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ELECTROSTATICS

1. INTRODUCTION

The branch of physics which deals with electric effect of static charge is called electrostatics.

2. ELECTRIC CHARGE

Charge of a material body or particle is the property (acquired or natural) due to which it produces and experiences electrical and magnetic effects. Some of naturally charged particles are electron, proton, α-particle etc.

2.1 Types of Charge

(i) Positive charge : It is the deficiency of electrons compared to protons.

(ii) Negative charge : It is the excess of electrons compared to protons.

2.2 Units of Charge

Charge is a derived physical quantity. Charge is measured in coulomb in S.I. unit. In practice we use mC (10^{-3}C), μC (10^{-6}C), nC(10^{-9}C) etc.

C.G.S. unit of charge = electrostatic unit = esu.

1 coulomb = 3×10^9 esu of charge

Dimensional formula of charge = $[M^0L^0T^1I^1]$

2.3 Properties of Charge

(i) **Charge is a scalar quantity :** It adds algebrically and represents excess, or deficiency of electrons.

(ii) **Charge is transferable :** Charging a body implies transfer of charge (electrons) from one body to another. Positively charged body means loss of electrons, i.e., deficiency of electrons. Negatively charged body means excess of electrons. This also shows that **mass of a negatively charged body > mass of a positively charged identical body.**

(iii) **Charge is conserved :** In an isolated system, total charge (sum of positive and negative) remains constant whatever change takes place in that system.

(iv) **Charge is quantized :** Charge on any body always exists in integral multiples of a fundamental unit of electric charge. This unit is equal to the magnitude of charge on electron (1e = 1.6×10^{-19} coulomb). So charge on anybody Q = $\pm$ ne, where n is an integer and e is the charge of the electron. **Millikan's oil drop** experiment proved the quantization of charge or atomicity of charge

Note : Recently, the existence of particles of charge $\pm\frac{1}{3}$e and $\pm\frac{2}{3}$e has been postulated. These particles are called quarks but still this is not considered as the quantum of charge because These are unstable (They have very short span of life).

(v) Like point charges repel each other while unlike point charges attract each other.

(vi) A charged body may attract a neutral body or an oppositely charged body but it always repels a similarly charged body.
Note : Repulsion is a sure test of electrification whereas attraction may not be.

(vii) Charge is always associated with mass, i.e., charge can not exist without mass though mass can exist without charge. The particle such as photon or neutrino which have no (rest) mass can never have a charge. As charge can not exist without mass, the presence of charge itself is a convincing proof of existence of mass.

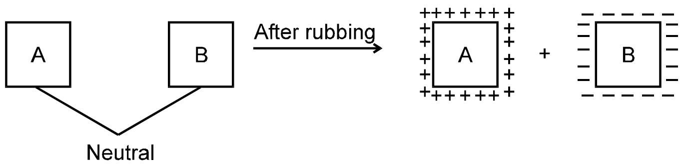
(viii) **Charge is relativistically invariant:** This means that charge is independent of frame of reference, i.e., charge on a body does not change whatever be its speed. This property is worth mentioning as in contrast to charge, the mass of a body depends on its speed and increases with increase in speed.

(ix) A charge at rest produces only electric field around itself; a charge having uniform motion produces electric as well as magnetic field around itself while a charge having accelerated motion emits electromagnetic radiation also in addition to producing electric and magnetic fields.

2.4 Charging of a body

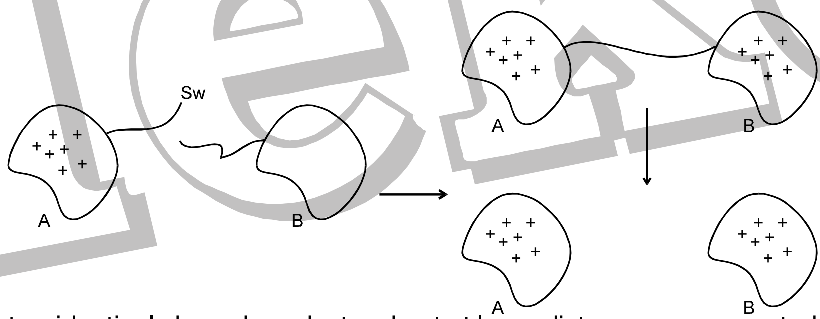
A body can be charged by means of (a) friction, (b) conduction, (c) induction, (d) thermoinic ionisation, (e) photoelectric effect and (f) field emission.

(a) Charging by Friction :
When a neutral body is rubbed with other neutral body (at least one of them should be insulator) then some electrons are transferred from one body to other. The body which gains electrons becomes negatively charged and other becomes positively charged.



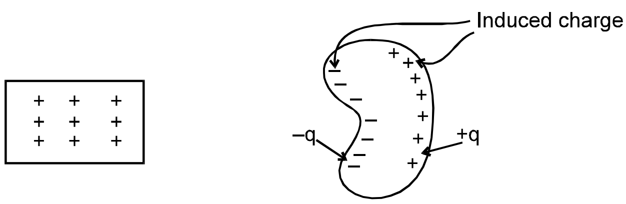
(b) Conduction (flow): There are three types of material in nature
(1) Conductor : Materials which have large number of free electrons.
(2) Insulator or Dielectric or Nonconductors : Materials which do not have free electrons.

When a charged conductor is connected with a neutral conductor then charge flows from one body to other body. In case of two charged conductors charge flows from higher potential energy to lower potential energy. The charge stops flowing when the potential of the two bodies become same.

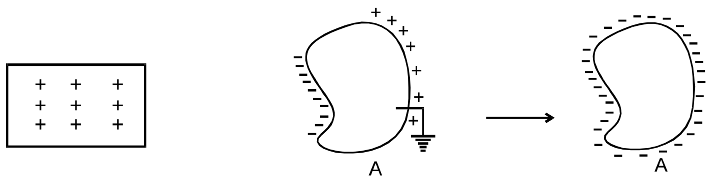


Note: If two identical shaped conductors kept at large distance are connected to each other then they will have equal charges finally.

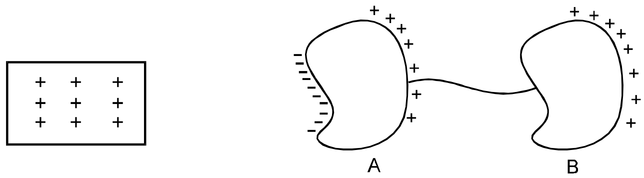
(c) Induction : When a charged particle is taken near to neutral object then the electrons move to one side and there is excess of electrons on that side making it negatively charged and deficiency on the other side making that side positively charged. Hence charges appear on two sides of the body (although total charge of the body is still zero). This phenomenon is called induction and the charge produced by it is called induced charge.



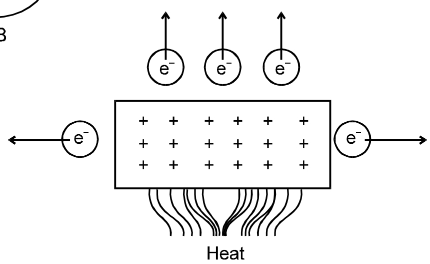
A body can be charged by in duction in following two ways.
Method I :
The potential of conductor A becomes zero after earthing. To make potential zero some electrons flow from the Earth to the conductor A and now connection is removed making it negatively charged.



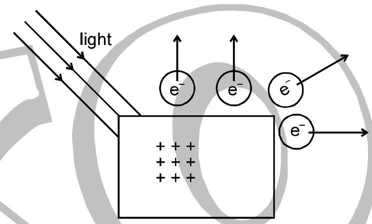
Method II :
The conductor which has induced charge on it, is connected to a neutral conductor which makes the flow of charge such that their potentials become equal and now they are disconnected making the neutral conductor charged.



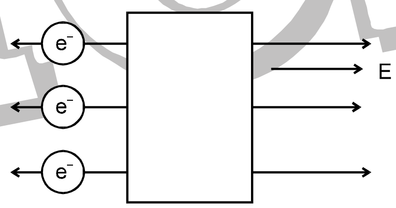
(d) Thermionic emission : When the metal is heated at a high temperature then some electrons of metals are ejected and the metal gets ionised. It becomes positively charged.



(e) Photoelectric effect : When light of sufficiently high frequency is incident on metal surface then some electrons come out and metal gets ionized



(f) Field emission : When electric field of large magnitude is applied near the metal surface then some electrons come out from the metal surface and hence the metal gets positively charged.



Ex.1 Charge conservation is always valid. Is it also true for mass ?

Sol. No, mass conservation is not valid. Mass can be converted into energy.

Ex.2 What are the differences between charging by induction and charging by conduction ?

Sol. Major differences between two methods of charging are as follows :

(i) In induction, two bodies are close to each other but do not touch each other while in conduction they touch each other.

(ii) In induction, total charge of body remains unchanged while in conduction it changes.

(iii) In induction, induced charge is always opposite in nature to that of source charge while in conduction charge on two bodies is of same nature.

Q.1 If a glass rod is rubbed with silk it acquires a positive charge because :
(A) protons are added to it (B) protons are removed from it
(C) electrons are added to it (D) electrons are removed from it.

Q.2 A positively charged body 'A' attracts a body 'B' then charge on body 'B' may be:
(A) positive (B) negative (C) zero (D) can't say

Answers : **1.** D **2.** B, C

3. COULOMB'S LAW (INVERSE SQUARE LAW)

On the basis of experiments Coulomb established the following law known as Coulamb's law.
The magnitude of electrostatic force between two point charges is directly proportional to the product of charges and inversely proportional to the square of the distance between them.

i.e. $F \propto q_1q_2$

$$F \propto \frac{1}{r^2} \quad \Rightarrow F \propto \frac{q_1q_2}{r^2} \quad \Rightarrow F = \frac{Kq_1q_2}{r^2}$$

Important points regarding Coulamb's law :

(i) It is applicable only for point charges.

(ii) The constant of proportionality K in SI units in vacuum is expressed as $\frac{1}{4\pi\varepsilon_0}$ and in any other medium expressed as $\frac{1}{4\pi\varepsilon}$. If charges are dipped in a medium then electrostatic force on one charge is $\frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1q_2}{r^2}$. ε_0 and ε are called permittivity of vacuum and absolute permittivity of the medium respectively. The ratio $\varepsilon/\varepsilon_0 = \varepsilon_r$ is called relative permittivity of the medium, which is a dimensionless quantity.

(iii) The value of relative permittivity ε_r varies between 1 to ∞. For vacuum, by definition it is equal to 1. For air it is nearly equal to 1 and may be taken to be equal to 1 for calculations. For metals the value of ε_r is ∞.

(iv) The value of $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ Nm^2 C^{-2}.

(v) The force acting on one point charge due to the other point charge is always along the line joining these two charges. It is equal in magnitude and opposite in direction on two charges, irrespective of the medium, in which they lie.

(vi) The force is conservative in nature i.e., work done by electrostatic force in moving a point charge along a close loop of any shape is zero.

(vii) Since the force is a central force, in the absence of any other external force, angular momentum of one particle w.r.t. the other particle (in two particle system) is conserved,

(x) In vector form formula can be given as below.

$$\vec{F} = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1q_2}{|\vec{r}|^3}\vec{r} = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1q_2}{|\vec{r}|^2}\hat{r}$$

here $\vec{r}$ is position vector of the test charge with respect to the source charge.

Ex.3 If the distance between two equal point charges is doubled and their individual charges are also doubled, what would happen to the force between them?

Sol. $F = \frac{1}{4\pi\varepsilon_0}\frac{q\times q}{r^2}$ ....(1) ; Again, $F' = \frac{1}{4\pi\varepsilon_0}\frac{(2q)(2q)}{(2r)^2}$

or $F' = \frac{1}{4\pi\varepsilon_0}\frac{4q^2}{4r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} = F$

So, the force will remain the same.

Ex.4 A particle of mass m carrying charge q_1 is revolving around a fixed charge $-q_2$ in a circular path of radius r. Calculate the period of revolution and its speed also.

Sol. $\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} = mr\omega^2 = \frac{4\pi^2mr}{T^2}$,

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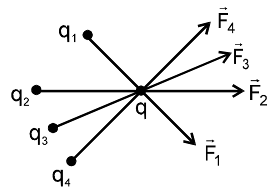
$$T^2 = \frac{(4\pi\varepsilon_0)r^2(4\pi^2 mr)}{q_1q_2} \qquad \text{or} \qquad T = 4\pi r\sqrt{\frac{\pi\varepsilon_0 mr}{q_1q_2}}$$

and also we can say that $\dfrac{q_1q_2}{4\pi\varepsilon_0 r^2} = \dfrac{mv^2}{r} \Rightarrow V = \sqrt{\dfrac{q_1q_2}{4\pi\varepsilon_0 mr}}$

4. PRINCIPLE OF SUPERPOSITION

The electrostatic force is a two body interaction, i.e., electrical force between two point charges is independent of presence or absence of other charges and so the principle of superposition is valid, i.e., force on charged particle due to number of point charges is the resultant of forces due to individual point charges, therefore, force on a point test charge due to many charges is given by

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots\ldots\ldots.$$



5. ELECTROSTATIC EQUILIBRIUM

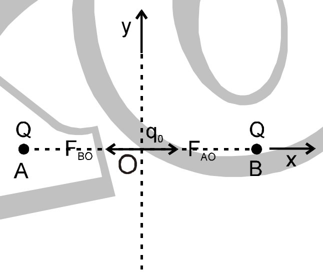
The point where the resultant force becomes zero is called equilibrium position.

5.1 Stable Equilibrium : If charge is initially in equilibrium position and is displaced by a small distance. If the charge tries to return back to the same equilibrium position then this equilibrium is called position of stable equilibrium.

5.2 Unstable Equilibrium : If charge is displaced by a small distance from its equilibrium position and the charge has no tendency to return to the same equilibrium position. Instead it goes away from the equilibrium position.

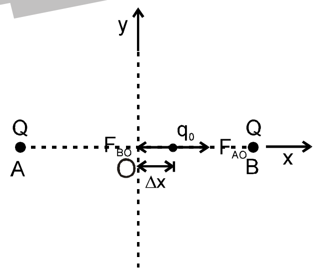
Ex. 5 Two equal positive point charges 'Q' are placed at points A(a, 0) and B(–a, 0). Another test charge q_0 is also placed at O(0, 0). Show that the equilibrium at 'O' is
(i) stable for displacement along X-axis.
(ii) unstable for displacement along Y-axis.

Sol. (i) Initially $\vec{F}_{AO} + \vec{F}_{BO} = 0 \Rightarrow |\vec{F}_{AO}| = |\vec{F}_{BO}| = \dfrac{KQq_0}{a^2}$



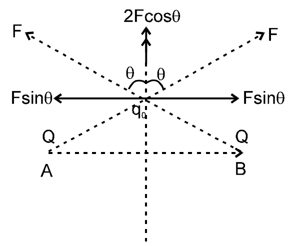
When charge is slightly shifted towards + x axis by a small distance Δx, then.

$$|\vec{F}_{AO}| < |\vec{F}_{BO}|$$



Therefore the particle will move towards origin (its original position) hence the equilibrium is stable.

(ii) When charge is shifted along y axis



After resolving components net force will be along y axis so the particle will not return to its original position so it is unstable equilibrium. Finally the charge will move to infinity.

Q.3 In example number 5 if q_0 is negative point charge then prove that the equilibrium at 'O' is

(i) stable for displacement in Y-direction.
(ii) unstable for displacement in X-direction.

5.3 Neutral Equilibrium : If charge is displaced by a small distance and it is still in equilibrium condition then it is called neutral equilibrium.

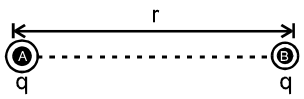
Ex. 6 Two point charges of charge q_1 and q_2 (both of same sign) and each of mass m are placed such that gravitation attraction between them balances the electrostatic repulsion. Are they in stable equilibrium? If not then what is the nature of equilibrium?

Sol. In given example : $\frac{Kq_1q_2}{r^2} = \frac{Gm^2}{r^2}$

We can see that irrespective of distance between them charges will remain in equilibrium. If now distance is increased or decreased then there is no effect in their equilibrium. Therefore it is a neutral equilibrium.

Ex. 7 Two equally charged identical metal sphere A and B repel each other with a force 2×10^{-5}N . Another identical uncharged sphere C is touched to B and then placed at the mid point between A and B. What is the net electric force on C?

Sol. Let initially the charge on each sphere be q and separation between their centres be r; then according to given problem.

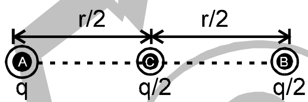


$$F = \frac{1}{4\pi\varepsilon_0}\frac{q\times q}{r^2} = 2 \times 10^{-5}\ \text{N}$$

When sphere C touches B, the charge of B, q will distribute equally on B and C as sphere are identical conductors, i.e., now charges on spheres;

$$q_B = q_C = (q/2)$$

So sphere C will experience a force



$$F_{CA} = \frac{1}{4\pi\varepsilon_0}\frac{q(q/2)}{(r/2)^2} = 2F \text{ along } \overrightarrow{AB} \text{ due to charge on A}$$

and, $$F_{CB} = \frac{1}{4\pi\varepsilon_0}\frac{(q/2)(q/2)}{(r/2)^2} = F \text{ along } \overrightarrow{BA} \text{ due to charge on B}$$

So the net force F_C on C due to charges on A and B,

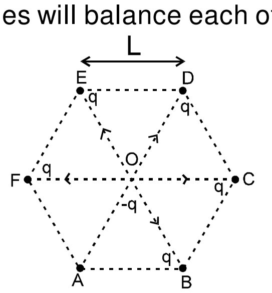
$$F_C = F_{CA} - F_{CB} = 2F - F = 2 \times 10^{-5}\ \text{N along } \overrightarrow{AB}.$$

Ex. 8 Five point charges, each of value q are placed on five vertices of a regular hexagon of side L. What is the magnitude of the force on a point charge of value –q coulomb placed at the centre of the hexagon?

