.76$_{43/mat}$ The number of solution of the matrix equation $X^2 = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$ is

(A*) more than 2 (B) 2 (C) 1 (D) 0

[Sol. Let $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$X^2 = \begin{pmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{pmatrix}$$

$a^2 + bc = 1$

$ab + bd = 1 \Rightarrow b(a + d) = 1$

$ac + cd = 2 \Rightarrow c(a + d) = 2$

$\Rightarrow \frac{b}{c} = \frac{1}{2} \Rightarrow c = 2b$

$bc + d^2 = 3 \Rightarrow (d^2 - a^2) = 2 \Rightarrow (d - a)(a + d) = 2$

$d - a = 2b$ (using $bc = 1 - a^2$)

$a + d = 1/b$

$2d = 2b + 1/b$ $2a = 1/b - 2b$

$d = b + 1/2b$ $a = 1/2b - b$

$c = 2b$

$$\left(b^2 + \frac{1}{4b^2} + 1\right) + 2b^2 = 3 \Rightarrow 3b^2 + \frac{1}{4b^2} = 2$$

$$3x + \frac{1}{4x} = 2 \Rightarrow b = \pm\frac{1}{\sqrt{6}} \text{ or } b = \pm\frac{1}{\sqrt{2}}$$

Matrices are $\begin{pmatrix} 0 & 1/\sqrt{2} \\ \sqrt{2} & \sqrt{2} \end{pmatrix}$; $\begin{pmatrix} 0 & -1/\sqrt{2} \\ -\sqrt{2} & -\sqrt{2} \end{pmatrix}$; $\begin{pmatrix} 2/\sqrt{6} & -1/\sqrt{6} \\ 2/\sqrt{6} & 4/\sqrt{6} \end{pmatrix}$

Alternative: n linear equation in n variable have exactly one solution but n equation not linear in n variables will have more than one solution here we will have 4 equations in 4 variable of degree higher than 1 hence more than 2 solutions.]

Q.77$_{92/qe}$ If both the roots of the equation, $(3a+1)x^2-(2a+3b)x+3=0$ are infinite then :
(A) $a=\infty$; $b=0$ (B) $a=0$; $b=\infty$
(C*) $a=-1/3$; $b=2/9$ (D) $a=\infty$; $b=\infty$

Q.78$_{36/det}$ If x, y, z are not all simultaneously equal to zero, satisfying the system of equations
$(\sin 3\theta)x-y+z=0$
$(\cos 2\theta)x+4y+3z=0$
$2x+7y+7z=0$
then the number of principal values of θ is
(A) 2 (B) 4 (C*) 5 (D) 6

[Hint: $\theta=n\pi+(-1)^n\cdot\frac{\pi}{6}$ or $n\pi$] [IIT '86 , 5]

Q.79$_{44/mat}$ Let $A+2B=\begin{bmatrix}1&2&0\\6&-3&3\\-5&3&1\end{bmatrix}$ and $2A-B=\begin{bmatrix}2&-1&5\\2&-1&6\\0&1&2\end{bmatrix}$

then Tr (A) – Tr (B) has the value equal to
(A) 0 (B) 1 (C*) 2 (D) none

[Hint: $t_r(A)+2t_r(B)=-1$ (from the given matrix)
and $2t_r(A)-t_r(B)=3$ (from the given matrix)
Let $t_r(A)=x$ and $t_r(B)=y$
$x+2y=-1$
$2x-y=3$
solving $x=1$ and $y=-1$
Hence $t_r(A)-t_r(B)=x-y=2$]

Q.80$_{93/qe}$ If $\tan\alpha$, $\tan\beta$, $\tan\gamma$ are the roots of the equation, $x^3-(a+1)x^2+(b-a)x-b=0$, $(b-a\neq 1)$
Where $\alpha+\beta+\gamma$ lies between 0 & π then $\alpha+\beta+\gamma$ is equal to
(A*) $\pi/4$ (B) $\pi/2$ (C) $3\pi/4$ (D) None

[Hint: Use $\tan(\alpha+\beta+\gamma)=\frac{S_1-S_3}{1-S_2}=\frac{(a+1)-b}{1-(b-a)}=\frac{1+a-b}{1+a-b}$]

Q.81$_{37/det}$ For a non - zero, real a, b and c $\begin{vmatrix}\frac{a^2+b^2}{c}&c&c\\a&\frac{b^2+c^2}{a}&a\\b&b&\frac{c^2+a^2}{b}\end{vmatrix}=\alpha\,abc$, then the values of α is

(A) – 4 (B) 0 (C) 2 (D*) 4

[Hint : $\frac{1}{abc}\begin{vmatrix}a^2+b^2&c^2&c^2\\a^2&b^2+c^2&a^2\\b^2&b^2&c^2+a^2\end{vmatrix}$

use $R_1\to R_1-(R_2+R_3)$

$$\frac{1}{abc}\begin{vmatrix} 0 & -2b^2 & -2a^2 \\ a^2 & b^2+c^2 & a^2 \\ b^2 & b^2 & c^2+a^2 \end{vmatrix}$$

$R_2 \to R_2 + 1/2R_1$ and $R_3 \to R_3 + 1/2\ R_1$

$$\frac{1}{abc}\begin{vmatrix} 0 & -2b^2 & -2a^2 \\ a^2 & c^2 & 0 \\ b^2 & 0 & c^2 \end{vmatrix}$$

$$\frac{1}{abc}\ [\ 2b^2\ (a^2 c^2) - 2a^2\ (-\ b^2 c^2)\ ]\ =\ \frac{4a^2 b^2 c^2}{abc} = 4abc\ \ ]$$

Q.82$_{46/mat}$ Given $A = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$; $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. If $A - \lambda I$ is a singular matrix then

(A) $\lambda \in \phi$ (B*) $\lambda^2 - 3\lambda - 4 = 0$ (C) $\lambda^2 + 3\lambda + 4 = 0$ (D) $\lambda^2 - 3\lambda - 6 = 0$

[Hint: $A - \lambda I$

$= \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 1-\lambda & 3 \\ 2 & 2-\lambda \end{bmatrix}$

since $A - \lambda I$ is singular $\Rightarrow$ det. $(A - \lambda I) = 0$;

now $\begin{vmatrix} 1-\lambda & 3 \\ 2 & 2-\lambda \end{vmatrix} = (1-\lambda)(2-\lambda) - 6 = \lambda^2 - 3\lambda - 4$

hence $\lambda^2 - 3\lambda - 4 = 0$]

Q.83$_{95/qe}$ The values of 'a' for which the quadratic equation $(a^2 - a - 2)x^2 + 2ax + a^3 - 27 = 0$ has roots of opposite signs are

(A) $(-1, 2) \cup (3, \infty)$ (B*) $(-\infty, -1) \cup (2, 3)$

(C) $R/(-1, 2)$ (D) $R/(2, 3)$

Q.84$_{38/det}$ If the system of equations, $a^2 x - ay = 1 - a$ & $bx + (3 - 2b)y = 3 + a$ possess a unique solution $x = 1$, $y = 1$ then :

(A*) $a = 1$; $b = -1$ (B) $a = -1$, $b = 1$

(C) $a = 0$, $b = 0$ (D) none

[Hint : put $x = 1$ & $y = 1$ and solve for a & b . In B & C system has infinite solutions]

Q.85$_{47/mat}$ Let $A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$, where $0 \le \theta < 2\pi$, then

(A) Det (A) = 0 (B) Det $A \in (0, \infty)$ (C*) Det $(A) \in [2, 4]$ (D) Det $A \in [2, \infty)$

[Sol. $|A| = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix} = 1(1 + \sin^2\theta) - \sin\theta\,(-\sin\theta + \sin\theta) + (1 + \sin^2\theta) = 2\,(1 + \sin^2\theta)$

