

SHORT REVESION

CAPACITANCE OF AN ISOLATED SPHERICAL CONDUCTOR :

 $C = 4\pi \in C_{0} \in R$ in a medium $C = 4\pi \in R$ in air

- * This sphere is at infinite distance from all the conductors.
- * The capacitance $C = 4\pi \in R$ exists between the surface of the sphere & earth.

SPHERICAL CAPACITOR :

It consists of two concentric spherical shells as shown in figure. Here capacitance of region between the two shells is C_1 and that outside the shell is C_2 . We have

$$C_1 = \frac{4\pi \in_0 ab}{b-a} \quad \text{and} \quad C_2 = 4\pi \in_0 b$$

Depending on connection, it may have different combinations of C_1 and C_2 .

PARALLEL PLATE CAPACITOR :

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(i) UNIFORM DI-ELECTRIC MEDIUM :

If two parallel plates each of area A & separated by a distance d are charged with equal & opposite charge Q, then the system is called a parallel plate capacitor & its capacitance is given by,

 $C = \frac{\epsilon_0 \epsilon_r A}{d}$ in a medium ; $C = \frac{\epsilon_0 A}{d}$ with air as medium

This result is only valid when the electric field between plates of capacitor is constant.

(ii) MEDIUM PARTLY AIR :
$$C = \frac{\epsilon_0 A}{d - t}$$

When a di-electric slab of thickness t & relative permittivity \in_{r} is introduced between the plates of an air capacitor, then the distance between

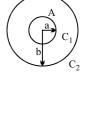
the plates is effectively reduced by $\left(t - \frac{t}{\epsilon_r}\right)$ irrespective of the position of the di-electric slab.

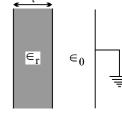
(iii) Composite Medium : $C = \frac{\epsilon_0 A}{\frac{t_1}{\epsilon_{r_1} + \epsilon_{r_2} + \epsilon_{r_3}}}$

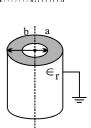
Cylindrical Capacitor :

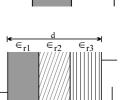
It consist of two co-axial cylinders of radii a & b, the outer conductor is earthed . The di-electric constant of the medium filled in the space between the cylinder is

$$\in_{\mathbf{r}}$$
. The capacitance per unit length is $C = \frac{2\pi \in_0 \in_{\mathbf{r}}}{\ell n(\frac{b}{a})} \frac{Farad}{m}$.









5. **CONCEPT OF VARIATION OF PARAMETERS:**

As capacitance of a parallel plate capacitor is $C = \frac{\epsilon_0 kA}{d}$, if either of k, A or d varies in the region between the plates, we choose a small dc in between the plates and for total capacitance of system.

If all dC's are in series $\frac{1}{C_T} = \int \frac{dx}{\epsilon_0 k(x) A(x)}$, If all dC's are in parallel $C_T = \int dC$

COMBINATION OF CAPACITORS :

CAPACITORS IN SERIES : (i)

In this arrangement all the capacitors when uncharged get the same charge Q but the potential difference across each will differ (if the capacitance are unequal).

$$\frac{1}{C_{eq.}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_n}$$

When one plate of each capacitor is connected to the positive terminal of the battery & the other plate of each capacitor is connected to the negative terminals of the battery, then the capacitors are said to be in parallel connection. The capacitors have the same potential difference, V but the

charge on each one is different (if the capacitors are unequal).

$$C_{eq} = C_1 + C_2 + C_3 + \dots + C_n$$
.

ENERGY STORED IN A CHARGED CAPACITOR :

Capacitance C, charge Q & potential difference V; then energy stored is

Bhopal Phone: 0 903 903 7779, $U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$. This energy is stored in the electrostatic field set up in the di-electric Sir), I

medium between the conducting plates of the capacitor.

8. HEAT PRODUCED IN SWITCHING IN CAPACITIVE CIRCUIT

Due to charge flow always some amount of heat is produced when a switch is closed in a circuit which a can be obtained by energy conservation as –

Heat = Work done by battery – Energy absorbed by capacitor.

SHARING OF CHARGES : 9.

