GEOMETRICAL OPTICS

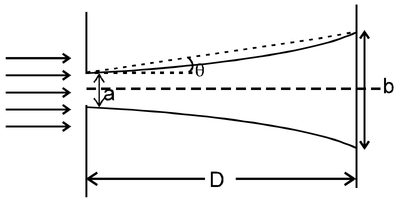
1. CONDITION FOR RECTILINEAR PROPAGATION OF LIGHT : (ONLY FOR INFORMATION NOTE IN JEE SYLLABUS)

Some part of the optics can be understood if we assume that light travels in a straight line and it bends abruptly when it suffers reflection or refraction.

The assumption that the light travels in a straight line is correct if

(i) the medium is isotropic, i.e. its behavior is same in all directions and (ii) the obstacle past which the light moves or the opening through which the light moves is not very small.

Consider a slit of width 'a' through which monochromatic light rays pass and strike a screen, placed at a distance D as shown.



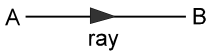
It is found that the light strikes in a band of width 'b' more than 'a'. This bending is called **diffraction**. Light bends by (b-a)/2 on each side of the central line .It can be shown by wave theory of light that $\sin\theta = \frac{\lambda}{a}$ ....(A),

where θ is shown in figure.

This formula indicates that the **bending is considerable only when a $\simeq$ λ** . Diffraction is more pronounced in sound because its wavelength is much more than that of light and it is of the order of the size of obstacles or apertures. Formula (A) gives $\frac{b-a}{2D} \approx \frac{\lambda}{a}$.

It is clear that the bending is negligible if $\frac{D\lambda}{a} << a$ or $a >> \sqrt{D\lambda}$ .If this condition is fulfilled , light is said to move rectilinearly. In most of the situations including geometrical optics the conditions are such that we can safely assume that light moves in straight line and bends only when it gets reflected or refracted. Thus geometrical optics is an approximate treatment in which the light waves can be represented by straight lines which are called rays. A **ray** of light is the straight line path of transfer of light energy. Arrow represents the direction of propagation of light.

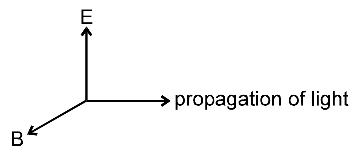
Figure shows a ray which indicates light is moving from A to B.



2. PROPERTIES OF LIGHT

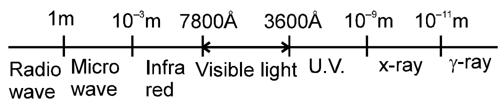
(i) Speed of light in vacuum, denoted by c, is equal to 3×10^8 m/s approximately.

(ii) Light is electromagnetic wave (proposed by Maxwell). It consists of varying electric field and magnetic field.



(iii) Light carries energy and momentum.

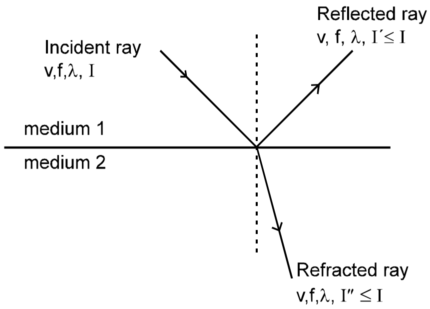
(iv) The formula v = fλ is applicable to light.



Electromagnetic spectrum

(v) When light gets reflected in same medium, it suffers no change in frequency, speed and wavelength.

(vi) Frequency of light remains unchanged when it gets reflected or refracted.

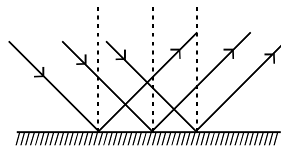


3. REFLECTION OF LIGHT

When light rays strike the boundary of two media such as air and glass, a part of light is turned back into the same medium. This is called *Reflection of Light.*

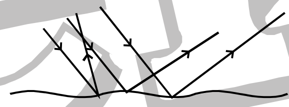
(a) Regular Reflection:

When the reflection takes place from a perfect plane surface it is called *Regular Reflection*. In this case the reflected light has large intensity in one direction and negligibly small intensity in other directions.


Regular Reflection

(b) Diffused Reflection

When the surface is rough, we do not get a regular behavior of light. Although at each point light ray gets reflected irrespective of the overall nature of surface, difference is observed because even in a narrow beam of light there are many rays which are reflected from different points of surface and it is quite possible that these rays may move in different directions due to irregularity of the surface . This process enables us to see an object from any position. Such a reflection is called as diffused reflection. **For example** reflection from a wall, from a news paper etc. This is why you can not see your face in news paper and in the wall.

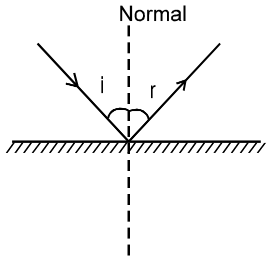

Diffused Reflection

3.1 Laws of Reflection

(a) The incident ray, the reflected ray and the normal at the point of incidence lie in the same plane. This plane is called the *plane of incidence* (or *plane of reflection*). This condition can be expressed mathematically as $\vec{R} \cdot (\vec{I} \times \vec{N}) = \vec{N} \cdot (\vec{I} \times \vec{R}) = \vec{I} \cdot (\vec{N} \times \vec{R}) = 0$ where $\vec{I}$, $\vec{N}$ and $\vec{R}$ are vectors of any magnitude along incident ray, the normal and the reflected ray respectively.

(b) The angle of incidence (the angle between normal and the incident ray) and the angle of reflection (the angle between the reflected ray and the normal) are equal, i.e.

$$\angle \mathbf{i} = \angle \mathbf{r}$$

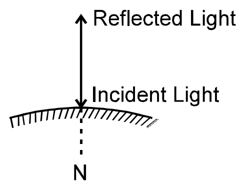


Special Cases :

Normal Incidence : In case light is incident normally,

$i = r = 0$

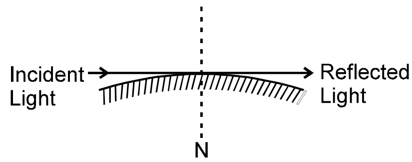
$\delta = 180^\circ$



Grazing Incidence : In case light strikes the reflecting surface tangentially,

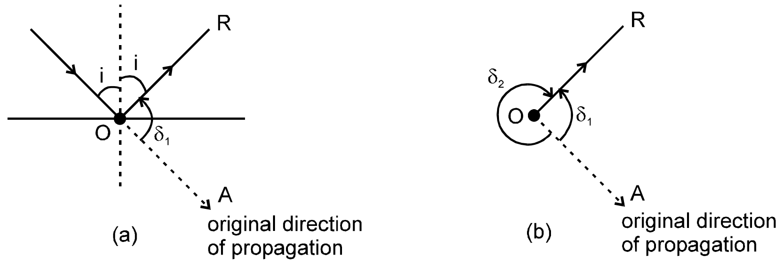
$i = r = 90$

$\delta = 0^\circ$ or 360°



Ex.1 Show that for a light ray incident at an angle 'i' on getting reflected the angle of deviation is $\delta = \pi - 2i$ or $\pi + 2i$.

Sol.

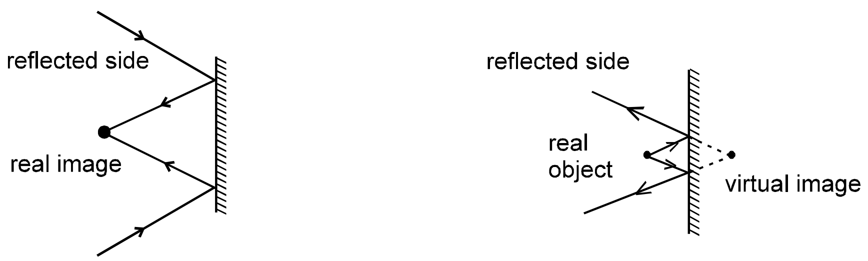


From figure (b) it is clear that light ray bends either by δ_1 anticlockwise or by δ_2 $(= 2\pi - \delta_1)$ clockwise.
From figure (a) $\delta_1 = \pi - 2i$.
$\therefore \quad \delta_2 = \pi + 2i$.

3.2 Object and Image

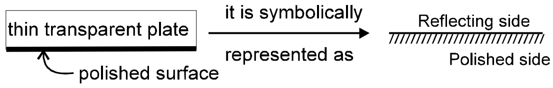
Object is defined as point of intersection of **incident** rays. **Image** is defined as point of intersection of **reflected** rays (in case of reflection) or **refracted** rays (in case of refraction).
Let us call the side in which incident rays are present as incident side and the side in which reflected (refracted) rays are present, as reflected (refracted) side.
An object is called **real** if it lies on incident side otherwise it is called **virtual** .

An image is called **real** if it lies on reflected or refracted side otherwise it is called **virtual.**



4. PLANE MIRROR

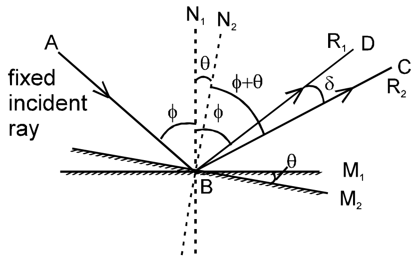
Plane mirror is formed by polishing one surface of a plane thin glass plate .It is also said to be silvered on one side.



PLANE MIRROR

A beam of parallel rays of light, incident on a plane mirror will get reflected as a beam of parallel reflected rays.

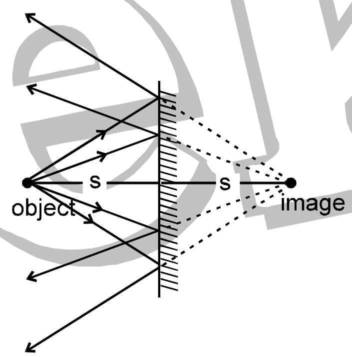
Ex.2 For a fixed incident light ray, if the mirror be rotated through an angle θ (about an axis which lies in the plane of mirror and perpendicular to the plane of incidence), show that the reflected ray turns through an angle 2θ in same sense.



Sol: See figure M_1, N_1 and R_1 indicate the initial position of mirror, initial normal and initial direction of reflected light ray respectively. M_2, N_2 and R_2 indicate the final position of mirror, final normal and final direction of reflected light ray respectively.

From figure it is clear that $\angle ABC = 2\phi + \delta = 2(\phi + \theta)$ or $\delta = 2\theta$.

4.1 Point object



Characteristics of image due to Reflection by a Plane Mirror :

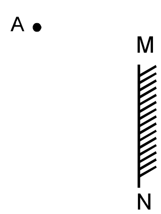
(i) Distance of object from mirror = Distance of image from the mirror.
All the incident rays from a point object will meet at a single point after reflection from a plane mirror which is called image.

(ii) The line joining a point object and its image is normal to the reflecting surface.

(iii) The size of the image is the same as that of the object.

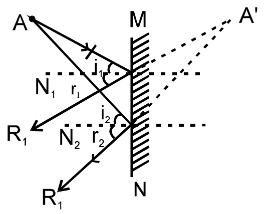
(iv) For a real object the image is virtual and for a virtual object the image is real

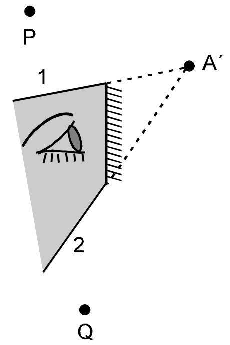
Ex.3 Figure shows a point object A and a plane mirror MN. Find the position of image of object A, in mirror MN, by drawing ray diagram. Indicate the region in which observer's eye must be present in order to view the image. (This region is called **field of view**).



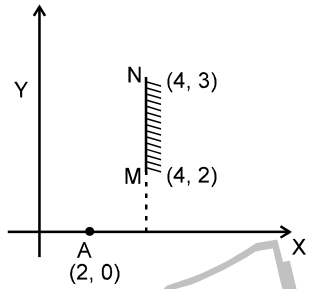
Sol. See figure, consider any two rays emanating from the object. N_1 and N_2 are normals; $i_1 = r_1$ and $i_2 = r_2$
The meeting point of reflected rays R_1 and R_2 is image A'. Though only two rays are considered it must be understood that all rays from A reflect from mirror such that their meeting point is A´. To obtain the region in

which reflected rays are present, join A´ with the ends of mirror and extend. The following figure shows this region as shaded. In figure there are no reflected rays beyond the rays 1 and 2, therefore the observers P and Q cannot see the image because they do not receive any reflected ray.]