$|\sin\theta| \le 1 \Rightarrow -1 \le \sin\theta \le 1 \Rightarrow 0 \le \sin^2\theta \le 1$

$\Rightarrow 1 \le 1 + \sin^2\theta \le 2 \Rightarrow 2 \le 2(1 + \sin^2\theta) \le 4$

$\Rightarrow |A| \in [2, 4]$]

Q.86$_{96/qe}$ Three roots of the equation, $x^4 - px^3 + qx^2 - rx + s = 0$ are $\tan A, \tan B$ & $\tan C$ where A, B, C are the angles of a triangle . The fourth root of the biquadratic is :

(A*) $\frac{p-r}{1-q+s}$ (B) $\frac{p-r}{1+q-s}$ (C) $\frac{p+r}{1-q+s}$ (D) $\frac{p+r}{1+q-s}$

[Hint : Let the fourth root be $\tan D$

Now $\tan(\Sigma A) = \frac{\Sigma \tan A - \Sigma \tan A \tan B \tan C}{1 - \Sigma \tan A \tan B + \Pi \tan A}$ $\tan D = \frac{p-r}{1-q+s}$]

Q.87$_{39/det}$ Number of value of 'a' for which the system of equations,

$a^2 x + (2 - a) y = 4 + a^2$

$a x + (2a - 1) y = a^5 - 2$ possess no solution is

(A) 0 (B) 1 (C*) 2 (D) infinite

[Hint : $\frac{a^2}{a} = \frac{2-a}{2a-1} \neq \frac{4+a^2}{a^5-2}$] [Ans. $\{-1, 1\}$]

Q.88$_{48/mat}$ If $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & a & 1 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} 1/2 & -1/2 & 1/2 \\ -4 & 3 & c \\ 5/2 & -3/2 & 1/2 \end{bmatrix}$, then

(A*) $a = 1, c = -1$ (B) $a = 2, c = -\frac{1}{2}$ (C) $a = -1, c = 1$ (D) $a = \frac{1}{2}, c = \frac{1}{2}$

[Sol. $AA^{-1} = I$ $\Rightarrow$ $R_2C_3 = 0$

$\frac{1}{2} + 2c + \frac{3}{2} = 0$

$2c = -2$ $\Rightarrow$ $c = -1$

also $R_3C_2 = 0$

$-\frac{3}{2} + 3a - \frac{3}{2} = 0$

$3a = 3$ $\Rightarrow$ $a = 1$

hence $a = 1$; $c = -1$]

Q.89$_{99/qe}$ Number of real values of x satisfying the equation

$\sqrt{x^2 - 6x + 9} + \sqrt{x^2 - 6x + 6} = 1$ is

(A*) 0 (B) 1 (C) 2 (D) more than 2

[Hint: $(x^2 - 6x + 9) - (x^2 - 6x + 6) = 3$

$\sqrt{x^2 - 6x + 9} - \sqrt{x^2 - 6x + 6} = 3$

adding $\sqrt{x^2 - 6x + 9} = 2$

$x^2 - 6x + 9 = 4$

$x^2 - 6x + 5 = 0$

$(x - 5)(x - 1) = 0$ $\Rightarrow$ $x = 1$ or 5

but none satisfies.]

Q.90$_{40/det}$ Number of triplets of a, b & c for which the system of equations,

$$ax - by = 2a - b \text{ and } (c + 1)x + cy = 10 - a + 3b$$

has infinitely many solutions and $x = 1,\ y = 3$ is one of the solutions, is :

(A) exactly one (B*) exactly two
(C) exactly three (D) infinitely many

[Hint : put $x = 1$ & $y = 3$ in 1[st] equation $\Rightarrow$ $a = -2b$ & from 2[nd] equation

$c = \dfrac{9+5b}{4}$; Now use $\dfrac{a}{c+1} = -\dfrac{b}{c} = \dfrac{2a-b}{10-a+3b}$; from first two $b = 0$ or $c = 1$;

if $b = 0 \Rightarrow a = 0$ & $c = 9/4$; if $c = 1$; $b = -1$; $a = 2$]

Q.91$_{100/qe}$ If the roots of the equation $x^3 - px^2 - r = 0$ are $\tan\alpha$, $\tan\beta$ and $\tan\gamma$ then the value of $\sec^2\alpha \cdot \sec^2\beta \cdot \sec^2\gamma$ is

(A) $p^2 + r^2 + 2rp + 1$ (B*) $p^2 + r^2 - 2rp + 1$ (C) $p^2 - r^2 - 2rp + 1$ (D) None

[Sol. $\sum \tan\alpha = p$; $\sum \tan\alpha \cdot \tan\beta = 0$; $\prod \tan\alpha = r$

now $\sec^2\alpha \cdot \sec^2\beta \cdot \sec^2\gamma = (1 + \tan^2\alpha)(1 + \tan^2\beta)(1 + \tan^2\gamma)$

$$= 1 + \sum \tan^2\alpha + \sum \tan^2\alpha \cdot \tan^2\beta + \tan^2\alpha \cdot \tan^2\beta \cdot \tan^2\gamma$$

now $\sum \tan^2\alpha = \left(\sum \tan\alpha\right)^2 - 2\sum \tan\alpha \cdot \tan\beta = p^2$

$$\sum \tan^2\alpha \cdot \tan^2\beta = \left(\sum \tan\alpha \cdot \tan\beta\right)^2 - 2\tan\alpha \cdot \tan\beta \cdot \tan\gamma \left(\sum \tan\alpha\right)$$
$$= 0 - 2rp$$

$$\prod \tan^2\alpha = r^2$$

$\therefore$ $\prod \sec^2\alpha = 1 + p^2 - 2rp + r^2 = 1 + (p - r)^2$]

Q.92$_{69/det}$ Let $A = \begin{bmatrix} 1+x^2-y^2-z^2 & 2(xy+z) & 2(zx-y) \\ 2(xy-z) & 1+y^2-z^2-x^2 & 2(yz+x) \\ 2(zx+y) & 2(yz-x) & 1+z^2-x^2-y^2 \end{bmatrix}$ then det. A is equal to

(A) $(1 + xy + yz + zx)^3$ (B*) $(1 + x^2 + y^2 + z^2)^3$
(C) $(xy + yz + zx)^3$ (D) $(1 + x^3 + y^3 + z^3)^2$

[Hint: multiply R_2 by z and R_3 by y and use $R_1 \to R_1 - R_2 + R_3$
Objective approach : put $z = y = 0$ then choices are $A = 1$; $B = (1 + x^2)^3$; $C = 0$; $D = (1 + x^3)^2$ and determinant comes out to be $(1 + x^2)^3 \Rightarrow$ (B)]

Q.93$_{102/qe}$ If the equation $a(x - 1)^2 + b(x^2 - 3x + 2) + x - a^2 = 0$ is satisfied for all $x \in R$ then the number of ordered pairs of (a, b) can be

(A) 0 (B*) 1 (C) 2 (D) infinite

[Sol. equation is an identity $\Rightarrow$ coefficient of $x^2 = 0$ = coefficient of x = constant term

$\therefore$ $a + b = 0$ ....(1)
$-2a - 3b + 1 = 0$ ....(2)
and $a + 2b - a^2 = 0$ ....(3)
from (1) and (2) $a = -1$ and $b = 1$
which also satisfies (3) $\Rightarrow$ $(a, b) = (-1, 1)$ $\Rightarrow$ (B)]

Q.94$_{44/det}$ The following system of equations $3x - 7y + 5z = 3$; $3x + y + 5z = 7$ and $2x + 3y + 5z = 5$ are
(A) consistent with trivial solution (B*) consistent with unique non trivial solution
(C) consistent with infinite solution (D) inconsistent with no solution
[Hint: $D \neq 0 \Rightarrow$ consistency]

