PERMUTATION & COMBINATION

Some questions (Assertion–Reason type) are given below. Each question contains **Statement – 1 (Assertion)** and **Statement – 2 (Reason)**. Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct. So select the correct choice :

(A) **Statement – 1** is True, **Statement – 2** is True; **Statement – 2 is** a correct explanation for **Statement – 1**.
(B) **Statement – 1** is True, **Statement – 2** is True; **Statement – 2 is NOT** a correct explanation for **Statement – 1**.
(C) **Statement – 1** is True, **Statement – 2** is False.
(D) **Statement – 1** is False, **Statement – 2** is True.

399. **Statement-1:** $51 \times 52 \times 53 \times 54 \times 55 \times 56 \times 57 \times 58$ is divisible by 40320
Statement-2: The product of r consecutive natural numbers is always divisible by r!

400. **Statement-1:** Domain is $\{d_1, d_2, d_3, d_4\}$, range is $\{r_1, r_2, r_3\}$. Number of into functions which can be made is 45.
Statement-2: Numbers of into function = number of all functions – number of onto functions.
$= \mathbf{3^4} - 3({}^4C_2 \,.\, {}^2C_1) = 81 - 36 = 45$ of d_1, d_2, d_3, d_4 any two correspond to r_1, remaining two to r_2, r_3 one with each
$\therefore {}^4C_2 \times {}^2C_1 = 12$, total $= 12 \times 3 = 36$ = number of onto functions.

401. **Statement-1:** The smallest number which has 24 divisors is 420.
Statement-2: $24 = 3 \times 2 \times 2 = (2 + 1)(1 + 1)(1 + 1)(1 + 1)$, therefore, prime factors of the number are 2, 2, 3, 5, 7 & their product is 420.

402. Consider the word 'SMALL'
Statement–1 : Total number of 3 letter words from the letters of the given word is 13.
Statement–2 : Number of words having all the letters distinct = 4 and number of words having two are alike and third different = 9

403. **Statement–1 :** Number of non integral solution of the equation $x_1 + x_2 + x_3 = 10$ is equal to 34.
S–2 : Number of non integral solution of the equation $x_1 + x_2 + x_3 + \ldots x_n = r$ is equal to ${}^{n+r-1}C_r$

404. **Statement–1 :** ${}^{10}C_r = {}^{10}C_4 \Rightarrow r = 4$ or 6 **Statement–2 :** ${}^nC_r = {}^nC_{n-r}$

405. **Statement–1 :** The number of ways of arranging n boys and n girls in a circle such that no two boys are consecutive, is $(\underline{|n-1})^2$.
Statement–2 : The number of ways of arranging n distinct objects in a circle is $\underline{|n-1}$

406. **Statement–1 :** The number of ways of selecting 5 students from 12 students (of which six are boys and six are girls), such that in the selection there are at least three girls is ${}^6C_3 \times {}^9C_2$.
Statement–2 : If a work has two independent parts, of which first part can be done in m way and for each choice of first part, the second part can be done in n ways, then the work can be completed in $m \times n$ ways.

407. **Statement–1 :** The number of ways of writing 1400 as a product of two positive integers is 12.
Statement–2 : 1400 is divisible by exactly three prime numbers.

408. **Statement–1 :** The number of selections of four letters taken from the word 'PARALLEL' must be 15.
Statement–2 : Coefficient of x^4 in the expansion of $(1 - x)^{-3}$ is 15.

409. **Statement–1 :** Total number of permutation of n things of which p are alike of one kind, q are alike of 2nd kind, r are alike of 3rd kind and rest are all difference is $\frac{n!}{p!q!r!}$.
Statement–2 : Total number of selection from n identical object is n.

410. **Statement–1 :** A polygon has 44 diagonals and number of sides are 11.
Statement–2 : From n distinct object r object can be selected in ^nC_r ways.

411. Let y = x + 3, y = 2x +3, y = 3x + 2 and y + x = 3 are four straight lines
Statement-1 : The number of triangles formed is $^4C_3$
Statement-2 : Number of distinct point of intersection between various lines will determine the number of possible triangle.