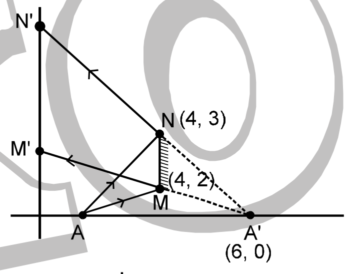




Ex.4 Find the region on Y axis in which reflected rays are present. Object is at A (2, 0) and MN is a plane mirror, as shown.

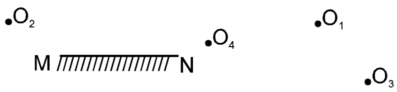


Sol. The image of point A, in the mirror is at A' (6, 0).
Join A' M and extend to cut Y axis at M' (Ray originating from A which strikes the mirror at M gets reflected as the ray MM' which appears to come from A'). Join A'N and extend to cut Y axis at N' (Ray originating from A which strikes the mirror at N gets reflected as the ray NN' which appears to come from A') .

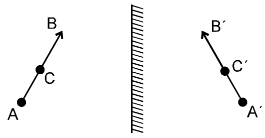


From Geometry.
M' ≡ (0, 6)
N' ≡ (0, 9) . M'N' is the region on Y axis in which reflected rays are present.

Q.1 See the following figure. Which of the object(s) shown in figure will not form its image in the mirror.



4.2 Extended object :



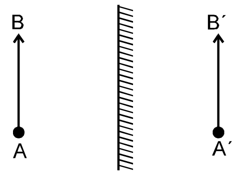
An extended object like AB shown in figure is a combination of infinite number of point objects from A to B. Image of every point object will be formed individually and thus infinite images will be formed. A´ will be image of A, C´ will be image of C, B´ will be image of B etc. All point images together form extended image. Thus extended image is formed of an extended object.

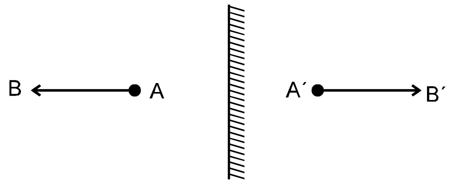
Properties of image of an extended object, formed by a plane mirror :

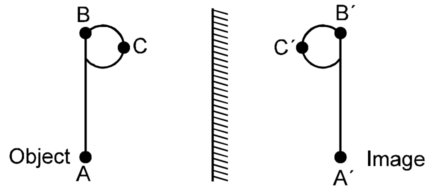
(1) Size of extended object = size of extended image.

(2) The image is upright, if the extended object is placed parallel to the mirror.

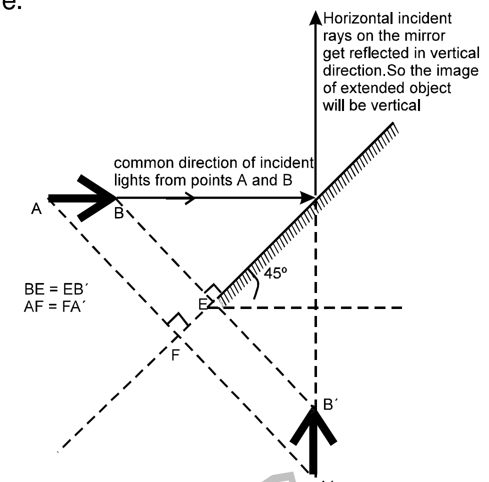
(3) The image is inverted if the extended object lies perpendicular to the plane mirror.





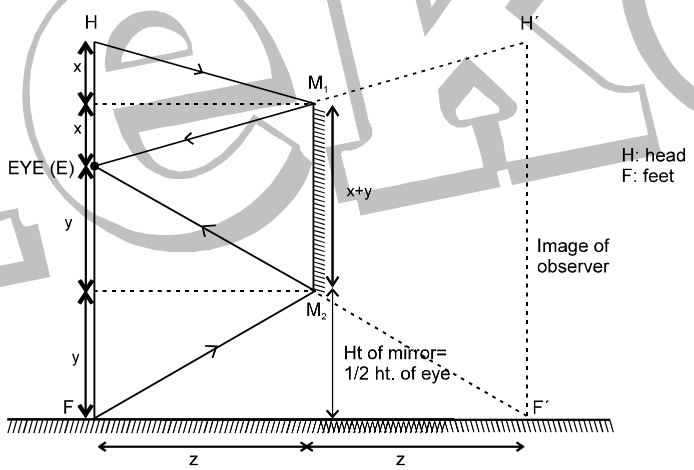


(4) If an extended horizontal object is placed infront of a mirror inclined 45º with the horizontal, the image formed will be vertical. See figure.



Ex.5 Show that the minimum size of a plane mirror, required to see the full image of an observer is half the size of that observer.

Sol. See the following figure. It is self explanatory if you consider lengths 'x' and 'y' as shown in figure.



Aliter :

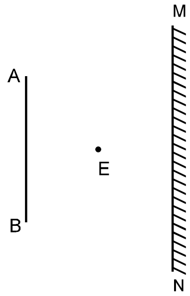
$\Delta E\ M_1, M_2$ and $\Delta E\ H'F'$ are similar

$\therefore \quad \dfrac{M_1M_2}{H'F'} = \dfrac{z}{2z}$

or $\quad M_1 M_2 = H'F' / 2 = HF / 2$

Note that the height of the mirror is half the height of eye as shown in figure.

Q.2 Figure shows an object AB and a plane mirror MN placed parallel to object.



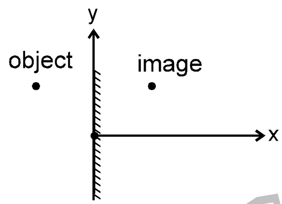
Indicate the mirror length required to see the image of object if observer's eye is at E.

4.3 Relation between velocity of object and image :

From mirror property : $x_{im} = - x_{om}$, $y_{im} = y_{om}$ and $z_{im} = z_{om}$

Here x_{im} means 'x' coordinate of image with respect to mirror. Similarly others have meaning.

Differentiating w.r.t time , we get



$v_{(im)x} = -v_{(om)x}$; $v_{(im)y} = v_{(om)y}$; $v_{(im)z} = v_{(om)z}$,

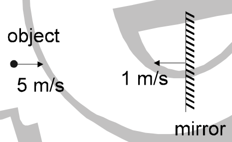
$\Rightarrow$ $v_{iG} - v_{mG} = -(v_{oG} - v_{mG})$ for x axis

but $v_{iG} - v_{mG} = (v_{oG} - v_{mG})$ or $v_{iG} = v_{oG}$ for y axis and z axis.

here: v_{iG} = velocity of image with respect to ground.

Ex.6 An object moves with 5 m/s towards right while the mirror moves with 1m/s towards the left as shown. Find the velocity of image.

Sol.



Take $\rightarrow$ as + direction. $v_i - v_m = v_m - v_0$

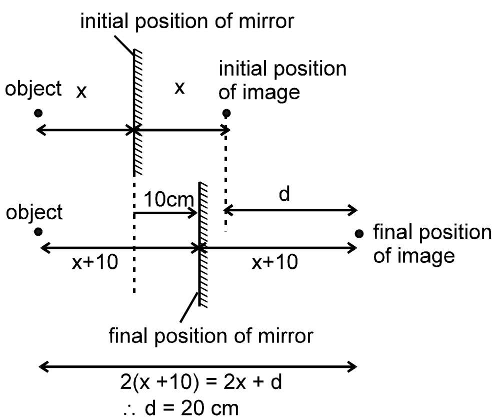
$v_i - (-1) = (-1) - 5$

$\therefore$ $v_i = - 7$m/s. $\Rightarrow$ 7 m/s and direction towards left.

Ex.7 There is a point object and a plane mirror. If the mirror is moved by 10 cm away from the object find the distance which the image will move.

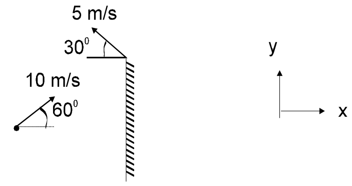
Sol. We know that $x_{im} = - x_{om}$ or $x_i - x_m = x_m - x_o$ or $\Delta x_i - \Delta x_m = \Delta x_m - \Delta x_o$. In this Q. $\Delta x_o = 0$; $\Delta x_m = 10$ cm. Therefore $\Delta x_i = 2\Delta x_m - \Delta x_o = 20$ cm.

or



Q.3 An object is kept fixed in front of a plane mirror which is moved by 10 m/s away from the object, find the velocity of the image.

Ex.8 In the situation shown in figure ,find the velocity of image.



Sol: Along x direction, applying $v_i - v_m = -(v_0 - v_m)$

$$v_i - (-5\cos 30^\circ) = -(10\cos 60^\circ - (-5\cos 30^\circ))$$

$$\therefore\ v_i = -5\,(1+\sqrt{3})\ \text{m/s}$$

Along y direction $v_0 = v_i$

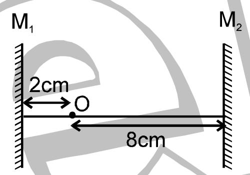
$$\therefore\ v_i = 10\sin 60^\circ = 5\ \text{m/s}$$

$$\therefore\ \text{Velocity of the image} = -5\,(1+\sqrt{3})\,\hat{i} + 5\,\hat{j}\ \text{m/s.}$$

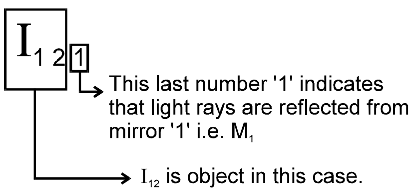
4.4 Images formed by two plane mirrors :

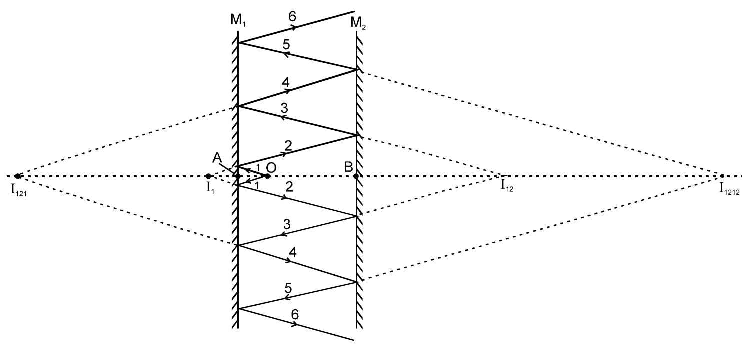
If rays after getting reflected from one mirror strike second mirror, the image formed by first mirror will function as an object for second mirror, and this process will continue for every successive reflection.

Ex.9 Figure shows a point object placed between two parallel mirrors. Its distance from M_1 is 2 cm and that from M_2 is 8 cm. Find the distance of images from the two mirrors considering reflection on mirror M_1 first.



Sol. To understand how images are formed see the following figure and table. You will require to know what symbols like I_{121} stands for. See the following diagram.





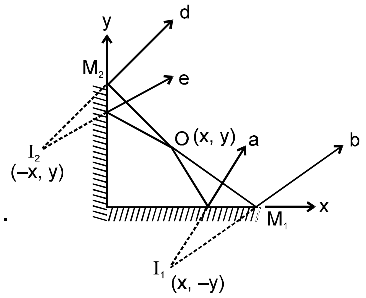
Incident rays	Reflected by	Reflected rays	Object	Image	Object distance	Image distance
Rays 1	M_1	Rays 2	O	I_1	AO = 2cm	AI_1 = 2 cm
Rays 2	M_2	Rays 3	I_1	I_{12}	BI_1 = 12 cm	BI_{12} = 12 cm
Rays 3	M_1	Rays 4	I_{12}	I_{121}	AI_{12} = 22cm	AI_{121} = 22cm
Rays 4	M_2	Rays 5	I_{121}	I_{1212}	BI_{121} =32cm	BI_{1212}=32cm

And so on …..

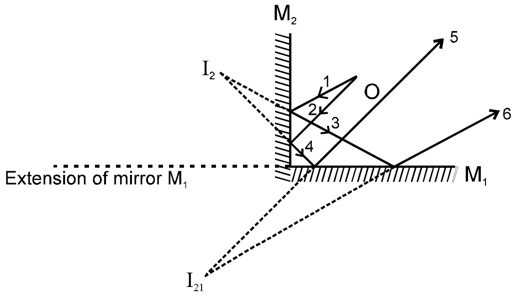
Similarly images will be formed by the rays striking mirror M_2 first. Total number of images = ∞.

Ex.10 Consider two perpendicular mirrors. M_1 and M_2 and a point object O. Taking origin at the point of intersection of the mirrors and the coordinate of object as (x, y), find the position and number of images.