Q.95$_{56/mat}$ If $A_1, A_3, \ldots\ldots A_{2n-1}$ are n skew symmetric matrices of same order then $B = \sum_{r=1}^{n}(2r-1)(A_{2r-1})^{2r-1}$ will be
(A) symmetric (B*) skew symmetric
(C) neither symmetric nor skew symmetric (D) data is adequate
[Sol. $B = A_1 + 3A_3^3 + \ldots\ldots (2n-1)(A_{2n-1})^{2n-1}$
$B^T = -[A_1 + 3A_3^3 + \ldots\ldots (2n-1)(A_{2r-1})^{2r-1}]$
$= -B$ so skew symmetric]

Q.96$_{104/qe}$ If $f(x) = x^2 + 6x + c$, where 'c' is an integer, then $f(0) + f(-1)$ is
(A) an even integer (B) an odd integer always divisible by 3
(C) an odd integer not divisible by 3 (D*) an odd integer may or not be divisible by 3
[Sol. $f(0) + f(-1) = c + c - 5 = 2c - 5$
Now 2c is always even so $2c - 5$ is odd. may or not be divisible by 3]

Q.97$_{47/det}$ The number of real values of x satisfying $\begin{vmatrix} x & 3x+2 & 2x-1 \\ 2x-1 & 4x & 3x+1 \\ 7x-2 & 17x+6 & 12x-1 \end{vmatrix} = 0$ is
(A) 3 (B) 0 (C*) more than 3 (D) 1
[Hint : Use $R_3 \to R_3 - (2R_2 + 3R_1)$ we get $\begin{vmatrix} x & 3x+2 & 2x-1 \\ 2x-1 & 4x & 3x+1 \\ 0 & 0 & 0 \end{vmatrix}$ which is equal to zero for all x.]

Q.98$_{57/mat}$ Number of real values of λ for which the matrix $A = \begin{bmatrix} \lambda-1 & \lambda & \lambda+1 \\ 2 & -1 & 3 \\ \lambda+3 & \lambda-2 & \lambda+7 \end{bmatrix}$ has no inverse
(A) 0 (B) 1 (C) 2 (D*) infinite
[Hint: $|A| = 0$ for all $\lambda \in R \Rightarrow A$ is singular. Hence inverse can not be found for any value of $\lambda \in R \Rightarrow$ (D)
Use $R_2 \to R_2 - \frac{1}{2}(R_3 - R_1)$]

Q.99$_{107/qe}$ Number of quadratic equations with real roots which remain unchanged even after squaring their roots, is :
(A) 1 (B) 2 (C*) 3 (D) 4
[Hint : $\alpha\beta = \alpha^2\beta^2$ ....(1)
and $\alpha^2 + \beta^2 = \alpha + \beta$ ....(2)
Hence $\alpha\beta(1-\alpha\beta) = 0 \Rightarrow \alpha = 0$ or $\beta = 0$ or $\alpha\beta = 1$
if $\alpha = 0$ then from (2) $\beta = 0$ or $\beta = 1 \Rightarrow$ roots are (0,0) or (0,1)
if $\beta = 0$ then $\alpha = 0$ or $\beta = 1$
if $\beta = \frac{1}{\alpha}$ then $\alpha^2 + \frac{1}{\alpha^2} = \alpha + \frac{1}{\alpha} \Rightarrow \left[\alpha + \frac{1}{\alpha}\right]^2 - 2 = \alpha + \frac{1}{\alpha}$
hence $t^2 - t - 2 = 0 \Rightarrow (t-2)(t+1) = 0 \Rightarrow t = 2$ or $t = -1$
if $t = 2 \Rightarrow \alpha = 1$ & $\beta = 1$, if $t = -1$ roots are imaginary (ω or ω^2)]

Q.100$_{48/det}$ If $D = \begin{vmatrix} \frac{1}{z} & \frac{1}{z} & -\frac{(x+y)}{z^2} \\ -\frac{(y+z)}{x^2} & \frac{1}{x} & \frac{1}{x} \\ -\frac{y(y+z)}{x^2 z} & \frac{x+2y+z}{xz} & -\frac{y(x+y)}{xz^2} \end{vmatrix}$ then, the incorrect statement is

(A) D is independent of x (B) D is independent of y
(C) D is independent of z (D*) D is dependent on x, y, z

[Hint : Multiply c_1 by x; c_2 by y and c_3 by z and divide the determinant by xyz.
Use $c_1 \to c_1 + c_2 + c_3 \Rightarrow$ values of determinant is zero.]

Q.101$_{59/mat}$ If every element of a square non singular matrix A is multiplied by k and the new matrix is denoted by B then $|A^{-1}|$ and $|B^{-1}|$ are related as

(A) $|A^{-1}| = k\,|B^{-1}|$ (B) $|A^{-1}| = \frac{1}{k}\,|B^{-1}|$ (C*) $|A^{-1}| = k^n\,|B^{-1}|$ (D) $|A^{-1}| = k^{-n}\,|B^{-1}|$

where n is order of matrices.

[Hint: verify by taking a square matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ or $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$]

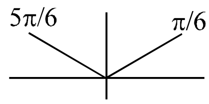
Q.102$_{109/qe}$ If the quadratic polynomial, $y = (\cot\alpha)\,x^2 + 2\left(\sqrt{\sin\alpha}\right)x + \frac{1}{2}\tan\alpha$, $\alpha \in [0, 2\pi]$, can take negative values for all $x \in R$, then the value of α must in the interval :

(A*) $\alpha \in \left(\frac{5\pi}{6}, \pi\right)$ (B) $\alpha \in \left(\frac{5\pi}{6}, \pi\right) \cup \left(\frac{11\pi}{6}, 2\pi\right)$

(C) $\alpha \in \left(\frac{11\pi}{6}, 2\pi\right)$ (D) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, \pi\right)$

[Sol. $b^2 - 4ac < 0$ and $a < 0$
hence $\cot\alpha < 0$ i.e. $\alpha \in 2^{nd}$ and 4^{th} quadrant
and $4\sin\alpha - 2\tan\alpha\cot\alpha < 0$

$2\sin\alpha < 1 \Rightarrow \sin\alpha < \frac{1}{2}$



$\therefore$ $\alpha \in 2^{nd}$ and 4^{th} quadrant

hence $a \in \left(\frac{5\pi}{6}, \pi\right)$]

Q.103$_{50/det}$ If $f'(x) = \begin{vmatrix} mx & mx-p & mx+p \\ n & n+p & n-p \\ mx+2n & mx+2n+p & mx+2n-p \end{vmatrix}$ then $y = f(x)$ represents

(A*) a straight line parallel to x- axis (B) a straight line parallel to y- axis
(C) parabola (D) a straight line with negative slope

[Hint: $R_3 \to R_3 - 2R_2$ hence 2 identical rows $\Rightarrow f'(x) = 0 \Rightarrow f(x) = \text{constant}$]

Q.104$_{60/mat}$ Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ and $10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$. If B is the inverse of matrix A, then α is