Sol. **Method : I**

If there had been a sixth charge +q at the remaining vertex of hexagon force due to all the six charges on –q at O would be zero (as the forces due to individual charges will balance each other), i.e.,

$$\overrightarrow{F_R} = 0$$



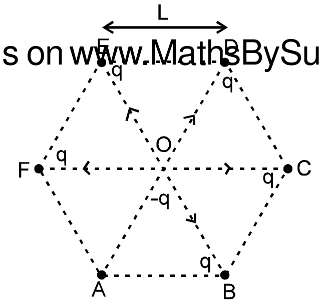
Now if $\vec{f}$ is the force due to sixth charge and $\vec{F}$ due to remaining five charges.

$$\vec{F} + \vec{f} = 0 \qquad \text{i.e.} \qquad \vec{F} = -\vec{f}$$

or, $$F = f = \frac{1}{4\pi\varepsilon_0}\frac{q\times q}{L^2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{L^2}$$ **Ans.**

Method : II

In the diagram we can see that force due to charge A and D are opposite to each other



$\vec{F}_{DO} + \vec{F}_{AO} = 0$ ....(i)

Similarly $\vec{F}_{BO} + \vec{F}_{EO} = 0$ ....(ii)

So $\vec{F}_{AO} + \vec{F}_{BO} + \vec{F}_{CO} + \vec{F}_{DO} + \vec{F}_{EO} = \vec{F}_{Net}$

Using (i) and (ii) $\vec{F}_{Net} = \vec{F}_{CO} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{L^2}$ along CO.

Ex.9 Two identical charged spheres are suspended by strings of equal length. Each string makes an angle θ with the vertical. When suspended in a liquid of density σ = 0.8 gm/cc, the angle remains the same. What is the dielectric constant of the liquid? (Density of the material of sphere is 1.6 gm/cc.)

Sol. Initially as the forces acting on each ball are tension T, weight mg and electric force F, for its equilibrium along vertical,

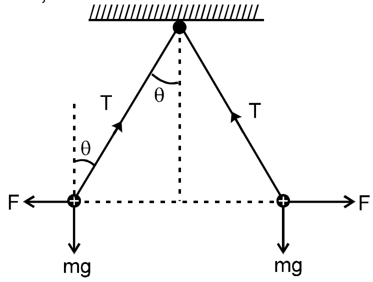
$T \cos\theta = mg$ ...(1)

and along horizontal

$T \sin\theta = F$ ...(2)

Dividing Eqn. (2) by (1), we have

$\tan\theta = \dfrac{F}{mg}$ ...(3)



When the balls are suspended in a liquid of density σ and dielectric constant K, the electric force will become (1/K) times, i.e., F' = (F/K) while weight

$mg' = mg - F_B = mg - V\sigma g$ [as $F_B = V\sigma g$, where σ is density of material of sphere]

i.e., $mg' = mg\left[1-\dfrac{\sigma}{\rho}\right]$ $\left[\text{as } V = \dfrac{m}{\rho}\right]$

So for equilibrium of ball,

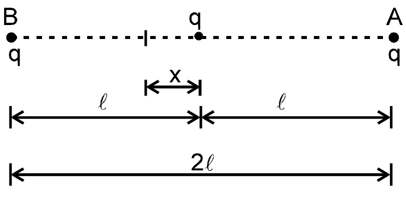
$\tan\theta' = \dfrac{F'}{mg'} = \dfrac{F}{Kmg[1-(\sigma/\rho)]}$ ... (4)

According to given information $\theta' = \theta$; so from equations (4) and (3), we have

$K = \dfrac{\rho}{(\rho-\sigma)} = \dfrac{1.6}{(1.6-0.8)} = 2$ **Ans.**

Ex.10 A particle of mass m and charge q is located midway between two fixed charged particles each having a charge q and a distance 2ℓ apart. Prove that the motion of the particle will be SHM if it is displaced slightly along the line connecting them and released. Also find its time period.

Sol. Let the charge q at the mid-point the displaced slightly to the left.



The force on the displaced charge q due to charge q at A,

$$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(\ell+x)^2}$$

The force on the displaced charge q due to charge at B,

$$F_2 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(\ell-x)^2}$$

Net force on the displaced charge q.

$$F = F_2 - F_1 \text{ or } F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(\ell-x)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(\ell+x)^2}$$

or $$F = \frac{q^2}{4\pi\varepsilon_0}\left[\frac{1}{(\ell-x)^2} - \frac{1}{(\ell+x)^2}\right] = \frac{q^2}{4\pi\varepsilon_0}\frac{4\ell x}{(\ell^2-x^2)^2}$$

Since $\ell >> x$, $\therefore$ $F = \dfrac{q^2 \ell x}{\pi\varepsilon_0 \ell^4}$ or $F = \dfrac{q^2 x}{\pi\varepsilon_0 \ell^3}$

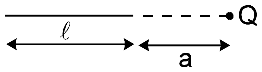
We see that $F \propto x$ and it is opposite to the direction of displacement. Therefore, the motion is SHM.

$$T = 2\pi\sqrt{\frac{m}{k}} \text{ , here } k = \frac{q^2}{\pi\epsilon_0\, \ell^3} \qquad = 2\pi\sqrt{\frac{m\pi\epsilon_0\, \ell^3}{q^2}}$$

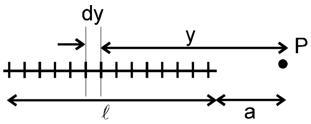
Q.4 A particle of mass m and charge –q is located midway between two fixed charged particles each having a charge q and a distance 2ℓ apart. Prove that the motion of the particle will be SHM if it is displaced slightly along perpendicular bisector and released. Also find its time period.

Ans. $2\pi\sqrt{\dfrac{m\ell^3}{2Kq^2}}$, where $K = \dfrac{1}{4\pi\varepsilon_0}$.

Ex.11 A thin straight rod of length l carrying a uniformly distributed change q is located in vacuum. Find the magnitude of the electric force on a point charge 'Q' kept as shown in the figure.



Sol.



As the charge on the rod is not point charge, therefore, first we have to find force on charge Q due to charge over a very small part on the length of the rod. This part called element of length dy can be considered as point charge.

Charge on element $dq = \lambda dy = \dfrac{q}{\ell}dy$

Electric force on 'Q' due to element $= \dfrac{K.dq.Q}{y^2} = \dfrac{K.Q.q.dy}{y^2.\ell}$

All forces are along the same direction

$\therefore$ $F = \sum dF$ This sum can be calculated using integration, therefore $F = \displaystyle\int_{y=a}^{a+\ell} \frac{KQqdy}{y^2\ell}$

$$= \frac{KqQ}{\ell}\left[-\frac{1}{y}\right]_a^{a+\ell} = \frac{KQ.q}{\ell}\left[\frac{1}{a} - \frac{1}{a+\ell}\right] = \frac{KQq}{a(a+\ell)}$$

Note : If a >> l then $F = \dfrac{KQq}{a^2}$

behaviour of the rod is just like a point charge.

Q.5 Three identical spheres each having a charge q (uniformly distributed) and radius R, are kept in such a way that each touches the other two. Find the magnitude of the electric force on any sphere due to other two.

Ans. $\dfrac{1}{4\pi\varepsilon_0}\dfrac{\sqrt{3}}{r^2}$

Q.6 Two charges of Q each are placed at two opposite corners of a square. A charge q is placed at each of the other two corners.
(a) If the resultant force on Q is zero, how are Q and q related ?
(b) Could q be chosen to make the resultant force on each charge zero ?

Ans. (a) $Q = -2\sqrt{2}\,q$, (b) No.

6. ELECTRIC FIELD

Electric field is the region around charged particle or charged body in which if another charge is placed, it experiences electrostatic force.

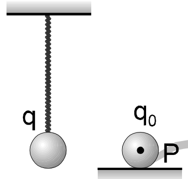
6.1 Electric field intensity $\vec{E}$: Electric field intensity at a point is equal to the electrostatic force experienced by a unit positive point charge both in magnitude and direction.

If a test charge q_0 is placed at a point in an electric field and experiences a force $\vec{F}$, the electric field intensity at that point is given by $\vec{E} = \dfrac{\vec{F}}{q_0}$;

If the $\vec{E}$ is to be determined practically then the test charge q_0 should be small otherwise it will affect the charge distribution which is producing the electric field and hence modify the quantity which is measured.

Ex.12 A positively charged ball hangs from a long silk thread. We wish to measure E at a point in the same horizontal plane as that of the hanging charge. To do so, we put a positive test charge q_0 at the point and measure F/q_0. Will F/q_0 be less than, equal to, or greater than E at the point in question?

Sol.



When we try to measure the electric field at point P then after placing the test charge at P it repels the source charge (suspended charge) and the measured value of electric field $E_{measured} = \dfrac{F}{q_0}$ will be less than the actual value E_{act}.

6.2 Properties of electric field intensity $\vec{E}$:

(i) It is a vector quantity. Its direction is the same as the force experienced by positive charge.

(ii) Electric field due to positive charge is always away from it while due to negative charge always towards it.

(iii) S.I. unit is Newton/Coulomb.

(iv) Its dimensional formula is $[MLT^{-3}A^{-1}]$

(v) Electric force on a charge q placed in a region of electric field at a point where the electric field intensity is $\vec{E}$ is given by $\vec{F} = q\vec{E}$.

Electric force on point charge is in the same direction of electric field on positive charge and in opposite direction on a negative charge.

(vi) It obeys the superposition principle, that is, the field intensity at a point due to a point charge distribution is vector sum of the field intensities due to individual point charges.

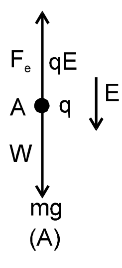
$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ...$$

Ex.13 Calculate the electric field intensity which would be just sufficient to balance the weight of a particle of charge $-10\ \mu c$ and mass 10 mg.

Sol. As force on a charge q in an electric field $\vec{E}$ is

$$\vec{F}_q = q\vec{E}$$

So according to given problem



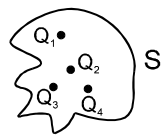
$$|\vec{F}_q| = |\vec{W}| \qquad \text{i.e.,} \qquad |q|E = mg$$

i.e., $E = \dfrac{mg}{|q|} = 10$ N/C., in downward direction.

Ex.14 Electrostatic force experienced by $-3\mu C$ charge placed at point

P due to a point charge system S as shown in figure is

$\vec{F} = 21\hat{i} + 9\hat{j}$ N.



(i) Find out electric field intensity at point P due to S.

(ii) If now 2μC charge is placed and –3 μC is removed at point P then force expereniced by it will be.

Sol. (i) $\vec{F} = q\vec{E}$

$21\hat{i} + 9\hat{j} = -3\mu C(\vec{E}) \quad \Rightarrow \quad \vec{E} = -7\hat{i} - 3\hat{j}$

(ii) Since the source charges are not disturbed the electric field intensity at 'P' will remain same.

$\vec{F}_{2\mu C} = +2(\vec{E}) \quad = 2(-7\hat{i} - 3\hat{j}) \quad = -14\hat{i} - 6\hat{j}$ N

Ex.15 Find out electric field intensity at point A (0, 1m, 2m) due to a point charge –20μC situated at point B($\sqrt{2}$ m, 0, 1m).

Sol.
$$E = \frac{KQ}{|\vec{r}|^3}\vec{r} = \frac{KQ}{|\vec{r}|^2}\hat{r}$$

$\vec{r}$ = P.V. of A – P.V. of B (P.V. = Position vector)

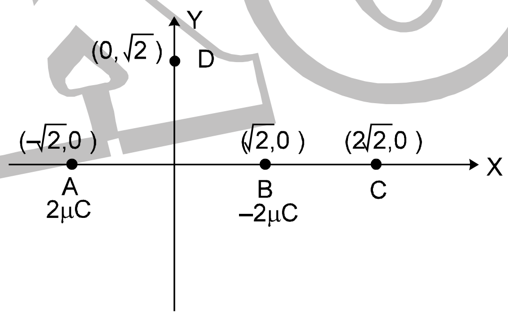
$= (-\sqrt{2}\ \hat{i} + \hat{j} + \hat{k})$

$$|\vec{r}| = \sqrt{(\sqrt{2})^2 + (1)^2 + (1)^2} = 2$$

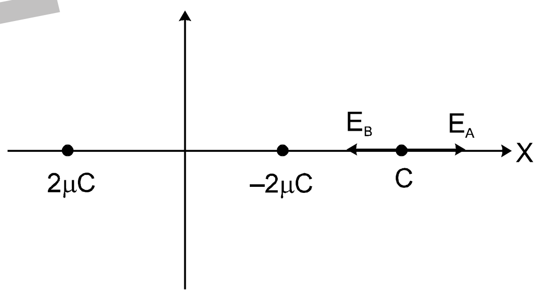
$$E = \frac{9\times10^9 \times (-20\times10^{-6})}{8}(-\sqrt{2}\,\hat{i} + \hat{j} + \hat{k})$$

$= -22.5 \times 10^3 (-\sqrt{2}\ \hat{i} + \hat{j} + \hat{k})$ N/C.

Ex.16 Two point charges 2μc and – 2μc are placed at point A and B as shown in figure. Find out electric field intensity at points C and D. [All the distances are measured in metre].



Sol. Electric field at point C



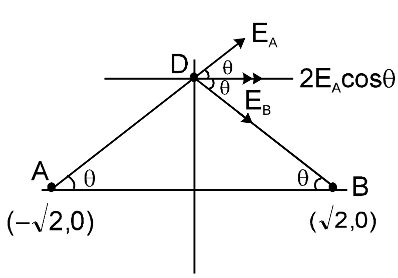
(E_A, E_B are magnitudes only and arrows respresent directions)

Electric field due to positive charge is away from it while due to negative charge it is towards the charge. It is it is clear that $E_B > E_A$.

$\therefore \quad E_{Net} = (E_B - E_A)$ towards negative X-axis

$$= \frac{K(2\mu c)}{(\sqrt{2})^2} - \frac{K(2\mu c)}{(3\sqrt{2})^2} \text{ towards negative X-axis}$$

$= \quad 8000\ (-\hat{i})$ N/C



Electric field at point D :

SInce magnitude of charges are same and also AD = BD

So $E_A = E_B$

Verticle components of $\vec{E}_A$ and $\vec{E}_B$ cancel each other while horizontal components are in the same direction.

So, $E_{net} = 2E_A \cos\theta \qquad = \dfrac{2.K(2\mu c)}{2^2}\cos 45^0 \qquad = \dfrac{K \times 10^{-6}}{\sqrt{2}} = \dfrac{9000}{\sqrt{2}}\hat{i}$ N/C.

Q.7 Three charges, each equal to q, are placed at the three corners of a square of side a. Find the electric field at the fourth corner.

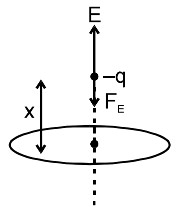
Ans. $(2\sqrt{2}+1)\dfrac{q}{8\pi\varepsilon_0 a^2}$.

Ex.17 Positive charge Q is distributed uniformly over a circular ring of radius R. A point particle having a mass m and a negative charge –q, is placed on its axis at a distance x from the centre. Find the force on the particle. Assuming x << R, find the time period of oscillation of the particle if it is released from there. (Neglect gravity)

Ans. When the negative charge is shifted at a distance x from the centre of the ring along its axis then force acting on the point charge due to the ring:

$F_E = qE$ (towards centre)

$$= q\left[\frac{KQx}{(R^2+x^2)^{3/2}}\right]$$



if R >>x then $R^2 + x^2 \simeq R^2$

$$F_E = \frac{1}{4\pi\varepsilon_0}\frac{Qqx}{R^3} \qquad \text{(Towards centre)}$$

Since restoring force $F_E \propto x$, therefore motion of charge the particle will be S.H.M.

Time period of SHM.

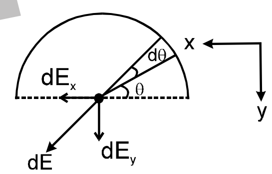
$$T = 2\pi\sqrt{\frac{m}{k}} \qquad = 2\pi\sqrt{\frac{m}{\left(\dfrac{Qq}{4\pi\varepsilon_0 R^3}\right)}} = \left[\frac{16\pi^3\varepsilon_0 mR^3}{Qq}\right]^{1/2}$$

Ex.18 Find out electric field intensity at the centre of uniformly charged semicircular ring of radius R and linear charge density λ.

Sol. λ = linear charge density.

The arc is the collection of large no. of point charges.

Consider a part of ring as an element of length $Rd\theta$ which substends an angle $d\theta$ at centre of ring and it lies between θ and $\theta + d\theta$

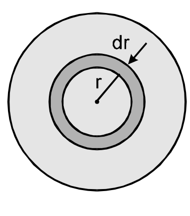


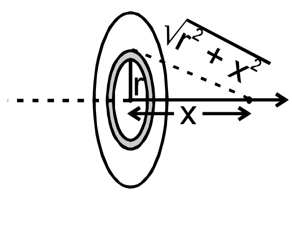
$$\overrightarrow{dE} = dE_x\,\hat{i} + dE_y\,\hat{j}$$

$$E_x = \int dE_x = 0 \quad \text{(due to symmatry)}$$

$$E_y = \int dE_y = \int_0^{\pi} dE\sin\theta$$

$$E_y = \frac{K\lambda}{R}\int_0^{\pi}\sin\theta.d\theta \qquad = \frac{2K\lambda}{R}$$





Ex.19 Derive the expression of electric field intensity at a point 'P' which is situated at a distance x on the axis of uniformly charged disc of radius R and surface charge density σ. Also derive results for

(i) x >> R (ii) x << R

Sol. The disc can be considered to be a collection of large number of concentric rings.