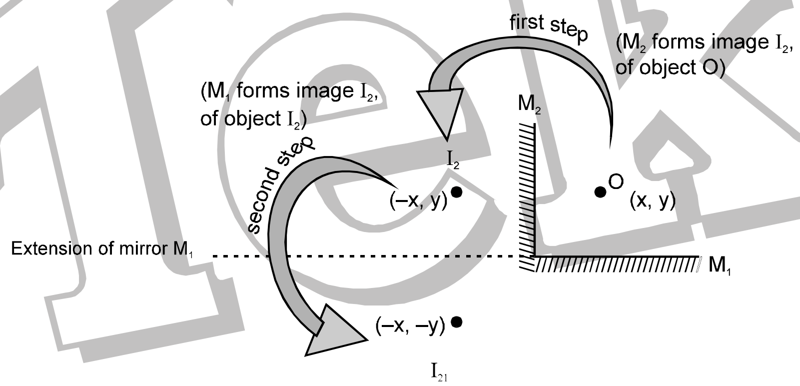
Sol. Rays 'a' and 'b' strike mirror M_1 only and these rays will form image I_1 at (x, –y), such that O and I_1 are equidistant from mirror M_1. These rays donot form further image because they do not strike any mirror again. Similarly rays 'd' and 'e' strike mirror M_2 only and these rays will form image I_2 at (–x, y), such that O and I_2 are equidistant from mirror M_2.



Now consider those rays which strike mirror M_2 first and then the mirror M_1.

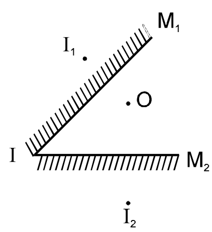


For incident rays 1, 2 object is O, and reflected rays 3, 4 form image I_2.
Now rays 3, 4 incident on M_1 (object is I_2) which reflect as rays 5, 6 and form image I_{21}. Rays 5, 6 do not strike any mirror, so image formation stops.
I_2 and I_{21}, are equidistant from M_1. To summarize see the following figure
Now rays 3, 4 incident on M_1 (object is I_2) which reflect as rays 5, 6 and form image I_{21}. Rays 5, 6 do not strike any mirror, so image formation stops.



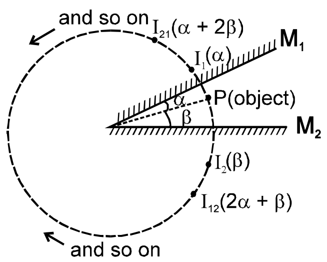
For rays reflecting first from M_1 and then from M_2, first image I_1 (at (x, –y)) will be formed and this will function as object for mirror M_2 and then its image I_{12} (at (–x, –y)) will be formed.
I_{12} and I_{21} coincide.
$\therefore$ Three images are formed

Q.4 Figure shows two inclined plane mirrors M_1 and M_2 and an object O. Its images formed in mirrors M_1 and M_2 individually are I_1 and I_2 respectively. Show that I_1 and I_2 and O lie on the circumference of a circle with centre at O. [This result can be extended to show that all the images will also lie on the same circle. Note that this result is independent of the angle of inclination of mirrors.]



4.5 Locating all the Images formed by two Plane Mirrors

Consider two plane mirrors M_1 and M_2 inclined at an angle $\theta = \alpha+\beta$ as shown in figure



Point P is an object kept such that it makes angle α with mirror M_1 and angle β with mirror M_2. Image of object P formed by M_1 ,denoted by I_1 ,will be inclined by angle α on the other side of mirror M_1. This angle is written in bracket in the figure besides I_1. Similarly image of object P formed by M_2 ,denoted by I_2 ,will be inclined by angle β on the other side of mirror M_2. This angle is written in bracket in the figure besides I_2.
Now I_2 will act as an object for M_1 which is at an angle $(\alpha+2\beta)$ from M_1 . Its image will be formed at an angle $(\alpha+2\beta)$ on the opposite side of M_1. This image will be denoted as I_{21}, and so on. Think when this will process stop. Hint: The virtual image formed by a plane mirror must not be in front of the mirror or its extension.

Number of images formed by two inclined mirrors

(i) If $\frac{360º}{\theta}$ = even number; number of image = $\frac{360º}{\theta} - 1$

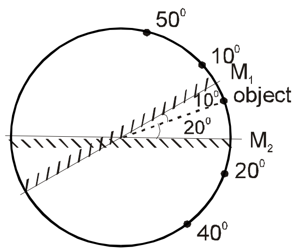
(ii) If $\frac{360º}{\theta}$ = odd number; number of image = $\frac{360º}{\theta} - 1$, if the object is placed on the angle bisector.

(iii) If $\frac{360º}{\theta}$ = odd number; number of image = $\frac{360º}{\theta}$, if the object is not placed on the angle bisector.

(iv) If $\frac{360º}{\theta} \neq$ integer, then count the number of images as explained above.

Ex.11 Two mirrors are inclined by an angle 30^0. An object is placed making 10^0 with the mirror M_1. Find the positions of first two images formed by each mirror. Find the total number of images using (i) direct formula and (ii) counting the images.

Sol. Figure is self explanatory.



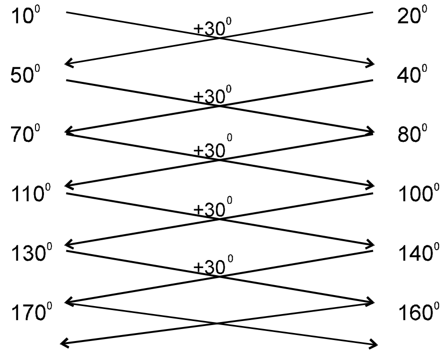
Number of images

(i) Using direct formula : $\frac{360º}{30º}$ = 12 (even number)

$\therefore$ number of images = 12 – 1 = 11

(ii) By counting . See the following table

Image formed by Mirror M_1 (angles are measured from the mirror M_1.)	Image formed by Mirror M_2 (angles are measured from the mirror M_2.)

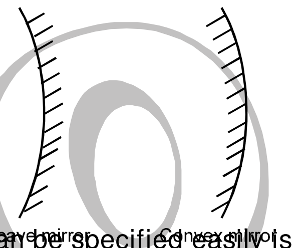


Stop because next angle will be more than 180º   Stop because next angle will be more than 180º

To check whether the final images made by the two mirrors coincide or not : add the last angles and the angle between the mirrors. If it comes out to be exactly 360º, it implies that the final images formed by the two mirrors coincide. Here last angles made by the mirrors + the angle between the mirrors = 160º + 170º + 30º = 360º. Therefore in this case the last images coincide. Therefore the number of images = number of images formed by mirror M_1 + number of images formed by mirror M_2 -1 (as the last images coincide)= 6 + 6 –1 = 11.

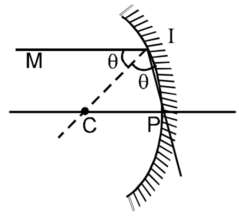
5. SPHERICAL MIRRORS

Spherical Mirror is formed by polishing one surface of a part of sphere. Depending upon which part is shining the spherical mirror is classified as (a) Concave mirror, if the side towards center of curvature is shining and (b) Convex mirror if the side away from the center of curvature is shining.



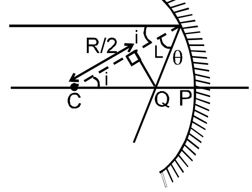
A point on the surface on the mirror from where the position of the object can be specified easily is called pole. The pole is generally taken at the mid point of reflecting surface**.** The centre of the sphere of which the mirror is a part, is called **centre of curvature** .The radius of the sphere of which the mirror is a part is called **Radius of curvature .**The straight line connecting pole P and centre of curvature C is **Principal Axis.**

Ex.12 Find the angle of incidence of ray for which it passes through the pole, given that MI || CP.



Sol. $\angle$ MIC = $\angle$ CIP = θ
MI || CP $\angle$ MIθ = $\angle$ ICP = θ
CI = CP
$\angle$ CIP = $\angle$CPI = θ
$\therefore$ In ΔCIP all angle are equal
$3\theta = 180^\circ$
$\theta = 60^\circ$

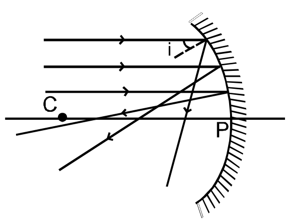
Ex.13 Find the distance CQ if incident light ray parallel to principal axis is incident at an angle i. Also find the distance CQ if i → 0.



Sol. $\cos i = \dfrac{R}{2CQ}$

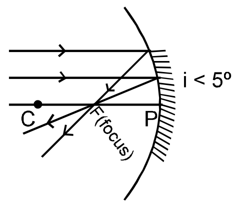
$CQ = \dfrac{R}{2\cos i}$

As i increases cos i decreases.
Hence CQ increases



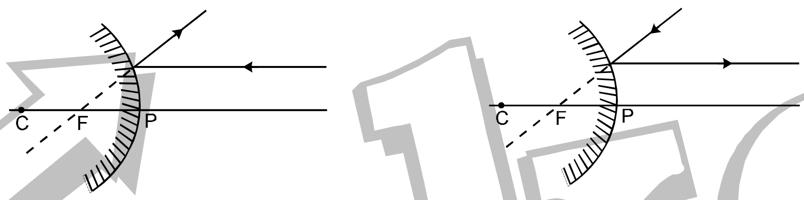
If i is a small angle cos i $\approx$ 1

$\therefore$ CQ = R/2



So, paraxial rays meet at a distance equal to R / 2 from center of curvature, which is called focus.
Principal focus (F) is the point of intersection of all the reflected rays for which the incident rays strike the mirror (with small aperture) parallel to the principal axis. In concave mirror it is real and in the convex mirror it is virtual. The distance from pole to focus is called **focal length. Aperture** (related to the size of mirror) is the diameter of the mirror.

Concave mirror



Convex mirror

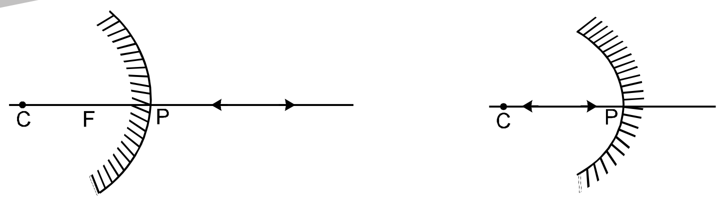
5.1 Ray tracing :

Following facts are useful in ray tracing.

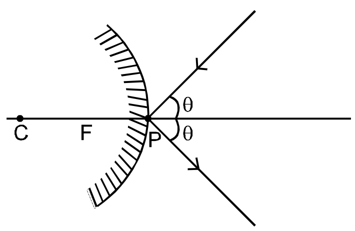
(i). If the incident ray is parallel to the principal axis, the reflected ray passes through the focus.

(ii). If the incident ray passes through the focus, then the reflected ray is parallel to the principal axis.

(iii). Incident ray passing through centre of curvature will be reflected back through the centre of curvature (because it is a normally incident ray).



(iv). It is easy to make the ray tracing of a ray incident at the pole as shown in below.



5.2 Sign Convention

We are using co–ordinate sign convention.

(i). Take origin at pole (in case of mirror)or at optical centre (in case of lens)
Take X axis along the Principal Axis ,taking **positive direction along the incident light .**

u, v, R and **f** indicate the x coordinate of object, image, centre of curvature and focus respectively.

(ii). y-coordinates are taken positive above Principal Axis and negative below Principal Axis'

$\mathbf{h_1}$ and $\mathbf{h_2}$ denote the y coordinate of object and image respectively.

Note : This sign convention is used for reflection from mirror, reflection through flat or curved surfaces or lens.

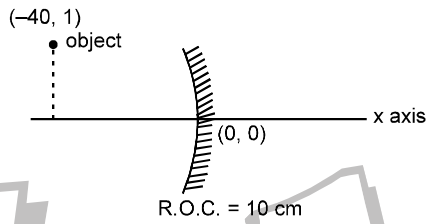
5.3 Formulae for Reflection from spherical mirrors :

5.3.1 Mirror formula : $\frac{1}{v} + \frac{1}{u} = \frac{2}{R} = \frac{1}{f}$

X-coordinate of centre of Curvature and focus of Concave mirror are negative and those for Convex mirror are positive.

In case of mirrors since light rays reflect back in X-direction, therefore **-ve sign of v indicates real image and +ve sign of v indicates virtual image.**

Ex.14 Figure shows a spherical concave mirror with its pole at (0, 0) and principal axis along x axis.



There is a point object at (–40 cm, 1cm), find the position of image.

Sol. According to sign convention,

$u = -40$ cm

$h_1 = +1$ cm

$f = -5$ cm.

