विघ्न विचारत भीरु जन, नहीं आरम्भे काम, विपति देख छोड़े तुरंत मध्यम मन कर श्याम।
पुरुष सिंह संकल्प कर, सहते विपति अनेक, 'बना' न छोड़े ध्येय को, रघुबर राखे टेक।।

रचितः मानव धर्म प्रणेता
सद्‌गुरु श्री रणछोड़दासजी महाराज

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Question bank on Vector, 3D and Complex Number & Misc. Subjective Problem

Select the correct alternative ***: (Only one is correct)***

Q.1$_{2/vec}$ If $|\vec{a}| = 11$, $|\vec{b}| = 23$, $|\vec{a}-\vec{b}| = 30$, then $|\vec{a}+\vec{b}|$ is :

(A) 10 (B*) 20 (C) 30 (D) 40

Q.2$_{5/vec}$ The position vector of a point P moving in space is given by $\overrightarrow{OP} = \vec{R} = (3\cos t)\hat{i} + (4\cos t)\hat{j} + (5\sin t)\hat{k}$. The time 't' when the point P crosses the plane $4x - 3y + 2z = 5$ is

(A) $\frac{\pi}{2}$ sec (B*) $\frac{\pi}{6}$ sec (C) $\frac{\pi}{3}$ sec (D) $\frac{\pi}{4}$ sec

[Hint: put $x = 3\cos t$; $y = 4\cos t$; $z = 5\sin t$ in the equation of the plane, we get

$12\cos t - 12\cos t + 10\sin t = 5$

$\sin t = \frac{1}{2} \Rightarrow t = \frac{\pi}{6}$ sec]

Q.3$_{6/vec}$ Indicate the correct order sequence in respect of the following :

I. The lines $\frac{x-4}{-3} = \frac{y+6}{-1} = \frac{y+6}{-1}$ and $\frac{x-1}{-1} = \frac{y-2}{-2} = \frac{z-3}{2}$ are orthogonal.

II. The planes $3x - 2y - 4z = 3$ and the plane $x - y - z = 3$ are orthogonal.

III. The function $f(x) = ln(e^{-2} + e^x)$ is monotonic increasing $\forall$ x $\in$ R.

IV. If g is the inverse of the function, $f(x) = ln(e^{-2} + e^x)$ then $g(x) = ln(e^x - e^{-2})$.

(A) FTFF (B) TFTT (C*) FFTT (D) FTTT

[Sol. **I.** L_1 is | | to $-3\hat{i} - \hat{j} - \hat{k} = \vec{V}_1$

L_2 is | | to $-\hat{i} - 2\hat{j} + 2\hat{k} = \vec{V}_2$

$\vec{V}_1 \cdot \vec{V}_2 = 3 + 2 - 2 = 3 \Rightarrow$ L is not perpendicular to $L_2 \Rightarrow$ False

II. $3\cdot 1 - (2)(-1) - (4)(-1) = 3 + 2 + 4 \neq 0 \Rightarrow$ planes are not perpendicular $\Rightarrow$ False

III. $f(x) = ln(e^{-2} + e^x)$

$f'(x) = \frac{1\cdot e^x}{e^{-2} + e^x} > 0 \Rightarrow$ f is increasing $\forall$ x $\in$ R $\Rightarrow$ True

IV. $y = ln(e^{-2} + e^x)$

$e^{-2} + e^x = e^y$

$e^x = e^y - e^{-2}$

$\therefore$ $f^{-1}(y) = ln(e^y - e^{-2})$

$g(x) = ln(e^x - e^{-2}) \Rightarrow$ True]

Q.4$_{2/complex}$ If $\frac{z_1}{z_2}$ is purely imaginary then $\left|\frac{z_1 + z_2}{z_1 - z_2}\right|$ is equal to :

(A*) 1 (B) 2 (C) 3 (D) 0

[Hint: $E = \left|\frac{1 + (z_1/z_2)}{(z_1/z_2) - 1}\right| = \left|\frac{1 + xi}{xi - 1}\right| = 1$]

Teko Classes, Maths : Suhag R. Kariya (S. R. K. Sir), Bhopal Phone : 0 903 903 7779, 0 98930 58881.

Q.5$_{7/vec}$ In a regular tetrahedron, the centres of the four faces are the vertices of a smaller tetrahedron. The ratio of the volume of the smaller tetrahedron to that of the larger is $\frac{m}{n}$, where *m* and *n* are relatively prime positive integers. The value of $(m+n)$ is

(A) 3 (B) 4 (C) 27 (D*) 28

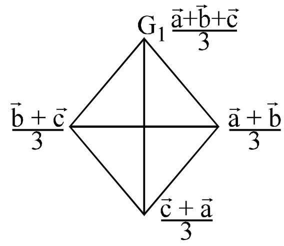
[Hint: $V_l = \frac{1}{6}[\vec{a}\ \vec{b}\ \vec{c}]$; $V_s = \frac{1}{6}\cdot\frac{1}{27}[\vec{a}\ \vec{b}\ \vec{c}]$

Hence $\frac{V_s}{V_l} = \frac{1}{27} = \frac{m}{n}$ or $\frac{n}{27} = \frac{m}{1} = k$

$\therefore$ *m* and *n* are relatively prime $\Rightarrow$ k = 1, $(m+n) = 28$

further hint for

$$V_s = \frac{1}{6}\left[\frac{\vec{a}}{3}\cdot\frac{\vec{b}}{3}\cdot\frac{\vec{c}}{3}\right] = \frac{1}{6}\cdot\frac{1}{27}[\vec{a}\ \vec{b}\ \vec{c}]$$



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Q.6$_{9/vec}$ If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{1}{\sqrt{2}}(\vec{b}+\vec{c})$ then the angle between $\vec{a}$ & $\vec{b}$ is

(A*) $3\pi/4$ (B) $\pi/4$ (C) $\pi/2$ (D) π

[JEE '95, 2]

Q.7$_{12/vec}$ The sine of angle formed by the lateral face ADC and plane of the base ABC of the tetrahedron ABCD where A ≡ (3, –2, 1); B ≡ (3, 1, 5); C ≡ (4, 0, 3) and D ≡ (1, 0, 0) is

(A) $\frac{2}{\sqrt{29}}$ (B*) $\frac{5}{\sqrt{29}}$ (C) $\frac{3\sqrt{3}}{\sqrt{29}}$ (D) $\frac{-2}{\sqrt{29}}$

Q.8$_{6/complex}$ Let z be a complex number, then the region represented by the inequality $|z+2| < |z+4|$ is given by :

(A*) $\text{Re}(z) > -3$ (B) $\text{Im}(z) < -3$

(C) $\text{Re}(z) < -3$ & $\text{Im}(z) > -3$ (D) $\text{Re}(z) < -4$ & $\text{Im}(z) > -4$

Q.9$_{14/vec}$ The volume of the parallelpiped whose edges are represented by the vectors $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$, $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$, $\vec{c} = \hat{i} + 2\hat{j} - \hat{k}$ is :

(A*) 7 (B) 5 (C) 4 (D) none

Q.10$_{15/vec}$ Let $\vec{u}, \vec{v}, \vec{w}$ be the vectors such that $\vec{u}+\vec{v}+\vec{w} = 0$, if $|\vec{u}| = 3, |\vec{v}| = 4$ & $|\vec{w}| = 5$ then the value of $\vec{u}.\vec{v} + \vec{v}.\vec{w} + \vec{w}.\vec{u}$ is :

(A) 47 (B*) –25 (C) 0 (D) 25

[JEE '95,1]

Q.11$_{16/vec}$ Let $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = \hat{j} - \hat{k}$, $\vec{c} = \hat{k} - \hat{i}$. If $\vec{d}$ is a unit vector such that $\vec{a}.\vec{d} = 0 = [\vec{b}, \vec{c}, \vec{d}]$ then $\vec{d}$

(A*) $\pm\frac{1}{\sqrt{6}}(\hat{i}+\hat{j}-2\hat{k})$ (B) $\pm\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}-\hat{k})$ (C) $\pm\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$ (D) $\pm\ \hat{k}$

Q.12$_{8/complex}$ If z be a complex number for which $\left|z+\frac{1}{z}\right| = 2$, then the greatest value of $|z|$ is :

(A*) $\sqrt{2}+1$ (B) $\sqrt{3}+1$ (C) $2\sqrt{2}-1$ (D) none

[Hint : $\left||z|-\frac{1}{|z|}\right| \le \left|z+\frac{1}{z}\right| \le |z|+\frac{1}{|z|}$

$\left|r-\frac{1}{r}\right| \le 2 \le r+\frac{1}{r}$

Now consider all inequalities]

Q.13$_{22/vec}$ If the non – zero vectors $\vec{a}$ & $\vec{b}$ are perpendicular to each other, then the solution of the equation, $\vec{r}\times\vec{a}=\vec{b}$ is :

(A*) $\vec{r}=x\vec{a}+\frac{1}{\vec{a}\cdot\vec{a}}\left(\vec{a}\times\vec{b}\right)$ (B) $\vec{r}=x\vec{b}-\frac{1}{\vec{b}\cdot\vec{b}}\left(\vec{a}\times\vec{b}\right)$

(C) $\vec{r}=x\vec{a}\times\vec{b}$ (D) $\vec{r}=x\vec{b}\times\vec{a}$

where x is any scalar.