412. **Statement-1 :** The total number of positive integral solutions (zero included) of $x + y + z + \omega = 20$ without restriction is $^{23}C_{20}$
Statement-2 : Number of ways of distributing n identical items among m persons when each person gets zero or more items = $^{m+n-1}C_n$

413. **Statement-1 :** The total ways of selection of 5 objects out of $n(n \ge 5)$ identical objects is one.
Statement-2: If objects are identical then total ways of selection of any number of objects from given objects is one.

414. **Statement-1:** The total number of different 3-digits number of type N = abc, where a < b < c is 84.
Statement-2: O cannot appear at any position, so total numbers are $^9C_3$.

415. **Statement-1:** The number of positive integral solutions of the equation $x_1x_2x_3x_4x_5 = 1050$ is 1875.
Statement-2: The total number of divisor of 1050 is 25.

416. **Statement-1:** $\left(\sum_{r=0}^{100} {}^{500-r}C_3\right) + {}^{400}C_4 = {}^{501}C_4$ **Statement-2 :** $^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

417. **Statement-1 :** $\frac{(n^2)!}{(n!)^n}$ is a natural number for all $n \in N$
S-2 : The number of ways of distributing mn things in m groups each containing n things is $\frac{(mn)!}{(n!)^m}$.

418. **Statement-1:** The number of divisors of 10, 800 is 60.
Statement-2: The number of odd divisors of 10, 800 is 12.

419. **Statement-1:** Number of onto functions from $A \to B$ where A contains n elements 2B contains m elements (where $n \ge m$) = $m^n - {}^mC_1 (m-1)^n + {}^mC_2 (m-2)^n + ...$
Statement-2: Number of ways of putting 5 identical balls in 3 different boxes when empty boxes are not allowed are 6.

420. **Statement-1 :** 4 persons can be seated in a row containing 12 chairs, such that no two of them are consecutive in $^9C_4 \times 4!$ ways
S-2: Number of non-negative integral solutions of equation $x_1+x_2+...+ x_r = n$ is = $^{n+r-1}C_{r-1}$.

421. **Statement-1:** The number of selections of four letters taken from the word PARALLEL must be 22.
Statement-2: Coefficient of x^4 in the expansion of $(1-x)^3$ is 10.

422. **Statement-1:** Number of permutations of n dissimilar things taken 'n' at a time is ^nP_n.
Statement-2: n(A) = n(B) = n then the total number of functions from A to B are n!

423. **Statement-1:** Number of permutations of n dissimilar things taken n at a time in ^nP_n .
Statement-2: n(A) = n(B) = n then the total number of functions from A to B are n!

424. **Statement-1:** $^nC_r = {}^nC_p \Rightarrow r = p$ or $r + p = n$ **Statement-2:** $^nC_r = {}^nC_{n-r}$

425. **S-1:** The total number of words with letters of the word civilization (all taken at a time) is 19958393.
Statement-2: The number of permutations of n distinct objects (r taken at a time) is $^np_{r+1}$.

426. **S-1:** The number of ways in which 81 different beads can be arranged to form a necklace is $\frac{80}{2!}$

Statement-2: Number of circular arrangements of n different objects is (n – 1)!.

427. **Statement-1:** There are 9^n, n digit numbers in which no two consecutive digits are same.
Statement-2: The n digits number in which no two consecutive digits are equal cannot contain zero.

428. **Statement-1:** $\frac{(n+2)!}{(n-1)!}$ is divisible by 6.**S-2:** : Product of three consecutive integer is divisible by 6.

Answer

399. A	**400. A**	**401. C**	**402. A**	**403. D**	**404. A**	**405. D**
406. D	**407. B**	**408. D**	**409. C**	**410. A**	**411. A**	**412. A**
413. A	**414. A**	**415. C**	**416. A**	**417. A**	**418. B**	**419. B**
420. A	**421. C**	**422. C**	**423. C**	**424. A**	**425. C**	**426. A**
427. C	**428. A**					

Details Solution

Number of words having all the letters distinct = $^4P_1 = 4$

Number of words having two are alike and third different = $^1C_1 \cdot {}^3C_1 \cdot \frac{3!}{2!} = 9$

$\therefore$ (A) is the correct option.

403. (D) Number of solution = $^{12}C_{10} = 66$.