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v} + \frac{1}{-40} = \frac{1}{-5} \; ; \; v = \frac{-40}{7} \text{ cm.} \; ; \; \frac{h_2}{h_1} = \frac{-v}{u}$$

$$\Rightarrow \quad h_2 = -\frac{-v}{u} \times h_1 = \frac{-\left(-\frac{40}{7}\right) \times 1}{-40}$$

$$= -\frac{1}{7} \text{ cm.}$$

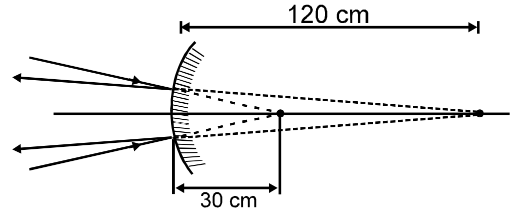
$\therefore$ The position of image is $\left(\frac{-40}{7}\text{cm}, -\frac{1}{7}\text{cm}\right)$

Ex.15 Converging rays are incident on a convex spherical mirror so that their extensions intersect 30 cm behind the mirror on the optical axis. The reflected rays form a diverging beam so that their extensions intersect the optical axis 1.2 m from the mirror. Determine the focal length of the mirror.

Sol. In this case

$u = +30$

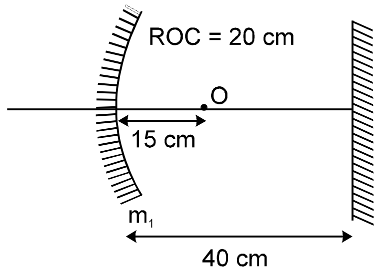
$v = +120$



$\therefore \quad \frac{1}{f} = \frac{1}{v} + \frac{1}{u} = \frac{1}{120} + \frac{1}{30}$

f = 24 cm

Ex.16 Find the position of final image after three successive reflections taking first reflection on m_1.



Sol. **I reflection :**

Focus of mirror $= -10$ cm

$u = -15$ cm

Applying mirror formula :

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$v = -30$ cm.

For II reflection on plane mirror :

$u = -10$ cm $\quad \therefore \quad v = 10$ cm

For III reflection on curved mirror again :

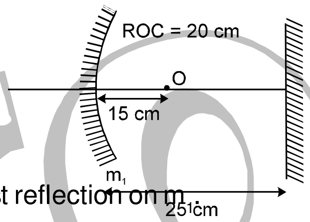
$u = -50$ cm $\quad f = -10$ cm

Applying mirror formula :

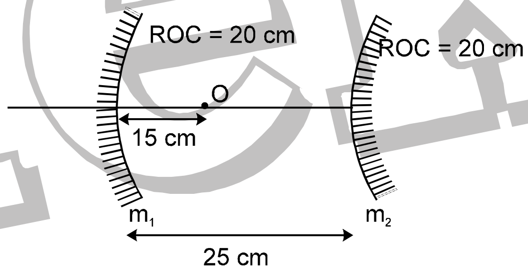
$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$v = -12.5$ cm.

Q.5 Find the position of final image after three successive reflections taking first reflection on m_1.



Q.6 Find the position of final image after three successive reflections taking first reflection on m_1.



5.3.2 **Lateral magnification** (or transverse magnification) denoted by m is defined as m= $\frac{h_2}{h_1}$ and is related as

m = $-\frac{v}{u}$.From the definition of m positive sign of m indicates erect image and negative sign indicates inverted image. From the definition of m positive sign of m indicates erect image and negative sign indicates inverted image.

5.3.3 In case of successive reflection from mirrors, the overall lateral magnification is given by $\mathbf{m_1 \times m_2 \times m_3}$ ......, where m_1 , m_2 etc. are lateral magnifications produced by individual mirrors.

Note : Using (**5.3.1**) and (**5.3.2**) the following conclusions can be made (check yourself).

Nature of Object	Nature of Image	Inverted or erect
Real	Real	Inverted
Real	Virtual	Erect
Virtual	Real	Erect
Virtual	Virtual	Inverted

5.3.4 From (**5.3.1**) and (**5.3.2**); we get $m = \frac{f}{f-u} = \frac{f-v}{f}$ ....(just a time saving formula)

Ex.17 An extended object is placed perpendicular to the principal axis of a concave mirror of radius of curvature 20 cm at a distance of 15 cm from pole. Find the lateral magnification produced.

Sol. $u = -15$ cm $\qquad f = -10$ cm

Using $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ we get, $\qquad v = -30$ cm

$\therefore \qquad m = -\frac{v}{u} = -2.$

Aliter : $m = \frac{f}{f-u} = \frac{-10}{-10-(-15)} = -2$

Ex.18 A person looks into a spherical mirror. The size of image of his face is twice the actual size of his face. If the face is at a distance 20 cm then find the nature of radius of curvature of the mirror.

Sol. Person will see his face only when the image is virtual. Virtual image of real object is erect.
Hence $m = 2$

$\therefore \qquad \frac{-v}{u} = 2$

$v = 40$ cm

Applying $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$; $f = -40$ cm or $R = 80$ cm.

Aliter : $m = \frac{f}{f-u}$

$\Rightarrow \qquad 2 = \frac{f}{f-(-20)} \qquad \Rightarrow \qquad f = -40$ cm

or $\qquad R = 80$ cm

Ex.19 An image of a candle on a screen is found to be double its size. When the candle is shifted by a distance 5 cm then the image become triple its size. Find the nature and ROC of the mirror.

Sol. Since the images formed on screen it is real. Real object and real image implies concave mirror.

Applying $m = \frac{f}{f-u}$ or $-2 = \frac{f}{f-(u)}$ ....(1)

After shifting $-3 = \frac{f}{f-(u+5)}$ ....(2)

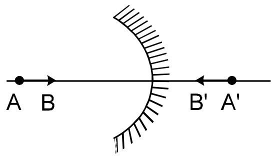
[Why u + 5 ? , why not u – 5 : In a concave mirror are size of real image will increase, only when the real object is brought closer to the mirror. In doing so, its x coordinate will increase]
From (1) & (2) we get,
$f = -30$ cm
or $\qquad R = 60$ cm

Q.7 A coin is placed 10 cm in front of a concave mirror . The mirror produces a real image that has diameter 4 times that of the coin. What is the image distance.

Q.8 A small statue has a height of 1 cm and is placed in front of a spherical mirror . The image of the statue is inverted and is 0.5cm tall and located 10 cm in front of the mirror. Find the focal length and nature of the mirror.

5.3.5 On differentiating the mirror formula we get $\frac{dv}{du} = -\frac{v^2}{u^2}$.

Mathematically 'du' implies small change in position of object and 'dv' implies corresponding small change in position of image. If a small object lies along principal axis, du may indicate the size of object and dv the size of its image along Principal axis (Note that the focus should not lie in between the initial and final points of object). In this case $\frac{dv}{du}$ is called longitudinal magnification. **Negative sign indicates inversion of image irrespective of nature of image and nature of mirror.**



Ex.20 A point object is placed 60 cm from pole of a concave mirror of focal length 10 cm on the principal axis. Find

(a) the position of image

(b) If object is shifted 1 mm towards the mirror along principal axis find the shift in image. Explain the result.

Sol. (a) $u = -60$ cm

$f = -10$cm

$$v = \frac{fu}{u-f} = \frac{-10(-60)}{-60-(-10)} = \frac{600}{-50} = -12 \text{ cm.}$$

(b) $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

Differentiating , we get $dv = -\frac{v^2}{u^2}du = -\left(\frac{-12}{-60}\right)^2 [1 \text{ mm}] = -\frac{1}{25}$ mm

[$\because$ du = 1mm; sign of du is + because it is shifted in +ve direction defined by sign convention.]

(A) –ve sign of dv indicates that the image will shift towards negative direction.

(B) The sign of v is negative. Which implies the image is formed on negative side of pole. (A) and (B) together imply that the image will shift away from pole.

Note that differentials dv and du denote small changes only.

5.3.6 Velocity of image

(a) Object moving perpendicular to principal axis : From the relation in 5.3.2,

we have $\frac{h_2}{h_1} = -\frac{v}{u}$ or $h_2 = -\frac{v}{u} \cdot h_1$

If a point object moves perpendicular to the principal axis, x coordinate of both the object & the image become constant. On differentiating the above relation w.r.t. time , we get,

$$\frac{dh_2}{dt} = -\frac{v}{u}\frac{dh_1}{dt}$$

Here, $\frac{dh_1}{dt}$ denotes velocity of object perpendicular to the principal axis and $\frac{dh_2}{dt}$ denotes velocity of image perpendicular to the principal axis.

(b) Object moving along principal axis : On differentiating the mirror formula with respect to time we get

$\frac{dv}{dt} = -\frac{v^2}{u^2}\frac{du}{dt}$, where $\frac{dv}{dt}$ is the velocity of image along Principal axis and $\frac{du}{dt}$ is the velocity of object along Principal axis. Negative sign implies that the image , in case of mirror, always moves in the direction opposite to that of object. This discussion is for velocity with respect to mirror and along the x axis.

(c) Object moving at an angle with the principal axis : Resolve the velocity of object along and perpendicular to the principal axis and find the velocities of image in these directions separately and then find the reusltant.

5.3.7 Newton's Formula: $XY = f^2$

X and Y are the distances (along the principal axis) of the object and image respectively from the principal focus. This formula can be used when the distances are mentioned or asked from the focus.

5.3.8 Optical power of a mirror (in Diopters) = $\frac{1}{f}$

f = focal length with sign and in meters.

5.3.9 If object lying along the principal axis is not of very small size, the **longitudinal magnification** $= \frac{v_2 - v_1}{u_2 - u_1}$ (it will always be inverted)

6. REFRACTION OF LIGHT

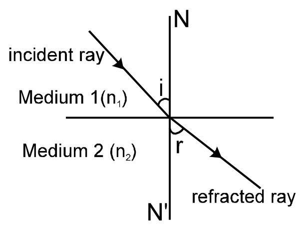
Deviation or bending of light rays from their original path while passing from one medium to another is called *refraction*. It is due to change in speed of light as light passes from one medium to another medium. If the light is incident normally then it goes to the second medium without bending, but still it is called refraction. Refractive index of a medium is defined as the factor by which speed of light reduces as compared to the speed of light in vacuum. $\mu = \frac{c}{v} = \frac{\textbf{speed of light in vacuum}}{\textbf{speed of light in medium}}$.

More (less) refractive index implies less (more) speed of light in that medium, which therefore is called denser (rarer) medium.

6.1 Laws of Refraction

(a) The incident ray , the normal to any refracting surface at the point of incidence and the refracted ray all lie in the same plane called the plane of incidence or plane of refraction.

(b) $\frac{\text{Sin}\, i}{\text{Sin}\, r}$ = Constant for any pair of media and for light of a given wave length. This is known as *Snell's* Law.



Also, $\frac{\text{Sin i}}{\text{Sin r}} = \frac{n_2}{n_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$

For applying in problems remember

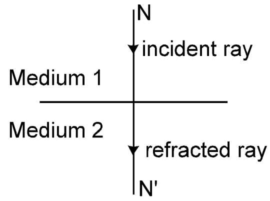
$n_1 \sin i = n_2 \sin r$

$\frac{n_2}{n_1} = {}_1n_2$ = Refractive Index of the second medium with respect to the first medium.

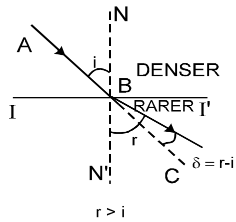
C = speed of light in air (or vacuum) = 3×10^8 m/s.

Special cases :

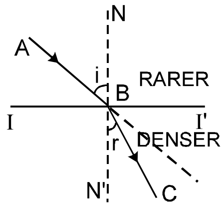
(i) Normal incidence : i = 0
from snell's law : r = 0



(ii) When light moves from denser to rarer medium it bends away from normal.



(iii) When light moves from rarer to denser medium it bends towards the normal.

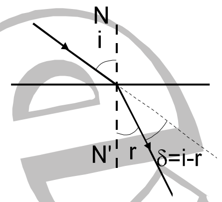


Note: (i) Higher the value of R.I., denser (optically) is the medium.
(ii) Frequency of light does not change during refraction.
(iii) Refractive index of the medium relative to vacuum = $\sqrt{\mu_r \in_r}$

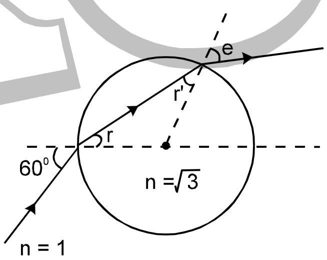
$n_{vacuum} = 1$; $n_{air} = \gtrsim 1$; n_{water} (average value) = 4/3 ; n_{glass} (average value)= 3/2

6.2 Deviation of a Ray Due to Refraction

Deviation (δ) of ray incident at ∠ i and refracted at ∠ r is given by **δ = |*i*–r|**.