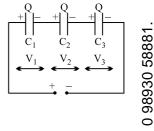
R. Kariya (S. When two charged conductors of capacitance $C_1 \& C_2$ at potential $V_1 \& V_2$ respectively are when two charged conductors of capacitance $C_1 & C_2$ at potential $V_1 & V_2$ respectively are connected by a conducting wire, the charge flows from higher potential conductor to lower potential conductor, until the potential of the two condensers becomes equal. The common potential (V) after sharing of charges; $V = \frac{\text{net charge}}{\text{net capacitance}} = \frac{q_1 + q_2}{C_1 + C_2} = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}.$ charges after sharing $q_1 = C_1 V \& q_2 = C_2 V$. In this process energy is lost in the connecting wire as heat . This loss of energy is $U_{\text{initial}} - U_{\text{real}} = \frac{C_1 C_2}{2(C_1 + C_2)} (V_1 - V_2)^2$. **REMEMBER :** (i) The energy of a charged conductor resides outside the conductor in its EF, where as in a condenser in its EF.

$$V = \frac{\text{net charge}}{\text{net capacitan ce}} = \frac{q_1 + q_2}{C_1 + C_2} = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

as heat. This loss of energy is
$$U_{initial} - U_{real} = \frac{C_1 C_2}{2(C_1 + C_2)} (V_1 - V_2)^2$$
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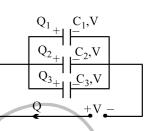
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- it is stored within the condenser in its EF.
- (ii) The energy of an uncharged condenser = 0.
- (iii) The capacitance of a capacitor depends only on its size & geometry & the di-electric between the conducting surface. (i.e. independent of the conductor, like, whether it is copper, silver, gold etc)