(A) – 2 (B) – 1 (C) 2 (D*) 5

[Sol. $B = A^{-1} \quad \Rightarrow \quad AB = I$

$$B = \begin{bmatrix} \frac{2}{5} & \frac{2}{10} & \frac{1}{5} \\ -\frac{1}{2} & 0 & \frac{\alpha}{10} \\ 1 & -2 & 3 \end{bmatrix}; \qquad \text{now} \quad \begin{bmatrix} \frac{2}{5} & \frac{2}{10} & \frac{1}{5} \\ -\frac{1}{2} & 0 & \frac{\alpha}{10} \\ \frac{1}{10} & 0 & \frac{3}{10} \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

hence product of 2nd and 1st column

$$-\frac{1}{2} + 0 + \frac{\alpha}{10} = 0$$

$$\alpha = \frac{10}{2} = 5 \quad \Rightarrow \quad \text{(D)]}$$

Q.105$_{113/qe}$ If the equation $\sin^4 x - (k+2)\sin^2 x - (k+3) = 0$ has a solution then k must lie in the interval :
(A) $(-4, -2)$ (B) $[-3, 2)$ (C) $(-4, -3)$ (D*) $[-3, -2]$

[Sol. $\sin^2 x = \dfrac{(k+2) \pm \sqrt{(k+2)^2 + 4(k+3)}}{2}$

$$= \frac{(k+2) \pm \sqrt{k^2 + 8k + 16}}{2}$$

$$= \frac{(k+2) \pm (k+4)}{2}$$

$\sin^2 x = k+3$ or -1 (rejected)

$\therefore \quad 0 \le k+3 \le 1 \;\Rightarrow\; -3 \le k \le -2$]

Q.106$_{51/det}$ If $D(x) = \begin{vmatrix} x-1 & (x-1)^2 & x^3 \\ x-1 & x^2 & (x+1)^3 \\ x & (x+1)^2 & (x+1)^3 \end{vmatrix}$ then the coefficient of x in D(x) is

(A*) 5 (B) – 2 (C) 6 (D) 0

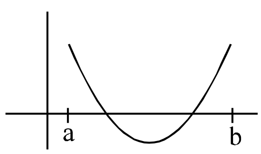
[Hint: Coefficient of x in $D(x) = D'(0)$]

Q.107$_{115/qe}$ If $b > a$, then the equation $(x-a)(x-b) + 1 = 0$, has :
(A*) both roots in (a, b) (B) both roots in $(-\infty, a)$
(C) both roots in (b, ∞) (D) one root in $(-\infty, a)$ & other in (b, ∞)

[Hint : consider $f(x) = (x-a)(x-b) + 1$

$f(a) = f(b) = 1$

$\Rightarrow$ both roots in (a, b)



]

Q.108$_{53/det}$ The set of equations

$$\lambda x - y + (\cos\theta) z = 0$$
$$3x + y + 2z = 0$$
$$(\cos\theta)x + y + 2z = 0$$

$0 \le \theta < 2\pi$, has non- trivial solution(s)

(A*) for no value of λ and θ (B) for all values of λ and θ

(C) for all values of λ and only two values of θ

(D) for only one value of λ and all values of θ

[Hint : $D = \cos\theta - \cos^2\theta + 6 \neq 0$ since $D \neq 0 \Rightarrow$ only trivial solution is possible $\Rightarrow$ (A)]

Q.109$_{61/mat}$ Matrix A satisfies $A^2 = 2A - I$ where I is the identity matrix then for $n \ge 2$, A^n is equal to ($n \in N$)

(A) $nA - I$ (B) $2^{n-1}A - (n-1)I$ (C*) $nA - (n-1)I$ (D) $2^{n-1}A - I$

[Sol. $A^2 = 2A - I \Rightarrow A^3 = 2A^2 - IA$

$= 2(2A - I) - A \quad (A^2 = 2A - I)$

$A^3 = 3A - 2I$

$A^4 = 3A^2 - 2A$

$= 3(2A - I) - 2A \quad (A^2 = 2A - I)$

$A^4 = 4A - 3I$

$A^5 = 5A - 4I$

$\vdots$

$A^n = nA - (n-1)I$]

Q.110$_{116/qe}$ The value of x satisfying the equation $\dfrac{6x + 2a + 3b + c}{6x + 2a - 3b - c} = \dfrac{2x + 6a + b + 3c}{2x + 6a - b - 3c}$ is :

(A*) $x = ab/c$ (B) $2ab/c$ (C) $ab/3c$ (D) $ab/2c$

[Hint : put $6x + 2a = A$; $3b + c = B$; $2x + 6a = C$; $b + 3c = D \Rightarrow$

$\dfrac{A+B}{A-B} = \dfrac{C+D}{C-D}$. Now add unity on both sides ; subtract unity from both sides and divide to get $A/B = C/D \Rightarrow x = ab/c$]

Q.111$_{54/det}$ If a, b, c are real then the value of determinant $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1$ if

(A) $a + b + c = 0$ (B) $a + b + c = 1$ (C) $a + b + c = -1$ (D*) $a = b = c = 0$

[Hint: Multiply R_1 by a, R_2 by b & R_3 by c & divide the determinant by abc. Now take a, b & c common from c_1, c_2 & c_3. Now use $C_1 \to C_1 + C_2 + C_3$ to get]

$$(a^2 + b^2 + c^2 + 1)\begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix} = 1.$$ Now use $c_1 \to c_1 - c_2$ & $c_2 \to c_2 - c_3$

we get $1 + a^2 + b^2 + c^2 = 1 \Rightarrow a = b = c = 0 \Rightarrow$ (D)]

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Q.112$_{63/mat}$ Read the following mathematical statements carefully:

I. There can exist two triangles such that the sides of one triangle are all less than 1 cm while the sides of the other triangle are all bigger than 10 metres, but the area of the first triangle is larger than the area of second triangle.

II. If x, y, z are all different real numbers, then

$$\frac{1}{(x-y)^2}+\frac{1}{(y-z)^2}+\frac{1}{(z-x)^2}=\left(\frac{1}{x-y}+\frac{1}{y-z}+\frac{1}{z-x}\right)^2.$$

III. $\log_3 x\cdot\log_4 x\cdot\log_5 x=(\log_3 x\cdot\log_4 x)+(\log_4 x\cdot\log_5 x)+(\log_5 x\cdot\log_3 x)$ is true for exactly for one real value of x.

IV. A matrix has 12 elements. Number of possible orders it can have is six.

Now indicate the correct alternatively.

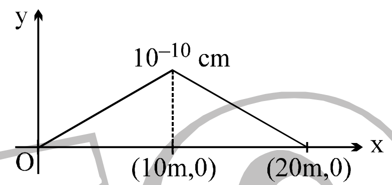
(A*) exactly one statement is INCORRECT.
(B) exactly two statements are INCORRECT.
(C) exactly three statements are INCORRECT.
(D) All the four statements are INCORRECT.

[**Explanation :**

I. Consider $2\Delta_1 = b_1h_1$

$2\Delta_2 = b_2h_2$

$b_1h_1 > b_2h_2$



first Δ is each with 1 cm, $2\Delta_1 = \frac{\sqrt{3}}{2}$

2nd triangle

$$2\Delta_2 = \frac{20\times10^3\times10^{-10}}{2}\,10^{-6} \quad\Rightarrow\quad \Delta_1 > \Delta_2 \qquad]$$

II. $$\text{RHS} = \frac{1}{(x-y)^2}+\frac{1}{(y-z)^2}+\frac{1}{(z-x)^2}+\frac{2}{(x-y)(y-z)}+\frac{2}{(y-z)(z-x)}+\frac{2}{(x-y)(z-x)}$$

$$=\frac{1}{(x-y)^2}+\frac{1}{(y-z)^2}+\frac{1}{(z-x)^2}+\frac{2[\cancel{z}-\cancel{x}+\cancel{x}-\cancel{y}+\cancel{y}-\cancel{z}]}{(x-t)(z-x)} = \text{LHS} \quad\Rightarrow\quad \text{True}]$$

III. x = 1 and 60 are two solutions.

IV. Possible orders

(1 × 12) ; (12 × 1) ; (2 × 6) ; (6 × 2) ; (3 × 4) ; (4 × 3)]

Q.113$_{118/qe}$ The quadratic equation $ax^2+bx+c=0$ has imaginary roots if :

(A) $a<-1,\ 0<c<1,\ b>0$ (B) $a<-1,\ -1<c<0,\ 0<b<1$
(C) $a<-1,\ c<0,\ b>1$ (D*) none

Q.114$_{55/det}$ The system of equations $(\sin\theta)x+2z=0$, $(\cos\theta)x+(\sin\theta)y=0$, $(\cos\theta)y+2z=a$ has

(A) no unique solution
(B*) a unique solution which is a function of a and θ
(C) a unique solution which is independent of a and θ
(D) a unique solution which is independent of θ only

[Hint: $D=\begin{vmatrix}\sin\theta & 0 & 2\\ \cos\theta & \sin\theta & 0\\ 0 & \cos\theta & 2\end{vmatrix}$

$\sin\theta\,(2\sin\theta)+2\cos^2\theta=2$]

Q.115$_{64/mat}$ Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 5 \\ 0 & 2 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 0 \\ -3 \\ 1 \end{bmatrix}$. Which of the following is true?