Ex.21 A light ray is incident on a glass sphere at an angle of incidence 60^0 as shown. Find the angles r, r',e and the total deviation after two refractions.



Sol. Applying Snell's law $1\sin60^0 = \sqrt{3}\sin r \Rightarrow r = 30^0$

From symmetry $r' = r = 30^0$.

Again applying snell's law at second surface $1\sin e = \sqrt{3}\sin r$

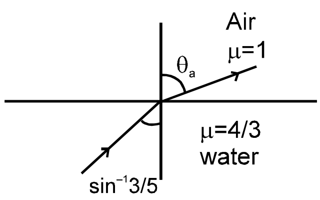
$\Rightarrow e = 60^0$

Deviation at first surface = $i - r = 60^0 - 30^0 = 30^0$

Deviation at second surface = $e - r' = 60^0 - 30^0 = 30^0$

Therefore total deviation = 60^0.

Ex.22 Find the angle θ_a made by the light ray when it gets refracted from water to air, as shown in figure.



Sol. Snell's Law

$$\mu_w \sin\theta_w = \mu_a \sin\theta_a \qquad \frac{4}{3}\times\frac{3}{5} = 1 \sin\theta_a$$

$$\sin\theta_a = \frac{4}{5} \qquad \theta_a = \sin^{-1}\frac{4}{5}$$

Ex.23 Find the speed of light in medium 'a' if speed of light in medium 'b' is $\frac{c}{3}$ where c = speed of light in vacuum and light refracts from medium 'a' to medium 'b' making 45º and 60º respectively with the normal.

Snell's Law

$$\mu_a \sin\theta_a = \mu_b \sin\theta_b$$

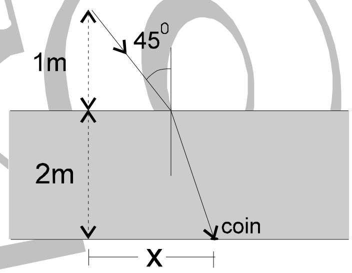
$$\frac{c}{v_a}\sin\theta_a = \frac{c}{v_b}\sin\theta_b.$$

$$\frac{c}{v_a}\sin 45^\circ = \frac{c}{c/3}\sin 60^\circ.$$

$$v_a = \frac{\sqrt{2}c}{3\sqrt{3}}$$

Q.9 A light ray deviates by 30^0 (which is one third of the angle of incidence) when it gets refracted from vacuum to a medium. Find the refractive index of the medium.

Q.10 A coin lies on the bottom of a lake 2m deep at a horizontal distance x from the spotlight (a source of thin parallel beam of light) situated 1 m above the surface of a liquid of refractive index $\mu = \sqrt{2}$ and height 2m. Find x.



6.3 Principle of Reversibility of Light Rays

(a) A ray travelling along the path of the reflected ray is reflected along the path of the incident ray.

(b) A refracted ray reversed to travel back along its path will get refracted along the path of the incident ray. Thus the incident and refracted rays are mutually reversible.

(c) According to this principle $_1n_2 = \frac{1}{_2n_1}$.

7. REFRACTION THROUGH A PARALLEL SLAB

When light passes through a parallel slab, having same medium on both sides, then

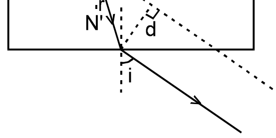
(a) Emergent ray is parallel to the incident ray.

Note: Emergent ray will not be parallel to the incident ray if the medium on both the sides of slab are different.

(b) Light is shifted laterally, given by
(student should be able to derive it)

$$\mathbf{d = \frac{t\sin(i-r)}{\cos r}}$$

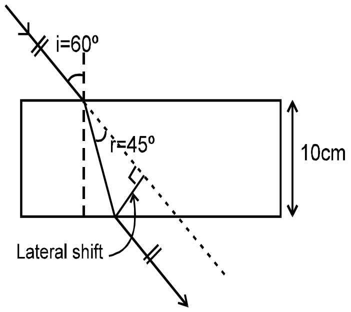
t = thickness of slab



Ex.24 Find the lateral shift of light ray while is passes through a parallel glass slab of thickness 10 cm placed in air. The angle of incidence in air is 60º and the angle of refraction in glass is 45º.

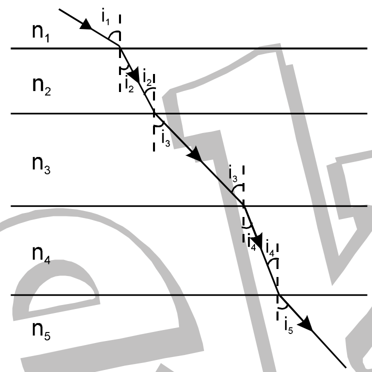
Sol. $d = \dfrac{t\sin(i-r)}{\cos r} = \dfrac{10\sin(60°-45°)}{\cos 45°}$

$= \dfrac{10\sin 15°}{\cos 45°} = 10\sqrt{5}\ \sin 15º.$



Q.11 A ray of light falls at an angle of 30º onto a plane-parallel glass plate and leaves it parallel to the initial ray. The refractive index of the glass is 1.5. What is the thickness d of the plate if the distance between the rays is 3.82 cm? [Given : $\sin^{-1}\left(\dfrac{1}{3}\right)$ = 19.5º ; cos 19.5º = 0.94 ; sin 10.5º = 0.18]

Q.12 A light passes through many parallel slabs one by one as shown in figure.



Prove that $n_1 \sin i_1 = n_2 \sin i_2 = n_3 \sin i_3 = n_4 \sin i_4 =$.............**[Remember this].** Also prove that if $n_1 = n_4$ then light rays in medium n_1 and in medium n_4 are parallel.

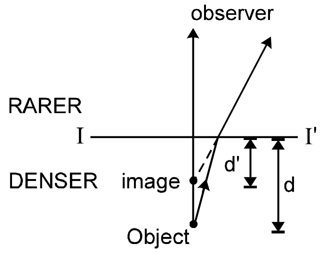
7.1 Apparent Depth and shift of Submerged Object

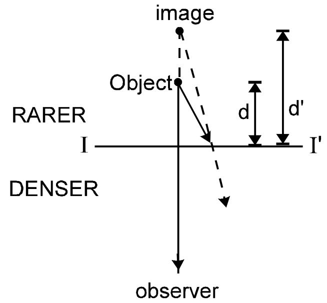
At near normal incidence (small angle of incidence i) apparent depth (d′) is given by:

$$\mathbf{d' = \frac{d}{n_{relative}}}$$

where $n_{relative} = \dfrac{n_i(\text{R.I. of medium of incidence})}{n_r(\text{R.I. of medium of refraction})}$

d = distance of object from the interface = real depth
d’ = distance of image from the interface = apparent depth
This formula can be easily derived using snell’s law and applying the condition of nearly normal incidence.... (try it).





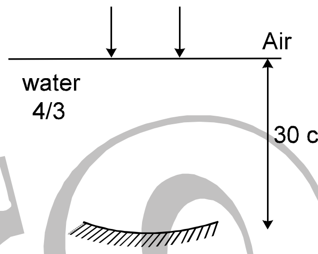
Apparent shift = $\mathbf{d\left(1-\frac{1}{n_{rel}}\right)}$

Ex.25 An object lies 100 cm inside water .It is viewed from air nearly normally. Find the apparent depth of the object.

Sol: $$d' = \frac{d}{n_{relative}} = \frac{100}{\frac{4/3}{1}} = 75 \text{ cm}$$

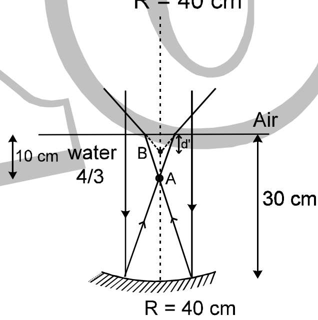
Q.13 An object lies 90 cm in air above water surface .It is viewed from water nearly normally. Find the apparent height of the object.

Ex.26 A concave mirror is placed inside water with its shining surface upwards and principal axis vertical as shown. Rays are incident parallel to the principal axis of concave mirror. Find the position of final image.

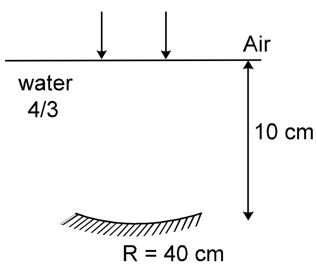


Sol. The incident rays will pass undeviated through the water surface and strike the mirror parallel to its principal axis. Therefore for the mirror, object is at ∞ . Its image A (in figure) will be formed at focus which is 20 cm from the mirror. Now for the interface between water and air, d = 10 cm.

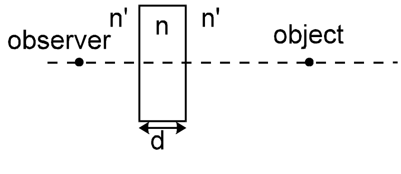
$$\therefore \quad d' = \frac{d}{\left(\frac{n_w}{n_a}\right)} = \frac{10}{\left(\frac{4/3}{1}\right)} = 7.5 \text{ cm.}$$



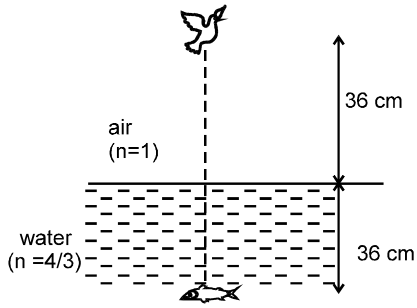
Q.14 A concave mirror is placed inside water with its shining surface upwards and principal axis vertical as shown. Rays are incident parallel to the principal axis of concave mirror. Find the position of final image.



Q.15 Prove that the shift in position of object due to parallel slab is given by shift = $d\left(1-\frac{1}{n_{rel}}\right)$ where $n_{rel} = \frac{n}{n'}$.



Ex.27 See the figure



(i) Find apparent height of the bird

(ii) Find apparent depth of fish

(iii) At what distance will the bird appear to the fish.

(iv) At what distance will the fish appear to the bird

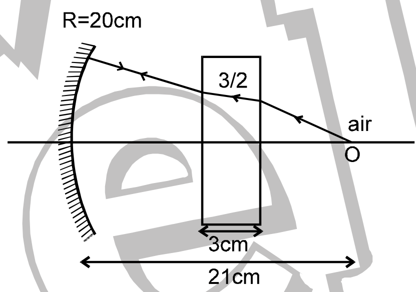
Sol. (i) $d'_B = \dfrac{36}{\dfrac{1}{\left(\dfrac{4}{3}\right)}} = \dfrac{36}{3/4} = 48$ cm

(ii) $d'_F = \dfrac{36}{4/3} = 27$ cm

(iii) For fish : $d_B = 36 + 48 = 84$ cm
$d_B = 36 + 48 = 84$ cm

(iv) For bird : $d_F = 27 + 36 = 63$ cm.
$d_F = 27 + 36 = 63$ cm.

Ex.28 See the figure.



Find the distance of final image formed by mirror

Sol. Shift = $3\left(1 - \dfrac{1}{3/2}\right)$

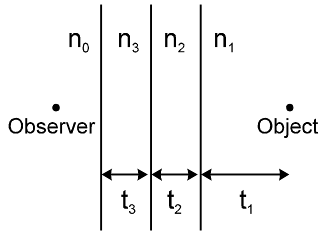
$= 3\left(1 - \dfrac{1}{3/2}\right)$

For mirror object is at a distance = $21 - 3\left(1 - \dfrac{1}{3/2}\right)$

= 20 cm

$\therefore$ Object is at the centre of curvature of mirror. Hence the light rays will retrace and image will formed on the object itself.