[Hint: $\vec{r}=x\vec{a}+y\vec{b}+2\vec{a}\times\vec{b}$

take cross with $\vec{a}$

$\vec{r}\times\vec{a}=y\vec{b}\times\vec{a}+z(\vec{a}\times\vec{b})\times\vec{a}$

$\vec{b}=y(\vec{b}\times\vec{a})+z[(\vec{a}\cdot\vec{a})\vec{b}-(\vec{b}\cdot a)\times\vec{a}]]$

$\vec{b}=y(\vec{b}\times\vec{a})+z(\vec{a}\cdot\vec{a})\vec{b}$

since $\vec{b} < a\times\vec{b}$ are non coplanar

$\therefore$ $z(\vec{a}\cdot\vec{a})=1$ & $y=0$

$z=\frac{1}{\vec{a}^2}$

$\therefore$ $\vec{r}=x\vec{a}+\frac{1}{\vec{a}^2}(\vec{a}\times\vec{b})$]

Q.14$_{23/vec}$ The lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar if

(A) k = 1 or – 1 (B*) k = 0 or – 3 (C) k = 3 or – 3 (D) k = 0 or – 1

Q.15$_{24/vec}$ Which one of the following statement is INCORRECT ?

(A) If $\vec{n}\cdot\vec{a}=0$, $\vec{n}\cdot\vec{b}=0$ & $\vec{n}\cdot\vec{c}=0$ for some non zero vector $\vec{n}$, then $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=0$

(B*) there exist a vector having direction angles $\alpha=30°$ & $\beta=45°$

(C) locus of point for which x = 3 & y = 4 is a line parallel to the z - axis whose distance from the z-axis is 5

(D) In a regular tetrahedron OABC where 'O' is the origin, the vector $\vec{OA}+\vec{OB}+\vec{OC}$ is perpendicular to the plane ABC.

[Explanation:

(A) $\therefore \vec{n}$ is perpendicular to $\vec{a}$, $\vec{b}$ as well as $\vec{c}$ $\Rightarrow \vec{a}, \vec{b}, \vec{c}$ must be in the same plane $\Rightarrow \left[\vec{a}\ \vec{b}\ \vec{c}\right] = 0$

(B) If one direction angle is θ then the remaining two cannot be less than $90 - \theta$

(D) verify that $\left(\vec{a} - \dfrac{\vec{a}+\vec{b}+\vec{c}}{3}\right) . \left(\vec{a}+\vec{b}+\vec{c}\right) = 0$ where $|\vec{a}| = |\vec{b}| = |\vec{c}|$]

Q.16$_{12/complex}$ Given that the equation, $z^2 + (p + iq)\,z + r + is = 0$ has a real root where p, q, r, s $\in$ R. Then which one is correct

(A) $pqr = r^2 + p^2s$ (B) $prs = q^2 + r^2p$ (C) $qrs = p^2 + s^2q$ (D*) $pqs = s^2 + q^2r$

[Hint: Let $z = \alpha$ be the real root $\Rightarrow \alpha^2 + (p + iq)\alpha + r + is = 0$
$(\alpha^2 + p\alpha + r) + i(q\alpha + s) = 0 + 0\,i \Rightarrow q\alpha + s = 0$ ....(1) and
$\alpha^2 + p\alpha + r = 0$ ....(2)

From (1) $\alpha = -\dfrac{s}{q}$. Put in (2) to get the result]

Q.17$_{27/vec}$ The distance of the point (3, 4, 5) from x-axis is

(A) 3 (B) 5 (C) $\sqrt{34}$ (D*) $\sqrt{41}$

[Hint: distance from x-axis of x, y, z = $\sqrt{y_1^2 + z_1^2}$]

Q.18$_{29/vec}$ Given non zero vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$, then which one of the following is false?

(A) A vector orthogonal to $\vec{A}\times\vec{B}$ and $\vec{C}$ is $\pm\,(\vec{A}\times\vec{B})\times\vec{C}$

(B) A vector orthogonal to $\vec{A}+\vec{B}$ and $\vec{A}-\vec{B}$ is $\pm\ \vec{A}\times\vec{B}$

(C) Volume of the parallelopiped determined by $\vec{A}$, $\vec{B}$ and $\vec{C}$ is $|\vec{A}\times\vec{B}\cdot\vec{C}|$

(D*) Vector projection of $\vec{A}$ onto $\vec{B}$ is $\dfrac{\vec{A}\cdot\vec{B}}{|\vec{B}|}$

[Hint: It should be $\dfrac{(\vec{A}\cdot\vec{B})}{\vec{B}^2}\vec{B}$]

Q.19$_{30/vec}$ Given three vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ such that they are non–zero, non–coplanar vectors, then which of the following are non coplanar.

(A*) $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$, $\vec{c}+\vec{a}$ (B) $\vec{a}-\vec{b}$, $\vec{b}+\vec{c}$, $\vec{c}+\vec{a}$

(C) $\vec{a}+\vec{b}$, $\vec{b}-\vec{c}$, $\vec{c}+\vec{a}$ (D) $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$, $\vec{c}-\vec{a}$

[Hint: Verify $\vec{v}_1 + \vec{v}_2 = \vec{v}_3$ in order to quickly answer]

Q.20$_{16/complex}$ The sum $i + 2i^2 + 3i^3 + \ldots\ldots.. + 2002i^{2002}$, where $i = \sqrt{-1}$ is equal to

(A) $-999 + 1002i$ (B) $-1002 + 999i$ (C) $-1001 + 1000i$ (D*) $-1002 + 1001i$

[Sol. $S = i + 2i^2 + 3i^3 + \ldots\ldots\ldots + 200i^{2002}$
$iS = \quad i^2 + 2i^3 + \ldots\ldots\ldots + 2001i^{2002} + 2002i^{2003}$
– – – – –

$S(1 - i) = i + i^2 + i^3 + \ldots\ldots\ldots + i^{2002} - 2002i^{2003}$

$$= \frac{i(1-i^{2002})}{1-i} + 2002\,i = \frac{2i}{1-i} + 2002\,i = i(1+i) + 2002i$$

$$S = \frac{-1+2003i}{1-i} = \frac{(-1+2003i)(1+i)}{2} = -1 - i + 2003i - 2003 = \frac{-2004+2002i}{2}$$

$= -1002 + 1001i$]

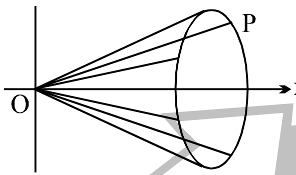
Q.21$_{31/vec}$ Locus of the point P, for which $\vec{OP}$ represents a vector with direction cosine $\cos\alpha = \frac{1}{2}$ ('O' is the origin) is :

(A) A circle parallel to y z plane with centre on the x − axis

(B*) a cone concentric with positive x − axis having vertex at the origin and the slant height equal to the magnitude of the vector

(C) a ray emanating from the origin and making an angle of 60º with x − axis

(D) a disc parallel to y z plane with centre on x − axis & radius equal to $|\vec{OP}|$ sin 60º

[Hint : 

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Q.22$_{38/vec}$ A line with direction ratios (2, 1, 2) intersects the lines $\vec{r} = -\hat{j} + \lambda(\hat{i} + \hat{j} + \hat{k})$ and $\vec{r} = -\hat{i} + \mu(2\hat{i} + \hat{j} + \hat{k})$ at A and B, then l (AB) is equal to

(A*) 3 (B) $\sqrt{3}$ (C) $2\sqrt{2}$ (D) $\sqrt{2}$

[Hint: $L_1 : \frac{x-1}{1} = \frac{y+1}{1} = \frac{z-0}{1} = \lambda$; $L_2 : \frac{x+1}{2} = \frac{y-0}{1} = \frac{z-0}{1} = \mu$

Hence any point on L_1 and L_2 can be $(\lambda, \lambda - 1, \lambda)$ and $(2\mu - 1, \mu, \mu)$

$\therefore \quad \frac{2\mu-1-\lambda}{2} = \frac{\mu-\lambda+1}{1} = \frac{\mu-\lambda}{1}$

solving $\mu = 1$ and $\lambda = 3$

A = (3, 2, 3) and B = (1, 1, 1)]

Q.23$_{47/vec}$ The vertices of a triangle are A (1, 1, 2), B (4, 3, 1) & C (2, 3, 5). The vector representing the internal bisector of the angle A is :

(A) $\hat{i} + \hat{j} + 2\hat{k}$ (B) $2\hat{i} - 2\hat{j} + \hat{k}$ (C) $2\hat{i} + 2\hat{j} - \hat{k}$ (D*) $2\hat{i} + 2\hat{j} + \hat{k}$

Q.24$_{28/complex}$ Lowest degree of a polynomial with rational coefficients if one of its root is, $\sqrt{2} + i$ is

(A) 2 (B*) 4 (C) 6 (D) 8

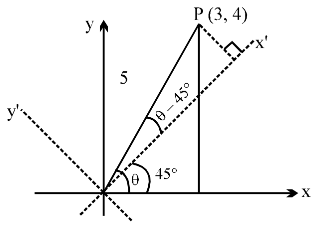
[Sol. Let $x = \sqrt{2} + i$

$\Rightarrow \quad (x - \sqrt{2})^2 = -1 \quad \Rightarrow \quad x^2 + 2 - 2\sqrt{2}\,x = -1$

$\Rightarrow \quad x^2 + 3 = 2\sqrt{2}\,x \quad \Rightarrow \quad x^4 + 9 + 6x^2 = 8x^2 \quad \Rightarrow \quad x^4 - 2x^2 + 9 = 0$]

Q.25$_{55/vec}$ A plane vector has components 3 & 4 w.r.t. the rectangular cartesian system. This system is rotated through an angle $\frac{\pi}{4}$ in anticlockwise sense. Then w.r.t. the new system the vector has components :