Consider an element of the shape of ring of radius r and of width dr. Electric field due to this ring at P is

$$dE = \frac{K.\sigma 2\pi r.dr.x}{(r^2+x^2)^{3/2}}$$

Put $r^2 + x^2 = y^2$

$2rdr = 2ydy$

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$$dE = \frac{K.\sigma 2\pi y.dy.x}{y^3} = 2K\sigma\pi.x\ \frac{ydy}{y^3}$$

Electric field at P due to all rings is along the axis.

$\therefore \qquad E = \int dE$

$$\Rightarrow \qquad E = 2K\sigma\pi x \int_x^{\sqrt{R^2+x^2}} \frac{1}{y^2}dy \qquad = 2K\rho\pi x.\left[-\frac{1}{y}\right]_x^{\sqrt{R^2+x^2}}$$

$$= 2K\sigma\pi x\left[+\frac{1}{x}-\frac{1}{\sqrt{R^2+x^2}}\right] \qquad = 2K\sigma\pi\left[1-\frac{x}{\sqrt{R^2+x^2}}\right]$$

$$= \frac{\sigma}{2\varepsilon_0}\left[1-\frac{x}{\sqrt{R^2+x^2}}\right], \text{ along the axis}$$

Cases :

(i) If x >> R

$$E = \frac{\sigma}{2\varepsilon_0}\ [1-\frac{x}{x\sqrt{\frac{R^2}{x^2}+1}}] \qquad = \frac{\sigma}{2\varepsilon_0}\ [1-\left(1+\frac{R^2}{x^2}\right)^{-1/2}]$$

$$= \frac{\sigma}{2\varepsilon_0}\ [1-1+\frac{1}{2}\ \frac{R^2}{x^2} + \text{higher order terms}]$$

$$= \frac{\sigma}{4\varepsilon_0}\ \frac{R^2}{x^2} = \frac{\sigma\pi R^2}{4\pi\varepsilon_0 x^2} \qquad = \frac{Q}{4\pi\varepsilon_0 x^2}$$

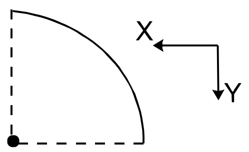
i.e. behaviour of the disc is like a point charge.

(ii) If x << R

$$E = \frac{\sigma}{2\varepsilon_0}\ [1-0] = \frac{\sigma}{2\varepsilon_0}$$

i.e. behaviour of the disc is like infinite sheet.

Q.8 Find out electric field intensity at the centre of uniformly charged circular arc (quarter ring) of radius R and linear charge density λ.

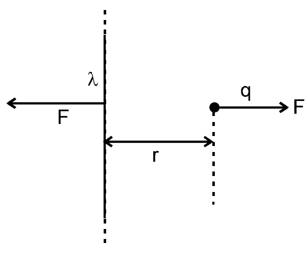


Ans. $\frac{K\lambda}{R}(\hat{i}+\hat{j})$

Ex.20 A point charge q is placed at a distance r from a very long charge thread of uniform linear charge density λ. Find out total electric force experienced by the line charge due to the point charge. (Neglect gravity).

Sol. Force on charge q due to the thread,

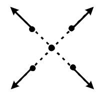
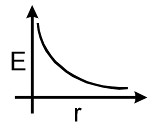
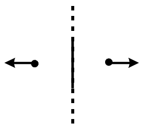
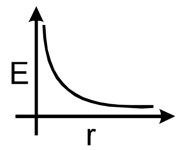
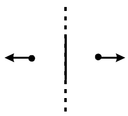
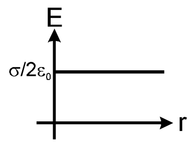
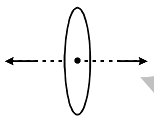
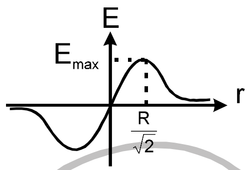
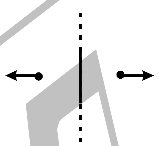
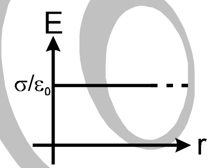
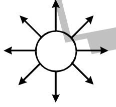
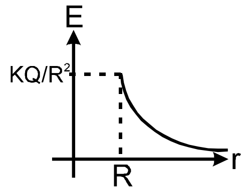
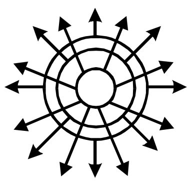
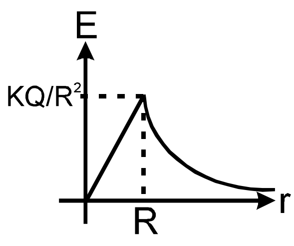
$$F = \left(\frac{2K\lambda}{r}\right).q$$



By Newton's III law, every action has equal and opposite reaction so force on the thread = $\frac{2K\lambda}{r}.q$ (away from point charge)

Ex.21 A block having mass m and charge –q is resting on a frictionless plane at a distance L from fixed large non-conducting infinite sheet of uniform charge density σ as shown in Figure. Discuss the motion of the

6.3 Electric field intensities due to various charge distributions are given in table.

Name/Type	Formula	Note	Graph
Point charge 	$\frac{Kq}{\lvert\vec{r}\rvert^2}\cdot\hat{r} = \frac{Kq}{r^3}\vec{r}$	* q is source charge. * $\vec{r}$ is vector drawn from source charge to the test point. * Electric field is nonuniform.	
Infinitely long line charge 	$\frac{\lambda}{2\pi\varepsilon_0 r}\hat{r} = \frac{2K\lambda\hat{r}}{r}$	* λ is linear charge density (assumed uniform) * r is perpendicular distance of point from line charge. * $\hat{r}$ is radial unit vector drawn from the charge to test point.	
Infinite non-conducting thin sheet 	$\frac{\sigma}{2\varepsilon_0}\hat{n}$	* σ is surface charge density. (assumed uniform) * $\hat{n}$ is unit normal vector. * Electric field intensity is independent of distance.	
Uniformly charged ring 	$E = \frac{KQx}{(R^2+x^2)^{3/2}}$ $E_{centre} = 0$	* Q is total charge of the ring. * x = distance of point on the axis from centre of the ring. * electric field is always along the axis.	
Infinitely large charged conducting sheet 	$\frac{\sigma}{\varepsilon_0}\hat{n}$	* σ is the surface charge density (assumed uniform) * $\hat{n}$ is the unit vector perpendicular is the surface. * Electric field intensity is independent of distance	
Uniformly charged hollow conducting/ nonconducting /solid conducting sphere 	(i) for r $\geq$ R $\vec{E} = \frac{kQ}{\lvert\vec{r}\rvert^2}\hat{r}$ (ii) for r < R $\vec{E} = 0$	* R is radius of the sphere. * $\vec{r}$ is vector drawn from centre of sphere to the point. * Sphere acts like a point charge. placed at centre for points outside the sphere. * $\vec{E}$ is always along radial direction. * Q is total charge ($= \sigma 4\pi R^2$). (σ = surface charge density)	
Uniformly charged solid nonconducting sphere (insulating material) 	(i) for r $\geq$ R $\vec{E} = \frac{kQ}{\lvert\vec{r}\rvert^2}\hat{r}$ (ii) for r $\leq$ R $\vec{E} = \frac{KQ\vec{r}}{R^3} = \frac{\rho\vec{r}}{3\varepsilon_0}$	* $\vec{r}$ is vector drawn from centre of sphere to the point * Sphere acts like a point charge placed at the centre for points outside the sphere * $\vec{E}$ is always along radial dir^n * Q is total charge $(\rho.\frac{4}{3}\pi R^3)$. ($\rho$ = volume charge density) * Inside the sphere E $\propto$ r. * Outside the sphere E $\propto$ 1/r^2.	

Note : **(i) Net charge on a conductor remains only on the outer surface of a conductor. This property will be discussed in the article of the conductor. (article no.15)**
(ii) On the surface of spherical conductors charge is uniformly distributed.

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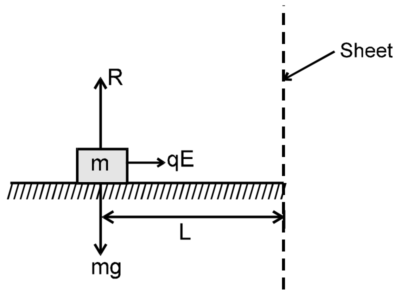
block assuming that collision of the block with the sheet is perfectly elastic. Is it SHM?

Sol. : The situation is shown in Figure. Electric force produced by sheet will accelerate the block towards the sheet producing an acceleration. Acceleration will be uniform because electric field E due to the sheet is uniform.



$a = \frac{F}{m} = \frac{qE}{m}$, where $E = \sigma/2\varepsilon_0$

As initially the block is at rest and acceleration is constant, from second equation of motion, time taken by the block to reach the wall

$$L = \frac{1}{2} at^2 \qquad \text{i.e.,} \qquad t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2mL}{aE}} = \sqrt{\frac{4mL\varepsilon_0}{a\sigma}}$$

As collision with the wall is perfectly elastic, the block will rebound with same speed and as now its motion is opposite to the acceleration, it will come to rest after travelling same distance L in same time t. After stopping it will be again accelerated towards the wall and so the block will execute oscillatory motion with 'span' L and time period.

$$T = 2t = 2\sqrt{\frac{2mL}{aE}} = 2\sqrt{\frac{4mL\varepsilon_0}{a\sigma}}$$

However, as the restoring force F = qE is constant and not proportional to displacement x, the motion is not simple harmonic.

Ex.22 Determine and draw the graph of electric field due to infinitely large nonconducting sheet of thickness t and uniform volume charge density ρ as a function of distance x from its symmetry plane.

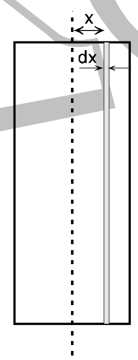
(a) $x \le \frac{t}{2}$ (b) $x \ge \frac{t}{2}$

Sol. We can assume thick sheet to be made of large number of uniformly charged thin sheets.Consider an elementry thin sheet of width dx at a distance x from symmatry plane.

Charge in sheet = $\rho A dx$

(A : assumed area of sheet)

surface charge density



$$s = \frac{\rho A dx}{A}$$

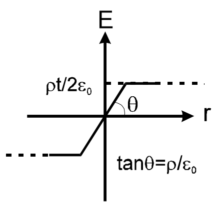
so, electric field intensity due to elementry sheet.

$$dE = \frac{\rho dx}{2\varepsilon_0}$$

(a) When $x < \frac{t}{2}$

$$E_{Net} = \int_{-t/2}^{x} \frac{\rho dx}{2\varepsilon_0} - \int_{x}^{t/2} \frac{\rho dx}{2\varepsilon_0} = \frac{\rho x}{\varepsilon_0}$$

(b) When $x > \frac{t}{2}$

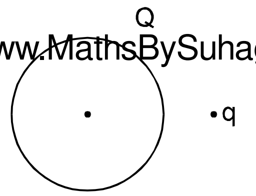


$$E_{Net} = \int_{-t/2}^{t/2} \frac{\rho dx}{2\varepsilon_0} = \frac{\rho t}{2\varepsilon_0}$$

Q.9 In the previous question if left half of the sheet contains charge density ρ and right half contains charge density 2ρ then find the electric field at the symmetry plane.

Ans. $E_{net} = \frac{\rho t}{4\varepsilon_0}$ (towards left)

Ex.23 Figure shows a uniformly charged sphere of radius R and total charge Q. A point charge q is situated outside the sphere at a distance r from centre of sphere. Find out the following :



(i) Force acting on the point charge q due to the sphere.

(ii) Force acting on the sphere due to the point charge.

Sol. (i) Electric field at the position of point charge

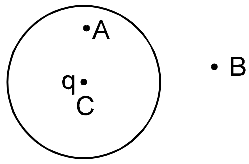
$$\vec{E} = \frac{KQ}{r^2}\hat{r} \qquad \text{so,} \qquad \vec{F} = \frac{KqQ}{r^2}\hat{r}$$

$$|\vec{F}| = \frac{KqQ}{r^2}$$

(ii) Since we know that every action has equal and opposite reaction so

$$\vec{F}_{\text{sphere}} = -\frac{KqQ}{r^2}\hat{r} \qquad \Rightarrow \quad |\vec{F}_{\text{sphere}}| = \frac{KqQ}{r^2}.$$

Ex.24 Figure shows a uniformly charged sphere of total charge Q and radius R. A point charge q is also situated at the centre of the sphere. Find out the following :



(i) Force on charge q

(ii) Electric field intensity at A.

(iii) Electric field intensity at B.

Sol. (i) Electric field at the centre of the uniformly charged hollow sphere = 0

So force on charge q = 0

(ii) Electric field at A

$$\vec{E}_A = \vec{E}_{\text{Sphere}} + \vec{E}_q \qquad = 0 + \frac{Kq}{r^2} \qquad ; \; r = CA$$

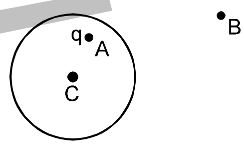
E due to sphere = 0 , because point lies inside the charged hollow sphere.

(iii) Electric field $\vec{E}_B$ at point B = $\vec{E}_{\text{Sphere}} + \vec{E}_q$

$$= \frac{KQ}{r^2}.\hat{r} + \frac{Kq}{r^2}.\hat{r} = \frac{K(Q+q)}{r^2}.\hat{r} \qquad ; \; r = CB$$

Note : Here we can also assume that the total charge of sphere is concentrated at the centre, for calculation of electric field at B.

Q.10 In the given example if point charge q is situated at point 'A' which is at a distance r < R from the centre of the sphere then find out following



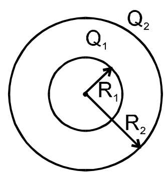
(i) Force acting on charge q.

(ii) Electric field at centre of sphere.

(iii) Electric field at point B.

Ans. (i) 0 (ii) $\frac{Kq}{(AC)^3}\overrightarrow{AC}$, AC = r (iii) $\frac{KQ}{CB^3}\overrightarrow{CB} + \frac{Kq}{AB^3}\overrightarrow{AB}$.

Ex.25 Two concentric uniformly charged spherical shells of radius R_1 and R_2 ($R_2 > R_1$) have total charges Q_1 and Q_2 respectively. Derive an expression of electric field as a function of r for following positions.



(i) $r < R_1$ (ii) $R_1 \le r < R_2$ (iii) $r \ge R_2$

Sol. (i) for $r < R_1$,

therefore point lies inside both the spheres

$$E_{net} = E_{Inner} + E_{outer}$$
$$= 0 + 0$$

(ii) for $R_1 \le r < R_2$,

therefore point lies outside inner sphere but inside outer sphere:

$$E_{net} = E_{inner} + E_{outer} \qquad = \frac{KQ_1}{r^2}\hat{r} + 0 \qquad = \frac{KQ_1}{r^2}\hat{r}$$

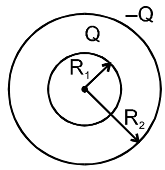
(iii) for $r \ge R_2$

point lies outside inner as well as outer sphere therefore.

$$E_{Net} = E_{inner} + E_{outer} \qquad = \frac{KQ_1}{r^2}\hat{r} + \frac{KQ_2}{r^2}\hat{r} \qquad = \frac{K(Q_1+Q_2)}{r^2}\hat{r}$$

Q.11 Figure shows two concentric sphere of radius R_1 and R_2 ($R_2 > R_1$) which contains uniformly distributed charges Q and –Q respectively. Find out electric field intensities at the following positions :

(i) $r < R_1$ (ii) $R_1 \le r < R_2$ (iii) $r \ge R_2$



Ans. (i) 0 (ii) $\frac{Kq}{r^2}\hat{r}$ (iii) 0.

Ex.26 A solid non conducting sphere of radius R and uniform volume charge density ρ has its centre at origin. Find out electric field intensity in vector form at following positions :

(i) (R/2, 0, 0) (ii) $\left(\frac{R}{\sqrt{2}}, \frac{R}{\sqrt{2}}, 0\right)$ (iii) (R, R, 0)

Sol. (i) at (R/2, 0, 0) : Distance of point from centre = $\sqrt{(R/2)^2+0^2+0^2}$ = R/2 < R, so point lies inside the sphere so $\vec{E} = \frac{\rho\vec{r}}{3\varepsilon_0} = \frac{\rho}{3\varepsilon_0}\left[\frac{R}{2}\hat{i}\right]$

(ii) At $\left(\frac{R}{\sqrt{2}}, \frac{R}{\sqrt{2}}, 0\right)$; distance of point from centre = $\sqrt{(R/\sqrt{2})^2+(R/\sqrt{2})^2+0^2}$ = R = R, so point lies at the surface of sphere, therefore

$$\vec{E} = \frac{KQ}{R^3}\vec{r} = \frac{K\frac{4}{3}\pi R^3\rho}{R^3}\left[\frac{R}{\sqrt{2}}\hat{i}+\frac{R}{\sqrt{2}}\hat{j}\right] = \frac{\rho}{3\varepsilon_0}\left[\frac{R}{\sqrt{2}}\hat{i}+\frac{R}{\sqrt{2}}\hat{j}\right]$$

(iii) The point is outside the sphere

so $$\vec{E} = \frac{KQ}{r^3}\vec{r} = \frac{K\frac{4}{3}\pi R^3\rho}{(\sqrt{2}R)^3}\left[R\hat{i}+R\hat{j}\right] = \frac{\rho}{6\sqrt{2}\varepsilon_0}\left[R\hat{i}+R\hat{j}\right]$$

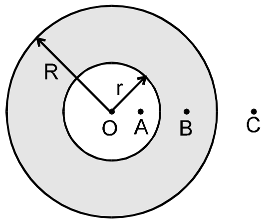
Q.12 A solid non conducting sphere of radius R and uniform volume charge density ρ has centre at origin. Find out electric field intensity in vector form at following positions.