7.2 Refraction through a Composite Slab (or Refraction through a number of parallel media, as seen from a medium of R. I. n_0)



Apparent depth (distance of final image from final surface)

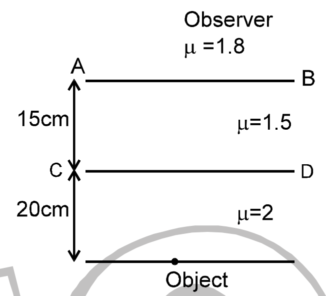
$$= \frac{t_1}{n_{1rel}} + \frac{t_2}{n_{2rel}} + \frac{t_3}{n_{3rel}} + \ldots\ldots\ldots + \frac{t_n}{n_{n\ rel}}$$

Apparent shift

$$= t_1\left[1 - \frac{1}{n_{1rel}}\right] + t_2\left[1 - \frac{1}{n_{2rel}}\right] + \ldots\ldots\ldots + \left[1 - \frac{n}{n_{n\ rel}}\right]t_n$$

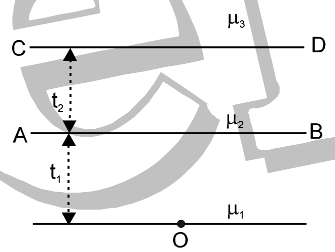
Where 't' represents thickness and 'n' represents the R.I. of the respective media, relative to the medium of observer. (i.e. $n_{1rel} = n_1/n_0$, $n_{2\ rel} = n_2/n_0$ etc.)

Ex.29 See figure. Find the apparent depth of object seen below surface AB.



Sol. $D_{app} = \sum \frac{d}{\mu} = \frac{20}{\left(\frac{2}{1.8}\right)} + \frac{15}{\left(\frac{1.5}{1.8}\right)} = 18 + 18 = 36$ cm.

Q.16 Find the apparent depth of object O below surface AB, seen by an observer in medium of refractive index μ_2



Q.17 In above question what is the depth of object corresponding to incident rays striking on surface CD in medium μ_2.

Q.18 In above question if observer is in medium μ_3 , what is the apparent depth of object seen below surface CD.

8 . CRITICAL ANGLE AND TOTAL INTERNAL REFLECTION (T. I. R.)

Critical angle is the angle made in denser medium for which the angle of refraction in rarer medium is 90º. When angle in denser medium is more then critical angle the light ray reflects back in denser medium following the laws of reflection and the interface behaves like a perfectly reflecting mirror.

In the figure

O = Object

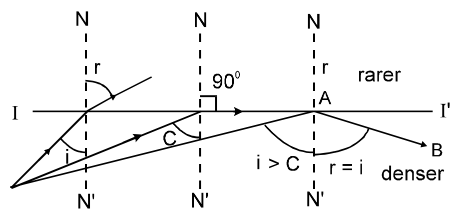
NN′ = Normal to the interface

II′ = Interface

C = Critical angle;

AB = reflected ray due to T. I. R.

When i = C then r = 90º

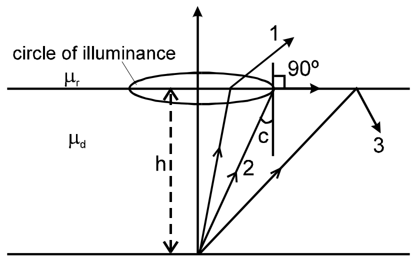


$\therefore$ **C = $\sin^{-1}\dfrac{n_r}{n_d}$**

8.1 Conditions of T. I. R.

(a) light is incident on the interface from denser medium.

(b) Angle of incidence should be greater than the critical angle (i > c). Figure shows a luminous object placed in denser medium at a distance h from an interface separating two media of refractive indices μ_r and μ_d. Subscript r &d stand for rarer and denser medium respectively.



In the figure ray 1 strikes the surface at an angle less than critical angle C and gets refracted in rarer medium . Ray 2 strikes the surface at critical angle and grazes the interface. Ray 3 strikes the surface making an angle more than critical angle and gets internally reflected. The locus of points where ray strikes at critical angle is a circle, called **circle of illuminance**. All light rays striking inside the circle of illuminance get refracted in rarer medium. If an observer is in rarer medium, he/she will see light coming out only from within the circle of illuminance. If a circular opaque plate covers the circle of illuminance , no light will get refracted in rarer medium and then the object can not be seen from the rarer medium. Radius of C.O.I can be easily found.

Ex.30 Find the max. angle that can be made in glass medium (μ = 1.5) if a light ray is refracted from glass to vacuum.

Sol. 1.5 sin C = 1 sin 90º, where C = critical angle.

sin C = 2/3

C = $\sin^{-1}$ 2/3

Ex.31 Find the angle of refraction in a medium (m = 2) if light is incident in vacuum, making angle equal to twice the critical angle.

Sol. Since the incident light is in rarer medium. Total Internal Reflection can not take place.

$$C = \sin^{-1}\frac{1}{\mu} = 30º$$

$\therefore$ i = 2C = 60º

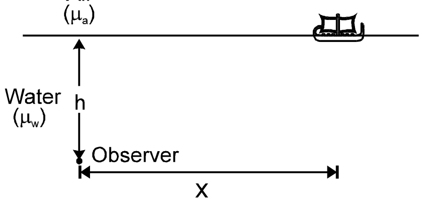
Applying Snell's Law. 1 sin 60º = 2 sin r

$$\sin r = \frac{\sqrt{3}}{4}$$

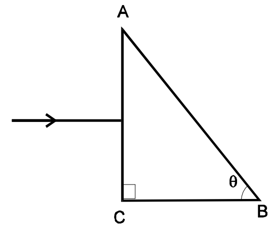
$$r = \sin^{-1}\left(\frac{\sqrt{3}}{4}\right).$$

Q.19 Find the radius of C.O.I., If a luminous object is placed at a distance h from the interface in denser medium.

Q.20 A ship is sailing in river. An observer is situated at a depth h in water (μ_w). If x >> h, find the angle made from vertical, of the line of sight of ship.



Ex.32 What should be the value of angle θ so that light entering normally through the surface AC of a prism (n=3/2) does not cross the second refracting surface AB.



Sol. Light ray will pass the surface AC without bending since it is incident normally. Suppose it strikes the surface AB at an angle of incidence i.

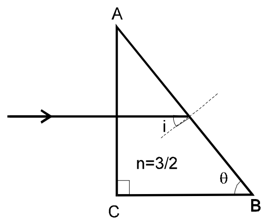
$i = 90 - \theta$

For the required condition: $90° - \theta > C$

or $\sin(90° - \theta) > \sin C$

or $\cos\theta > \sin C = \dfrac{1}{3/2} = \dfrac{2}{3}$

or $\theta < \cos^{-1}\dfrac{2}{3}$.



Ex.33 What should be the value of refractive index n of a glass rod placed in air, so that the light entering through the flat surface of the rod does not cross the curved surface of the rod.

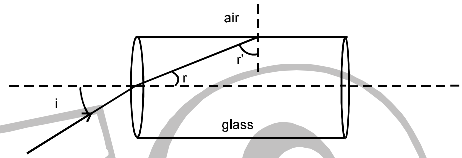
Sol: It is required that all possible r' should be more than critical angle. This will be automatically fulfilled if minimum r' is more than critical angle ....(A)

Angle r' is minimum when r is maximum i.e. C(why ?).Therefore the minimum value of r'is 90-C.

From condition (A) :

$90° - C > C$ or $C < 45°$

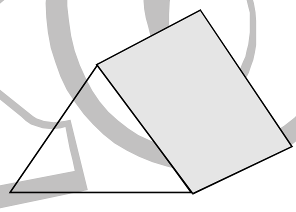
$\sin C < \sin 45°$; $\dfrac{1}{n} < \dfrac{1}{\sqrt{2}}$ or $n > \sqrt{2}$.



9. CHARACTERISTICS OF A PRISM

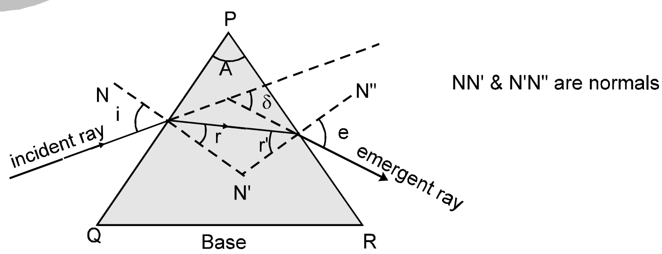
(a) A homogeneous solid transparent and refracting medium bounded by two plane surfaces inclined at an angle is called a prism.

3-D view



Refraction through a prism:

View from one side



(b) PQ and PR are refracting surfaces.

(c) $\angle QPR = A$ is called refracting angle or the angle of prism (also called Apex angle).

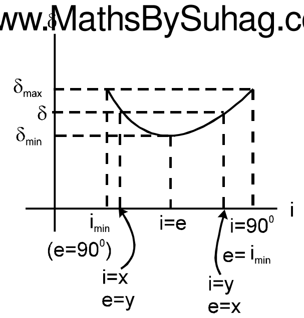
(d) δ = angle of deviation

(e) For refraction of a monochromatic (single wave length) ray of light through a prism;

$\delta = (i + e) - (r_1 + r_2)$ and $r_1 + r_2 = A$

$\therefore$ $\boldsymbol{\delta = i + e - A.}$

(f) Variation of δ versus i (shown in diagram).
For one δ (except δ min) there are two values of angle of incidence. If i and e are interchanged then we get the same value of δ because of reversibility principle of light



(g) There is one and only one angle of incidence for which the angle of deviation is minimum.

(h) When $\delta = \delta_{min}$, the angle of minimum deviation, then i = e and $r_1 = r_2$, the ray passes symmetrically w.r.t. the refracting surfaces. We can show by simple calculation that $\delta_{min} = 2i_{min} - A$ where i_{min} = angle of incidence for minimum deviation, and r = A/2.

$$\therefore \quad n_{rel} = \frac{\sin\left[\frac{A+\delta_m}{2}\right]}{\sin\left[\frac{A}{2}\right]}, \text{ where } n_{rel} = \frac{n_{prism}}{n_{surroundings}}$$

Also $\delta_{min} = (n-1)A$ (for small values of $\angle A$)

(i) For a thin prism ($A \le 10^o$) and for small value of i, all values of

$$\delta = (n-1)A$$

Ex.34 Show that if $A > A_{max}$ (= 2 C), then Total internal reflection occurs at second refracting surface PR for any value of 'i'.

Sol. For T.I.R. at second surface
r' > C
$\Rightarrow$ (A – r) > C
or A > (C + r)
The above relation will be fullfilled if
or $A > C + r_{max}$
or A > C + C
or A > 2C

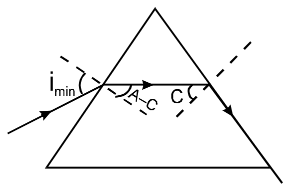
(j) On the basis of above example and similar reasoning, it can be shown that (you should try the following cases (ii) and (iii) yourself.)

(i) If A > 2C, all rays are reflected back from the second surface.

(ii) If $A \le C$, no rays are reflected back from the second surface i.e. all rays are refracted from second surface.

(iii) If $2C \ge A > C$, some rays are reflected back from the second surface and some rays are refracted from second surface, depending on the angle of incidence..

(k) δ is maximum for two values of $i \Rightarrow i_{min}$ (corresponding to e = 90º) and i = 90º (corresponding to e_{min}).
For i_{min} : $n_s \sin i_{min} = n_p \sin(A - C)$
If $i < i_{min}$ then T.I.R. takes place at second refracting surface PR.



Ex.35 Refracting angle of a prism A = 60º and its refractive index is, n = 3/2, what is the angle of incidence i to get minimum deviation. Also find the minimum deviation. Assume the surrounding medium to be air (n = 1).

Sol. For minimum deviation,

$$r_1 = r_2 = \frac{A}{2} = 30º.$$

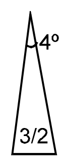
applying snell's law at I surface

$$1 \times \sin i = \frac{3}{2} \sin 30º \qquad \Rightarrow \qquad i = \sin^{-1}\left(\frac{3}{4}\right)$$

$$\delta_{min} = 2\sin^{-1}\left(\frac{3}{4}\right) - \frac{\pi}{3}$$

Ex.36 See the figure

Find the deviation caused by a prism having refracting angle 4º and refractive index $\frac{3}{2}$.



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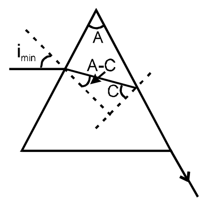
Sol. $\delta = (\frac{3}{2} - 1) \times 4^0 = 2^0$

Ex.37 For a prism, A = 60º, n = $\sqrt{\frac{7}{3}}$. Find the minimum possible angle of incidence, so that the light ray is refracted from the second surface. Also find δ_{max}.