(A) 4, 3 (B*) $\frac{7}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ (C) $\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}$ (D) none



[Hint : $\cos\theta = \frac{3}{5}$ $\sin\theta = \frac{4}{5}$

now w.r.t. new system X′Y′

X component is $5\cos(\theta - 45°)$

Y component is $5\sin(\theta - 45°)$]

Q.26$_{56/vec}$ Let $\vec{a} = xi + 12j - k$; $\vec{b} = 2i + 2xj + k$ & $\vec{c} = i + k$. If the ordered set $\left[\vec{b}\ \vec{c}\ \vec{a}\right]$ is left handed , then:

(A) $x \in (2, \infty)$ (B) $x \in (-\infty, -3)$ (C*) $x \in (-3, 2)$ (D) $x \in \{-3, 2\}$

[Sol. For a right hand set $[\vec{a}\ \vec{b}\ \vec{c}\ ] > 0$ and for a left handed system $[\vec{a}\ \vec{b}\ \vec{c}\ ] < 0$]

Q.27$_{73/vec}$ If $\cos\alpha\hat{i} + \hat{j} + \hat{k}$, $\hat{i} + \cos\beta\hat{j} + \hat{k}$ & $\hat{i} + \hat{j} + \cos\gamma\hat{k}$ $(\alpha \neq \beta \neq \gamma \neq 2n\pi)$ are coplanar then the value of $\left[\text{cosec}^2\frac{\alpha}{2} + \text{cosec}^2\frac{\beta}{2} + \text{cosec}^2\frac{\gamma}{2}\right]$ equal to

(A) 1 (B*) 2 (C) 3 (D) none of these

[Hint : $\begin{vmatrix} \cos\alpha & 1 & 1 \\ 1 & \cos\beta & 1 \\ 1 & 1 & \cos\gamma \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} \cos\alpha - 1 & 1-\cos\beta & 0 \\ 0 & \cos\beta - 1 & 1-\cos\gamma \\ 1 & 1 & \cos\gamma \end{vmatrix} = 0$ — (2)

$$\begin{vmatrix} -2\sin^2\frac{\alpha}{2} & 2\sin^2\frac{\beta}{2} & 0 \\ 0 & -2\sin^2\frac{\beta}{2} & 2\sin^2\frac{\gamma}{2} \\ 1 & 1 & \cos\gamma \end{vmatrix} = 0$$

$$+2\sin^2\frac{\alpha}{2}\left(+2\sin^2\frac{\beta}{2}\cos\gamma + 2\sin^2\frac{\gamma}{2}\right) + 2\sin^2\frac{\beta}{2}\ 2\sin^2\frac{\gamma}{2}$$

$$\sin^2\frac{\alpha}{2}\left[\sin^2\frac{\beta}{2}\left(1 - 2\sin^2\frac{\gamma}{2}\right) + \sin^2\frac{\gamma}{2}\right] + \sin^2\frac{\beta}{2}\ \sin^2\frac{\gamma}{2} = 0$$

multiply by $\text{cosec}^2\frac{\alpha}{2}\ \text{cosec}^2\frac{\beta}{2}\ \text{cosec}^2\frac{\gamma}{2}$ $\text{cosec}^2\frac{\gamma}{2} - 2 + \text{cosec}^2\frac{\beta}{2} + \text{cosec}^2\frac{\alpha}{2} = 0$

Alternatively : Expand number 2

$(\cos\alpha - 1)\left[\cos\gamma(\cos\beta - 1) - (1 - \cos\gamma)\right] + (1 - \cos\beta)(1 - \cos\gamma) = 0$

or $(1 - \cos\alpha)(1 - \cos\beta)\cos\gamma + (1 - \cos\beta)(1 - \cos\gamma) + (1 - \cos\gamma)(1 - \cos\alpha) = 0$

Now proceed]

Q.28$_{34/complex}$ The straight line $(1 + 2i)z + (2i - 1)\bar{z} = 10i$ on the complex plane, has intercept on the imaginary axis equal to

(A*) 5 (B) $\frac{5}{2}$ (C) $-\frac{5}{2}$ (D) -5

[Hint: put z = iy $(1 + 2i)\,iy - (2i - 1)\,i\,y = 10\,i$
$y + 0\,y = 10 \Rightarrow y = 5$]

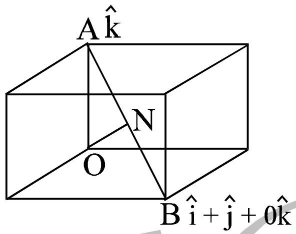
Q.29$_{75/vec}$ The perpendicular distance of an angular point of a cube of edge 'a' from the diagonal which does not pass that angular point, is

(A) $\sqrt{3}\,a$ (B) $\sqrt{2}\,a$ (C) $\sqrt{\frac{3}{2}}\,a$ (D*) $\sqrt{\frac{2}{3}}\,a$

[Sol. Consider a unit cube

equation of AB is $\vec{r} = \hat{k} + \lambda(\hat{i} + \hat{j} - \hat{k})$

p.v. of N $\lambda, \lambda, (-1 - \lambda)$



$\vec{ON} = \lambda\hat{i} + \lambda\hat{j} - (1 + \lambda)\hat{k}$

now $\vec{ON} \cdot \vec{AB} = 0$

$\lambda + \lambda + 1 + \lambda = 0 \Rightarrow \lambda = -1/3$

Hence $\vec{ON} = -\frac{1}{3}\hat{i} - \frac{1}{3}\hat{j} - \frac{2}{3}\hat{k}$; $|\vec{ON}| = \sqrt{\frac{1}{9} + \frac{1}{9} + \frac{4}{9}} = \sqrt{\frac{2}{3}}$]

Q.30$_{88/vec}$ Which one of the following does not hold for the vector $\vec{V} = \vec{a} \times (\vec{b} \times \vec{a})$?

(A) perpendicular to $\vec{a}$ (B*) perpendicular to $\vec{b}$
(C) coplanar with $\vec{a}$ & $\vec{b}$ (D) perpendicular to $\vec{a} \times \vec{b}$.

Q.31$_{53/complex}$ Let z_1, z_2 & z_3 be the complex numbers representing the vertices of a triangle ABC respectively. If P is a point representing the complex number z_0 satisfying ;
$a(z_1 - z_0) + b(z_2 - z_0) + c(z_3 - z_0) = 0$, then w.r.t. the triangle ABC, the point P is its :
(A) centroid (B) orthocentre (C) circumcentre (D*) incentre

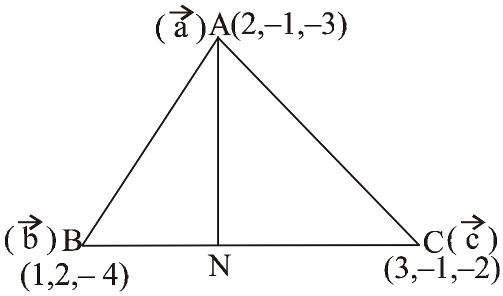
[Hint : $az_1 + bz_2 + cz_3 = z_0(a + b + c) \Rightarrow z_0 = \frac{az_1 + bz_2 + cz_3}{a + b + c} \Rightarrow z_0$ is the incentre]

Q.32$_{96/vec}$ Given the position vectors of the vertices of a triangle ABC, $A \equiv (\bar{a})$; $B \equiv (\bar{b})$; $C \equiv (\bar{c})$. A vector $\vec{r}$ is parallel to the altitude drawn from the vertex A, making an obtuse angle with the positive Y-axis. If $|\vec{r}| = 2\sqrt{34}$; $\vec{a} = 2\hat{i} - \hat{j} - 3\hat{k}$; $\vec{b} = \hat{i} + 2\hat{j} - 4\hat{k}$; $\vec{c} = 3\hat{i} - \hat{j} - 2\hat{k}$, then $\vec{r}$ is

(A*) $-6\hat{i} - 8\hat{j} - 6\hat{k}$ (B) $6\hat{i} - 8\hat{j} + 6\hat{k}$ (C) $-6\hat{i} - 8\hat{j} + 6\hat{k}$ (D) $6\hat{i} + 8\hat{j} + 6\hat{k}$

[Sol. $|\vec{r}| = 2\sqrt{34}$

Equation of line BC: $\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\underbrace{2i - 3j + 2k}_{\overrightarrow{BC}})$



p.v. of N is $2\lambda + 1$, $2 - 3\lambda$, $2\lambda - 4$

vector $\overrightarrow{AN} = (2\lambda - 1)\hat{i} + (3 - 3\lambda)\hat{j} + (2\lambda - 1)\hat{k}$

now $\overrightarrow{AN} \cdot \overrightarrow{BC} = 0$

$2(2\lambda - 1) - 3(3 - 3\lambda) + 2(2\lambda - 1)$

$(4\lambda + 9\lambda + 4\lambda) = 2 + 9 + 2 = 13 \Rightarrow \lambda = 13/17$

$\overrightarrow{AN} = \dfrac{9\hat{i} + 12\hat{j} + 9\hat{k}}{17}$; $\left|\overrightarrow{AN}\right| = \dfrac{\sqrt{306}}{17} = \dfrac{3\sqrt{34}}{17}$

$\vec{r} = P(\overrightarrow{AN}) \Rightarrow |\vec{r}| = |P| . |\overrightarrow{AN}|$ hence $2\sqrt{34} = |P| \dfrac{3\sqrt{34}}{17}$