(i) (R, 0, 0) (ii) $(0, 0, \frac{R}{2})$ (iii) (R, R, R)

Ans. (i) $\vec{E} = \frac{\rho R\hat{i}}{3\varepsilon_0}$ (ii) $\vec{E} = \frac{\rho R\hat{k}}{6\varepsilon_0}$ (iii) $\vec{E} = \frac{\rho(R\hat{i}+R\hat{j}+R\hat{k})}{9\sqrt{3}\varepsilon_0}$

Ex.27 A Uniformly charged solid nonconducting sphere of uniform volume charge density ρ and radius R is having a concentric spherical cavity of radius r. Find out electric field intensity at following points, as shown in the figure :

(i) Point A
(ii) Point B
(iii) Point C
(iv) Centre of the sphere



Sol. **Method I :**

(i) For point A :

We can consider the solid part of sphere to be made of large number of spherical shells which have uniformly distributed charge on its surface.Now since point A lies inside all spherical shells so electric field intensity due to all shells will be zero.

$$\vec{E_A} = 0$$

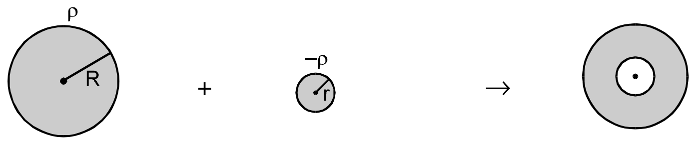
(ii) For point B :
All the spherical shells for which point B lies inside will make electric field zero at point B. So electric field will be due to charge present from radius r to OB.

So, $$\vec{E_B} = \frac{K\frac{4}{3}\pi(OB^3 - r^3)\rho}{OB^3}\vec{OB} = \frac{\rho}{3\varepsilon_0}\frac{[OB^3 - r^3]}{OB^3}\vec{OB}$$

(iii) For point C, similarly we can say that for all the shells point C lies outside the shell

$$\vec{E_C} = \frac{K[\frac{4}{3}\pi(R^3 - r^3)]}{[OC]^3}\vec{OC} = \frac{\rho}{3\varepsilon_0}\frac{R^3 - r^3}{[OC]^3}\vec{OC}$$

Method : II
We can consider that the spherical cavity is filled with charge density ρ and also –ρ, thereby making net charge density zero after combining. We can consider two concentric solid spheres one of radius R and charge density ρ and other of radius r and charge density –ρ. Applying superposition principle.



(i) $$\vec{E_A} = \vec{E_\rho} + \vec{E_{-\rho}} = \frac{\rho(\vec{OA})}{3\varepsilon_0} + \frac{\left[-\rho(\vec{OA})\right]}{3\varepsilon_0} = 0$$

(ii) $$\vec{E_B} = \vec{E_\rho} + \vec{E_{-\rho}}$$

$$= \frac{\rho(\vec{OB})}{3\varepsilon_0} + \frac{K\left[\frac{4}{3}\pi r^3(-\rho)\right]}{(OB)^3}\vec{OB} = \left[\frac{\rho}{3\varepsilon_0} - \frac{r^3\rho}{3\varepsilon_0(OB)^3}\right]\vec{OB} = \frac{\rho}{3\varepsilon_0}\left[1 - \frac{r^3}{OB^3}\right]\vec{OB}$$

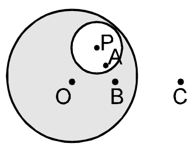
(iii) $$\vec{E_C} = \vec{E_\rho} + \vec{E_{-\rho}} = \frac{K\left(\frac{4}{3}\pi R^3\rho\right)}{OC^3}\vec{OC} + \frac{K\left(\frac{4}{3}\pi r^3(-\rho)\right)}{OC^3}\vec{OC}$$

$$= \frac{\rho}{3\varepsilon_0(OC)^3}\left[R^3 - r^3\right]\vec{OC}$$

(iv) $$\vec{E_O} = \vec{E_\rho} + \vec{E_{-\rho}}$$
$$= 0 + 0$$
$$= 0$$

Ex.28 In above question if cavity is not concentric and centred at point P then repeat all the steps.
Sol. Again assume ρ and –ρ in the cavity, similar to the previous example.

(i) $$\vec{E_A} = \vec{E_\rho} + \vec{E_{-\rho}}$$



$$= \frac{\rho[\vec{OA}]}{3\varepsilon_0} + \frac{(-\rho)\vec{PA}}{3\varepsilon_0}$$

$$= \frac{\rho}{3\varepsilon_0}[\vec{OA} - \vec{PA}] = \frac{\rho}{3\varepsilon_0}\left[\vec{OP}\right]$$

Note : Here we can see that the electric field intensity at point P is independent of position of point P inside the cavity. Also the electric field is along the line joining the centres of the sphere and the spherical cavity.

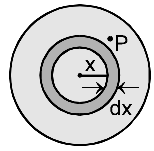
(ii) $\overrightarrow{E_B} = \overrightarrow{E_\rho} + \overrightarrow{E_{-\rho}}$ $= \dfrac{\rho(\overrightarrow{OB})}{3\varepsilon_0} + \dfrac{K\left[\frac{4}{3}\pi r^3(-\rho)\right]}{[PB]^3}\overrightarrow{PB}$

(iii) $\overrightarrow{E_C} = \overrightarrow{E_\rho} + \overrightarrow{E_{-\rho}}$ $= \dfrac{K\left[\frac{4}{3}\pi R^3\rho\right]}{[OC]^3}\overrightarrow{OC} + \dfrac{K\left[\frac{4}{3}\pi r^3(-\rho)\right]}{[PC]^3}\overrightarrow{PC}$

(iv) $\overrightarrow{E_O} = \overrightarrow{E_\rho} + \overrightarrow{E_{-\rho}}$ $= 0 + \dfrac{K\left[\frac{4}{3}\pi r^3(-\rho)\right]}{[PO]^3}\overrightarrow{PO}$

Ex.29 A nonconducting solid sphere has volume charge density that varies as $\rho = \rho_0$ r, where ρ_0 is a constant and r is distance from centre. Find out electric field intensities at following positions.
(i) $r < R$ (ii) $r \geq R$

Sol. **Method I :**
(i) for $r < R$
The sphere can be considered to be made of large number of spherical shells. Each shell has uniform charge density on its surface. So the previous results of the spherical shell can be used. Consider a shell of radius x and thickness dx as an element. Charge on shell $dq = (4\pi x^2 dx)\rho_0 x$
Electric field intensity at point P due to shell



$$dE = \frac{Kdq}{x^2}$$

Since all the shell will have electric field in same direction

$$E = \int_0^R dE = \int_0^r dE + \int_r^R dE$$

Due to shells which lie between region $r < x \leq R$, electric field at point P will be zero.

$$\vec{E} = \int_0^r \frac{Kdq}{r^2} + 0 = \int_0^r \frac{K.4\pi x^2 dx \rho_0 x}{r^2} = \frac{4\pi K\rho_0}{r^2}\left[\frac{x^4}{4}\right]_0^r = \frac{\rho_0 r^2}{4\varepsilon_0}\hat{r}$$

(ii) $r \geq R$

$$E = \int_0^R dE = \int_0^R \frac{K.4\pi x^2 dx \rho_0 x}{r^2} = \frac{\rho_0 R^4}{4\varepsilon_0 r^2}\hat{r}$$

Method II :

(i) The sphere can be considered to be made of large number of spherical shells. Each shell has uniform charge density on its surface. So the previous results of the spherical shell can be used. we can say that all the shells for which point lies inside will make electric field zero at that point,

so $$\vec{E}_{(r<R)} = \frac{K\int_0^r (4\pi x^2 dx)\rho_0 x}{r^2} = \frac{\rho_0 r^2}{4\varepsilon_0}\hat{r}$$

(ii) similarly for $r \geq R$, all the shells will contribute in electric field, therefore

$$\vec{E}_{(r<R)} = \frac{K\int_0^R (4\pi x^2 dx)\rho_0 x}{r^2} = \frac{\rho_0 R^4}{4\varepsilon_0 r^2}\hat{r}$$

7. ELECTRIC POTENTIAL

In electrostatic field the electric potential (due to some source charges) at a point is defined as the work done by external agent in taking a point unit positive charge from a reference point (generally taken at infinity) to that point without acceleration.

7.1 Mathematical representation :

If $(W_{\infty P})_{ext}$ is the work required in moving a point charge q from infinity to a point P, the electric potential of the point P is

$$V_p = \left.\frac{W_{\infty p})_{ext}}{q}\right]_{acc=0}$$

Note **(i)** W can also be called as the work done by external agent against the electric field produced by the source charge.

(ii) Write both W and q with proper sign.

7.2 Properties :

(i) Potential is a scalar quantity, its value may be positive, negative or zero.

(ii) S.I. Unit of potential is volt = $\frac{\text{joule}}{\text{coulmb}}$ and its dimensional formula is $[M^1L^2T^{-3}I^{-1}]$.

(iii) Electric potential at a point is also equal to the negative of the work done by the electric field in taking the point charge from reference point (i.e. infinity) to that point.

(iv) Electric potential due to a positive charge is always positive and due to negative charge it is always negative except at infinite. (taking $V_\infty = 0$).

(v) Potential decreases in the direction of electric field.

7.3 Use of potential :

If we know the potential at some point (interms of numerical value or interms of formula) then we can find out the work done by electric force when charge moves from point 'P' to ∞ by the formula

$$\mathbf{W_{el})_{p\,\infty} = qV_p}$$

Ex.30 A charge 2μC is taken from infinity to a point in an electric field, without changing its velocity. If work done against electrostatic forces is –40μJ then find the potential at that point.

Sol. $V = \frac{W_{ext}}{q} = \frac{-40\mu J}{2\mu C} = -20\ V$

Ex.31 When charge 10 μC is shifted from infinity to a point in an electric field, it is found that work done by electrostatic forces is 10 μJ. If the charge is doubled and taken again from infinity to the same point without accelerating it, then find the amount of work done by electric field and against electric field.

Sol. $W_{ext})_{\infty\,p} = -w_{el})_{\infty\,p} = w_{el})_{p\,\infty} = 10\ \mu J$

because $\Delta KE = 0$

$$V_p = \frac{W_{ext})_{\infty p}}{q} = \frac{10\mu J}{10\mu C} = 1V$$

So if now the charge is doubled and taken from infinity then

$$1 = \frac{w_{ext})_{\infty p}}{20\mu C} \Rightarrow W_{ext})_{\infty\,P} = 20\ \mu J$$

$$W_{el})_{\infty\,P} = -20\ \mu J$$

Ex.32 A charge 3μC is released at rest from a point P where electric potential is 20 V then its kinetic energy when it reaches to infinite is :

Sol. $W_{el} = \Delta K = K_f - 0$

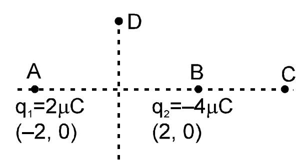
$W_{el})_{P\to\infty} = qV_P = 60\ \mu J$

so, $K_f = 60\ \mu J$

Q.13 A charge 10 μC is taken from infinity to a point in an electric field without acceleration. If work done by electrostatic forces is 30 μJ then find out potential at that point.

Ans. – 3 volt

Ex.33 Two point charges 2μC and – 4μC are situated at points (–2m, 0m) and (2m, 0m) respectively. Find out potential at point C. (4m, 0m) and. D (0m, $\sqrt{5}$ m).



Sol. Potential at point C

$$V_C = V_{q_1} + V_{q_2}$$

$= \frac{K(2\mu C)}{6} + \frac{K(-4\mu C)}{2} \qquad = \frac{9\times10^9\times2\times10^{-6}}{6} - \frac{9\times10^9\times4\times10^{-6}}{2}$

$= -15000$ V.

Similarly

$$V_D = V_{q_1} + V_{q_2} \qquad = \frac{K(2\mu C)}{\sqrt{(\sqrt{5})^2+2^2}} + \frac{K(-4\mu C)}{\sqrt{(\sqrt{5})^2+2^2}}$$

$$= \frac{K(2\mu C)}{3} + \frac{K(-4\mu C)}{3} \qquad = -6000 \text{ V.}$$

Ex.34 A point charge q_0 is placed at the centre of uniformly charged ring of total charge Q and radius R. If the point charge is slightly displaced with negligible force along axis of the ring then find out its speed when it reaches to a large distance.

Sol. Only electric force is acting on q_0

$\therefore \quad W_{el} = \Delta K \qquad = \frac{1}{2}mv^2 - 0$

$\Rightarrow \quad$ Now $W_{el})_{c\to\infty} = q_0 V_c = q_0 \cdot \frac{KQ}{R}$

$\therefore \quad \frac{Kq_0Q}{R} = \frac{1}{2}mv^2 \qquad \Rightarrow v = \sqrt{\frac{2Kq_0Q}{mR}}$

Ex.35 Two concentric spherical shells of radius R_1 and R_2 $(R_2 > R_1)$ are having uniformly distributed charges Q_1 and Q_2 respectively. Find out potential

(i) at point A

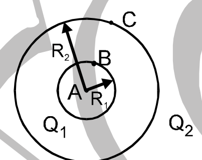
(ii) at surface of smaller shell (i.e. at point B)

(iii) at surface of larger shell (i.e. at point C)

(iv) at $r \le R_1$

(v) at $R_1 \le r \le R_2$

(vi) at $r \ge R_2$



Sol. Using the results of hollow sphere as given in the table 7.4.

(i) $\quad V_A = \frac{KQ_1}{R_1} + \frac{KQ_2}{R_2}$

(ii) $\quad V_B = \frac{KQ_1}{R_1} + \frac{KQ_2}{R_2}$

(iii) $\quad V_C = \frac{KQ_1}{R_2} + \frac{KQ_2}{R_2}$

(iv) $\quad$ for $r \le R_1$

$V = \frac{KQ_1}{R_1} + \frac{KQ_2}{R_2}$

(v) $\quad$ for $R_1 \le r \le R_2$

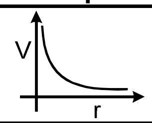
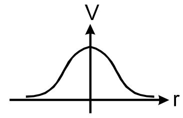
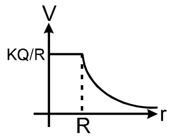
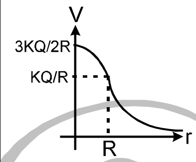
$V = \frac{KQ_1}{r} + \frac{KQ_2}{R_2}$

(vi) $\quad$ for $r \ge R_2$

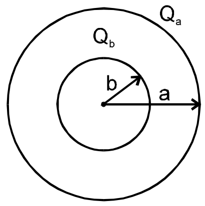
$V = \frac{KQ_1}{r} + \frac{KQ_2}{r}$

Ex.36 Two hollow concentric nonconducting spheres of radius a and b (a > b) contains charges Q_a and Q_b respectively. Prove that potential difference between two spheres is independent of charge on outer sphere. If outer sphere is given an extra charge, is there any change in potential difference?

7.4 Electric Potential due to various charge distributions are given in table.

Name/Type	Formula	Note	Graph
Point charge	$\frac{Kq}{r}$	* q is source chage. * r is the distance of the point from the point charge.	
Ring (uniform/nonuniform charge distribution)	at centre $\frac{KQ}{R}$ at the axis $\frac{KQ}{\sqrt{R^2+x^2}}$	* Q is source chage. * x is the distance of the point from centre.	
Uniformly charged hollow conducting/ nonconducting / solid conducting sphere	for $r \geq R$ $V = \frac{kQ}{r}$ for $r \leq R$ $V = \frac{kQ}{R}$	* R is radius of sphere * r is the distance from centre of sphere to the point * Q is total charge = $\sigma 4\pi R^2$.	
Uniformly charged solid nonconducting sphere (insulating material)	for $r \geq R$ $V = \frac{kQ}{r}$ for $r \leq R$ $\frac{KQ(3R^2 - r^2)}{2R^3}$ $= \frac{\rho}{6\varepsilon_0}(3R^2 - r^2)$	* R is radius of sphere * r is distance from centre. to the point * $V_{centre} = \frac{3}{2} V_{surface}$. * Q is total charge = $\rho \frac{4}{3}\pi R^3$. * Inside sphere potentail varies parabolically * outside potential varies hyperbolically.	
Line charge	Not defined	* Absolute potential is not defined. * Potential difference between two points is given by formula $v_B - v_A = -2K\lambda \ \ln (r_B / r_A)$	
Infinite nonconducting thin sheet	Not defined	* Absolute potential is not defined. * Potential difference between two points is given by formula $v_B - v_A = -\frac{\sigma}{2\varepsilon_0}(r_B - r_A)$	
Infinite charged conducting thin sheet	Not defined	* Absolute potential is not defined. * Potential difference between two points is given by formula $v_B - v_A = -\frac{\sigma}{\varepsilon_0}(r_B - r_A)$	

Sol. $V_{\text{inner sphere}} = \dfrac{KQ_b}{b} + \dfrac{KQ_a}{a}$



$V_{\text{outer sphere}} = \dfrac{KQ_b}{a} + \dfrac{KQ_a}{a}$

$V_{\text{inner sphere}} - V_{\text{outer sphere}} = \dfrac{KQ_b}{b} - \dfrac{KQ_b}{a}$

$\Delta V = KQ_b \left[\dfrac{1}{b} - \dfrac{1}{a}\right]$

Which is independent of charge on outer sphere.
If outer sphere in given any extra charge then there will be no change in potential differece.

7.5 Potential difference

The potential difference between two points A and B is work done by external agent against electric field in taking a unit positive charge from A to B without acceleration (or keeping Kinetic Energy constant or $K_i = K_f$))

(a) Mathematical representation :

If $(W_{BA})_{ext}$ = work done by external agent against electric field in taking the unit charge from A to B

$$V_A - V_B = \left.\frac{(W_{BA})_{ext}}{q}\right)_{acc=0,\text{ or keeping KE constant or } K_i = K_f}.$$

Note : Take W and q both with sign

(b) Properties :

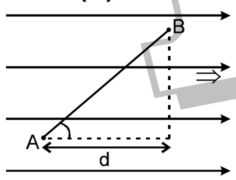
(i) The difference of potential between two points is called potential difference. It is also called voltage.