(A*) Ax = b has a unique solution. (B) Ax = b has exactly three solutions.
(C) Ax = b has infinitely many solutions. (D) Ax = b is inconsistent.

[Hint: $|A| = 1(0 - 10) - 2(2 - 6)$
$= -10 + 8 = -2 \quad \Rightarrow \quad |A| \neq 0 \quad \Rightarrow$ unique solution]

Q.116$_{57/det}$ The number of positive integral solutions of the equation

$$\begin{vmatrix} x^3+1 & x^2y & x^2z \\ xy^2 & y^3+1 & y^2z \\ xz^2 & yz^2 & z^3+1 \end{vmatrix} = 11 \text{ is}$$

(A) 0 (B*) 3 (C) 6 (D) 12

[Hint: Multiply R_1 by x; R_2 by y and R_3 by z and divide the determinant by xyz

$$\frac{1}{xyz}\begin{vmatrix} x^4+x & x^3y & x^3z \\ xy^3 & y^4+y & y^3z \\ xz^3 & yz^3 & z^4+z \end{vmatrix} = 11$$

$$= \frac{xyz}{xyz}\begin{vmatrix} x^3+1 & x^3 & x^3 \\ y^3 & y^3+1 & y^3 \\ z^3 & z^3 & z^3+1 \end{vmatrix} = 11$$

use $R_1 \to R_1 + R_2 + R_3$

$$D = (x^3 + y^3 + z^3 + 1)\begin{vmatrix} 1 & 1 & 1 \\ y^3 & y^3+1 & y^3 \\ z^3 & z^3 & z^3+1 \end{vmatrix} = 11$$

hence $x^3 + y^3 + z^3 = 10$ (as the det. has the value 1)
(2, 1, 1) , (1, 2, 1) , (1, 1, 2) $\Rightarrow$ (B)]

Q.117$_{126/qe}$ The number of non-zero solutions of the equation , $x^2 - 5x - (\text{sgn } x)\, 6 = 0$ is :
(A*) 1 (B) 2 (C) 3 (D) 4

[Hint: If x = 0 we have $x^2 - 5x = 0 \Rightarrow x = 0$ or 5 $\Rightarrow$ no solution
if x > 0 we have $x^2 - 5x - 6 = 0 \Rightarrow (x - 6)(x + 1) \Rightarrow x = 6$ is the solution
if x < 0 we have $x^2 - 5x + 6 = 0 \Rightarrow (x - 3)(x - 2) = 0 \Rightarrow$ no solution]

Q.118$_{65/mat}$ If A, B and C are $n \times n$ matrices and det(A) = 2, det(B) = 3 and det(C) = 5, then the value of the $\det(A^2BC^{-1})$ is equal to

(A) $\frac{6}{5}$ (B*) $\frac{12}{5}$ (C) $\frac{18}{5}$ (D) $\frac{24}{5}$

[Hint: $|A| = 2$; $|B| = 3$; $|C| = 5$

$$\det(A^2BC^{-1}) = |A^2BC^{-1}| = \frac{|A|^2|B|}{|C|} = \frac{4\cdot 3}{5} = \frac{12}{5} \text{ Ans. }]$$

Q.119$_{58/det}$ The equation $\begin{vmatrix} (1+x)^2 & (1-x)^2 & -(2+x^2) \\ 2x+1 & 3x & 1-5x \\ x+1 & 2x & 2-3x \end{vmatrix} + \begin{vmatrix} (1+x)^2 & 2x+1 & x+1 \\ (1-x)^2 & 3x & 2x \\ 1-2x & 3x-2 & 2x-3 \end{vmatrix} = 0$

(A) has no real solution
(B) has 4 real solutions
(C) has two real and two non-real solutions
(D*) has infinite number of solutions , real or non-real

[Hint: 1st two columns of 1st determinant are same as 1st two rows of 2nd. Hence transpose the 2nd. Add the two determinants and use $C_1 \to C_1 + C_3 \Rightarrow D = 0$]

Q.120$_{130/qe}$ The quadratic equation with real co-efficients one of whose complex roots has the real part 12 and modulus 13 is :

(A) $x^2 - 12x + 13 = 0$
(B) $x^2 - 24x + 13 = 0$
(C*) $x^2 - 24x + 169 = 0$
(D) $x^2 - 24x - 169 = 0$

[Hint : Let the roots be $12 + \alpha i$; other roots is $12 - \alpha i$

now $z\bar{z} = |z|^2$

$\therefore \quad 144 + a^2 = 169 \quad \Rightarrow \quad \alpha = \pm 5$

Hence roots are $12 \pm 5i$]

Q.121$_{60/det}$ The value of the determinant $\begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix}$ is

(A) $9a^2(a+b)$
(B*) $9b^2(a+b)$
(C) $3b^2(a+b)$
(D) $7a^2(a+b)$

[Hint: Use $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$ and expand]

Q.122$_{66/mat}$ Let three matrices $A = \begin{bmatrix} 2 & 1 \\ 4 & 1 \end{bmatrix}$; $B = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}$ then

$t_r(A) + t_r\left(\frac{ABC}{2}\right) + t_r\left(\frac{A(BC)^2}{4}\right) + t_r\left(\frac{A(BC)^3}{8}\right) + \ldots\ldots + \infty =$

(A*) 6
(B) 9
(C) 12
(D) none

[Sol. $BC = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix} \Rightarrow BC = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$$t_r(A) + t_r\left(\frac{A}{2}\right) + t_r\left(\frac{A}{2^2}\right) + \ldots\ldots$$

$$= t_r(A) + \frac{1}{2}t_r(A) + \frac{1}{2^2}t_r(A) + \ldots\ldots$$

$$= \frac{t_r(A)}{1-(1/2)} = 2\,t_r(A) = 2(2+1) = 6 \text{ Ans. }]$$

Q.123$_{62/det}$ The number of positive integral solutions

$$\begin{vmatrix} 1-\lambda & 2 & 1 \\ -3 & \lambda & -2 \\ 2 & -2 & 1+\lambda \end{vmatrix} = 0 \quad \text{is}$$

(A) 0 (B) 2 (C) 3 (D*) 1

[Hint: $\lambda = 0,\ -3, 3 \ \Rightarrow$ no. of positive solution is 1 which is x = 3]

Q.124$_{67/mat}$ P is an orthogonal matrix and A is a periodic matrix with period 4 and $Q = PAP^T$ then $X = P^TQ^{2005}P$ will be equal to