$|P| = \dfrac{34}{3} \Rightarrow P = \dfrac{34}{3}$ or $-\dfrac{34}{3}$

$\vec{r} = \pm\dfrac{34}{3}\left(\dfrac{9\hat{i} + 12\hat{j} + 9\hat{k}}{17}\right) = \pm 2(3\hat{i} + 4\hat{j} + 3\hat{k})$

$\therefore$ angle with y axis is –ve $\Rightarrow$ +ve sign to be rejected

$\vec{r} = -6\hat{i} - 8\hat{j} - 6\hat{k} \Rightarrow$ (A)]

Q.33$_{72/complex}$ The complex number, $z = 5 + i$ has an argument which is nearly equal to :

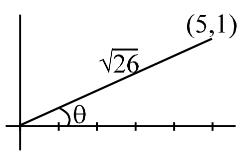
(A) $\pi/32$ (B*) $\pi/16$ (C) $\pi/12$ (D) $\pi/8$

[Hint : $z = 5 + i$

$5 + i = \sqrt{26}(\cos\theta + i\sin\theta)$

$+24 + 10i = 26(\cos 2\theta + i \sin 2\theta)$

$+476 + 480i = 676(\cos 4\theta + i \sin 4\theta)$



$\Rightarrow \left.\begin{matrix} 676 \sin 4\theta = 476 \\ \text{and } 676 \cos 4\theta = 480 \end{matrix}\right] \Rightarrow \tan 4\theta = \dfrac{476}{480} \simeq 1$

$4\theta = \dfrac{\pi}{4} \Rightarrow \theta = \dfrac{\pi}{16}$]

Q.34$_{97/vec}$ If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, $\vec{c} = \hat{i} + 2\hat{j} - \hat{k}$, then the value of $\begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix}$ equal to

(A) 2 (B) 4 (C*) 16 (D) 64

[Hint : $\left[\vec{a}\,\vec{b}\,\vec{c}\right]^2 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 2 & -1 \end{vmatrix}^2 = 16$]

Q.35$_{96/complex}$ If the equation $x^2 + ax + b = 0$ where a, b $\in$ R has a non real root whose cube is 343 then (7a + b) has the value

(A*) 98 (B) – 49 (C) – 98 (D) 49

[Sol. the cube root of 343 are the roots of $x^3 - 343 = 0$

$$(x - 7)(x^2 + 7x + 49) = 0$$

where a = 7 and b = 49 $\Rightarrow$ 7a + b = 98 Ans.]

Direction for Q.36 to Q.40.

Let $\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{B} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

Q.36$_{3(i)/vec}$ The value of the scalar $\sqrt{|\vec{A}\times\vec{B}|^2 + (\vec{A}\cdot\vec{B})^2}$ is equal to

(A) 8 (B) $7\sqrt{10}$ (C*) $10\sqrt{7}$ (D) 64

[Sol. $|\vec{a}|^2|\vec{b}|^2 = 50 \cdot 14 = 700 = 10\sqrt{7}$ Ans]

Q.37$_{3(ii)/vec}$ Equation of a line passing through the point with position vector $2\hat{i} + 3\hat{j}$ and orthogonal to the plane containing the vectors $\vec{A}$ and $\vec{B}$, is

(A*) $\vec{r} = (\lambda + 2)\hat{i} - (2\lambda - 3)\hat{j} + \lambda\hat{k}$ (B) $\vec{r} = (\lambda - 2)\hat{i} - (2\lambda - 3)\hat{j} + \lambda\hat{k}$

(C) $\vec{r} = \lambda\hat{i} + (2\lambda - 3)\hat{j} - \lambda\hat{k}$ (D) none

[Sol. $$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 3 & 4 & 5 \end{vmatrix} = (10-12)\hat{i} - (5-9)\hat{j} + (4-6)\hat{k} = -2\hat{i} + 4\hat{j} - 2\hat{k} = -2(\hat{i} - 2\hat{j} + \hat{k})$$

Here $\vec{r} = 2\hat{i} + 3\hat{j} + \lambda(\hat{i} - 2\hat{j} + \hat{k}) = \vec{r} = (2+\lambda)\hat{i} + (3 - 2\lambda)\hat{j} + \lambda\hat{k}$ Ans.]

Q.38$_{3(iii)/vec}$ Equation of a plane containing the point with position vector $(\hat{i} - \hat{j} + \hat{k})$ and parallel to the vectors $\vec{A}$ and $\vec{B}$, is

(A) x + 2y + z = 0 (B) x – 2y – z – 2 = 0

(C*) x – 2y + z – 4 = 0 (D) 2x + y + z – 1 = 0

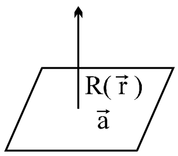
[Sol. $\vec{n} = \hat{i} - 2\hat{j} + \hat{k}$

$\vec{a} = \hat{i} - \hat{j} + \hat{k}$

$(\vec{r} - \vec{a})\cdot\vec{n} = 0$

$\vec{r}\cdot(\hat{i} - 2\hat{j} + \hat{k}) = \vec{a}\cdot\vec{n} = 4$

x – 2y + z = 4]



Q.39$_{3(iv)/vec}$ Volume of the tetrahedron whose 3 coterminous edges are the vectors $\vec{A}$, $\vec{B}$ and $\vec{C} = 2\hat{i} + \hat{j} - 4\hat{k}$ is

(A) 1 (B*) 4/3 (C) 8/3 (D) 8

[Sol. $$\frac{1}{6}[\vec{a}\,\vec{b}\,\vec{c}] = \frac{1}{6}\begin{vmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 2 & 1 & -4 \end{vmatrix}$$

$$= \frac{1}{6}[1(-16-5) - 2(-12-10) + 3(3-8)] = \frac{1}{6}[-21 + 44 - 15] = \frac{8}{6} = \frac{4}{3}]$$

Q.40$_{3(v)/vec}$ Vector component of $\vec{A}$ perpendicular to the vector $\vec{B}$ is given by

(A*) $\frac{\vec{B}\times(\vec{A}\times\vec{B})}{\vec{B}^2}$ (B) $\frac{\vec{A}\times(\vec{A}\times\vec{B})}{\vec{B}^2}$ (C) $\frac{\vec{B}\times(\vec{A}\times\vec{B})}{\vec{A}^2}$ (D) $\frac{\vec{A}\times(\vec{A}\times\vec{B})}{\vec{A}^2}$

[Sol. $\vec{x} = \vec{A} - \left(\frac{\vec{A}\cdot\vec{B}}{\vec{B}^2}\right)\vec{B}$ $\Rightarrow$ (A)]

__Select the correct alternatives__ : (More than one are correct)

Q.41$_{501/vec}$ If a, b, c are different real numbers and $a\hat{i} + b\hat{j} + c\hat{k}$; $b\hat{i} + c\hat{j} + a\hat{k}$ & $c\hat{i} + a\hat{j} + b\hat{k}$ are position vectors of three non-collinear points A, B & C then :

(A*) centroid of triangle ABC is $\frac{a+b+c}{3}\left(\hat{i} + \hat{j} + \hat{k}\right)$

(B*) $\hat{i} + \hat{j} + \hat{k}$ is equally inclined to the three vectors

(C*) perpendicular from the origin to the plane of triangle ABC meet at centroid

(D*) triangle ABC is an equilateral triangle.

Q.42$_{504/vec}$ The vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ are of the same length & pairwise form equal angles. If $\vec{a} = \hat{i}+\hat{j}$ & $\vec{b} = \hat{j}+\hat{k}$, the pv's of $\vec{c}$ can be :

(A*) (1, 0, 1) (B) $\left(-\frac{4}{3}, \frac{1}{3}, -\frac{4}{3}\right)$ (C) $\left(\frac{1}{3}, -\frac{4}{3}, \frac{1}{3}\right)$ (D*) $\left(-\frac{1}{3}, \frac{4}{3}, -\frac{1}{3}\right)$

[Hint : Let $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$ $x^2 + y^2 + z^2 = 2$ — (1)

now $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a}$ $\Rightarrow$ I = y + z = x + y — (2)

$\therefore$ z = x y = 1 – x

put z and y in terms of x in (1) to get x and then get y and z]

Q.43$_{512/complex}$ Which of the following locii of z on the complex plane represents a pair of straight lines?

(A*) Re $z^2 = 0$ (B*) Im $z^2 = 0$ (C) $|z| + z = 0$ (D) $|z-1| = |z-i|$

[Hint : C $\Rightarrow$ negative real axis ;

D $\Rightarrow$ perpendicular bisector of the line joining (0, 1) & (1, 0)]

Q.44$_{506/vec}$ If $\vec{a}$, $\vec{b}$, $\vec{c}$ & $\vec{d}$ are linearly independent set of vectors & $K_1\vec{a} + K_2\vec{b} + K_3\vec{c} + K_4\vec{d} = 0$ then :

(A*) $K_1 + K_2 + K_3 + K_4 = 0$ (B*) $K_1 + K_3 = K_2 + K_4 = 0$

(C*) $K_1 + K_4 = K_2 + K_3 = 0$ (D) none of these

[Hint : $k_1\vec{a} + k_2\vec{b} + k_3\vec{c} + k_4\vec{d} = 0$ $\vec{a}$, $\vec{b}$, $\vec{c}$, $\vec{d}$ are linearly independent

$\therefore$ $k_1 = k_2 = k_3 = k_4 = 0$]

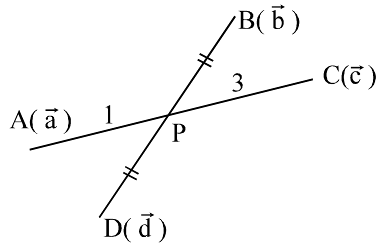
Q.45$_{507/vec}$ If $\vec{a}$, $\vec{b}$, $\vec{c}$ & $\vec{d}$ are the pv's of the points A, B, C & D respectively in three dimensional space & satisfy the relation $3\vec{a} - 2\vec{b} + \vec{c} - 2\vec{d} = 0$, then :

(A*) A, B, C & D are coplanar

(B) the line joining the points B & D divides the line joining the point A & C in the ratio 2 : 1.