(ii) Potential difference is a scaler quantity. Its S.I. unit is also volt.

(iv) If V_A and V_B be the potential of two points A and B, then work done by an external agent in taking the charge q from A to B is $(W_{ext})_{AB} = q\,(V_B - V_A)$ or $(W_{el})_{AB} = q\,(V_A - V_B)$.

(v) Potential difference between two points is independent of reference point.

(c) Potential difference in a uniform electric field :



$\vec{E}$ $\quad$ $\mathbf{V_B - V_A = -\,\vec{E}\cdot\overrightarrow{AB}}$

$\Rightarrow \quad V_B - V_A = |E|\,|AB| \cos\theta$

$= -\,|E|\,d$

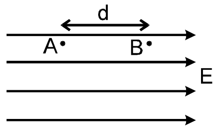
$= -\,Ed$

d = effective distance between A and B along electric field.

or we can also say that $E = \dfrac{\Delta V}{\Delta d}$

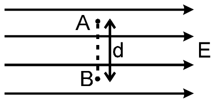
Special Cases :

Case 1. Line AB is parallel to electric field.



$\therefore \quad V_A - V_B = Ed$

Case 2. Line AB is perpendicular to electric field.



$\therefore \qquad V_A - V_B = 0 \qquad \Rightarrow \qquad V_A = V_D$

Note : In the direction of electric field potential always decreases.

Ex.37 1μC charge is shifted from A to B and it is found that work done by an external force is 40μJ in doing so against electrostatic forces then, find potential difference $V_A - V_B$

Sol. $(W_{AB})_{ext} = q(V_B - V_A)$

$40\ \mu J = 1\mu C\ (V_B - V_A)$

$\Rightarrow \quad V_A - V_B = -40$

Ex.38 A uniform electric field is present in the positive x-direction. If the intensity of the field is 5N/C then find the potential difference $(V_B - V_A)$ between two points A (0m, 2 m) and B (5 m ,3 m)

Sol. $V_B - V_A = -\vec{E} \cdot \overrightarrow{AB} \qquad = -(5\hat{i}) . (5\hat{i} + \hat{j}) \qquad = -25V.$

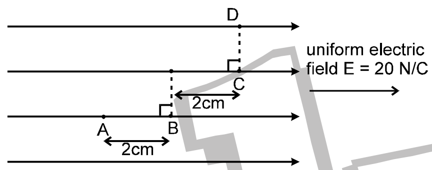
The electric field intensity in uniform electric field, $E = \dfrac{\Delta V}{\Delta d}$

Where ΔV = potential difference between two points.

Δd = effective distance between the two points.

(projection of the displacement along the direction of electric field.)

Ex.39 Find out following



(i) $V_A - V_B$ (ii) $V_B - V_C$ (iii) $V_C - V_A$ (iv) $V_D - V_C$ (v) $V_A - V_D$

(vi) Arrange the order of potential for points A, B, C and D.

Sol. (i) $|\Delta V_{AB}| = Ed = 20 \times 2 \times 10^{-2} = 0.4$

so, $V_A - V_B = 0.4$ V

because **In the direction of electric field potential always decreases.**

(ii) $|\Delta V_{BC}| = Ed = 20 \times 2 \times 10^{-2} = 0.4$

so, $V_B - V_C = 0.4$ V

(iii) $|\Delta V_{CA}| = Ed = 20 \times 4 \times 10^{-2} = 0.8$

so, $V_C - V_A = -0.8$ V

because **In the direction of electric field potential always decreases.**

(iv) $|\Delta V_{DC}| = Ed = 20 \times 0 = 0$

so, $V_D - V_C = 0$

because the effective distance between D and C is zero.

(v) $|\Delta V_{AD}| = Ed = 20 \times 4 \times 10^{-2} = 0.8$

so, $V_A - V_D = 0.8$ V

because **In the direction of electric field potential always decreases.**

(vi) The order of potential

$V_A > V_B > V_C = V_D$.

Q.15 A uniform electric field of 10 N/C exists in the vertically downward direction. Find the increases in the electric potential as one goes up through a height of 50cm.

Ans. 5V

Q.16 An electric field of 20 N/C exists along the x-axis in space. Calculate the potential difference $V_B - V_A$ where the point A and B are given by –

(a) A = (0,0) ; B = (4m , 2m) (b) A = (4m,2m) ; B = (6m , 5m)

(c) A = (0,0) ; B = (6m , 5m)

Answer : (a) –80V **(b)** –40V **(c)** – 120V

7.6 Equipotential Surface :

If potential of a surface is same throughout then such surface is known as a equipotential surface.

(i) Properties of equipotential surfaces :

(a) When a charge is shifted from one point to another point on an equipotential surface then work done against electrostatic forces is zero.

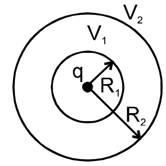
(b) Electric field is always perpendicular to equipotential surfaces.

(c) Two equipotential surfaces do not cross each other.

(ii) Examples of equipotential surfaces :

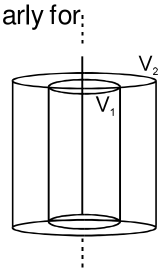
(a) Point charge :

Equipotential surfaces are concentric and spherical as shown in figure. In figure we can see that sphere of radius R_1 has potential V_1 throughout its surface and similarly for other concentric sphere potential is same.

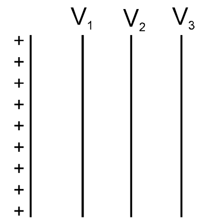


(b) Line charge :

Equipotential surfaces have curved surfaces as that of coaxial cylinders of different radii.

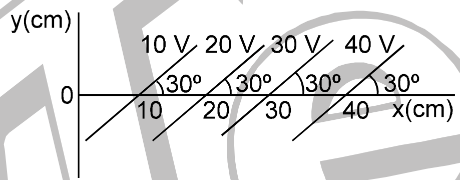


(c) Uniformly charged large conducting / non conducting sheets
Equipotential surfaces are parallel planes.



Note : In uniform electric field equipotential surfaces are always parallel planes.

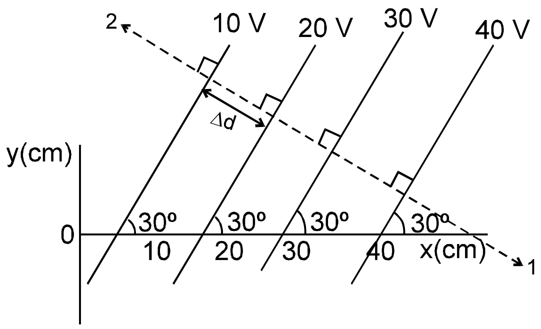
Ex.40 Some equipotential surfaces are shown in figure. What can you say about the magnitude and the direction of the electric field ?



Sol. Here we can say that the electric will be perpendicular to equipotential surfaces.

Also $|\vec{E}| = \dfrac{\Delta V}{\Delta d}$

where ΔV = potential difference between two equipotential surfaces.
Δd = perpendicular distance between two equipotential surfaces.

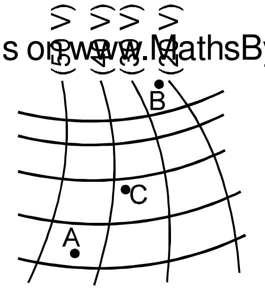


So $|\vec{E}| = \dfrac{10}{(10\sin 30^{\circ})\times 10^{-2}} = 200$ V/m

Now there are two perpendicular directions either direction 1 or direction 2 as shown in figure, but since we know that in the direction of electric field electric potential decreases so the correct direction is direction 2.

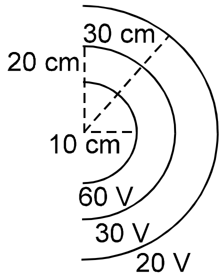
Hence E = 200 V/m, making an angle 120° with the x-axis

Ex.41 Figure shows the lines of constant potential in region in which an electric field is present. The values of potentials are written in brackets. The electric field is greatest



Sol. E is larger where equipotential surfaces are closer. ELOF are ⊥ to equipotential surfaces. In the figure we can see that for point B they are closer so E at point B is maximum

Q.17 Some equipotential surfaces are shown in figure. What can you say about the magnitude and the direction of the electric field ?



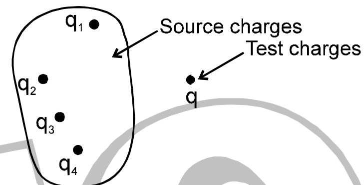
Ans. Radially outward, decreasing with distance as

$$E = \frac{6\,V - m}{r^2}\,.$$

8. ELECTROSTATIC POTENTIAL ENERGY

8.1 Electrostatic potential energy of a point charge due to many charges :

The electrostatic potential energy of a point charge at a point in electric field is the work done in taking the charge from reference point (generally at infinity) to that point without acceleration (or keeping KE const.or $K_i = K_f$)). Its Mathematical formula is U = qV



Here q is the charge whose potential energy is being calculated and V is the potential at its position due to the source charges.

Note : Always put q and V with sign.

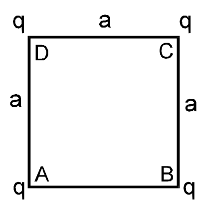
8.2 Properties :

(i) Electric potential energy is a scalar quantity but may be positive, negative or zero.

(ii) Its unit is same as unit of work or energy that is joule (in S.I. system).
Some times energy is also given in electron-volts.
$1\text{eV} = 1.6 \times 10^{-19}$ J

(iii) Electric potential energy depends on reference point. (Generally Potential Energy at r= ∞ is taken zero)

Ex.42 The four identical charges q each are placed at the corners of a square of side a. Find the potential energy of one of the charges due to the remaining charges.



The electric potential of point A due to the charges placed at B, C and D is

$$V = \frac{1}{4\pi\varepsilon_0}\,\frac{q}{a} + \frac{1}{4\pi\varepsilon_0}\,\frac{q}{\sqrt{2}a} + \frac{1}{4\pi\varepsilon_0}\,\frac{q}{a} = \frac{1}{4\pi\varepsilon_0}\left(2+\frac{1}{\sqrt{2}}\right)\frac{q}{a}$$

∴ Potential energy of the charge at A is = $qV = \dfrac{1}{4\pi\varepsilon_0}\left(2+\dfrac{1}{\sqrt{2}}\right)\dfrac{q^2}{a}$.

Ex.43 A particle of mass 40 mg and carrying a charge 5×10^{-9} C is moving directly towards a fixed positive point charge of magnitude 10^{-8} C. When it is at a distance of 10 cm from the fixed point charge it has spped of 50 cm/s. At what distance from the fixed point charge will the particle come momentarily to rest? Is the acceleration constant during the motion?

Sol. If the particle comes to rest momentarily at a distance r form the fixed charge, then from conservation of energy' we have

$$\frac{1}{2}mu^2 + \frac{1}{4\pi\varepsilon_0}\frac{Qq}{a} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r}$$

Substituting the given data, we get

$$\frac{1}{2} \times 40 \times 10^{-6} \times \frac{1}{2} \times \frac{1}{2} = 9 \times 10^9 \times 5 \times 10^{-8} \times 10^{-9}\left[\frac{1}{r} - 10\right]$$

or, $\frac{1}{r} - 10 = \frac{5\times10^{-6}}{9\times5\times10^{-8}} = \frac{100}{9}$ $\Rightarrow \frac{1}{r} = \frac{190}{9}$ $\Rightarrow r = \frac{9}{190}$ m

or, i.e., $r = 4.7 \times 10^{-2}$ m

As here, $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$ so acc. $= \frac{F}{m} \propto \frac{1}{r^2}$

i.e., acceleration is not constant during the motion.

Ex.44 A proton moves from a large distance with a speed u m/s directly towards a free proton originally at rest. Find the distance of closet approach for the two protons in terms of mass of proton m and its charge e.

Sol. As here the particle at rest is free to move, when one particle approaches the other, due to electrostatic repulsion other will also start moving and so the velocity of first particle will decrease while of other will increase and at closest approach both will move with same velocity. So if v is the common velocity of each particle at closest approach, then by 'conservation of momentum' of the two protons system.

$$mu = mv + mv \quad \text{i.e.,} \quad v = \frac{1}{2}u$$

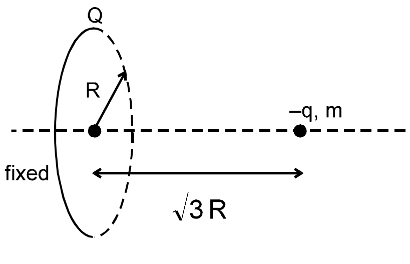
And by conservation of energy'

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

$$\Rightarrow \frac{1}{2}mu^2 - m\left(\frac{u}{2}\right)^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} \qquad [\text{as } v = \frac{u}{2}]$$

$$\Rightarrow \frac{1}{4}mu^2 = \frac{e^2}{4\pi\varepsilon_0 r} \qquad \Rightarrow \quad r = \frac{e^2}{\pi m\varepsilon_0 u^2}$$

Q.18 A point charge of charge –q and mass m is released with negligible speed from a distance $\sqrt{3}R$ on the axis of fixed uniformly charged ring of charge Q and radius R. Find out its velocity when it reaches at the centre of the ring.



Ans. $v = \sqrt{\frac{KQq}{Rm}}$

9. ELECTROSTATIC POTENTIAL ENERGY OF A SYSTEM OF CHARGES

(This will be used when more than one charges move.)

It is the work done by an external agent against the internal electric field required to make a system of charges in a particular configuration from infinite separation.

9.1 Types of system of charge

(i) Point charge system
(ii) Continuous charge system.

9.2 Derivation for a system of point charges:

(i) Keep all the charges at infinity. Now bring the charges one by one to its corresponding position and find work required. PE of the system is algebric sum of all the works.

Let W_1 = work done in bringing first charge
W_2 = work done in bringing second charge against force due to 1st charge.
W_3 = work done in bringing third charge against force due to 1st and 2nd charge.

PE = $W_1 + W_2 + W_3 + \ldots\ldots$. (This will contain $\frac{n(n-1)}{2} = {}^nC_2$ terms)

(ii) Method of calculation (to be used in problems)
U = sum of the interaction energies of the charges.
$= (U_{12} + U_{13} + \ldots\ldots\ldots + U_{1n}) + (U_{23} + U_{24} + \ldots\ldots\ldots + U_{2n}) + (U_{34} + U_{35} + \ldots\ldots\ldots + U_{3n}) \ldots.$

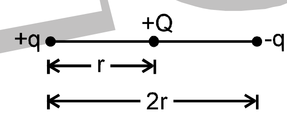
(iii) Method of calculation useful for symmetrical point charge systems.
Find PE of each charge due to rest of the charges.
If U_1 = PE of first charge due to all other charges.
$= (U_{12} + U_{13} + \ldots\ldots\ldots + U_{1n})$
U_2 = PE of second charges due to all other charges.
$= (U_{21} + U_{23} + \ldots\ldots\ldots + U_{2n})$

U = PE of the system = $\frac{U_1 + U_2 + \ldots}{2}$

Ex.45 Find out potential energy of the two point charge system having q_1 and q_2 charges separated by distance r.

Sol. Let both the charges be placed at a very large separation initially.
Let W_1 = work done in bringing charge q_1 in absence of $q_2 = q(V_f - V_i) = 0$
W_2 = work done in bringing charge q_2 in presence of $q_1 = q(V_f - V_i) = q_1(Kq_2/r - 0)$
$PE = W_1 + W_2 = 0 + Kq_1q_2 / r = Kq_1q_2 / r$

Ex.46 Figure shows an arrangement of three point charges.
The total potential energy of this arrangement is zero.



Calculate the ratio $\frac{q}{Q}$.

Sol.
$$U_{sys} = \frac{1}{4\pi\varepsilon_0}\left[\frac{-qQ}{r} + \frac{(+q)(-q)}{2r} + \frac{Q(-q)}{r}\right] = 0$$

$$-Q + \frac{q}{2} - Q = 0 \qquad \text{or} \qquad 2Q = \frac{q}{2} \text{ or } \quad \frac{q}{Q} = \frac{4}{1}.$$

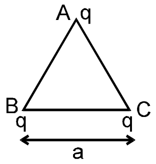
Ex.47 Two charged particles each having equal charges 2×10^{-5} C are brought from infinity to within a separation of 10 cm. Calculate the increase in potential energy during the process and the work required for this purpose.

Sol. $\Delta U = U_f - U_i = U_f - 0 = U_f$
We have to simply calculate the electrostatic potential energy of the given system of charges

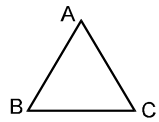
$$\Delta U = U_f = \frac{1}{4\pi\varepsilon_0}\ \frac{q_1q_2}{r} = \frac{9\times10^9 \times 2\times10^{-5} \times 2\times10^{-5} \times 100}{10} \text{ J} = 36 \text{ J}$$

work required = 36 J.

Ex. 48 Three equal charges q are placed at the corners of an equilateral triangle of side a.
(i) Find out potential energy of charge system.
(ii) Calculate work required to decrease the side of triangle to a/2.
(iii) If the charges are released from the shown position and each of them has same mass m then find the speed of each particle when they lie on triangle of side 2a.