Sol In minimum incidence case the angles will be as shown in figure
Applying snell's law :



$$1 \times \sin i_{min} = \sqrt{\frac{7}{3}} \sin (A - C)$$

$$= \sqrt{\frac{7}{3}} (\sin A \cos C - \cos A \sin C)$$

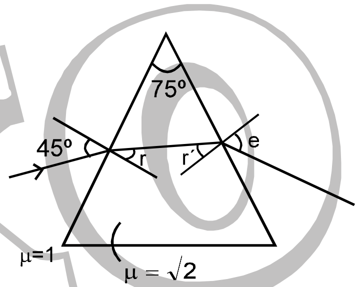
$$= \sqrt{\frac{7}{3}} \left(\sin 60\sqrt{1-\frac{3}{7}} - \cos 60\sqrt{\frac{3}{7}}\right) = \frac{1}{2}$$

$\therefore$ $i_{min} = 30º$

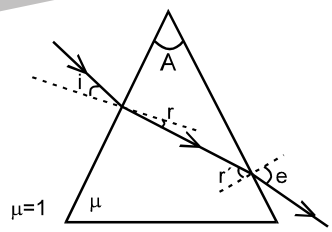
$\therefore$ $\delta_{max} = i_{min} + 90^0 - A$

$= 30^0 + 90^0 - 60^0 = 60^0$.

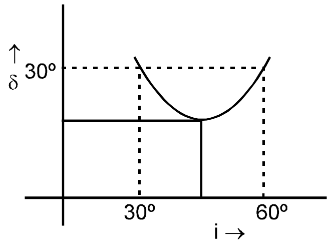
Q.21 Find r, r´, e, δ for the case shown in figure.



Q.22 For the case shown in figure prove the relations r´ – r = A and δ = | (i – e) + A | (do not try to remember these relations because the prism is normally not used in this way).



Q.23 From the graph of angle of deviation δ versus angle of incidence i, find the prism angle



10. DISPERSION OF LIGHT

The angular splitting of a ray of white light into a number of components and spreading in different directions is called *Dispersion of Light.* [It is for whole Electro Magnetic Wave in totality]. This phenomenon is because waves of different wavelength move with same speed in vacuum but with different speeds in a medium.

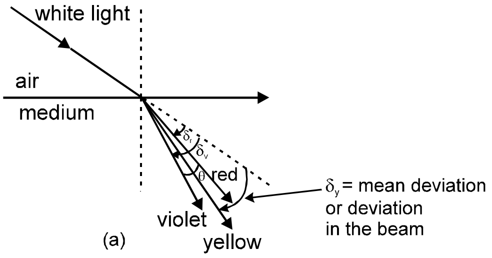
Therefore, the refractive index of a medium depends slightly on wavelength also. This variation of refractive index with wavelength is given by Cauchy's formula.

Cauchy's formula $n(\lambda) = a + \frac{b}{\lambda^2}$ where a and b are positive constants of a medium.

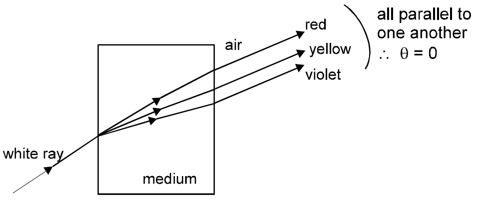
Note : Such phenomenon is not exhibited by sound waves.

Angle between the rays of the extreme colours in the refracted (dispersed) light is called **angle of dispersion.** $\theta = \delta_v - \delta_r$ (Fig. (a))

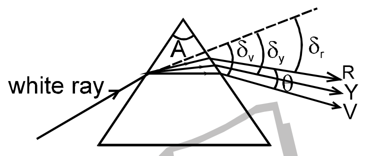
Fig (a) and (c) represents dispersion, whereas in fig. (b) there is no dispersion.



(a)



(b)



(c)

For prism of small 'A' and with small 'i' :

$\theta = \delta_v - \delta_r = (n_v - n_r)A$

Ex.38 The refractive indices of flint glass for red and violet light are 1.613 and 1.632 respectively. Find the angular dispersion produced by a thin prism of flint glass having refracting angle 5^0.

Sol. Deviation of the red light is $\delta_r = (\mu_r - 1)A$ and deviation of the violet light is $\delta_v = (\mu_v - 1)A$.

The disperation $= \delta_v - \delta_r = (\mu_v - \mu_r)A$

$= (1.632 - 1.613) \times 5^0$

$= 0.095^0$.

Deviation of beam (also called mean deviation)

$\delta = \delta_y = (n_y - 1)A$

n_v, n_r and n_y are R. I. of material for violet, red and yellow colours respectively.

Note : Numerical data reveals that if the average value of μ is small $\mu_v - \mu_r$ is also small and if theaverage value of μ is large $\mu_v - \mu_r$ is also large. Thus, larger the mean deviation, larger will be the angular dispersion.

Dispersive power (ω) of the medium of the material of prism is given by:

$$\omega = \frac{n_v - n_r}{n_y - 1}$$

Note : ω is the property of a medium.

For small angled prism ($A \le 10^o$) with light incident at small angle i : $\frac{n_v - n_r}{n_y - 1} = \frac{\delta_v - \delta_r}{\delta_y} = \frac{\theta}{\delta_y}$

$$= \frac{\text{angular dispersion}}{\text{deviation of mean ray (yellow)}}$$

[$n_y = \frac{n_v + n_r}{2}$ if n_y is not given in the problem]

Note: $n - 1$ = refractivity of the medium for the corresponding colour.

Ex.39 Refractive index of glass for red and violet colours are 1.50 and 1.60 respectively.

Find (A) the ref. index for yellow colour, approximately

(B) Dispersive power of the medium.

Sol. (A) $\mu_r \simeq \frac{\mu_v + \mu_R}{2} = \frac{1.50+1.60}{2} = 1.55$ (B) $\omega = \frac{\mu_v - \mu_R}{\mu_r - 1} = \frac{1.60-1.50}{1.55-1} = 0.18.$

10.1 Dispersion without deviation (Direct Vision Combination)

The condition for direct vision combination is :

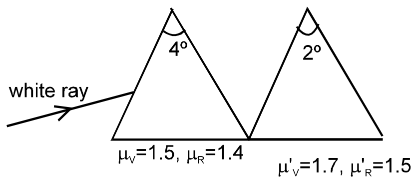
$$\left[n_y - 1\right] A = \left[n'_y - 1\right] A' \Leftrightarrow \left[\frac{n_v + n_r}{2} - 1\right] A = \left[\frac{n'_v + n'_r}{2} - 1\right] A'$$

Two or more prisms can be combined in various ways to get different combination of angular dispersion and deviation.6

10.2 Deviation without dispersion (Achromatic Combination)

Condition for achromatic combination is: $(n_v - n_r)\, A = (n'_v - n'_r)\, A'$

Ex.40 If two prisms are combined, as shown in figure, find the total angular dispersion and angle of deviation suffered by a white ray of light incident on the combination.



Sol. Both prisms will turn the light rays towards their bases and hence in same direction. Therefore turnings caused by both prisms are additive.

Total angular dispersion

$= \theta + \theta' = (\mu_V - \mu_R)\, A + (\mu'_V - \mu'_R)\, A'$

$= (1.5 - 1.4)\, 4^\circ + (1.7 - 1.5) 2^\circ = 0.8°$

Total derivation

$= \delta + \delta'$

$= \left(\frac{\mu_V + \mu_R}{2} - 1\right) A + \left(\frac{\mu'_V + \mu'_R}{2} - 1\right) A'$

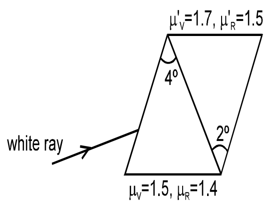
$= \left(\frac{1.5+1.4}{2} - 1\right) 0.4^\circ + \left(\frac{1.7+1.5}{2} - 1\right) 0.2^\circ$

$= (1.45 - 1)\, 0.4^\circ + (1.6 - 1)\, 0.2^\circ$

$= 0.45 \times 0.4^\circ + 0.6 \times 0.2^\circ$

$= 1.80 + 1.2 =$ **3.0º** **Ans.**

Q.24



If two prisms are combined, as shown in figure, find the net angular dispersion and angle of deviation suffered by a white ray of light incident on the combination.

Ex.41 Two thin prisms are combined to form an achromatic combination. For I prism A = 4º, $\mu_R = 1.35$, $\mu_Y = 1.40$, $\mu_v = 1.42$. for II prism $\mu'_R = 1.7$, $\mu'_Y = 1.8$ and $\mu'_R = 1.9$ find the prism angle of II prism and the net mean derivation.

Sol. Condition for achromatic combination.

$$\theta = \theta'$$

$$(\mu_V - \mu_R)A = (\mu'_V - \mu'_R)A'$$

$$\therefore \quad A' = \frac{(1.42 - 1.35)4º}{1.9 - 1.7} = 1.4°$$

$$\delta_{Net} = \delta \sim \delta' = (\mu_Y - 1)A \sim (\mu'_Y - 1)A'$$

$$= (1.40 - 1)\ 4º \sim (1.8 - 1)\ 1.4°$$
$$= 0.48º.$$

$$\theta = \theta'$$

$$(\mu_V - \mu_R)A = (\mu'_V - \mu'_R)A'$$

$$\therefore \quad A' = \frac{(1.42 - 1.35)4º}{1.9 - 1.7} = 1.4°$$

$$\delta_{Net} = \delta \sim \delta' = (\mu_Y - 1)A \sim (\mu'_Y - 1)A'$$

$$= (1.40 - 1)\ 4º \sim (1.8 - 1)\ 1.4°$$
$$= 0.48º.$$

Ex.42 A crown glass prism of angle 5^0 is to be combined with a flint prism in such a way that the mean ray passes undeviated. Find (a) the angle of the flint glass prism needed and (b) the angular dispersion produced by the combination when white light goes through it. Refractive indices for red, yellow and violet light are 1.514, 1.517 and 1.523 respectively for crown glass and 1.613,1.620 and 1.632 for flint glass.

Sol. The deviation produced by the crown prism is

$$\delta = (\mu - 1)A$$

and by the flint prism is :

$$\delta' = (\mu' - 1)A'.$$

The prisms are placed with their angles inverted with respect to each other. The deviations are also in opposite directions. Thus, the net deviation is :

$$D = \delta - \delta' = (\mu - 1)A - (\mu' - 1)A'. \qquad ....(1)$$

(a) If the net deviation for the mean ray is zero,

$$(\mu - 1)A = (\mu' - 1)A'.$$

$$\text{or,} \quad A' = \frac{(\mu - 1)}{(\mu' - 1)}A = \frac{1.517 - 1}{1.620 - 1} \times 5^0$$

(b) The angular dispersion produced by the crown prism is :

$$\delta_v - \delta_r = (\mu_v - \mu_r)A$$

and that by the flint prism is,

$$\delta'_v - \delta'_r = (\mu'_v - \mu'_r)A$$

The net angular dispersion is,

$$(\mu_v - \mu_r)A - (\mu'_v - \mu'_r)A$$
$$= (1.523 - 1.514) \times 5^0 - (1.632 - 1.613) \times 4.2^0$$
$$= -0.0348^0.$$

The angular dispersion has mangitude 0.0348^0.

(a) If the net deviation for the mean ray is zero,

$$(\mu - 1)A = (\mu' - 1)A'.$$

$$\text{or,} \quad A' = \frac{(\mu - 1)}{(\mu' - 1)}A = \frac{1.517 - 1}{1.620 - 1} \times 5^0$$

(b) The angular dispersion produced by the crown prism is :

$$\delta_v - \delta_r = (\mu_v - \mu_r)A$$

and that by the flint prism is,

$\delta'_v - \delta'_r = (\mu'_v - \mu'_r)A$

The net angular dispersion is,

$(\mu_v - \mu_r)A - (\mu'_v - \mu'_r)A$

$= (1.523 - 1.514) \times 5^0 - (1.632 - 1.613) \times 4.2^0$

$= - 0.0348^0$.

The angular dispersion has mangitude 0.0348^0 .

Q.25 The dispersive powers of crown and flint glasses are 0.03 and 0.05 respectively. The refractive indices for yellow light for these glasses are 1.517 and 1.621 respectively. It is desired to form an achromatic combination of prisms of crown and flint glasses which can produce a deviation of 1^0 in the yellow ray. Find the refracting angles of the two prisms needed.

11. SPECTRUM :

(Only for your knowledge and not of much use for JEE)

Ordered pattern produced by a beam emerging from a prism after refraction is called *Spectrum*. Types of spectrum:

11.1 Types of spectrum:

(a) Line spectrum:
Due to source in atomic state.

(b) Band spectrum:
Due to source in molecular state.

(c) Continuous spectrum:
Due to white hot solid.

11.2 In Emission Spectrum:

Bright colours or lines, emitted from source are observed.

The spectrum emitted by a given source of light is called emission spectrum. It is a wavelength-wise distribution of light emitted by the source. The emission spectra are given by incandescent solids, liquids and gases which are either burned directly as a flame (or a spark) or burnt under low pressure in a discharge tube.