(C*) the line joining the points A & C divides the line joining the points B & D in the ratio 1 : 1

(D*) the four vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ & $\vec{d}$ are linearly dependent.



[Hint : $\dfrac{3\vec{a}+\vec{c}}{4} = \dfrac{2\vec{b}+2\vec{d}}{4} = \dfrac{\vec{b}+\vec{d}}{2}$

Hence line joining A & C intersect line joining B & C]

Q.46$_{519/complex}$ If $z^3 - iz^2 - 2iz - 2 = 0$ then z can be equal to :

(A) $1 - i$ (B*) i (C*) $1 + i$ (D*) $-1 - i$

[Hint : $(z - i)(z^2 - 2i) = 0 \Rightarrow z = i$ or $z^2 = 2i = 2e^{i\pi/2} \Rightarrow z = 1 + i$ or $-1 - i$]

Q.47$_{509/vec}$ If $\vec{a}$ & $\vec{b}$ are two non collinear unit vectors & $\vec{a}$, $\vec{b}$, $x\vec{a} - y\vec{b}$ form a triangle, then :

(A*) $x = -1$; $y = 1$ & $|\vec{a} + \vec{b}| = 2\cos\left(\dfrac{\vec{a}\,\hat{}\,\vec{b}}{2}\right)$

(B*) $x = -1$; $y = 1$ & $\cos\left(\vec{a}\,\hat{}\,\vec{b}\right) + |\vec{a} + \vec{b}|\cos\left[\vec{a}, \hat{-}\left(\vec{a} + \vec{b}\right)\right] = -1$

(C) $|\vec{a} + \vec{b}| = -2\cot\left(\dfrac{\vec{a}\,\hat{}\,\vec{b}}{2}\right)\cos\left(\dfrac{\vec{a}\,\hat{}\,\vec{b}}{2}\right)$ & $x = -1$, $y = 1$ (D) none

[Hint : $\hat{a}$, $\hat{b}$ & $x\hat{a} - y\hat{b}$ form a triangle hence , $\hat{a} + \hat{b} + x\hat{a} - y\hat{b} = 0$

$\Rightarrow (x + 1)\hat{a} + (1 - y)\hat{b} = 0$ Since $\hat{a}$ & $\hat{b}$ are collinear $\Rightarrow x = -1$ & $y = 1$

Also $\left|\hat{a} + \hat{b}\right|^2 = 2 + 2\hat{a}\,.\,\hat{b} = 2(1 + \cos\theta)$. $\left|\hat{a} + \hat{b}\right| = 2\cos\dfrac{\theta}{2} = 2\cos\left(\dfrac{\vec{a}\,\hat{}\,\vec{b}}{2}\right) \Rightarrow$ A

Also $\cos\phi = -\dfrac{\hat{a}.(\hat{a}+\hat{b})}{|\hat{a}+\hat{b}|} = -\dfrac{1 + \cos\theta}{|\hat{a} + \hat{b}|}$ (where ϕ is the angle between $-\hat{a}$ & $\hat{a} + \hat{b}$)

$\therefore \left|\hat{a} + \hat{b}\right|\cos\phi = -(1 + \cos\theta) \Rightarrow \left|\hat{a} + \hat{b}\right|\cos\phi + \cos\theta = -1$]

Q.48$_{510/vec}$ The lines with vector equations are ; $\vec{r}_1 = -3\hat{i} + 6\hat{j} + \lambda\left(-4\hat{i} + 3\tilde{j} + 2\hat{k}\right)$ and

$\vec{r}_2 = -2\hat{i} + 7\hat{j} + \mu\left(-4\hat{i} + \tilde{j} + \hat{k}\right)$ are such that :

(A) they are coplanar (B*) they do not intersect

(C*) they are skew (D*) the angle between them is $\tan^{-1}(3/7)$

Q.49$_{523/complex}$ Given a, b, x, y $\in$ R then which of the following statement(s) hold good?

(A*) $(a + ib)(x + iy)^{-1} = a - ib \Rightarrow x^2 + y^2 = 1$

(B*) $(1 - ix)(1 + ix)^{-1} = a - ib \Rightarrow a^2 + b^2 = 1$

(C*) $(a + ib)(a - ib)^{-1} = x - iy \Rightarrow |x + iy| = 1$

(D*) $(y - ix)(a + ib)^{-1} = y + ix \Rightarrow |a - ib| = 1$

[Hint : Modulus of a complex number, which is the ratio of two conjugates is unity.

e.g. in A, $\dfrac{a+ib}{a-ib} = x + iy \quad \Rightarrow \quad \left|\dfrac{a+ib}{a-ib}\right| = |x+iy| \Rightarrow x^2 + y^2 = 1$]

Q.50$_{514/vec}$ The acute angle that the vector $2\hat{i} - 2\hat{j} + \hat{k}$ makes with the plane contained by the two vectors $2\hat{i} + 3\hat{j} - \hat{k}$ and $\hat{i} - \hat{j} + 2\hat{k}$ is given by :

(A) $\cos^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$ (B*) $\sin^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$ (C) $\tan^{-1}\left(\sqrt{2}\right)$ (D*) $\cot^{-1}\left(\sqrt{2}\right)$

[Hint : $\vec{n}_1 = \vec{a} \times \vec{b} = \left(2\hat{i} + 3\hat{j} - \hat{k}\right) \times \left(\hat{i} - \hat{j} + 2\hat{k}\right) = 5\left(\hat{i} - \hat{j} + \hat{k}\right)$

$\vec{n}_1 = \dfrac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}} \quad \vec{v} = 2\hat{i} - 2\hat{j} + \hat{k} \Rightarrow \vec{v} = \dfrac{2\hat{i} - 2\hat{j} + \hat{k}}{3}$

$\cos\left(\dfrac{\pi}{2} - \theta\right) = \sin\theta = \hat{v}\,.\,\hat{n} = \dfrac{1}{\sqrt{3}}$]

Q.51$_{518/vec}$ The volume of a right triangular prism $ABCA_1B_1C_1$ is equal to 3. If the position vectors of the vertices of the base ABC are A(1, 0, 1) ; B(2,0, 0) and C(0, 1, 0) the position vectors of the vertex A_1 can be:

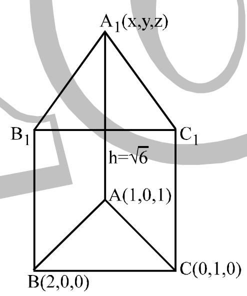
(A*) (2, 2, 2) (B) (0, 2, 0) (C) (0, –2, 2) (D*) (0, –2, 0)

[Hint : knowing the volume of the prism we find its altitude $H = (AA_1) = \sqrt{6}$ and designating the vertex $A_1(x_1, y_1, z_1)$ relate the co-ordinates of the vector

$\vec{AA_1} = (x-1, y, z-1)$ and its length. We get the other equation from the contition

$\vec{AA_1}$ perpendicular to $\vec{AC}$] **OR**

compute $\pm\dfrac{\vec{AB} \times \vec{AC}}{\left|\vec{AB} \times \vec{AC}\right|} = \hat{n}$



$\therefore \sqrt{6}\,\hat{n} = AA_1 = \pm\left(\hat{i} + 2\hat{j} + \hat{k}\right)$

$= (x_1 - 1)\hat{i} + (y_1 - 1)\hat{j} + (z_1 - 1)\hat{k}$

Compare to get at the possible coordinates of A]

Q.52$_{528/complex}$ If $x_r = \text{CiS}\left(\dfrac{\pi}{2^r}\right)$ for $1 \le r \le n \quad r, n \in N$ then :

(A*) $\underset{n\to\infty}{\text{Limit}}\ \text{Re}\left(\prod_{r=1}^{n} x_r\right) = -1$ (B) $\underset{n\to\infty}{\text{Limit}}\ \text{Re}\left(\prod_{r=1}^{n} x_r\right) = 0$

(C) $\underset{n\to\infty}{\text{Limit}}\ \text{Im}\left(\prod_{r=1}^{n} x_r\right) = 1$ (D*) $\underset{n\to\infty}{\text{Limit}}\ \text{Im}\left(\prod_{r=1}^{n} x_r\right) = 0$

Q.53$_{524/vec}$ If a line has a vector equation, $\vec{r} = 2\hat{i} + 6\hat{j} + \lambda\left(\hat{i} - 3\hat{j}\right)$ then which of the following statements holds good ?