Sol. **(i) Method I** (Derivation)



Assume all the charges are at infinity initially.
work done in putting charge q at corner A

$$W_1 = q\,(v_f - v_i) = q\,(0 - 0)$$

Since potential at A is zero in absence of charges, work done in putting q at corner B in presence of charge at A :

$$W_2 = \left(\frac{Kq}{a} - 0\right) = \frac{Kq^2}{a}$$

Similarly work done in putting charge q at corner C in presence of charge at A and B.

$$W_3 = q(v_f - v_i) \qquad = q\left[\left(\frac{Kq}{a} + \frac{Kq}{a}\right) - 0\right]$$

So net potential energy PE = $W_1 + W_2 + W_3$

$$= 0 + \frac{Kq^2}{a} + \frac{2Kq^2}{a} \qquad = \frac{3Kq^2}{a}$$

Method II (using direct formula)

$$U = U_{12} + U_{13} + U_{23}$$

$$= \frac{Kq^2}{a} + \frac{Kq^2}{a} + \frac{Kq^2}{a} \qquad = \frac{3Kq^2}{a}$$

(ii) Work required to decrease the sides

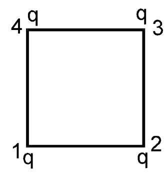
$$W = U_f - U_i \qquad = \frac{3Kq^2}{a/2} - \frac{3Kq^2}{a} = \frac{3Kq^2}{a}$$

(iii) Work done by electrostatic forces = change is kinetic energy of particles.

$$U_i - U_f = K_f - K_i$$

$$\Rightarrow \quad \frac{3Kq^2}{a} - \frac{3Kq^2}{2a} = 3\left(\frac{1}{2}mv^2\right) - 0 \quad \Rightarrow v = \sqrt{\frac{Kq^2}{am}}$$

Ex.49 Four identical point charges q are placed at four corners of a square of side a. Find out potential energy of the charge system



Sol. **Method 1 (using direct formula) :**

$$U = U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34}$$

$$= \frac{Kq^2}{a} + \frac{Kq^2}{a\sqrt{2}} + \frac{Kq^2}{a} + \frac{Kq^2}{a} + \frac{Kq^2}{a\sqrt{2}} + \frac{Kq^2}{a}$$

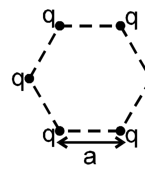
$$= \left[\frac{4Kq^2}{a} + \frac{2Kq^2}{a\sqrt{2}}\right] \qquad = \frac{2Kq^2}{a}\left[2 + \frac{1}{\sqrt{2}}\right]$$

Method 2 [using $U = \frac{1}{2}\,(U_1 + U_2 + ...)$] **:**

U_1 = total P.E. of charge at corner 1 due to all other charges
U_2 = total P.E. of charge at corner 2 due to all other charges
U_3 = total P.E. of charge at corner 3 due to all other charges
U_4 = total P.E. of charge at corner 4 due to all other charges
Since due to symmetry $U_1 = U_2 = U_3 = U_4$

$$U_{Net} = \frac{U_1+U_2+U_3+U_4}{2} = 2U_1 = 2\left[\frac{Kq^2}{a}+\frac{Kq^2}{a}+\frac{Kq^2}{\sqrt{2}a}\right] = \frac{2Kq^2}{a}\left[2+\frac{1}{\sqrt{2}}\right]$$

Q.19 Six equal point charges q are placed at six corners of a hexagon of side a. Find out potential energy of charge system



Ans. $\frac{3Kq^2}{a}\left[2+\frac{2}{\sqrt{3}}+\frac{1}{2}\right]$

9.3 Energy density :

Def: Energy density is defined as energy stored in unit volume in any electric field, its mathematical formula is given as following

Energy density = $\frac{1}{2}\varepsilon E^2$

where E = electric field intensity at that point
ε = electric permittivity of medium

Ex.50 Find out energy stored in an imaginary cubical volume of side a infront of a infinitely large nonconducting sheet of uniform charge density σ.

Sol. Energy stored

$$U = \int \frac{1}{2}\varepsilon_0 E^2\, dV \quad \text{where dV is small volume.}$$

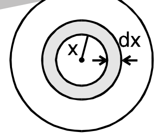
$$= \frac{1}{2}\varepsilon_0 E^2 \int dV \qquad \because \text{E is constant}$$

$$= \frac{1}{2}\varepsilon_0 \frac{\sigma^2}{4\varepsilon_0^2}\,.\, a^3\,. \qquad \Rightarrow \qquad U = \frac{\sigma^2 a^3}{8\varepsilon_0}$$

Ex.51 Find out energy stored inside a solid nonconducting sphere of total charge Q and radius R. [Assume charge is uniformly distributed in its volume.]

Sol. We can consider solid sphere to be made of large number of concentric spherical. Also electric field intensity at the location of any particular shell in constant.

$$U_{inside} = \int_0^R \frac{1}{2}\varepsilon_0 E^2 dV$$



Consider an elementry shell of thickness dx and radius x.
Volume of the shell = $(4\pi x^2 dx)$

$$U = \int_0^R \frac{1}{2}\varepsilon_0\left[\frac{KQx}{R^3}\right]^2 .\, 4\pi x^2\, dx = \frac{1}{2}\varepsilon_0 \frac{K^2Q^2 4\pi}{R^6}\int_0^R x^4 dx$$

$$= \frac{4\pi\varepsilon_0}{2R^6}\,\frac{Q^2}{(4\pi\varepsilon_0)^2}\,.\,\frac{R^5}{5} = \frac{Q^2}{40\pi\varepsilon_0 R} = \frac{KQ^2}{10R}.$$

9.4 For continues charge system :

This energy is also known as self energy.

(i) For hollow /solid uniformly charged conducting sphere or hollow uniformly charged non-conducting sphere :

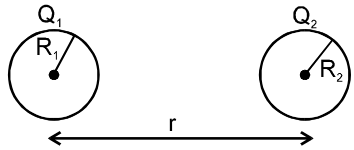
$$U_{self} = \frac{KQ^2}{2R} = \frac{Q^2}{8\pi\varepsilon_0 R}.$$

Q : charge on sphere,
R : Radius of sphere.

(ii) For uniformly charged solid nonconducting sphere:

$$U_{self} = \frac{3KQ^2}{5R} = \frac{3Q^2}{20\pi\varepsilon_0 R}$$

Ex.52 Two non-conducting hollow uniformly charged spheres of radii R_1 and R_2 with charge Q_1 and Q_2 respectively are placed at a distance r. Find out total energy of the system.



Sol. $U_{total} = U_{self} + U_{Interaction}$

$$= \frac{Q_1^2}{8\pi\varepsilon_0 R_1} + \frac{Q_2^2}{8\pi\varepsilon_0 R_2} + \frac{Q_1Q_2}{4\pi\varepsilon_0 r}$$

Ex.53 q_0 charge is placed at the centre of hollow conducting sphere of charge Q and radius R. Find out energy of system.

Sol. $$U_{total} = U_{self} + U_{interaction} = \frac{Q^2}{8\pi\varepsilon_0 R} + q_0\left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{R}\right) = \frac{Q}{4\pi\varepsilon_0 R}\left[\frac{Q}{2}+q_0\right]$$

10. ELECTRIC DIPOLE

If two point charges equal in magnitude q and opposite in sign separated by a distance a such that the distance of field point r>>a, the system is called a dipole. The electric dipole moment is defined as a vector quantity having magnitude p = (q × a) and direction from negative charge to positive charge.

Note: [In chemistry, the direction of dipole moment is assumed to be from positive to negative charge.] The C.G.S unit of electric dipole moment is **debye** which is defined as the dipole moment of two equal and opposite point charges each having charge 10^{-10} frankline and separation of 1 Å, i.e.,

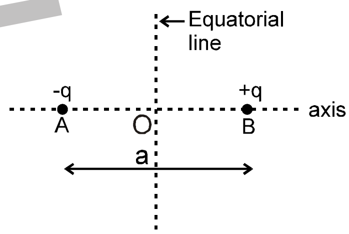
1 debye (D) = $10^{-10} \times 10^{-8} = 10^{-18}$ Fr × cm

$$1\ D = 10^{-18} \times \frac{C}{3\times10^9} \times 10^{-2}\ m = 3.3 \times 10^{-30}\ C \times m.$$

S.I. Unit is coulomb × metre = C . m

10.1 Electric Field Intensity Due to Dipole :

O the centre of the dipole is mid point of line AB.



(i) On the axis (except points between A and B)

$$\vec{E} = \frac{\vec{p}\,r}{2\pi\varepsilon_0[r^2-(a^2/4)]^2} \approx \frac{\vec{p}}{2\pi\varepsilon_0 r^3} \text{ (if } r >> a) = \frac{2K\vec{P}}{r^3}$$

$\vec{p} = q\vec{a}$ = Dipole moment,

r = distance of the point from the centre of dipole

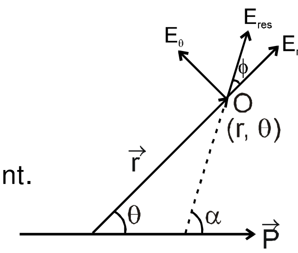
(ii) On the equatorial position :

$$\vec{E} = \frac{\vec{p}}{4\pi\varepsilon_0[r^2+(a^2/4)]^{3/2}} \approx -\frac{\vec{p}}{4\pi\varepsilon_0 r^3} \text{ (if } r >> a) = -\frac{K\vec{P}}{r^3}$$

(iii) Total electric field at general point O (r,θ) is $E_{res} = \frac{KP}{r^3}\sqrt{1+3\cos^2\theta}$,

$$E_r = \frac{2KP\cos\theta}{r^3}$$

$$E_\theta = \frac{KP\sin\theta}{r^3} \quad ; \qquad K = \frac{1}{4\pi\varepsilon_0}$$



At an angle $\alpha = \theta + \phi$ with the direction of dipole moment.

where $\tan\phi = \frac{\tan\theta}{2}$

10.2 Potential Energy of an Electric Dipole in External Electric Field :

$U = -\vec{p}.\vec{E}$

10.3 Electric Dipole in Uniform Electric Field :

torque $\vec{\tau} = \vec{p} \times \vec{E}$; $\vec{F} = 0$

10.4 Electric Dipole in Nonuniform Electric Field :

torque $\vec{\tau} = \vec{p} \times \vec{E}$; $U = -\vec{p}\cdot\vec{E}$, force and torque can be found by finding forces on individual charges.

10.5 Electric Potential Due to Dipole at General Point (r, θ) :

$$V = \frac{P\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{\vec{p}\,.\,\vec{r}}{4\pi\varepsilon_0 r^3} \text{ ; } \vec{p} = \text{electric dipole moment.}$$

Ex.54 A system has two charges $q_A = 2.5 \times 10^{-7}$ C and $q_B = -2.5 \times 10^{-7}$ C located at points A : (0, 0, – 0.15 m) and B ; (0, 0, + 0.15 m) respectively. What is the net charge and electric dipole moment of the system ?

Sol. Net charge $= 2.5 \times 10^{-7} - 2.5 \times 10^{-7} = 0$

Electric dipole moment,

P = (Magnitude of charge) × (Separation between charges)

$= 2.5 \times 10^{-7}[0.15 + 0.15]$ C m

$= 7.5 \times 10^{-8}$ C m

The direction of dipole moment is from B to A.

Ex.55 The electric field due to a short dipole at a distance r, on the axial line, from its mid point is the same as that of electric field at a distance r', on the equatorial line, from its mid-point. Determine the ratio $\frac{r}{r'}$.

Sol. $$\frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r'^3}$$

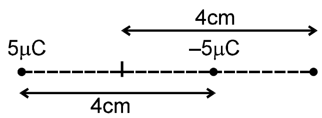
or $\frac{2}{r^3} = \frac{1}{r'^3}$ or $\frac{r^3}{r'^3} = 2$ or $\frac{r}{r'} = 2^{1/3}$

Ex.56 Two charges, each of 5 μC but opposite in sign, are placed 4 cm apart. Calculate the electric field intensity of a point that is at a distance 4 cm from the mid point on the axial line of the dipole.

Sol. We can not use formula of short dipole here because distance of the point is comparable to the distance between the two point charges.

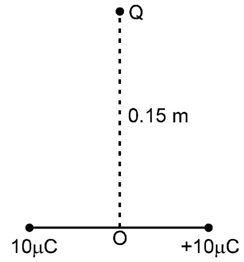
$q = 5 \times 10^{-6}$ C, $a = 4 \times 10^{-2}$ m, $r = 4 \times 10^{-2}$ m

$$E_{res} = E_+ + E_- = \frac{K(5\mu C)}{(2cm)^2} - \frac{K(5\mu C)}{(6cm)^2}$$



$$= \frac{144}{144\times10^{-8}} \text{ NC}^{-1} = 10^8 \text{ N C}^{-1}$$

Ex.57 Two charges ± 10 μC are placed 5×10^{-3} m apart. Determine the electric field at a point Q which is 0.15 m away from O, on the equitorial line.



Sol. In the given problem, r >> a

$$\therefore \qquad E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3} = \frac{1}{4\pi\varepsilon_0}\frac{q(a)}{r^3}$$

or $$E = 9 \times 10^9 \ \frac{10\times10^{-6}\times5\times10^{-3}}{0.15\times0.15\times0.15}\ NC^{-1}$$

$$= 1.33 \times 10^5\ NC^{-1}$$

Q.20 An electric dipole with dipole moment 4×10^{-9} C m makes an angle 30º with the direction of a uniform electric field of magnitude $5 \times 10^4\ NC^{-1}$. Calculate the magnitude of the torque acting on the dipole. Also find out work required to rotate the dipole to make angle 90º with the direction of the electric field.

Ans. 10^{-4} N m, $W_{req} = \Delta U = \sqrt{3} \times 10^{-4} J$.

Q.21 An electric dipole consists of two opposite charges, each of 1μC, separated by 0.02 m. The dipole is placed in an external uniform electric field of $10^5\ N C^{-1}$. Calculate the maximum torque exerted by the electric field on the dipole.

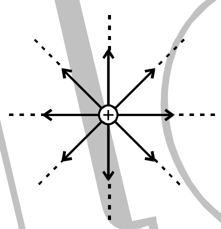
Ans. 0.002 Nm.

11. ELECTRIC LINES OF FORCE (ELOF)

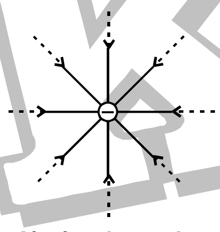
The line of force in an electric field is an imaginary line, the tangent to which at any point on it represents the direction of electric field at the given point.

11.1 Properties :

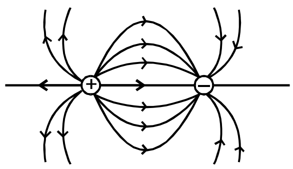
(i) Line of force originates out from a positive charge and terminates on a negative charge. If there is only one positive charge then lines start from positive charge and terminate at ∞. If there is only one negative charge then lines start from ∞ and terminates at negative charge.



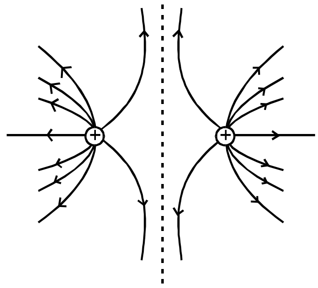
ELOF of Isolated positive charge



ELOF of Isolated negative charge

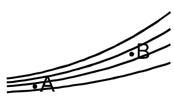


ELOF due to positive and negative charge



ELOF due to two positive charges

(ii) The electric intensity at a point is the number of lines of force streaming through per unit area normal to the direction of the intensity at that point. The intensity will be more where the density of lines is more.



$E_A > E_B$

(iii) Number of lines originating (terminating) from (on) is directly proportional to the magnitude of the charge.

Note:– A charge particle need not follow an ELOF.

Ex.58 If number of electric lines of force from charge q are 10 then find out number of electric lines of force from 2q charge.

Sol. No. of ELOF $\propto$ charge

$10 \propto q$

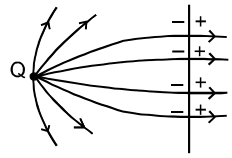
$\Rightarrow \quad 20 \propto 2q$

So number of ELOF will be 20.

(iv) ELOF of resultant electric field can never intersect with each other.
(v) Electric lines of force produced by static charges do not form close loop.
(vi) Electric lines of force end or start perpendicularly on the surface of a conductor.
(vi) Electric lines of force never enter in to conductors.

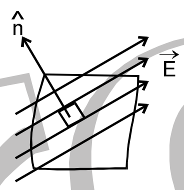
Ex.59 A charge + Q is fixed at a distance of d in front of an infinite metal plate. Draw the lines of force indicating the directions clearly.

Sol. There will be induced charge on two surfaces of conducting plate, so ELOF will start from +Q charge and terminate at conductor and then will again start from other surface of conductor.



12. ELECTRIC FLUX

Consider some surface in an electric field $\vec{E}$. Let us select a small area element $\overrightarrow{dS}$ on this surface. The electric flux of the field over the area element is given by $d\phi_E = \vec{E}.\overrightarrow{dS}$



Direction of $\overrightarrow{dS}$ is normal to the surface. It is along $\hat{n}$

or $\quad d\phi_E = EdS \cos\theta$

or $\quad d\phi_E = (E\cos\theta)\, dS$

or $\quad d\phi_E = E_n\, dS$

where E_n is the component of electric field in the direction of $\overrightarrow{dS}$.

The electric flux over the whole area is given by $\phi_E = \int_S \vec{E}.\overrightarrow{dS} = \int_S E_n dS$

If the electric field is uniform over that area then $\phi_E = \vec{E}\cdot\vec{S}$

12.1 Physical Meaning :

The electric flux through a surface inside an electric field represents the total number of electric lines of force crossing the surface in a direction normal the surface. It is a property of electric field

12.2 Unit

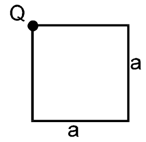
(i) The SI unit of electric flux is $Nm^2\, C^{-1}$ (gauss) or $J\, m\, C^{-1}$.
(ii) Electric flux is a scalar quantity. (It can be positive, negative or zero)

Ex.60 The electric field in a region is given by $\vec{E} = \frac{3}{5}E_0\vec{i} + \frac{4}{5}E_0\vec{j}$ with $E_0 = 2.0 \times 10^3$ N/C. Find the flux of this field through a rectangular surface of area $0.2m^2$ parallel to the Y–Z plane.