(A*) A (B) A^2 (C) A^3 (D) A^4

[Sol. $X = P^T[(PAP^T)(PAP^T).........(PAP^T)]\ P$

$= A^{2005} = A^{2004} \cdot A = A$ **Ans.**

Note : If k is the period of A $\Rightarrow A^{nk+1} = A$ for $n \in I$]

Q.125$_{138/qe}$ The equations $x^3 + 5x^2 + px + q = 0$ and $x^3 + 7x^2 + px + r = 0$ have two roots in common. If the third root of each equation is represented by x_1 and x_2 respectively, then the ordered pair (x_1, x_2) is :

(A*) $(-5, -7)$ (B) $(1, -1)$ (C) $(-1, 1)$ (D) $(5, 7)$

[Hint : The common roots must be roots of the equation $2x^2 + (r - q) = 0$

$\Rightarrow$ sum is zero . Hence third root of first is -5 and third root of 2nd is -7]

Q.126$_{63/det}$ If $x = a + 2b$ satisfies the cubic (a, b$\in$R) $f(x) = \begin{vmatrix} a-x & b & b \\ b & a-x & b \\ b & b & a-x \end{vmatrix} = 0$, then its other two roots are

(A) real and different (B*) real and coincident

(C) imaginary (D) such that one is real and other imaginary

[Hint : Other roots are each equal to $(a - b) \Rightarrow$ (B)]

Q.127$_{68/mat}$ A is a 2×2 matrix such that $A\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $A^2\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. The sum of the elements of A, is

(A) –1 (B) 0 (C) 2 (D*) 5

[Sol. $A\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ ....(1)

and $A^2\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ ....(2)

Let A be given by $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

The first equation gives

$a - b = -1$ ....(3) and $c - d = 2$ ....(4)

For second equation, $A^2\begin{bmatrix} 1 \\ -1 \end{bmatrix} = A\left(A\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = A\left(\begin{bmatrix} -1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

This gives $-a + 2b = 1$ ....(5) and $-c + 2d = 0$ ....(6)

(3) + (5) $\Rightarrow$ $b = 0$ and $a = -1$

(4) + (6) $\Rightarrow$ $d = 2$ and $c = 4$

so the sum $a + b + c + d = 5$ Ans.]

Q.128$_{64/det}$ Three digit numbers x17, 3y6 and 12z where x, y, z are integers from 0 to 9, are divisible by a fixed constant k. Then the determinant $\begin{vmatrix} x & 3 & 1 \\ 7 & 6 & z \\ 1 & y & 2 \end{vmatrix}$ must be divisible by

(A*) k (B) k^2 (C) k^3 (D) None

Q.129$_{69/mat}$ In a square matrix A of order 3, a_{ii}'s are the sum of the roots of the equation $x^2 - (a + b)x + ab = 0$; $a_{i, i+1}$'s are the product of the roots, $a_{i, i-1}$'s are all unity and the rest of the elements are all zero. The value of the det. (A) is equal to

(A) 0 (B) $(a + b)^3$ (C) $a^3 - b^3$ (D*) $(a^2 + b^2)(a + b)$

[Sol. Given $a_{11} = a_{22} = a_{33} = a + b$
$a_{12} = a_{23} = ab$

$$\text{Det (A)} = \begin{vmatrix} a+b & ab & 0 \\ 1 & a+b & ab \\ 0 & 1 & a+b \end{vmatrix} = (a^2 + b^2)(a + b)]$$

Q.130$_{65/det}$ Let $N = \begin{vmatrix} 28 & 25 & 38 \\ 42 & 38 & 65 \\ 56 & 47 & 83 \end{vmatrix}$, then the number of ways is which N can be resolved as a product of two divisors which are relatively prime is

(A) 4 (B*) 8 (C) 9 (D) 16

[Hint: $N = 770 = 2 \times 5 \times 7 \times 11$
Hence no. of ways $= 2^{n-1} = 2^3 = 8$]

Q.131$_{66/det}$ If A, B, C are the angles of a triangle and $\begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin A+\sin^2 A & \sin B+\sin^2 B & \sin C+\sin^2 C \end{vmatrix} = 0$, then the triangle is

(A) a equilateral (B*) an isosceles
(C) a right angled triangle (D) any triangle

Q.132$_{71/mat}$ Let $a = \lim_{x\to1} \frac{x}{\ln x} - \frac{1}{x \ln x}$; $b = \lim_{x\to0} \frac{x^3 - 16x}{4x + x^2}$; $c = \lim_{x\to0} \frac{\ln(1+\sin x)}{x}$ and

$d = \lim_{x\to-1} \frac{(x+1)^3}{3(\sin(x+1) - (x+1))}$, then the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

(A) Idempotent (B) Involutary (C) Non singular (D*) Nilpotent

[Sol. $a = +2$; $b = -4$; $c = 1$; $d = -2$

Let $A = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}$

now $\begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ = null matrix

hence A is nilpotent $\Rightarrow$ (D)

note that any matrix of the form $\begin{bmatrix} a & -a^2 \\ 1 & -a \end{bmatrix}$ is a nilpotent]

Q.133$_{67/det}$ If the system of linear equations

$$x + 2ay + az = 0$$
$$x + 3by + bz = 0$$
$$x + 4cy + cz = 0$$

has a non-zero solution, then a, b, c

(A) are in G..P. (B*) are in H.P.

(C) satisfy $a + 2b + 3c = 0$ (D) are in A.P.

Q.134$_{68/det}$ Give the correct order of initials **T** or **F** for following statements. Use **T** if statement is true and **F** if it is false.

Statement-1 : If the graphs of two linear equations in two variables are neither parallel nor identical, then there is a unique solution to the system.

Statement-2 : If the system of equations $ax + by = 0$, $cx + dy = 0$ has a non-zero solution, then it has infinitely many solutions.

Statement-3 : The system $x + y + z = 1$, $x = y$, $y = 1 + z$ is inconsistent.

Statement-4 : If two of the equations in a system of three linear equations are inconsistent, then the whole system is inconsistent.

(A) FFTT (B*) TTFT (C) TTFF (D) TTTF

***Select the correct alternatives* : (More than one are correct)**

Q.135$_{512/mat}$ D is a 3 x 3 diagonal matrix. Which of the following statements is not true?

(A) $D' = D$ (B*) AD = DA for every matrix A of order 3 x 3

(C*) D^{-1} if exists is a scalar matrix (D) none of these

[Sol. Let $D = \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}$. Clearly $D' = D$ $\Rightarrow$ A is correct

Also, $AD = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix} = \begin{bmatrix} d_1a_{11} & d_2a_{12} & d_3a_{13} \\ d_1a_{21} & d_2a_{22} & d_3a_{23} \\ d_1a_{31} & d_2a_{32} & d_3a_{33} \end{bmatrix}$

and, $DA = \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} d_1a_{11} & d_1a_{12} & d_1a_{13} \\ d_2a_{21} & d_2a_{22} & d_2a_{23} \\ d_3a_{31} & d_3a_{32} & d_3a_{33} \end{bmatrix}$

This shows that in general $AD \neq DA$

If $d_1d_2d_3 \neq 0$, then $D^{-1} = \begin{bmatrix} d_1^{-1} & 0 & 0 \\ 0 & d_2^{-1} & 0 \\ 0 & 0 & d_3^{-1} \end{bmatrix} \Rightarrow$ (C) is correct]

Q.136$_{501/det}$ The set of equations $x - y + 3z = 2$, $2x - y + z = 4$, $x - 2y + \alpha z = 3$ has

(A) unique soluton only for $\alpha = 0$ (B*) unique solution for $\alpha \neq 8$

(C) infinite number of solutions for $\alpha = 8$ (D*) no solution for $\alpha = 8$

[Hint : $D = \alpha - 8 \Rightarrow$ (B) ; If $\alpha = 8$, $D = D_1 = D_2 = D_3$]