(A) the line is parallel to $2\hat{i} + 6\hat{j}$ (B*) the line passes through the point $3\hat{i} + 3\hat{j}$

(C*) the line passes through the point $\hat{i} + 9\hat{j}$ (D*) the line is parallel to xy plane

[Hint : Line is parallel to $\hat{i} - 3\hat{j} \Rightarrow$ D

Also put $\vec{r}_1 = 3\hat{i} + 3\hat{j}$ for which $\lambda = 1$ and

$\vec{r}_1 = \hat{i} + 9\hat{j}$ for which $\lambda = -1$ $\Rightarrow$ B & C]

Q.54$_{525/vec}$ If $\vec{a}$, $\vec{b}$, $\vec{c}$ are non-zero, non-collinear vectors such that a vector

$\vec{p} = a\,b\cos\left(2\pi - \left(\vec{a} \wedge \vec{b}\right)\right)\vec{c}$ and a vector $\vec{q} = a\,c\cos\left(\pi - \left(\vec{a} \wedge \vec{c}\right)\right)\vec{b}$ then $\vec{p} + \vec{q}$ is

(A) parallel to $\vec{a}$ (B*) perpendicular to $\vec{a}$

(C*) coplanar with $\vec{b}$ & $\vec{c}$ (D) coplanar with $\vec{a}$ and $\vec{c}$

[Sol. $\vec{p} = a\,b\cos(2\pi - \theta)\vec{c}$ where θ is the angle between $\vec{a}$ and $\vec{b}$ and

$\vec{q} = a\,c\cos(\pi - \phi)\vec{b}$ where ϕ is the angle between $\vec{a}$ and $\vec{c}$

now $\vec{p} + \vec{q} = (a\,b\cos\theta)\vec{c} - a\,c\cos\phi\,\vec{b} = (\vec{a}.\vec{b})\vec{c} - (\vec{a}.\vec{c})\vec{b} = \vec{a} \times (\vec{c} \times \vec{b}) \Rightarrow$ B and C]

Q.55$_{539/complex}$ The greatest value of the modulus of of the complex number 'z' satisfying the equality $\left|z + \frac{1}{z}\right| = 1$ is

(A) $\frac{-1+\sqrt{5}}{2}$ (B*) $\sqrt{\frac{3+\sqrt{5}}{2}}$ (C) $\sqrt{\frac{3-\sqrt{5}}{2}}$ (D*) $\frac{\sqrt{5}+1}{2}$

SUBJECTIVE:

Q.1$_{90/5}$ Let $\vec{a} = \sqrt{3}\,\hat{i} - \hat{j}$ and $\vec{b} = \frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}$ and $\vec{x} = \vec{a} + (q^2 - 3)\vec{b}$, $\vec{y} = -p\,\vec{a} + q\vec{b}$. If $\vec{x} \perp \vec{y}$, then express *p* as a function of *q*, say *p* = *f* (*q*), (*p* ≠ 0 & *q* ≠ 0) and find the intervals of monotonicity of *f* (q).

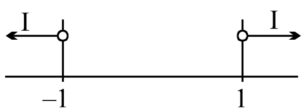
[Sol. $\vec{x} = \left(\sqrt{3}\,\hat{i} - \hat{j}\right) + (q^2 - 3)\left(\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right) = \left(\sqrt{3} + \frac{q^2 - 3}{2}\right)\hat{i} - \left(1 - \frac{\sqrt{3}}{2}(q^2 - 3)\right)\hat{j}$

$\vec{y} = -p\left(3\,\hat{i} - \hat{j}\right) + q\left(\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right)$

$\vec{x} \cdot \vec{y} = 0$ gives

$p = \frac{q(q^3 - 3)}{4}$ **Ans.**

$\frac{dp}{dq} = \frac{1}{4}[3q^2 - 3] > 0$



$q^2 - 1 > 0$

$q > 1$ or $q < -1$
and decreasing in $q \in (-1, 1)$, $q \neq 0$ **Ans.**]

Q.$2_{25/3}$ Using only the limit theorems $\lim\limits_{x \to 1} \frac{\ln x}{x-1} = 1$ and $\lim\limits_{x \to 0} \frac{e^x - 1}{x} = 1$. Evaluate $\lim\limits_{x \to 1} \frac{x^x - x}{\ln x - x + 1}$.

[Ans. – 2]

[Sol. $\lim\limits_{x \to 1} \frac{x^x - x}{\ln x - x + 1}$

$$l = \lim_{x \to 1} \frac{e^{x \ln x} - e^{\ln x}}{\ln x - x + 1} = \lim_{x \to 1} e^{\ln x} \cdot \frac{[e^{x \ln x - \ln x} - 1]}{(x \ln x - \ln x)} \cdot \frac{x \ln x - \ln x}{\ln x - x + 1}$$

$$= (1)\,(1) \cdot \lim_{x \to 1} \frac{\ln x (x-1)(x-1)}{(x-1)(\ln x - x + 1)} = (1)\,(1)\,(1) \cdot \lim_{x \to 1} \frac{(x-1)^2}{\ln x - x + 1}$$

put $x = 1 + h$, as $x \to 1$, $h \to 0$

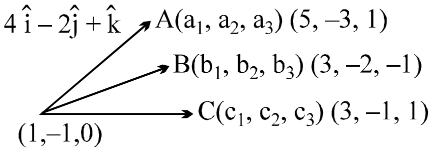
$$= \lim_{h \to 0} \frac{h^2}{\ln(1+h) - h}$$

put $\ln(1 + h) = y \quad \Rightarrow \quad 1 + h = e^y$

$$= \lim_{y \to 0} \frac{(e^y - 1)^2}{y - (e^y - 1)} = \lim_{y \to 0} \left(\frac{e^y - 1}{y}\right)^2 \cdot \lim_{y \to 0} \frac{y^2}{y - e^y + 1} = -(1) \lim_{y \to 0} \frac{y^2}{y - e^y - 1}$$

$l = -2$ and $\lim\limits_{y \to 0} \frac{e^y - y - 1}{y^2} = \frac{1}{2}$ Ans.]

Q.$3_{92/5}$ The three vectors $\vec{a} = 4\hat{i} - 2\hat{j} + \hat{k}$; $\vec{b} = 2\hat{i} - \hat{j} - \hat{k}$ and $\vec{c} = 2\hat{i} + \hat{k}$ are all drawn from the point with p.v. $\hat{i} - \hat{j}$. Find the equation of the plane containing their end point in scalar dot product form.



[Ans. $(2\hat{i} + 2\hat{j} - \hat{k}) \cdot \vec{r} = 3$]

Q.$4_{222/3}$ $\displaystyle\int_{-\pi/2}^{\pi/2} \sqrt{(\cos^{2n-1} x - \cos^{2n+1} x)}\; dx$ where $n \in N$

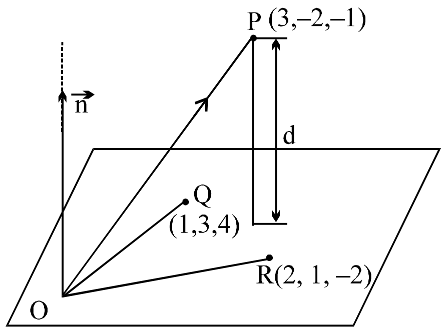
[Sol. $I = 2 \displaystyle\int_0^{\pi/2} \sqrt{(\cos x)^{2n-1} (1 - \cos^2 x)}\; dx$ as f is even

$$= 2\int_0^{\frac{\pi}{2}} (\cos x)^{\frac{2n-1}{2}} . \sin x \ dx = 2\int_0^1 t^{\frac{2n-1}{2}} . dt \quad \text{when } \cos x = t = \frac{2 . 2}{2n+1}\left[t^{\frac{2n+1}{2}}\right]_0^1 = \frac{4}{2n+1} \ ]$$

Q.5$_{97/5}$ Let points P, Q & R have position vectors, $\vec{r}_1 = 3i - 2j - k$; $\vec{r}_2 = i + 3j + 4k$ & $\vec{r}_3 = 2i+j-2k$ respectively, relative to an origin O. Find the distance of P from the plane OQR.

[Ans : 3 units]

[Sol. $\vec{n} = \vec{r}_2 \times \vec{r}_3 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 4 \\ 2 & 1 & -2 \end{vmatrix}$



$\hat{i}(-6-4) - \hat{j}(-2-8) + \hat{k}(1-6)$

$= -10\hat{i} + 10\hat{j} - 5\hat{k}$

$\hat{n} = \dfrac{2\hat{i} - 2\hat{j} + \hat{k}}{3}$

$\therefore \ d = \left|\text{Projection of } \overrightarrow{OP} \text{ on } \vec{n}\right| = \left|\dfrac{(3\hat{i} - 2\hat{j} + \hat{k})\cdot(2\hat{i} - 2\hat{j} + \hat{k})}{3}\right| = \dfrac{6+4-1}{3} = 3$ units]

Q.6$_{228/3}$ Evaluate: $\displaystyle\int_1^3 \left|(x-1)(x-2)(x-3)\right| dx$ [Ans. 1/2]

[Sol. $I = \displaystyle\int_1^3 \left|(x-1)(3-x)(x-2)\right| dx$

let $x = \cos^2\theta + 3\sin^2\theta$

$dx = 2\sin 2\theta \, d\theta$

$x - 1 = 2\sin^2\theta$; $3 - x = 2\cos^2\theta$ and $x - 2 = \cos^2\theta + 3\sin^2\theta - 2 = 2\sin^2\theta - 1 = -\cos 2\theta$

$$I = \int_0^{\pi/2} \left|2\sin\theta \cdot 2\cos^2\theta \cdot \cos 2\theta\right| 2\sin 2\theta \, d\theta = \int_0^{\pi/2} 4\sin^2\theta \cdot \cos^2\theta \cdot 2\sin 2\theta \left|\cos 2\theta\right| d\theta$$

$$= \int_0^{\pi/2} 2\sin^3 2\theta \left|\cos 2\theta\right| d\theta$$

put $2\theta = t$

$$I = \int_0^{\pi} 2\sin^3 t \left|\cos t\right| \frac{dt}{2} = 2\int_0^{\pi/2} (\sin^3 t \cdot \cos t) \, dt$$

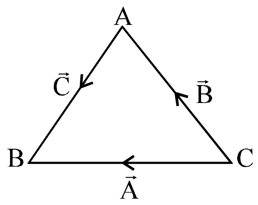
put $\sin t = y$

$$I = 2\int_0^1 y^3\,dy = 2\cdot\left.\frac{y^4}{4}\right|_0^1 = \frac{1}{2} \text{ Ans.]}$$

Q.7$_{98/5}$ Given that vectors $\vec{A}, \vec{B}, \vec{C}$ form a triangle such that $\vec{A} = \vec{B} + \vec{C}$, find a,b,c,d such that the area of the triangle is $5\sqrt{6}$ where $\vec{A} = ai + bj + ck$; $\vec{B} = di + 3j + 4k$ & $\vec{C} = 3i + j - 2k$.