Sol. $\phi_E = \vec{E}\cdot\vec{S} \quad = \left(\frac{3}{5}E_0\vec{i} + \frac{4}{5}E_0\vec{j}\right) \cdot \left(0.2\,\hat{i}\right)$

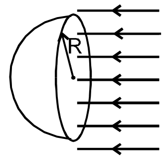
$= 240\,\frac{N-m^2}{C}$

Ex.61 A point charge Q is placed at the corner of a square of side a, then find the flux through the square.



Sol. The electric field due to Q at any point of the square will be along the plane of square and the electric field line are perpendicular to square ; so $\phi = 0$.
In other words we can say that no line is crossing the square so flux = 0.

Ex.62 Find out flux through the curved surface of the hemisphere of radius R if it is placed in uniform electric field E as shown in figure.



Sol The electric lines which are passing through area πR^2 are also the same which will pass through hemisphere.
so, $\phi = E\pi R^2$

13. GAUSS'S LAW IN ELECTROSTATICS OR GAUSS'S THEOREM

This law was stated by a mathematician Karl F Gauss. This law gives the relation between the electric field at a point on a closed surface and the net charge enclosed by that surface. This surface is called Gaussian surface. It is a closed hypothetical surface. Its validity is shown by experiments. It is used to determine the electric field due to some symmetric charge distributions.

13.1 Statement and Details :

Gauss's law is stated as given below.
The surface integral of the electric field intensity over any closed hypothetical surface (called Gaussian surface) in free space is equal to $\frac{1}{\varepsilon_0}$ times the total charge enclosed within the surface. Here, ε_0 is the permittivity of free space.

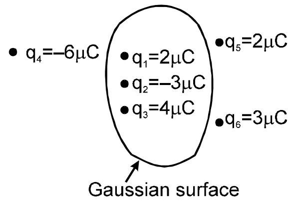
If S is the Gaussian surface and $\sum_{i=1}^{n} q_i$ is the total charge enclosed by the Gaussian surface, then according to Gauss's law,

$$\phi_E = \oint \vec{E} \cdot \overrightarrow{dS} = \frac{1}{\varepsilon_0} \sum_{i=1}^{n} q_i \,.$$

The circle on the sign of integration indicates that the integration is to be carried out over the closed surface.

Note : (i) Flux through gaussian surface is independent of its shape.
(ii) Flux through gaussian surface depends only on total charge present inside gaussian surface.
(iii) Flux through gaussian surface is independent of position of charges inside gaussian surface.
(iv) Electric field intensity at the gaussian surface is due to all the charges present inside as well as out side the gaussian surface.
(v) In a close surface incoming flux is taken negative while outgoing flux is taken positive, because $\hat{n}$ is taken positive in outward direction.
(vi) In a gaussian surface $\phi = 0$ does not imply E = 0 at every point of the surface but E = 0 at every point implies $\phi = 0$.

Ex.63 Find out flux through the given gaussian surface.



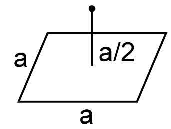
Sol. $\phi = \frac{Q_{in}}{\varepsilon_0} = \frac{2\mu C - 3\mu C + 4\mu C}{\varepsilon_0} = \frac{3\times 10^{-6}}{\varepsilon_0}$ Nm²/C

Ex.64 If a point charge q is placed at the centre of a cube then find out flux through any one surface of cube.

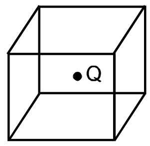
Sol. Flux through 6 surfaces = $\frac{q}{\varepsilon_0}$. Since all the surfaces are symmetrical

so, flux through one surfaces = $\frac{1}{6}\frac{q}{\varepsilon_0}$

Ex.65. A charge Q is placed at a distance a/2 above the centre of a horizontal, square surface of edge a as shown in figure. Find the flux of the electric field through the square surface.



Sol.



We can consider imaginary faces of cube such that the charge lies at the centre of the cube. Due to symmatry we can say that flux through the given area (which is one face of cube) $\phi = \frac{Q}{6\varepsilon_0}$

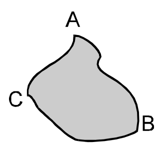
Q.22 A charge Q is uniformy distributed over a rod of length ℓ. Consider a hypothetical cube of edge ℓ with the centre of the cube at one end of the rod. Find the minimum possible flux of the electric field through the entire surface of the cube.
Ans. $Q/(2\varepsilon_0)$

Q.23 A charge Q is placed at a corner of a cube. Find the flux of the electric field through the six surfaces of the cube.
Ans. $Q/8\varepsilon_0$

14. CONDUCTOR AND IT'S PROPERTIES [FOR ELECTROSTATIC CONDITION]

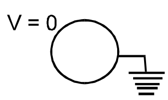
(i) Conductors are materials which contains large number of free electrons which can move freely inside the conductor.
(ii) In electrostatics conductors are always equipotential surfaces.
(iii) Charge always resides on outer surface of conductor.
(iv) If there is a cavity inside the conductor having no charge then charge will always reside only on outer surface of conductor.
(v) Electric field is always perpendicular to conducting surface.
(vi) Electric lines of force never enter into conductors.
(vii) Electric field intensity near the conducting surface is given by formula

$$\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$$



$$\vec{E_A} = \frac{\sigma_A}{\varepsilon_0}\hat{n} \;;\; \vec{E_B} = \frac{\sigma_B}{\varepsilon_0}\hat{n} \text{ and } \vec{E_C} = \frac{\sigma_C}{\varepsilon_0}\hat{n}$$

(viii) When a conductor is grounded its potential becomes zero.



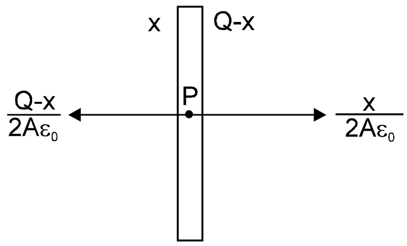
(ix) When an isolated conductor is grounded then its charge becomes zero.
(x) When two conductors are connected there will be charge flow till their potential becomes equal.

(xi) Electric pressure : Electric pressure at the surface of a conductor is givey by formula

$$P = \frac{\sigma^2}{2\varepsilon_0}$$ where σ is the local surface charge density.

Ex.66 Prove that if an isolated (isolated means no charges are near the sheet) large conducting sheet is given a charge then the charge distributes equally on its two surfaces.

Sol. Let there is x charge on left side of sheet and Q–x charge on right side of sheet.
Since point P lies inside the conductor so
$E_P = O$



$$\frac{x}{2A\varepsilon_O} - \frac{Q-x}{2A\varepsilon_O} = 0 \Rightarrow \frac{2x}{2A\varepsilon_O} = \frac{Q}{2A\varepsilon_O}$$

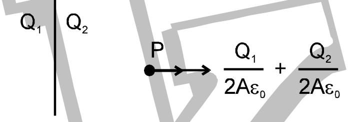
$$\Rightarrow x = \frac{Q}{2}$$

$$Q - x = \frac{Q}{2}$$

So charge in equally distributed on both sides

Ex.67 If an isolated infinite sheet contains charge Q_1 on its one surface and charge Q_2 on its other surface then prove that electric field intensity at a point in front of sheet will be $\frac{Q}{2A\varepsilon_O}$, where $Q = Q_1 + Q_2$

Sol.. Electric field at point P :



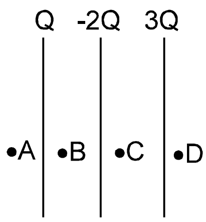
$$\vec{E} = \vec{E}_{Q_1} + \vec{E}_{Q_2}$$

$$= \frac{Q_1}{2A\varepsilon_0}\hat{n} + \frac{Q_2}{2A\varepsilon_0}\hat{n}$$

$$= \frac{Q_1 + Q_2}{2A\varepsilon_0}\hat{n} \quad = \frac{Q}{2A\varepsilon_0}\hat{n}$$

[This shows that the resultant field due to a sheet depends only on the total charge of the sheet and not on the distribution of charge on individual surfaces].

Ex.68 Three large conducting sheets placed parallel to each other at finite distance contains charges Q, –2Q and 3Q respectively. Find electric field at points A, B , C, and D



Sol. $E_A = E_Q + E_{-2Q} + E_{3Q}$.
Here E_Q means electric field due to 'Q'.

$$E_A = \frac{(Q-2Q+3Q)}{2A\varepsilon_0} = \frac{2Q}{2A\varepsilon_0} = \frac{Q}{A\varepsilon_0}\text{, towards left}$$

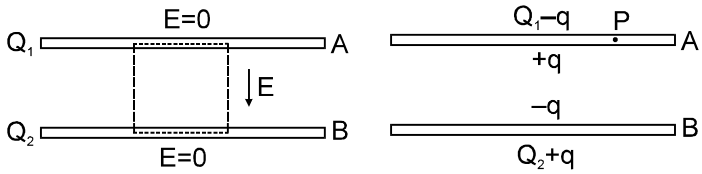
(ii) $$E_B = \frac{Q-(-2Q+3Q)}{2A\varepsilon_0}\text{, towards right} \qquad = 0$$

(iii) $$E_C = \frac{(Q-2Q)-(3Q)}{2A\varepsilon_0} = \frac{-4Q}{2A\varepsilon_0} = \frac{-2Q}{A\varepsilon_0}\text{, towards right} \Rightarrow \frac{2Q}{A\varepsilon_0}\text{ towards left}$$

(iv) $$E_D = \frac{(Q-2Q+3Q)}{2A\varepsilon_0} = \frac{2Q}{2A\varepsilon_0} = \frac{Q}{A\varepsilon_0}\text{, towards right}$$

Ex.69 Two conducting plates A and B are placed parallel to each other. A is given a charge Q_1 and B a charge Q_2. Prove tht the charges on the inner facing surfaces are of equal magnitude and opposite sign.

Sol. Consider a Gaussian surface as shown in figure. Two faces of this closed surface lie completely inside the conductor where the electric field is zero. The flux through these faces is, therefore, zero. The other parts of the closed surface which are outside the conductor are parallel to the electric field and hence the flux on these parts is also zero. The total flux of the electric field through the closed surface is, therefore zero. From Gauss's law, the total charge inside this closed surface should be zero. The charge on the inner surface of A should be equal and opposite to that on the inner surface of B.



The distribution should be like the one shown in figure. To find the value of q, consider the field at a point P inside the plate A. Suppose, the surface area of the plate (one side) is A. Using the equation $E = \sigma / (2\varepsilon_0)$, the electric field at P

due to the charge $Q_1 - q = \dfrac{Q_1 - q}{2A\varepsilon_0}$ (downward)

due to the charge $+ q = \dfrac{q}{2A\varepsilon_0}$ (upward),

due to the charge $- q = \dfrac{q}{2A\varepsilon_0}$ (downward),

and due to the charge $Q_2 + q = \dfrac{Q_2 + q}{2A\varepsilon_0}$ (upward).

The net electric field at P due to all the four charged surfaces is (in the downward direction)

$$E_p = \frac{Q_1 - q}{2A\varepsilon_0} - \frac{q}{2A\varepsilon_0} + \frac{q}{2A\varepsilon_0} - \frac{Q_2 + q}{2A\varepsilon_0}$$

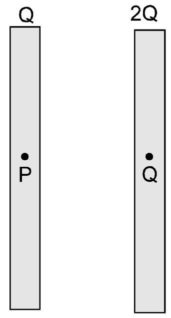
As the point P is inside the conductor, this field should be zero. Hence,

$$Q_1 - q - q + q - Q_2 - q = 0$$

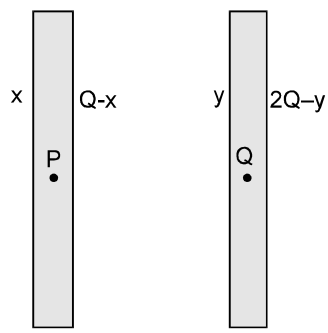
or, $$q = \frac{Q_1 - Q_2}{2}$$

This result is a special case of the following result. When charged conducting plates are placed parallel to each other, the two outermost, surfaces get equal charges and the facing surfaces get equal and opposite charges.

Ex.70 Two large parallel conducting sheets (placed at finite distance) are given charges Q and 2Q respectively. Find out charges appearing on all the surfaces.



Sol. Let there is x amount of charge on left side of first plate, so on its right side charge will be Q–x, similarly for second plate there is y charge on left side and 2Q – y charge is on right side of second plate



$E_p = 0$ (By property of conductor)

$$\Rightarrow \frac{x}{2A\varepsilon_o} - \left\{\frac{Q - x}{2A\varepsilon_o} + \frac{y}{2A\varepsilon_o} + \frac{2Q - y}{2A\varepsilon_o}\right\} = 0$$

we can also say that charge on left side of P = charge on right side of P

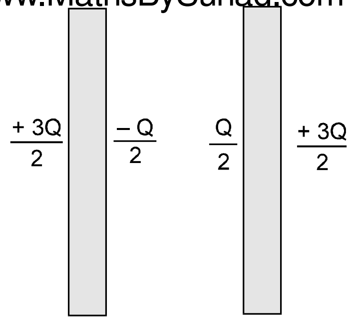
$$x = Q - x + y + 2Q - y \quad \Rightarrow x = \frac{3Q}{2}\ ,\ Q - x = \frac{-Q}{2}$$

Similarly for point Q:

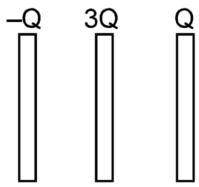
$x + Q - x + y = 2Q - y$

$\Rightarrow y = Q/2\ ,\ 2Q - y = 3Q/2$

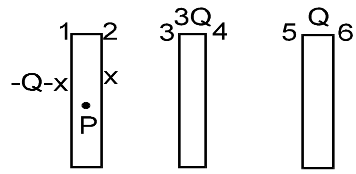
So final charge distribution of plates is : -



Ex.71 Figure shows three large metallic plates with charges – Q, 3Q and Q respectively. Determine the final charges on all the surfaces.



Sol. We assume that charge on surface 2 is x. Following conservation of charge, we see that surfaces 1 has charge $(-Q-x)$. The electric field inside the metal plate is zero so fields at P is zero.



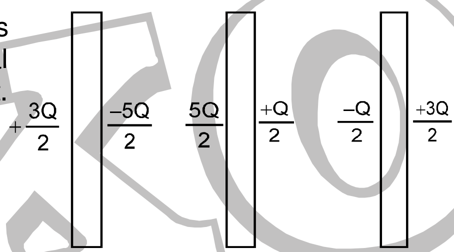
Resultant field at P -

$$E_P = 0$$

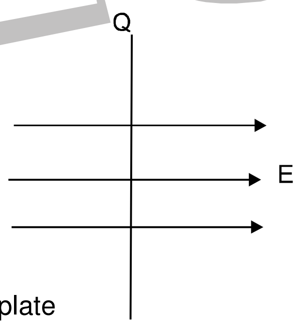
$$\Rightarrow \quad \frac{-Q-x}{2A\varepsilon_0} = \frac{x+3Q+Q}{2A\varepsilon_0} \quad \Rightarrow -Q - x = x + 4Q \quad \Rightarrow x = \frac{-5Q}{2}$$

Note : We see that charges on the facing surfaces of the plates are of equal magnitude and opposite sign. This can be in general proved by gauss theorem also. Remember this it is important result.

Thus the final charge distribution on all the surfaces is :



Ex.72 An isolated conducting sheet of area A and carrying a charge Q is placed in a uniform electric field E, such that electric field is perpendicular to sheet and covers all the sheet. Find out charges appearing on its two surfaces.



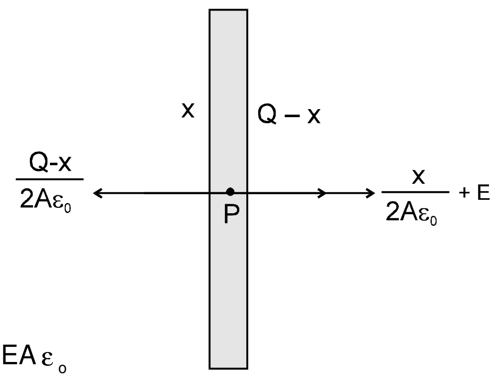
Sol.. Let there is x charge on left side of plate and Q – x charge on right side of plate

$$E_P = 0$$

$$\frac{x}{2A\varepsilon_0} + E = \frac{Q-x}{2A\varepsilon_0}$$

$$\Rightarrow \quad \frac{x}{A\varepsilon_0} = \frac{Q}{2A\varepsilon_0} - E$$

$$\Rightarrow \quad x = \frac{Q}{2} - EA\varepsilon_0 \ \text{ and } \ Q - x = \frac{Q}{2} + EA\varepsilon_0$$



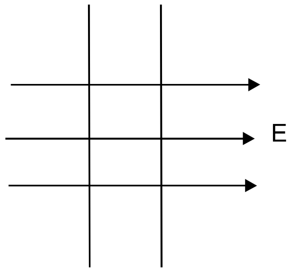
So charge on one side is $\frac{Q}{2} - EA\varepsilon_o$ and other side $\frac{Q}{2} + EA\varepsilon_o$

Note : Solve this question for Q = 0 without using the above answer and match that answers with the answers that you will get by putting Q = 0 in the above answer.

Q.24 In the above problem find the restant electric field on the left and right side of the plate.

Ans. [On right side E = $\frac{Q}{2A\varepsilon_0}$ + E towards right and on left side $\frac{Q}{2A\varepsilon_0}$ – E towards left.]