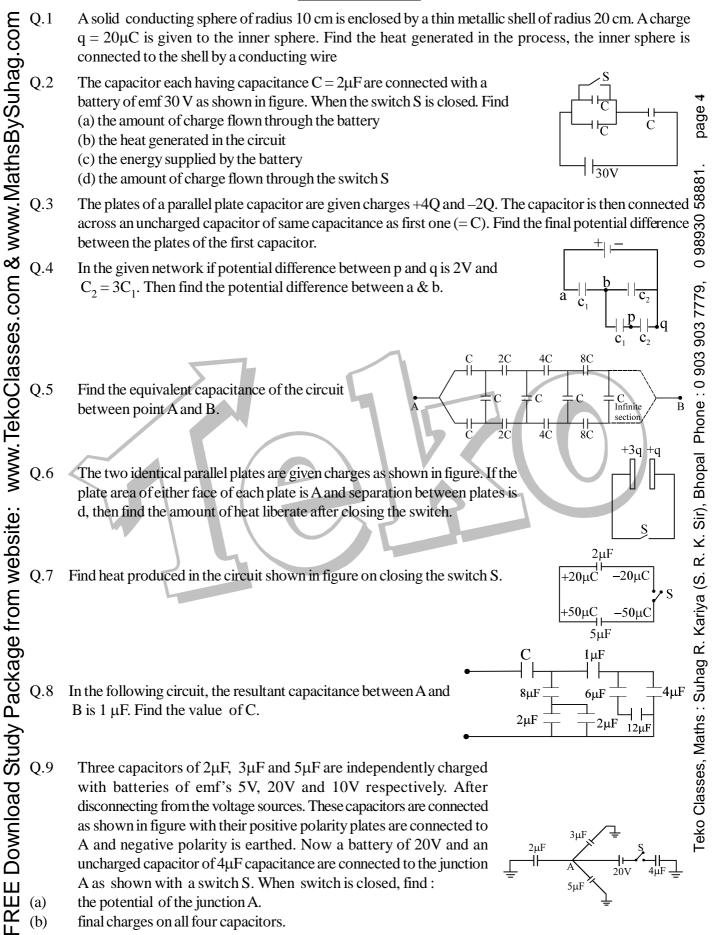


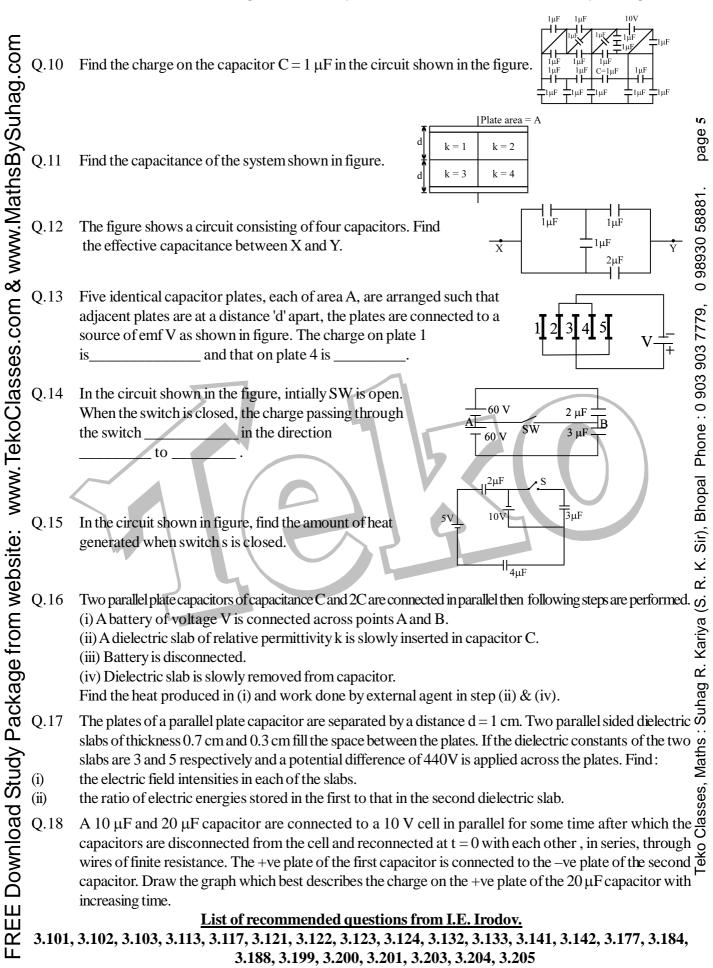
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Get Solution of These Packages & Learn by Video Tutorials on www.MathsBySuhag.com EXERCISE # II 6µF」

E Download Study Package from website: www.TekoClasses.com & www.MathsBySuhag.com Q.1 (a) For the given circuit. Find the potential difference across all the capacitors. (b) How should 5 capacitors, each of capacities, 1µF be connected so as to produce a total capacitance of $3/7 \,\mu\text{F}$. The gap between the plates of a plane capacitor is filled with an isotropic insulator whose di-electric $\mathbf{a}_{\mathbf{b}}$ Q.2 constant varies in the direction perpendicular to the plates according to the law $K = K_1 \left[1 + \sin \frac{\pi}{d} X \right]$ where d is the separation, between the plates & K_1 is a constant. The area of the plates is S. Determine the capacitance of the capacitor. Q.3 to and equidistant from each other (see figure). Fraces 2 & 3 are connected by a conductor while 1 & 3 are joined by another conductor . The junction of 1 & 3 and the plate 4 are connected to a source of constant e.m.f. V_0 . Find ; the effective capacity of the system between the terminals of the source. the charges on plates 3 & 5. Given d = distance between any 2 successive plates & A = area of either face of each plate . A potential difference of 300 V is applied between the plates of a plane capacitor spaced 1 cm apart. A plane parallel glass plate with a thickness of 0.5 cm and a plane parallel paraffin plate with a thickness of Q 5 cm are placed in the space between the capacitor plates find : (i) (ii) Q.4 0.5 cm are placed in the space between the capacitor plates find : Bhopal Intensity of electric field in each layer. (i) (ii) The drop of potential in each layer. The surface charge density of the charge on capacitor the plates. Given that : $k_{glass} = 6$, $k_{paraffin} = 2$ Sir), I (iii) A charge 200 μ C is imparted to each of the two identical parallel plate capacitors connected in parallel. Q.5 At t =0, the plates of both the capacitors are 0.1m apart. The plates of first capacitor move towards \simeq each other with relative velocity $0.001 \,\mathrm{m/s}$ and plates of second capacitor move apart with the same \mathcal{O} Kariya velocity. Find the current in the circuit at the moment. Q.6 A parallel plate capacitor has plates with area A & separation d . A battery charges the plates to a с. Ш potential difference of V₀. The battery is then disconnected & a di-electric slab of constant K & thickness Suhag d is introduced. Calculate the positive work done by the system (capacitor + slab) on the man who introduces the slab. A capacitor of capacitance C_0 is charged to a potential V_0 and then isolated. A small capacitor C is then charged from C_0 , discharged & charged again, the process being repeated n times. The potential of the large capacitor has now fallen to V. Find the capacitance of the small capacitor. If $V_0 = 100$ volt, V=35volt, find the value of n for $C_0 = 0.2 \ \mu F \& C = 0.01075 \ \mu F$. Is it possible to remove charge on C_0 this way? When the switch S in the figure is thrown to the left, the plates of capacitors Q.7 Q.8 When the switch S in the figure is thrown to the left, the plates of capacitors C_1 acquire a potential difference V. Initially the capacitors C_2C_3 are uncharged. Thw switch is now thrown to the right. What are the final charges $q_1, q_2 \& q_3$ on the corresponding capacitors.