[Ans: (–8, 4, 2, –11) or (8, 4, 2, 5)] [REE '90, 6]

[Sol. $\vec{A} = \vec{B} + \vec{C}$



$a\hat{i} + b\hat{j} + c\hat{k} = (d+3)\hat{i} + (3+1)\hat{j} + (4-2)\hat{k}$

$= (d+3)\hat{i} + 4\hat{j} + 2\hat{k}$

Hence d + 3 = a; b = 4 and c = 2

again $|\vec{B} \times \vec{C}| = 5\sqrt{6}$

$|\vec{B}|^2\,|\vec{C}|^2 - (\vec{B}\cdot\vec{C})^2 = 150$

$(25 + d^2)14 - (3d + 3 - 8)^2 = 150$

$14(25 + d^2) - (3d - 5)^2 = 150$ now proceed to get two values of d]

Q.8$_{246/3}$ $\displaystyle \lim_{n\to\infty} n\sum_{k=0}^{n-1} \int_{\frac{k}{n}}^{\frac{k+1}{n}} \sqrt{\left(x - \frac{k}{n}\right)\left(\frac{k+1}{n} - x\right)}\,dx$ [Ans. $\frac{\pi}{8}$]

[Sol. Let $\displaystyle\int_\alpha^\beta \sqrt{(x-\alpha)(\beta-x)}\,dx$ where $\alpha = \frac{k}{n}$; $\beta = \frac{k+1}{n}$

$x = \alpha\cos^2\alpha + \beta\sin^2\theta$

$dx = (\beta - \alpha)2\sin\theta\cos\theta$

$x - \alpha = (\beta - \alpha)\sin^2\theta$

$$I = 2(\beta-\alpha)^2 \int_0^{\pi/2} (\sin^2\theta\cos^2\theta)\,d\theta = \frac{(\beta-\alpha)^2}{2}\int_0^{\pi/2}\sin^2 2\theta\,d\theta$$

put $2\theta = t$

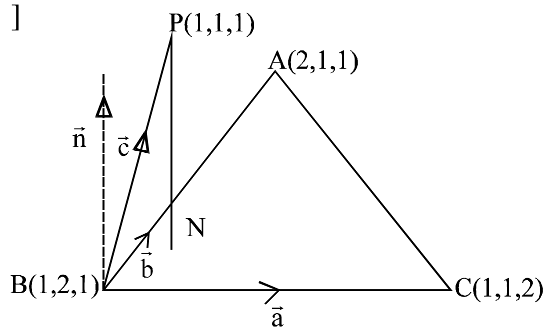
$$I = \frac{(\beta-\alpha)^2}{4}\int_0^{\pi}\sin^2 t\,dt = \frac{(\beta-\alpha)^2}{4}\cdot 2\cdot\int_0^{\pi/2}\sin^2 t\,dt$$

$$= \frac{(\beta-\alpha)^2}{8}\pi = \frac{\pi}{8}(\beta-\alpha)^2 = \frac{\pi}{8}\cdot\frac{1}{n^2}$$ which is independent of k.

$$\therefore \quad l = \lim_{n\to\infty} n\cdot\sum_{k=0}^{n-1}\frac{\pi}{8}\cdot\frac{1}{n^2} = \lim_{n\to\infty}\frac{1}{n}\frac{\pi}{8}\sum_{k=0}^{n-1}(1) = \lim_{n\to\infty}\frac{\pi}{8n}\cdot n = \frac{\pi}{8} \text{ Ans.]}$$

Q.9$_{114/5}$ Find the distance of the point $P(i + j + k)$ from the plane L which passes through the three points $A(2i + j + k)$, $B(i + 2j + k)$, $C(i + j + 2k)$. Also find the pv of the foot of the perpendicular from P on the plane L.

$$\left[\text{Ans : } \frac{1}{\sqrt{3}}, \left(\frac{4}{3},\frac{4}{3},\frac{4}{3}\right)\ \right]$$



[Sol.

$\vec{a} = 0\hat{i} - \hat{j} + \hat{k}$

$\vec{b} = \hat{i} - \hat{j} + 0\hat{k}$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -1 & 1 \\ 1 & -1 & 0 \end{vmatrix}$$

$= \hat{i}(1) - \hat{j}(-1) + \hat{k}(1)$

$\vec{a}\times\vec{b} = \hat{i} + \hat{j} + \hat{k} = \vec{n}$ (say)

$\overrightarrow{BP} = 0\hat{i} - \hat{j} + 0\hat{k} = \vec{c}$

$$\overrightarrow{PN} = \text{Projection of } \vec{c} \text{ on } \vec{n} = \left|\frac{\vec{c}.\vec{n}}{\vec{n}}\right| = \left|\frac{-1}{\sqrt{1+1+1}}\right| = \frac{1}{\sqrt{3}}$$

Now equation of a line through P and || $\vec{n}$ is $\vec{r} = \hat{i} + \hat{j} + \hat{k} + \lambda(\hat{i} + \hat{j} + \hat{k}) = 1 + \lambda\ [\hat{i} + \hat{j} + \hat{k}]$

Let the position vector of N = $(1+\lambda), (1+\lambda), (1+\lambda)$

$\overrightarrow{AN} = (\lambda - 1)\hat{i} + \lambda\hat{j} + \lambda\hat{k}$

Now $\vec{a}, \vec{b}$ and $\overrightarrow{AN}$ must be copalnar

$$\begin{vmatrix} 0 & -1 & 1 \\ 1 & -1 & 0 \\ \lambda - 1 & \lambda & \lambda \end{vmatrix} = 0$$

$1[\lambda] + 1[\lambda + \lambda - 1] = 0$

$3l = 1 \Rightarrow \lambda = 1/3$

$\therefore$ Position vector of N $\left(\frac{4}{3},\frac{4}{3},\frac{4}{3}\right)$]

Q.10 Evaluate: **(a)** $\int \frac{\sqrt{\sin^4 x + \cos^4 x}}{\sin^3 x \cos x}dx$, $x \in \left(0, \frac{\pi}{2}\right)$; **(b)** $\int \frac{\sqrt{\sin^4 x + \cos^4 x}}{\sin x \cos^3 x}dx$

[Sol.**(a)** $I = \int \frac{\sqrt{\sin^4 x + \cos^4 x}}{\sin^3 x \cos x} dx$, $\quad x \in \left(0, \frac{\pi}{4}\right)$ [

$$= \int \frac{\cos^2 x\sqrt{1+\tan^4 x}}{\sin^3 x \cos x} dx = \int \frac{\cos x\sqrt{1+\tan^4 x}}{\sin^3 x} dx = \int \frac{\sqrt{1+\cot^4 x}}{\cot^2 x} \cdot \cot x \cdot \operatorname{cosec}^2 x \, dx$$

put $\cot^2 x = t \Rightarrow 2\cot x \cdot \operatorname{cosec}^2 x \, dx = -dt$

$$I = -\frac{1}{2}\int \frac{\sqrt{1+t^2}}{t} dt$$

put $1 + t^2 = y^2 \Rightarrow t\,dt = y\,dy$

$$I = -\frac{1}{2}\int \frac{y \cdot y}{t^2} dy = -\frac{1}{2}\int \frac{y^2 - 1 + 1}{y^2 - 1} dy = -\frac{1}{2}\left(\int dy + \int \frac{dy}{y^2 - 1}\right) = C - \frac{y}{2} - \frac{1}{4} ln \frac{y-1}{y+1}$$

$$= C - \frac{\sqrt{1+t^2}}{2} - \frac{1}{4} ln \frac{\sqrt{t^2+1}-1}{\sqrt{t^2+1}+1} \quad \text{where } t = \cot^2 x \quad \textbf{Ans(a).}$$

(b) $$I = \int \frac{\sqrt{\sin^4 x + \cos^4 x}}{\sin x \cos^3 x} dx = \int \frac{\sin^2 x\sqrt{1+\cot^4 x}}{\sin x \cos^3 x} dx = \int \frac{\sqrt{1+\tan^4 x}}{\tan^2 x} \cdot \tan x \cdot \sec^2 x \, dx$$

put $\tan^2 x = t$

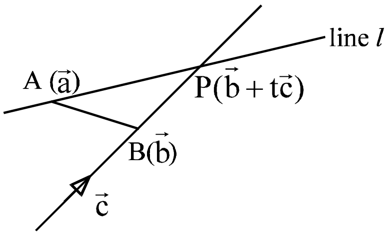
$$= \frac{1}{2}\int \frac{\sqrt{1+t^2}}{t} dt \quad \Rightarrow \quad \frac{\sqrt{1+t^2}}{2} + \frac{1}{4} ln \frac{\sqrt{t^2+1}-1}{\sqrt{t^2+1}+1} + C, \text{ where } t = \tan^2 x \quad \textbf{Ans(b).} \ ]$$

Q.11$_{115/5}$ Find the equation of the straight line which passes through the point with position vector $\vec{a}$, meets the line $\vec{r} = \vec{b} + t\vec{c}$ and is parallel to the plane $\vec{r}.\vec{n} = 1$.