404. (A) r = 4
or r = 10 – 4 = 6.

405. Statement – II is true as on fixing one object anywhere in the circle, the remaining n – 1 objects can be arranged in $\lfloor n-1$ ways

Statement – II is false, as after arranging boys on the circle in $\lfloor n-1$ ways, girls can be arranged in between the boys in $\lfloor n$ ways (for any arrangement of boys).

Hence number of arrangements is $\lfloor n \, \lfloor n-1$.

Hence (D) is the correct answer.

406. Statement – II is true, known as the rule of product.

Statement – I is not true, as the two parts of the work are not independent . Three girls can be chosen out of six girls in $^6C_3$ ways, but after this choosing 3 students out of remaining nine students depends on the first part.

Hence (D) is the correct answer.

407. Since, $1400 = 2^3.5^2.7^1$

$\Rightarrow$ Total no. of factors = (3 + 1) (2 + 1) (1 + 1) = 24

$\Rightarrow$ No. of ways of expressing 1400 as a product of two numbers = $\frac{1}{2}\times 24 = 12$.

But this does not follow from statement – II which is obviously true.

Hence (b) is the correct answer.

408. Statement – I is false since the number of selection of four letters from 'PARALLEL' is 22.
1. 3 alike, 1 diff. = $^1c_1 \times {}^4c_1 = 4$
2. 2 alike, 2 alike = $^2c_2 = 1$
3. 2 alike, 2 diff. = $^2c_1 \times {}^4c_2 = 12$
4. All diff. = $^5c_4 = 5$

Total selection = 22

Statement – II is true, since
$(1-x)^{-3} = 1 + 3x + 6x^2 + 10x^3 + 15x^4 + \ldots$ Hence (D) is the correct answer.

410. (A) Let no of sides are n.
$^nC_2 - n = 44$ $\Rightarrow n = -8$ or 11 $\Rightarrow n = 11$.

415. $x_1x_2x_3x_4 = 1050 = 2 \times 3 \times 5^2 \times 7$

Thus 5^2 can as sign in ${}^5C_1 + {}^5C_2 = 15$ ways
We can assign 2, 3, or 7 to any. of 5 variables.
Hence req. number of solutions.
$= 5 \times 5 \times 5 \times 15 = 1875$ Ans. (C)

416. $({}^{400}C_4 + {}^{400}C_3) + {}^{401}C_3 + ... + {}^{500}C_3$
$= ({}^{401}C_4 + {}^{401}C_3) + {}^{402}C_3 + ... + {}^{500}C_3$
$.... = ({}^{500}C_4 + {}^{500}C_3) = {}^{501}C_4$ Ans. (A)

417. The number of ways of distributing mn things in m groups each containing n things is $\frac{(mn)!}{(n!)^m}$

here if m = n, then $\frac{(n^2)!}{(n!)^n}$ which must be a natural number.
'A' is correct.

418. If n = 10, 800 $= 2^4 \times 3^3 \times 5^2$
Number of divisors depends upon all possible selection of prime factors. So clearly (4 + 1) (3 + 1) (2+1) = 5 ×4 ×3 = 60 for odd divisors, only selection of odd prime factors, (3 + 1) (2 + 1) = 12
b is correct.

421. (C) A is true since number of selection of four letters from PARALLEL is 22. (3 alike 1 different 4 cases; 2 alike and 2 alike one case;2 alike 2 different $2 \times {}^4C_2 = 12$ and all different ${}^5C_4 = 5$ total selections = 4 + 1 + 12 + 5 = 22). R is false since $(1 - x)^{-3} = 1 + 3x + 6x^2 + 10x^3 + 15x^4 + ...$

422. ${}^nP_n = n!$ but number of function from A to B is n^n. (C)

423. (C) ${}^nP_n = n!$, but the number of functions from A to B is n^n.

424. (A) Statement-1 is true,
Statement-2 is true, Also Statement-2 is the correct explanation of Statement-1.

425. (C)
In the given word 4 are there so required number of permutations is $\frac{12!}{4!} = 19958392$

426. (A) Since clockwise and anticlockwise arrangements are not different so required number of arrangements is $\frac{80}{2!}$.