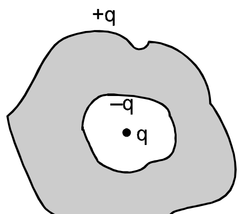
Ex.73 Two uncharged and parallel conducting sheets each of area A are placed in a uniform electric field E at a finite distance from each other. Such that electric field is perpendicular to sheets and covers all the sheets. Find out charges appearing on its two surfaces.



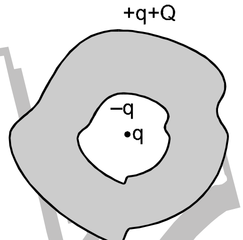
Ans. $-EA\varepsilon_0$, $+EA\varepsilon_0$, $-EA\varepsilon_0$, $+EA\varepsilon_0$,

14.1 Some other important results for a closed conductor.

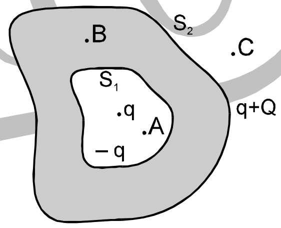
(i) If a charge q is kept in the cavity then –q will be induced on the inner surface and +q will be induced on the outer surface of the conductor (it can be proved using gauss theorem)



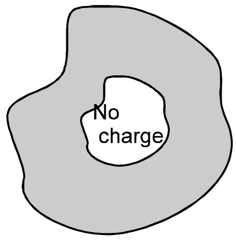
(ii) If a charge q is kept inside the cavity of a conductor and conductor is given a charge Q then –q charge will be induced on inner surface and total charge on the outer surface will be q + Q. (it can be proved using gauss theorem)



(iii) Resultant field, due to q (which is inside the cavity) and induced charge on S_1, at any point outside S_1 (like B,C) is zero. Resultant field due to **q + Q on S_2 and any other charge outside S_2**, at any point inside of surface S_2 (like A, B) is zero

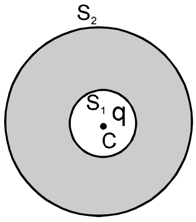


(iv) Resultant field in a charge free cavity in a closed conductor is zero. There can be charges outside the conductor and on the surface also. Then also this result is true. No charge will be induced on the inner most surface of the conductor.



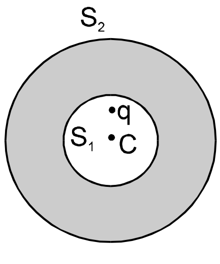
(v). Charge distribution for different types of cavities in conductors

(A)

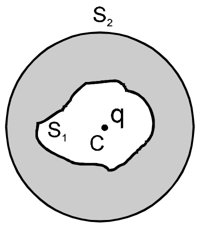


charge is at the common centre
(S_1, S_2 → spherical)

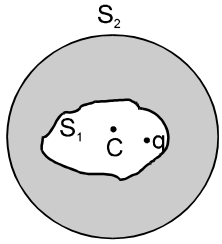
(B)



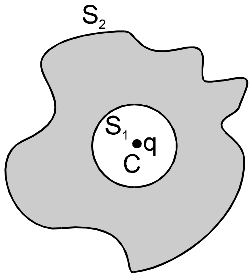
charge is not at the common centre
(S_1, S_2 → spherical)

(C)


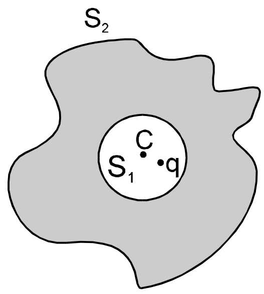
charge is at the centre of S_2
($S_2 \to$ spherical)

(D)


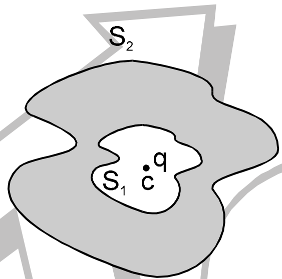
charge is not at the centre of S_2
($S_2 \to$ spherical)

(E)


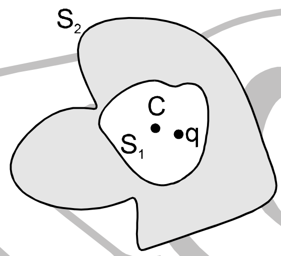
charge is at the centre of
S_1(Spherical)

(F)


charge not at the centre
of S_1(Spherical)

(G)


charge is at the geometrical centre

(H)


charge is at the geometrical centre

Using the result that $\vec{E}_{res}$ in the conducting material should be zero and using result (iii) We can show that

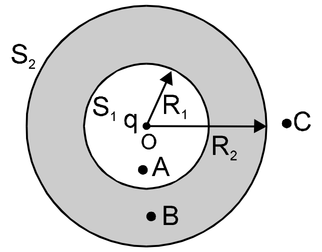
Case	A	B	C	D	E	F	G	H
S_1	Uniform	Nonuniform	Nonuniform	Nonuniform	Uniform	Nonuniform	Nonuniform	Nonuniform
S_2	Uniform	Uniform	Uniform	Uniform	Nonuniform	Nonuniform	Nonuniform	NonUniform

Note : In all cases charge on inner surface $S_1 = -q$ and on outer surface $S_2 = q$. The distribution of charge on 'S_1' will not change even if some charges are kept outside the conductor (i.e. outside the surface S_2). But the charge distribution on 'S_2' may change if some charges(s) is/are kept outside the conductor.

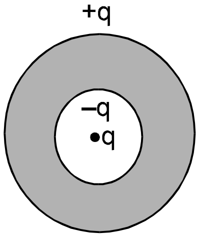
Ex.74 An uncharged conductor of inner radius R_1 and outer radius R_2 contains a point charge q at the centre as shown in figure

(i) Find $\vec{E}$ and V at points A,B and C

(ii) If a point charge Q is kept out side the sphere at a distance 'r' (>>R_2) from centre then find out resultant force on charge Q and charge q.



Sol. At point A :



$$V_A = \frac{Kq}{OA} + \frac{Kq}{R_2} + \frac{K(-q)}{R_1} \quad , \quad \vec{E}_A = \frac{Kq}{OA^3}\overrightarrow{OA}$$

(**Note :** Electric field due at 'A' due to –q of S_1 and +q of S_2 is zero individually because they are uniformly distributed)

At point B :

$$V_B = \frac{Kq}{OB} + \frac{K(-q)}{OB} + \frac{Kq}{R_2} = \frac{Kq}{R_2}, \; E_B = 0$$

At point C :

$$V_C = \frac{Kq}{OC}, \; \vec{E}_C = \frac{Kq}{OC^3}\overrightarrow{OC}$$

(ii) Force on point charge Q :

(**Note :** Here force on 'Q' will be only due to 'q' of S_2 see result (iii)

$$\vec{F}_Q = \frac{KqQ}{r^2}\hat{r} \text{ (r = distance of 'Q' from centre 'O')}$$

Force on point charge q:

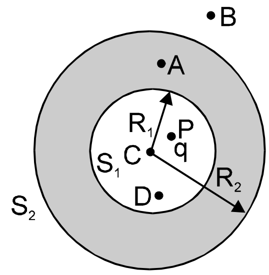
$$\vec{F}_q = 0 \text{ (using result (iii) \& charge on } S_1 \text{ uniform)}$$

Ex.75 An uncharged conductor of inner radius R_1 and outer radius R_2 contains a point charge q placed at point P (not at the centre) as shown in figure? Find out the following :

(i) V_C (ii) V_A

(iii) V_B (iv) E_A

(v) E_B (vi) force on charge Q if it is placed at B



Sol. (i) $V_C = \frac{Kq}{CP} + \frac{K(-q)}{R_1} + \frac{Kq}{R_2}$

Note : –q on S_1 is nonuniformly distributed still it produces potential $\frac{K(-q)}{R_1}$ at 'C' because 'C' is at distance 'R_1' from each points of 'S_1'.

(ii) $V_A = \frac{Kq}{R_2}$ (iii) $V_B = \frac{Kq}{CB}$

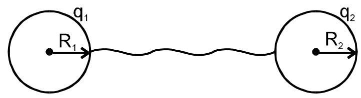
(iv) $E_A = O$ (point is inside metallic conductor) (v) $E_B = \frac{Kq}{CB^2}\hat{CB}$

(vi) $F_Q = \frac{KQq}{CB^2}\hat{CB}$

Note : (force on charge q ≠ 0, think . If you can think right you are extraordinary for varification of your answer you can send it to our office addressing to HOD physics).

(vi) Sharing of charges :

Two conducting hollow spherical shells of radii R_1 and R_2 having charges Q_1 and Q_2 respectively and seperated by large distance, are joined by a conducting wire



Let final charges on spheres are q_1 and q_2 respectively.

Potential on both spherical shell become equal after joining, therefore

$$\frac{Kq_1}{R_1} = \frac{Kq_2}{R_2}$$

$$\frac{q_1}{q_2} = \frac{R_1}{R_2} \quad ......(i)$$

and, $q_1 + q_2 = Q_1 + Q_2$ ......(ii)

from (i) and (ii)

$$q_1 = \frac{(Q_1+Q_2)R_1}{R_1+R_2} \Rightarrow q_2 = \frac{(Q_1+Q_2)R_2}{R_1+R_2}$$

ratio of charges $\frac{q_1}{q_2} = \frac{R_1}{R_2}$

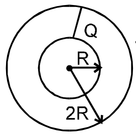
$$\frac{\sigma_1 4\pi R_1^2}{\sigma_2 4\pi R_2^2} = \frac{R_1}{R_2}$$

ratio of surface charge densities $\frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1}$

Ratio of final charges $\frac{q_1}{q_2} = \frac{R_1}{R_2}$

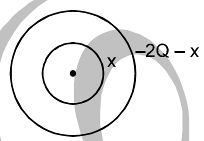
Ratio of final surface charge densities. $\frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1}$

Ex.76 The two conducting spherical shells are joined by a conducting wire and cut after some time when charge stops flowing.Find out the charge on each sphere after that.



Sol. After cutting the wire, the potential of both the shells is equal

Thus, potential of inner shell $V_{in} = \frac{Kx}{R} + \frac{K(-2Q-x)}{2R} = \frac{k(x-2Q)}{2R}$



and potential of outer shell $V_{out} = \frac{Kx}{2R} + \frac{K(-2Q-x)}{2R} = \frac{-KQ}{R}$

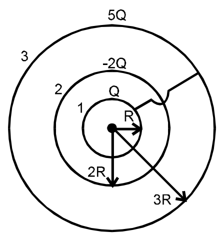
As $V_{out} = V_{in}$

$\Rightarrow \frac{-KR}{R} = \frac{K(x-2Q)}{2R}$ $\Rightarrow -2Q = x - 2Q$

$\Rightarrow x = 0$

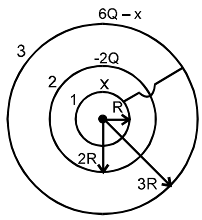
So charge on inner spherical shell = 0
and outer spherical shell = – 2Q.

Ex.77 Find charge on each spherical shell after joining the inner most shell and outer most shell by a contucting wire. Also find charges on each surface.



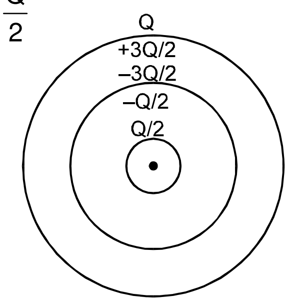
Sol. Let the charge on the innermost sphere be x.

Finally potential of shell 1 = Potential of shell 3



$$\frac{Kx}{R} + \frac{K(-2Q)}{2R} + \frac{K(6Q-x)}{3R} = \frac{KQ}{3R} + \frac{k(-2q)}{3R} + \frac{k(5Q)}{3R}$$

$3x - 3Q + 6Q - x = 4Q$; $2x = Q$; $x = \frac{Q}{2}$

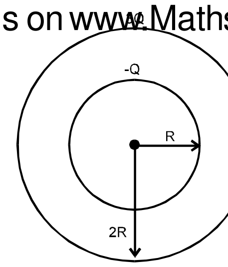


Charge on innermost shell = $\frac{Q}{2}$

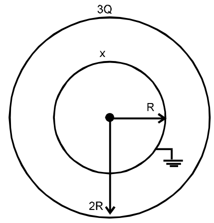
charge on outermost shell = $\frac{5Q}{2}$

middle shell = –2Q
Final charge distribution is as shown in figure.

Ex.78 Two conducting hollow spherical shells of radii R and 2R carry charges – Q and 3Q respectively. How much charge will flow into the earth if inner shell is grounded ?



Sol. When inner shell is grounded to the Earth then the potential of inner shell will bcome zero because potential of the Earth is taken to be zero.



$$\frac{Kx}{R} + \frac{K3Q}{2R} = 0$$

$$x = \frac{-3Q}{2}, \qquad \text{the charge that has increased}$$

$$= \frac{-3Q}{2} - (-Q) \qquad = \frac{-Q}{2} \quad \text{hence charge flows into the Earth} = \frac{Q}{2}$$

Q.25 An isolated conducting sphere of charge Q and radius R is grounded by using a high resistance wire. What is the amount of heat loss ?

Ans. $\frac{Q^2}{8\pi\varepsilon_0 R}$

Ex.79 An isolated conducting sphere of charge Q and radius R is connected to a similar uncharged sphere (kept at a large distance) by using a high resistance wire. After a long time what is the amount of heat loss ?

Sol. When two conducting spheres of equal radius are connected charge is equally deistributed on them (Result VI). So we can say that heat loss of system

$$\Delta H = U_i - U_f$$

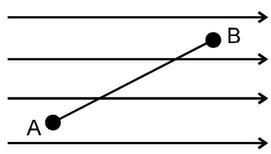
$$= \left(\frac{Q^2}{8\pi\varepsilon_0 R} - 0\right) - \left(\frac{Q^2/4}{8\pi\varepsilon_0 R} + \frac{Q^2/4}{8\pi\varepsilon_0 R}\right) \qquad = \frac{Q^2}{16\pi\varepsilon_0 R}$$

Q.26 A positive charge q is placed in front of a conducting solid cube at a distance d from its centre. Find the electric field at the centre of the cube due to the charges appearing on its surface.

Ans. $\left\{\frac{kq}{d^2}\right\}$

15. RELATION BETWEEN ELECTRIC FIELD INTENSITY AND ELECTRIC POTENTIAL

15.1 For uniform electric field :



(i) Potential difference between two points A and B

$$V_B - V_A = \vec{E} \,.\, \overrightarrow{AB}$$

15.2 Non uniform electric field

(i) $$E_x = -\frac{\partial V}{\partial x},\ E_y = -\frac{\partial V}{\partial y},\ E_z = -\frac{\partial V}{\partial z}$$

$$\Rightarrow \quad \vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$$

$$= -\left[\hat{i}\frac{\partial}{\partial x}V + \hat{j}\frac{\partial}{\partial x}V + \hat{k}\frac{\partial}{\partial z}V\right] \qquad = -\left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial x} + \hat{k}\frac{\partial}{\partial z}\right]V$$

$$= -\nabla V = -\text{grad } V$$

Where $\frac{\partial V}{\partial x}$ = derivative of V with respect to x (keeping y and z constant)

$\frac{\partial V}{\partial y}$ = derivative of V with respect to y (keeping z and x constant)

$\frac{\partial V}{\partial z}$ = derivative of V with respect to z (keeping x and y constant)

15.3 If electric potential and electric field depends only on one coordinate, say r :

(i) $\vec{E} = -\frac{\partial V}{\partial r}\hat{r}$

where $\hat{r}$ is a unit vector along increasing r.

(ii) $\int dV = -\int \vec{E}\cdot\vec{dr} \quad \Rightarrow \quad V_B - V_A = -\int_{r_A}^{r_B} \vec{E}.\ \vec{dr}$

$\vec{dr}$ is along the increasing direction of r.

(iii) The potential of a point

$$V = -\int_{\infty}^{r} \vec{E}.\ \vec{dr}$$

Ex.80 A uniform electric field is along x – axis . The potential difference $V_A - V_B$ = 10 V between two points A (2m , 3m) and B (4m, 8m). Find the electric field intensity.

Sol. $E = \frac{\Delta V}{\Delta d} = \frac{10}{2} = 5$ V / m.

It is along + ve x-axis.

Ex.81 $V = x^2 + y$, Find $\vec{E}$.

Sol. $\frac{\partial V}{\partial x} = 2x$, $\quad \frac{\partial V}{\partial y} = 1 \quad$ and $\quad \frac{\partial V}{\partial z} = 0$

$\vec{E} = -\left(\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right) = -(2x\ \hat{i} + \hat{j})$ Electric field is nonuniform.

Q.27 If $V = x^2y + y^2z$ then find $\vec{E}$ at (x, y, z)

Ans. $-2xy\ \hat{i} - (x^2 + 2yz)\,\hat{j} - y^2\hat{k}$

Ex.82 For given $\vec{E} = 2x\hat{i} + 3y\hat{j}$ find the potential at (x, y) if V at origin is 5 volts.

Sol. $\int_5^V dV = -\int \vec{E}\cdot\vec{dr} = -\int_0^x E_x dx - \int_0^y E_y dy$

$\Rightarrow \quad V - 5 = -\frac{2x^2}{2} - \frac{3y^2}{2} \quad \Rightarrow \quad V = -\frac{2x^2}{2} - \frac{3y^2}{2} + 5.$

Q.28 Magnitude of electric field depends only on the x – coordinate given $\vec{E} = \frac{20}{x^2}\hat{i}$ V / m . Find

(i) the potential difference between two point A (5m, 0) and B (10m, 0).

(ii) potential at x = 5 if V at ∞ is 10 volt.

(iii) inpart (i) does the potential difference between A and B depend on whether the potential at ∞ is 10 volt or something else.

Ans. (i) – 2V

Q.29 If $E = 2r^2$ then find V(r)

Ans $\frac{-2r^3}{3} + C$