Q.137$_{501/qe}$ $\cos\alpha$ is a root of the equation $25x^2 + 5x - 12 = 0$, $-1 < x < 0$, then the value of $\sin 2\alpha$ is :

(A*) 24/25 (B) $-12/25$ (C*) $-24/25$ (D) 20/25

Q.138$_{501/mat}$ Suppose $a_1, a_2, ...$ real numbers, with $a_1 \neq 0$. If $a_1, a_2, a_3, ...$are in A.P. then

(A*) $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_5 & a_6 & a_7 \end{bmatrix}$ is singular

(B*) the system of equations $a_1x + a_2y + a_3z = 0$, $a_4x + a_5y + a_6z = 0$, $a_7x + a_8y + a_9z = 0$ has infinite number of solutions

(C*) $B = \begin{bmatrix} a_1 & ia_2 \\ ia_2 & a_1 \end{bmatrix}$ is non singular ; where $i = \sqrt{-1}$

(D) none of these

[Sol. Let We have $|A| = \begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_5 & a_6 & a_7 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ 3d & 3d & 3d \\ d & d & d \end{vmatrix} = 0$

[Using $R_3 \to R_3 - R_2$, and $R_2 \to R_2 - R_1$]

$\Rightarrow$ A is singular

$\therefore$ The given system of homogeneous equations has infinite number of solutions.

Also $|B| = a_1^{\,2} + a_2^{\,2} \neq 0$. Thus B is non- singular]

Q.139$_{503/det}$ The determinant $\begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix}$ is divisible by :

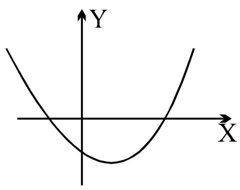
(A*) $a + b + c$ (B) $(a + b)(b + c)(c + a)$

(C*) $a^2 + b^2 + c^2$ (D*) $(a - b)(b - c)(c - a)$

[Hint : Use $C_2 \to C_2 - C_1 - 2C_3$ then $C_1 \to C_1 + C_2$ take $a^2 + b^2 + c^2$ common from first column]

Q.140$_{502/qe}$ The graph of the quadratic polynomial ;

$y = ax^2 + bx + c$ is as shown in the figure . Then :



(A*) $b^2 - 4ac > 0$ (B*) $b < 0$

(C*) $a > 0$ (D*) $c < 0$

Q.141$_{502/mat}$ If A and B are 3×3 matrices and $|A| \neq 0$, then which of the following are true?

(A*) $|AB| = 0 \Rightarrow |B| = 0$ (B) $|AB| = 0 \Rightarrow B = 0$

(C*) $|A^{-1}| = |A|^{-1}$ (D) $|A + A| = 2|A|$

[Sol. For $|AB| = 0 \Rightarrow |A| \cdot |B| = 0 \Rightarrow |A| = 0, |B| = 0$

$AA^{-1} = I \Rightarrow |A| \cdot |A|^{-1} = |I| = 1 \Rightarrow |A^{-1}| = \dfrac{1}{|A|} = |A|^{-1}$]

Q.142$_{504/det}$ The value of θ lying between $-\dfrac{\pi}{4}$ & $\dfrac{\pi}{2}$ and $0 \le A \le \dfrac{\pi}{2}$ and satisfying the equation

$$\begin{vmatrix} 1+\sin^2 A & \cos^2 A & 2\sin 4\theta \\ \sin^2 A & 1+\cos^2 A & 2\sin 4\theta \\ \sin^2 A & \cos^2 A & 1+2\sin 4\theta \end{vmatrix} = 0 \text{ are :}$$

(A*) $A = \frac{\pi}{4}$, $\theta = -\frac{\pi}{8}$ (B*) $A = \frac{3\pi}{8} = \theta$

(C*) $A = \frac{\pi}{5}$, $\theta = -\frac{\pi}{8}$ (D*) $A = \frac{\pi}{6}$, $\theta = \frac{3\pi}{8}$

[Hint : Use $R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$ and expand to get
$D = 2(1 + \sin 4\theta) = 0$]

$\Rightarrow \theta = \frac{n\pi}{4} - (-1)^n \frac{\pi}{8}$ $\Rightarrow$ independent of A $\Rightarrow$ A, B, C, D]

Q.143$_{505/qe}$ If the quadratic equations, $x^2 + abx + c = 0$ and $x^2 + acx + b = 0$ have a common root then the equation containing their other roots is/are :
(A) $x^2 + a(b + c)x - a^2bc = 0$ (B*) $x^2 - a(b + c)x + a^2bc = 0$
(C) $a(b + c)x^2 - (b + c)x + abc = 0$ (D*) $a(b + c)x^2 + (b + c)x - abc = 0$

Q.144$_{503/mat}$ If AB = A and BA = B, then
(A*) $A^2B = A^2$ (B*) $B^2A = B^2$ (C*) ABA = A (D*) BAB = B
[Sol. We have $A^2B = A(AB) = AA = A^2$, $B^2A = B(BA) = BB = B^2$,
ABA = A(BA) = AB = A, and BAB = B(AB) = BA = B]

Q.145$_{505/det}$ The solution(s) of the equation $\begin{vmatrix} x & a & b \\ a & x & a \\ b & b & x \end{vmatrix} = 0$ is/are :

(A*) $x = -(a+b)$ (B*) $x = a$ (C*) $x = b$ (D) $-b$
[Hint: Use $c_1 \to c_1 - c_2$ & then $R_1 \to R_1 + R_2$ to get

$$\begin{vmatrix} 0 & a+x & b+a \\ -(x-a) & x & a \\ 0 & b & x \end{vmatrix} = 0.$$ Now open by c_1 & factorize]

Q.146$_{511/qe}$ The value(s) of 'p' for which the equation $ax^2 - px + ab = 0$ and $x^2 - ax - bx + ab = 0$ may have a common root, given a, b are non zero real numbers, is
(A) $a + b^2$ (B*) $a^2 + b$ (C*) $a(1 + b)$ (D) $b(1 + a)$
[Sol. $x^2 - (a + b)x + ab = 0$ or $(x - a)(x - b) = 0$
$\Rightarrow$ x = a or b
if x = a is the root of other equation, $a^3 - ap + ab = 0$ $\Rightarrow$ $p = a(a + b)$
if x = b is the root of the other equation, then $ab^2 - pb + ab = 0$
$p = a(1 + b)$]

Q.147$_{505/mat}$ If D_1 and D_2 are two 3 x 3 diagonal matrices when none of the diagonal element is zero, then
(A*) D_1D_2 is a diagonal matrix (B*) $D_1D_2 = D_2D_1$

(C*) $D_1^2 + D_2^2$ is a diagonal matrix (D) none of these

[Sol. Let $D_1 = \begin{bmatrix} x_1 & 0 & 0 \\ 0 & y_1 & 0 \\ 0 & 0 & z_1 \end{bmatrix}$ and $D_2 = \begin{bmatrix} x_2 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & z_2 \end{bmatrix}$, when $x_1, y_1, z_1, x_2, y_2, z_2 \neq 0$

then $D_1D_2 = D_2D_1$]

Q.148$_{507/det}$ If $\begin{vmatrix} 1 & a & a^2 \\ 1 & x & x^2 \\ b^2 & ab & a^2 \end{vmatrix} = 0$, then :

(A*) $x = a$ (B) $x = b$ (C) $x = \frac{1}{a}$ (D*) $x = \frac{a}{b}$

[Hint: $R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$ gives

$(x-a)(b-1)\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & x+a \\ b+1 & a & 0 \end{vmatrix}$ open by c_1 & get the value of $x = a/b$, $x = a$]