- Q.9 A parallel plate capacitor with air as a dielectric is arranged horizontally. The lower plate is fixed and the other connected with a vertical spring. The area of each plate is A. In the steady position, the distance between the plates is d_0 . When the capacitor is connected with an electric source with the voltage V, a new equilibrium appears, with the distance between the plates as d_1 . Mass of the upper plates is m.
- Find the spring constant K. (i)
- (ii) What is the maximum voltage for a given K in which an equilibrium is possible?
- (iii) What is the angular frequency of the oscillating system around the equilibrium value d_1 . (take amplitude of oscillation \ll d₁)
- An insolated conductor initially free from charge is charged by repeated contacts with a plate which after $\frac{1}{80}$ each contact has a charge Q due to some mechanism. If q is the charge on the conductor after the first $\frac{2}{80}$ Q.10

operation, prove that the maximum charge which can be given to the conductor in this way is $\frac{Qq}{Q-q}$

- A parallel plate capacitor is filled by a di-electric whose relative permittivity varies with the applied \aleph voltage according to the law = α V, where α = 1 per volt. The same (but containing no di-electric) Q.11 capacitor charged to a voltage V = 156 volt is connected in parallel to the first "non-linear" uncharged $\bigotimes_{i=1}^{\infty} capacitor$ capacitor. Determine the final voltage V_f across the capacitors. 903
- A capacitor consists of two air spaced concentric cylinders. The outer of radius b is fixed, and the inner is of $\stackrel{\circ}{.}$ Q.12 A capacitor consists of two air spaced concentric cylinders. The outer of radius b is fixed, and the inner is of \Im radius a. If breakdown of air occurs at field strengths greater than E_b , show that the inner cylinder should have radius a = b/e if the potential of the inner cylinder is to be maximum radius $a = b/\sqrt{e}$ if the energy per unit length of the system is to be maximum. Find the charge flown through the switch from A to B when it is closed. Find the charge flown through the switch from A to B when it is closed. $a_{\mu F} = \frac{10V}{6\mu F}$ (i)
- (ii)
- Q.13
- Q.14 Figure shows three concentric conducting spherical shells with inner and outer shells earthed and the middle shell is given a charge q. Find the electrostatic energy of the system stored in the region I and II.
- Q.15 The capacitors shown in figure has been charged to a potential difference of V volts, so that it carries a charge CV with both the switches S_1 and S_2 remaining open. Switch S_1 is closed at t=0. At t= R_1C switch S_1 is opened and S_2 is closed. Find the charge on the capacitor at t=2R₁C + R₂C.
- Q.16 In the figure shown initially switch is open for a long time. Now the switch is closed at t = 0. Find the charge on the rightmost capacitor as a function of time given that it was intially unchanged.

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 S_2

EXERCISE # III

In the given circuit, the switch is closed in the position 1 at t = 0 and then moved to 2 after 250 µs. Derive an expression for current as a function of time for t > 0. Also plot the variation of current with time.