[Sol. Suppose the required line intersects the given line at P with p.v.$(\vec{b} + t\vec{c})$. As the line l is | | to the plane $\vec{r}.\vec{n} = 1$. Hence $\overrightarrow{AP}.\vec{n} = 0$

$$\left[(\vec{b} - \vec{a}) + t\vec{c}\right] \cdot \vec{n} = 0 \Rightarrow t = \frac{(\vec{a} - \vec{b}).\vec{n}}{\vec{c}.\vec{n}}$$

Hence equation of the line is



$$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a} + t\vec{c})$$

$$\vec{r} = \vec{a} + \lambda\left[\vec{b} - \vec{a} + \frac{(\vec{a} - \vec{b}).\vec{n}}{\vec{c}.\vec{n}}\vec{c}\right]$$

$$\vec{r} = \vec{a} + \lambda\left((\vec{a} - \vec{b}) - \frac{(\vec{a} - \vec{b}).\vec{n}}{\vec{c}.\vec{n}}\vec{c}\right) \text{Ans }]$$

Q.12 Integrate: $\int \frac{dx}{\cos^3 x - \sin^3 x}$. [Ans. $2\left[\tan^{-1}(\sin x + \cos x) + \frac{1}{2\sqrt{2}} ln\left|\frac{\sqrt{2} + \sin x + \cos x}{\sqrt{2} - \sin x - \cos x}\right| + C\right]$

[Sol. $I = \int \frac{dx}{\cos^3 x - \sin^3 x} = \int \frac{dx}{(\cos x - \sin x)(1 + \sin x \cos x)} = 2\int \frac{(\cos x - \sin x)dx}{(\cos x - \sin x)^2(2 + \sin 2x)}$

$= 2\int \frac{(\cos x - \sin x)dx}{(1 - \sin 2x)(2 + \sin 2x)}$

$I = \int \frac{(\cos x - \sin x)dx}{\left(2 - (\sin x + \cos x)^2\right)\left(1 + (\sin x + \cos x)^2\right)}$

put $\sin x + \cos x = t \Rightarrow (\cos x - \sin x)\, dx = dt$

hence $I = \int \frac{dt}{(2 - t^2)(1 + t^2)} = \int \frac{(2 - t^2) + (1 + t^2)}{(2 - t^2)(1 + t^2)} dt = \int \frac{dt}{1 + t^2} + \int \frac{dt}{2 - t^2}$

$= \tan^{-1}(t) + \frac{1}{2\sqrt{2}}\, ln \frac{\sqrt{2} + t}{\sqrt{2} - t} + C$

$= 2\, [\tan^{-1}(\sin x + \cos x) + \frac{1}{2\sqrt{2}}\, ln \left| \frac{\sqrt{2} + \sin x + \cos x}{\sqrt{2} - \sin x - \cos x} \right| + C$ **Ans.**]

Q.13$_{147/5}$ Find the equation of the line passing through the point (1, 4, 3) which is perpendicular to both of the lines

$\frac{x-1}{2} = \frac{y+3}{1} = \frac{z-2}{4}$ and $\frac{x+2}{3} = \frac{y-4}{2} = \frac{z+1}{-2}$

Also find all points on this line the square of whose distance from (1, 4, 3) is 357.

[Ans. $\frac{x-1}{-10} = \frac{y-4}{16} = \frac{z-3}{1}$, ; (–9, 20, 4) ; (11, –12, 2)]

[Sol. Equation of the line passing through (1, 4, 3)

$\frac{x-1}{a} = \frac{y-4}{b} = \frac{z-3}{c}$ ....(1)

since (1) is perpendicular to $\frac{x-1}{2} = \frac{y+3}{1} = \frac{z-2}{4}$ and $\frac{x+2}{3} = \frac{y-4}{2} = \frac{z+1}{-2}$

hence $2a + b + 4c = 0$
and $3a + 2b - 2c = 0$

$\therefore \frac{a}{-2-8} = \frac{b}{12+4} = \frac{c}{4-3} \Rightarrow \frac{a}{-10} = \frac{b}{16} = \frac{c}{1}$

hence the equation of the lines is $\frac{x-1}{-10} = \frac{y-4}{16} = \frac{z-3}{1}$ **....(2)** Ans.

now any point P on (2) can be taken as

$1 - 10\lambda$; $16\lambda + 4$; $\lambda + 3$

distance of P from Q (1, 4, 3)

$(10\lambda)^2 + (16\lambda)^2 + \lambda^2 = 357$

$(100 + 256 + 1)\lambda^2 = 357$

$\lambda = 1$ or -1 Hence Q is $(-9, 20, 4)$ or $(11, -12, 2)$ Ans.]

Q.14 $\displaystyle \lim_{n\to\infty}\left(\frac{\sqrt{n^2+n}-1}{n}\right)^{2\sqrt{n^2+n}-1}$ [Ans. e^{-1}]

[Sol. $L = e^{\lim_{n\to\infty} 2\sqrt{n^2+n}-1\left(\frac{\sqrt{n^2+n}-1}{n}-1\right)} = e^{l}$, where $\displaystyle l = \lim_{n\to\infty}\left(2\sqrt{n^2+n}+1\right)\left(\frac{\sqrt{n^2+n}-(1+n)}{n}\right)$

$$= \lim_{n\to\infty}\frac{n\left[2\sqrt{1+\frac{1}{n}}+\frac{1}{n}\right]}{n}\cdot\lim_{n\to\infty}\left(\sqrt{n^2+n}-(n+1)\right)$$

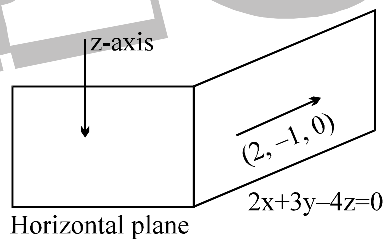
$$= 2\cdot\lim_{n\to\infty}\left(\frac{(n^2+n)-(n+1)^2}{\sqrt{n^2+n}+(n+1)}\right)\text{(rationalisation)} = 2\cdot\lim_{n\to\infty}\frac{n^2+n-n^2-2n-1}{\sqrt{n^2+n}+n+1}$$

$$= 2\cdot\lim_{n\to\infty}\frac{-(n+1)}{n\left[\left(\left(1+\frac{1}{n}\right)+1+\frac{1}{n}\right)\right]} = 2\cdot\lim_{n\to\infty}\frac{-n\left(1+\frac{1}{n}\right)}{n\left[\left(1+\frac{1}{n}\right)+1+\frac{1}{n}\right]} = -2\left(\frac{1}{2}\right) = -1$$

$\therefore$ $L = e^{-1}$ ans.]

Q.15[151/5] If z-axis be vertical, find the equation of the line of greatest slope through the point $(2, -1, 0)$ on the plane $2x + 3y - 4z = 1$.

[Sol. Equation of the line of greatest slope



$$\frac{x-2}{a} = \frac{y+1}{b} = \frac{z}{c}$$

where $2a + 3b - 4c = 0$ ....(1)

now equation of the horizontal plane is $z = 0$

i.e. $0\cdot x + 0\cdot y + 1\cdot z = 0$

now a vector along the line of intersection of given plane and horizontal plane is

$$\vec{v} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 0 & 1\\ 2 & 3 & -4\end{vmatrix} = -(3\hat{i}-2\hat{j}) = 3\hat{i}+2\hat{j}+0\hat{k}$$

since the line of greatest slope is also perpendicular to the vector $\vec{v}$ hence

$-3a + 2b + 0\cdot c = 0$ ....(2)

from (1) and (2)

$2a + 3b - 4c = 0$

$-3a + 2b + 0\cdot c = 0$

$$\frac{a}{0+8} = \frac{b}{12} = \frac{c}{4+9} \quad\Rightarrow\quad \frac{a}{8} = \frac{b}{12} = \frac{c}{13}$$

$\therefore$ equation of the line of greatest slope $= \dfrac{x-2}{8} = \dfrac{y+1}{12} = \dfrac{z}{13}$]

Q.16 Let $I = \int_0^{\pi/2} \frac{\cos x}{a\cos x + b\sin x} dx$ and $J = \int_0^{\pi/2} \frac{\sin x}{a\cos x + b\sin x} dx$, where a > 0 and b > 0.

Compute the values of I and J.

[Sol. $aI + bJ = \frac{\pi}{2}$ ....(1)

and $bI - aJ = \int_0^{\pi/2} \frac{b\cos x - a\sin x}{a\cos x + b\sin x} dx$

$\therefore$ $bI - aJ = ln\,[a\cos x + b\sin x]_0^{\pi/2}$ $\Rightarrow$ $bI - aJ = ln\left(\frac{b}{a}\right)$ .....(2)

from (1) and (2)

$a^2 I + abJ = \frac{a\pi}{2}$

$b^2 I - abJ = b\,ln(b/a)$

$I = \frac{1}{a^2+b^2}\left(\frac{a\pi}{2} + b\,ln\left(\frac{b}{a}\right)\right)$ **Ans.**

again $abI + b^2 I = \frac{b\pi}{2}$

and $abI - a^2 J = a\,ln\,(b/a)$

subtract

$J = \frac{1}{a^2+b^2}\left(\frac{b\pi}{2} - a\,ln\left(\frac{b}{a}\right)\right)$ **Ans.**

Alternatively: convert $a\cos x + b\sin x$ into a single cosine say cos(x + f) and put x – f = t]