Q.149$_{513/qe}$ Let Δ^2 be the discriminant and α, β be the roots of the equation $ax^2 + bx + c = 0$. Then $2a\alpha + \Delta$ and $2a\beta - \Delta$ can be the roots of the equation :

(A*) $x^2 + 2bx + b^2 = 0$ (B) $x^2 - 2bx + b^2 = 0$

(C*) $x^2 + 2bx - 3b^2 + 16ac = 0$ (D) $x^2 - 2bx - 3b^2 + 16ac = 0$

[Hint : $\alpha, \beta = \frac{-b \pm \sqrt{\Delta^2}}{2a} \Rightarrow \alpha = \frac{-b+\Delta}{2a}$ and $\beta = \frac{-b-\Delta}{2a}$

$\Rightarrow$ $2a\alpha - \Delta = -b$ and $2a\beta + \Delta = -b$ ....(1)

or $2a\alpha + \Delta = -b$ and $2a\beta - \Delta = -b$ ....(2)

From (1) $2a\alpha + \Delta = 2\Delta - b$ and $2a\beta - \Delta = -2\Delta - b$

sum $= -2b$; Product $= b^2 - 4\Delta^2 = b^2 - 4(b^2 - 4ac) = 16ac - 3b^2$ ....(3)

From (2) sum $= -2b$; Product $= b^2$ ....(4)

Hence QE is $x^2 + 2bx + b^2 = 0$

or $x^2 + 2bx + 16ac - 3b^2 = 0$ $\Rightarrow$ A and C]

Q.150$_{508/det}$ Which of the following determinant(s) vanish(es)?

(A*) $\begin{vmatrix} 1 & bc & bc(b+c) \\ 1 & ca & ca(c+a) \\ 1 & ab & ab(a+b) \end{vmatrix}$ (B*) $\begin{vmatrix} 1 & ab & \frac{1}{a}+\frac{1}{b} \\ 1 & bc & \frac{1}{b}+\frac{1}{c} \\ 1 & ca & \frac{1}{c}+\frac{1}{a} \end{vmatrix}$

(C*) $\begin{vmatrix} 0 & a-b & a-c \\ b-a & 0 & b-c \\ c-a & c-b & 0 \end{vmatrix}$ (D*) $\begin{vmatrix} \log_x xyz & \log_x y & \log_x z \\ \log_y xyz & 1 & \log_y z \\ \log_z xyz & \log_z y & 1 \end{vmatrix}$

Q.151$_{510/mat}$ If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ (where $bc \neq 0$) satisfies the equations $x^2 + k = 0$, then

(A*) $a + d = 0$ (B) $k = -|A|$ (C*) $k = |A|$ (D) none of these

[Sol. We have $A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a^2 + bc & ab + db \\ ac + cd & bc + d^2 \end{bmatrix} = 0$

As A satisfies, $x^2 + k = 0$, $A^2 + kI = \mathbf{O}$

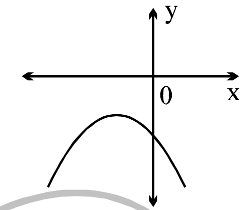
$\Rightarrow \begin{bmatrix} a^2 + bc + k & (a+d)b \\ (a+d)c & bc + d^2 + k \end{bmatrix} = \mathbf{O}$

$\Rightarrow$ $a^2 + bc + k = 0 = bc + d^2 + k = 0$ and $(a + d)b = (a + d)\,c = 0$

As $bc \neq 0$, $b \neq 0$, $c \neq 0 \Rightarrow a + d = 0 \Rightarrow a = -d$

Also, $k = -(a^2 + bc) = -(d^2 + bc) = -\,(\,(-ad) + bc\,) = |A|$]

Q.152$_{515/qe}$ The graph of a quadratic polynomial $y = ax^2 + bx + c$ ($a, b, c \in R$, $a \neq 0$) is as shown. Then the incorrect statement(s) is/are



(A*) $c > 0$ (B) $b < 0$

(C*) product of the roots is negative (D*) sum of the roots is positive

Q.153$_{510/det}$ The value of θ lying between $\theta = 0$ & $\theta = \pi/2$ & satisfying the equation :

$$\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0 \text{ are :}$$

(A*) $\frac{7\pi}{24}$ (B) $\frac{5\pi}{24}$ (C*) $\frac{11\pi}{24}$ (D) $\frac{\pi}{24}$

Q.154$_{513/det}$ If p, q, r, s are in A.P. and $f(x) = \begin{vmatrix} p + \sin x & q + \sin x & p - r + \sin x \\ q + \sin x & r + \sin x & -1 + \sin x \\ r + \sin x & s + \sin x & s - q + \sin x \end{vmatrix}$ such that $\int_0^2 f(x)dx = -4$

then the common difference of the A.P. can be :

(A*) -1 (B) $\frac{1}{2}$ (C*) 1 (D) none

[Start : $p = a$; $q = a + d$; $r = a + 2d$; $s = a + 3d$ $\Rightarrow$ $f(x) = -2d^2$

Also use $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$]

Q.155$_{517/qe}$ If one root of the quadratic equation $px^2 + qx + r = 0$ ($p \neq 0$) is a surd $\frac{\sqrt{a}}{\sqrt{a} + \sqrt{a-b}}$

where p, q, r ; a, b are all rationals then the other root is

(A*) $\frac{\sqrt{a}}{\sqrt{a} - \sqrt{a-b}}$ (B) $a + \frac{\sqrt{a(a-b)}}{b}$ (C*) $\frac{a + \sqrt{a(a-b)}}{b}$ (D) $\frac{\sqrt{a} - \sqrt{a-b}}{\sqrt{b}}$

[Hint: $\alpha = \dfrac{\sqrt{a}}{\sqrt{a}+\sqrt{a-b}} = \dfrac{\sqrt{a}\left(\sqrt{a}-\sqrt{a-b}\right)}{a-(a-b)} = \dfrac{a-\sqrt{a(a-b)}}{b}$

Conjugate of α is $\dfrac{a+\sqrt{a(a-b)}}{b}$ $\Rightarrow$ (C)]

Q.156$_{511/mat}$ Let $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then the correct statement is

(A*) $A^2 - 4A - 5I_3 = 0$
(B*) $A^{-1} = \dfrac{1}{5}(A - 4I_3)$
(C) A^3 is not invertible
(D*) A^2 is invertible

[Sol. $A^2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$

We have $A^2 - 4A - 5I_3$

$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - 4\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \mathbf{O}$

$\Rightarrow \quad 5I_3 = A^2 - 4A = A(A - 4I_3)$

$\Rightarrow \quad I_3 = A\left[\dfrac{1}{5}(A - 4I_3)\right] \Rightarrow A^{-1} = \dfrac{1}{5}(A - 4I_3)$

Note that $|A| = 5$. Since $|A^3| = |A|^3 = 5^3 \neq 0$, A^3 is invertible
Similarly, A^2 is invertible]

Q.157 Which of the following statements are **False**?
(A) If $0 < p < \pi$ then the quadratic equation, $(\cos p - 1)\,x^2 + \cos px + \sin p = 0$ has real roots.
(B*) If $2a + b + c = 0$ $(c \neq 0)$ then the quadratic equation, $ax^2 + bx + c = 0$ has no root in (0, 2).
(C*) The sum of the roots of the equation $\cos^2 x = 1$ which lie in the interval $[0, 314]$ is 5050π.
(D*) If x & y are positive real numbers & m, n are any positive integers then, $\dfrac{x^n \cdot y^m}{\left(1+x^{2n}\right)\left(1+y^{2m}\right)} > \dfrac{1}{4}$.

[Hint: (B) note that f(0) & f(2) have opposing signs under the given condition
(C) correct answer is $4950\,\pi$]