500Ω 0.5 uF

R

5μ̈́F

5μF

5μF

5μF

201

5μF

Q.17 Q.18 Find the charge which flows from point A to B, when switch is closed. Q.1 Two parallel plate capacitors A & B have the same separation d = 8.85×10^{-4} m between the plates. The plate areas of A & B are 0.04 m² & 0.02 m² respectively. A slab of di-electric constant (relative permittivity) K=9 has dimensions such that it can exactly fill the space between the plates of capacitor B. (i) the di-electric slab is placed inside A as shown in the figure (i) A is then charged to a potential difference of 110 volt. Calculate the capacitance of A and the energy stored in it. the battery is disconnected & then the di-electric slab is removed from A. Find the work done by the S (ii) external agency in removing the slab from A. (iii) the same di-electric slab is now placed inside B, filling it completely. The two capacitors A & B are then connected as shown in figure (iii). Calculate the energy stored in the system. Q.3

(A) zero

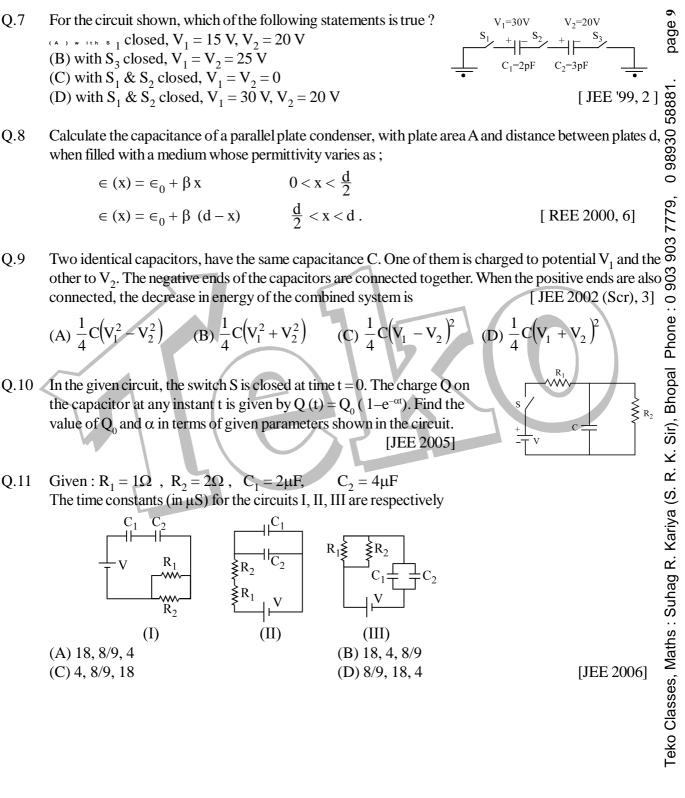
- shown in figure (iii). Calculate the energy stored in the system. a way that one of their edges is perpendicular, to an oil surface in a tank filled with an insulating oil. The $\overline{\mathbf{rg}}$ plates are connected to a battery of e.m.f. 500 volt. The plates are then lowered vertically into the oil at $\overline{\mathbf{rg}}$ a speed of 0.001 m/s. Calculate the current drawn from the battery during the process. a way that one of their edges is perpendicular, to an oil surface in a tank filled with an insulating oil. The [di-electric constant of oil = 11, $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N}^2 \text{ m}^2$] A parallel plate capacitor C is connected to a battery & is charged to a potential difference V. Another $\dot{\mathbf{x}}$ capacitor of capacitance 2C is similarly charged to a potential difference 2V volt. The charging battery is α now disconnected & the capacitors are connected in parallel to each other in such a way that the positive 9 terminal of one is connected to the negative terminal of other. The final energy of the configuration is : (B) $\frac{3}{2}$ CV² (C) $\frac{25}{6}$ CV² (D) $\frac{9}{2}$ CV² The capacitance of a parallel plate capacitor with plate area 'A' & separation d is C. The space between the plates is filled with two wedges of di-electric constant $K_1 \& K_2$ respectively. Find the capacitance of the resulting capacitor. [JEE '96, 2] Two capacitors A and B with capacities 3μ F and 2μ F are charged to a potential difference of 100 V and 180 V respectively. The plates of the capacitors are connected as shown in figure with one wire from each capacitor free. The upper plate of a is positive and that of B is negative. an 3µF uncharged 2 µF capacitor C with lead wires falls on the free ends to complete Ā 100V the circuit. Calculate :
- the final charges on the three capacitors (i) (ii)
 - The amount of electrostatic energy stored in the system before and after the completion of the circuit. [JEE '97 (cancelled)]

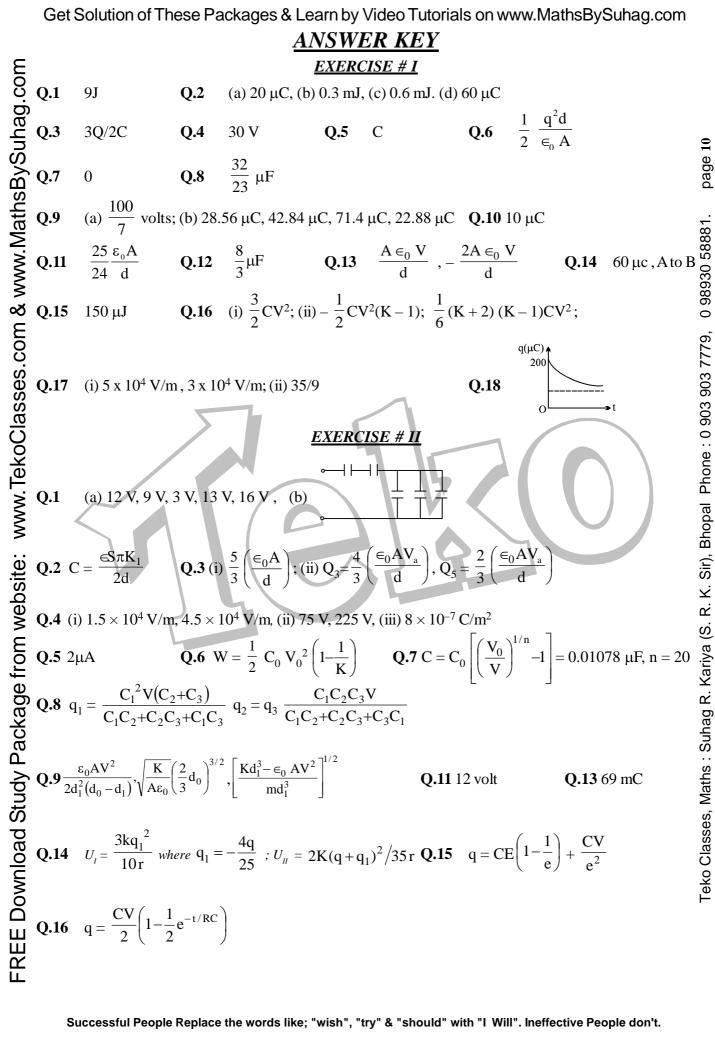
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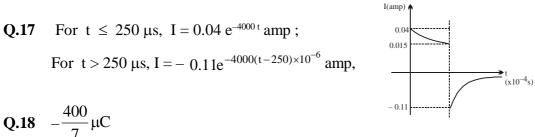
2μF

в ₁₈₀v

Q.6 An electron enters the region between the plates of a parallel plate capacitor at a point equidistant from either plate. The capacitor plates are 2×10^{-2} m apart & 10^{-1} m long. A potential difference of 300 volt is kept across the plates. Assuming that the initial velocity of the electron is parallel to the capacitor plates, calculate the largest value of the velocity of the electron so that they do not fly out of the capacitor at the other end. [JEE '97, 5]







EXERCISE # III

- (i) 0.2×10^{-8} F, 1.2×10^{-5} J ; (ii) 4.84×10^{-5} J ; (iii) 1.1×10^{-5} J Q.1
- **Q.4** $\frac{CK_1K_2}{(K_2-K_1)} \ln \frac{K_2}{K_1}$ $4.425\times10^{-9}\,Ampere$ **Q.3** B Q.2

Q.5
$$Q_A = 90 \ \mu\text{C}, \ Q_B = 150 \ \mu\text{C}, \ Q_C = 210 \ \mu\text{C}, \ U_i = 47.4 \ \text{mJ}, \ U_f = 18 \ \text{mJ}$$

Q.8 $\frac{\beta A}{2} / \ell n \left(\frac{2 \epsilon_0 + \beta d}{2 \epsilon_0} \right)$ **Q.6** $\frac{\sqrt{4.8}}{2\sqrt{9.1}} \times 10^8 \text{ m/s}$ **Q.7** D **Q.10** $Q_0 = \frac{CVR_2}{R_1 + R_2}$ and $a = \frac{R_1 + R_2}{CR_1R_2}$ **Q.11** D Q.9 С