11.3 In Absorption Spectrum:

Dark lines indicates frequencies absorbed.

When a beam of light from a hot source is passed through a substance (at a lower temperature), a part of the light is transmitted but rest of it is absorbed. With the help of a spectrometer, we can know the fraction of light obsorbed corresponding to each wavelength. The distribution of the wavelength absorptionof light by a substance is called an absorption spectrum. Every substance has its own characteristic absorption spectrum.

11.4 Spectrometer

Consists of a collimator (to collimate light beam), prism and telescope. It is used to observe the spectrum and also measure deviation.

12. REFRACTION AT SPHERICAL SURFACES

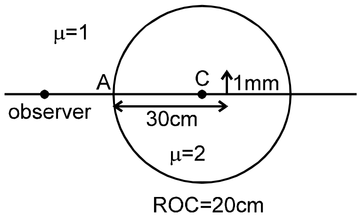
For paraxial rays incident on a spherical surface separating two media:

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R} \quad .... \text{(A)}$$

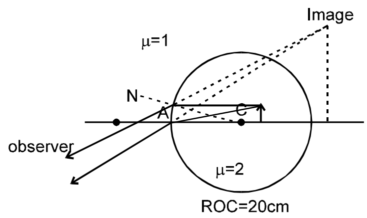
where light moves from the medium of refractive index n_1 to the medium of refractive index n_2.

Transverse magnification (m) (of dimension perpendicular to principal axis) due to refraction at spherical surface is given by $\mathbf{m} = \dfrac{v-R}{u-R} = \left(\dfrac{v/n_2}{u/n_1}\right)$

Ex.43 Find the position, size and nature of image, for the situation shown in figure. Draw ray diagram .



Sol. For refraction near point A, u = – 30 ; R = – 20; n_1 = 2 ; n_2 = 1.
Applying refraction formula



$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

$$\frac{1}{v} - \frac{2}{-30} = \frac{1-2}{-20}$$

$$v = -60 \text{ cm}$$

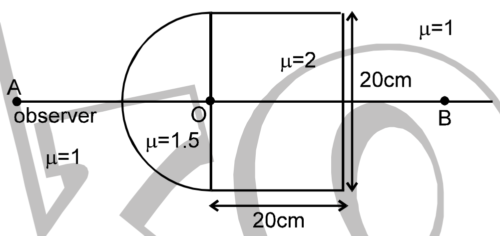
$$m = \frac{h_2}{h_1} = \frac{n_1 v}{n_2 u} = \frac{2(-60)}{1(-30)} = 4$$

∴ h_2 = 4 mm.

Q.26 See the situation shown in figure

(1) Find the position of image as seen by observer A.

(2) Find the position of image as seen by observer B.



<u>**Special case:**</u>

Refraction at plane Surfaces

Putting R = ∞ in the formula $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$, we get;

$$\mathbf{v = \frac{n_2 u}{n_1}}$$

The same sign of v and u implies that the object and the image are always on the same side of the interface separating the two media. If we write the above formula as

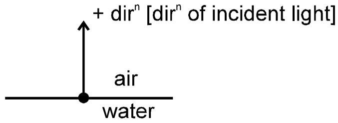
$$v = \frac{u}{n_{rel}},$$

it gives the relation between the apparent depth and real depth, as we have seen before.

Ex.44 Using formula of spherical surface or otherwise, find the apparent depth of an object placed 10 cm below the water surface, if seen near normally from air.

Sol: Put R = ∞ in the formula of the Refraction at Spherical Surfaces we get,

$$v = \frac{un_2}{n_1}$$



$$u = -10 \text{ cm}$$

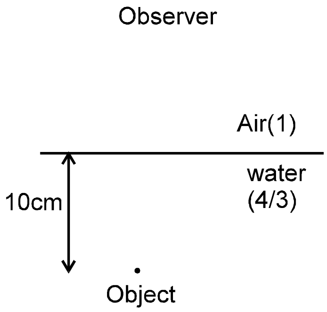
$$n_1 = \frac{4}{3}$$

$$n_2 = 1$$

$$v = -\frac{10 \times 1}{4/3} = -7.5 \text{ cm}$$

–ve sign implies that the image is formed in water.

Aliter:



$$d_{app} = \frac{d_{real}}{\mu_{rel}}$$

$$= \frac{10}{4/3} = \frac{30}{4} = 7.5 \text{ cm.}$$

13. Thin Lens

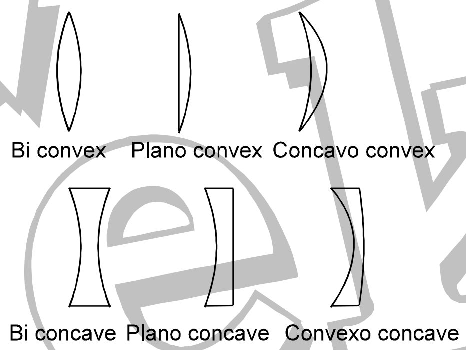
A thin lens is called convex if it is thicker at the middle and it is called concave if it is thicker at the ends. One surface of a convex lens is always convex . Depending on the other surface a convex lens is categorized as

(a) biconvex or convexo convex , if the other surface is also convex,

(b) Plano convex if the other surface is plane and

(c) Concavo convex if the other surface is concave.

Similarly concave lens is categorized as concavo-concave or biconcave, plano-concave and convexo-concave.



For a spherical, thin lens *having the same medium on both sides*:

$$\frac{1}{v} - \frac{1}{u} = (n_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{.........(a),}$$

where $n_{rel} = \dfrac{n_{lens}}{n_{medium}}$ and R_1 and R_2 are x coordinates of the centre of curvature of the 1st surface and 2nd surface respectively.

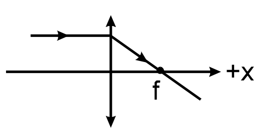
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \rightarrow \text{ Lens Maker's Formula................(b)}$$

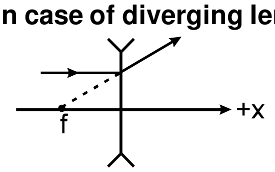
Lens has two Focii:

If $u = \infty$, then $\dfrac{1}{v} - \dfrac{1}{\infty} = \dfrac{1}{f}$ $\Rightarrow$ $v = f$

$\Rightarrow$ If incident rays are parallel to principal axis then its refracted ray will cut the principal axis at 'f'. It is called 2nd focus.

In case of converging lens it is positive and in case of diverging lens it is negative.

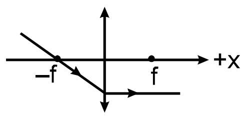


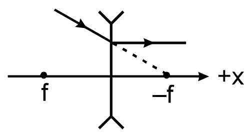


If $v = \infty$ that means

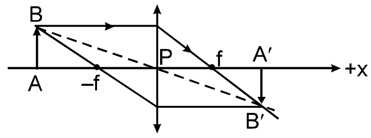
$$\frac{1}{\infty} - \frac{1}{u} = \frac{1}{f} \quad \Rightarrow \quad u = -f$$

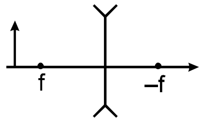
⇒ If incident rays cuts principal axis at – f then its refracted ray will become parallel to the principal axis. It is called 1st focus. In case of converging lens it is negative (∵ f is positive) and in the case of diverging lens it positive (∵ f is negative)





use of – f & + f is in drawing the ray diagrams.
Notice that the point B, its image B′ and the pole P of the lens are collinear. It is due to parallel slab nature of the lens at the middle. This ray goes straight. (Remember this)





From the relation $\frac{1}{f}=(n_{rel}-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ it can be seen that the second focal length depends on two factors. **(A)** The factor $\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ is

(a) **Positive** for all types of **convex** lenses and
(b) **Negative** for all types of **concave** lenses.

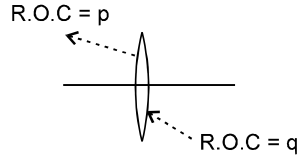
(B) The factor $(n_{rel}-1)$ is
(a) **Positive** when **surrounding medium is rarer than the medium of lens.**
(b) **Negative** when **surrounding medium is denser than the medium of lens.**

(C) So a lens is **converging** if **f is positive** which happens when **both the factors (A) and (B) are of same sign**.
(D) And a lens is **diverging** if **f is negative** which happens when the **factors (A) and (B) are of opposite signs**.

Ex.45 Find the behavior of a concave lens placed in a rarer medium.
Sol. Factor (A) is **negative** , because the lens is **concave**.
Factor (B) is **positive** , because the **lens is placed in a rarer medium**.
Therefore the focal length of the lens, which depends on the product of these factors , is negative and hence the lens will behave as diverging lens.

Ex.46 Show that the factor $\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$(and therefore focal length) does not depend on which surface of the lens light strike first.

Sol: Consider a convex lens of radii of curvature p and q as shown.



CASE 1:
Suppose light is incident from left side and strikes the surface with radius of curvature p, first.

Then $R_1 = +p$; $R_2 = -q$ and $\left(\frac{1}{R_1}-\frac{1}{R_2}\right) = \left(\frac{1}{p}-\frac{1}{-q}\right)=\left(\frac{1}{p}+\frac{1}{q}\right)$

CASE 2:
Suppose light is incident from right side and strikes the surface with radius of curvature q, first.

Then $R_1 = +q$; $R_2 = -p$ and $\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{1}{q} - \frac{1}{-p}\right) = \left(\frac{1}{p} + \frac{1}{q}\right)$

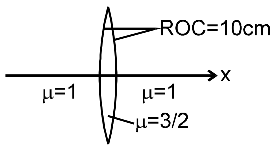
Though we have shown the result for biconvex lens , it is true for every lens.

Q.27 Find the focal length of a double-convex lens with R_1 = 15 cm and R_2 = –25 cm. The refractive index of the lens material n = 1.5.

Q.28 Find the focal length of a plano-convex lens with R_1 = 15 cm and $R_2 = \infty$. The refractive index of the lens material n = 1.5.

Q.29 Find the focal length of a concavo-convex lens (positive meniscus) with R_1 = 15 cm and R_2 = 25 cm. The refractive index of the lens material n = 1.5.

Ex.47 Find the focal length of the lens shown in the figure.



converging lens

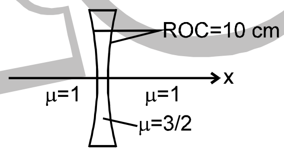
Sol. $\because$ $\frac{1}{f} = (n_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\frac{1}{f} = (3/2 - 1)\left(\frac{1}{10} - \frac{1}{(-10)}\right)$$

$$\frac{1}{f} = \frac{1}{2} \times \frac{2}{10}$$

f = + 10 cm.

Ex.48 Find the focal length of the lens shown in figure



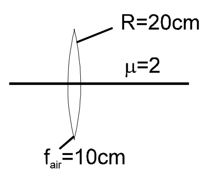
Sol. $\frac{1}{f} = (n_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{3}{2} - 1\right)\left(\frac{1}{-10} - \frac{1}{10}\right)$ f = – 10 cm

Ex: 49 Point object is placed on the principal axis of a thin lens with parallel curved boundaries i.e., having same radii of curvature. Discuss about the position of the image formed .

Sol: $\frac{1}{f} = (n_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = 0$ [$\because R_1 = R_2$]

$\frac{1}{v} - \frac{1}{u} = 0$ or v = u i.e. rays pass without appreciable bending.

Ex.50 Focal length of a thin lens in air, is 10 cm. Now medium on one side of the lens is replaced by a medium of refractive index μ=2. The radius of curvature of surface of lens, in contact with the medium, is 20 cm. Find the new focal length.



Sol: Let radius of I surface be R_1 and refractive index of lens be μ . Let parallel rays be incident on the lens. Applying refraction formula at first surface

$$\frac{\mu}{V_1} - \frac{1}{\infty} = \frac{\mu - 1}{R_1} \qquad ...(1)$$

At II surface $\dfrac{2}{V} - \dfrac{\mu}{V_1} = \dfrac{2-\mu}{-20}$ ...(2)

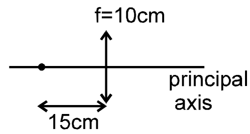
Adding (1) and (2)

$$\frac{\mu}{V_1} - \frac{1}{\infty} + \frac{2}{V} - \frac{\mu}{V_1} = \frac{\mu - 1}{R_1} + \frac{2-\mu}{-20}$$

$$= (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{-20}\right) - \frac{\mu - 1}{20} - \frac{2-\mu}{20} = \frac{1}{f}\ \text{(in air)} + \frac{1}{20} - \frac{2}{20}$$

$\Rightarrow$ v = 40 cm $\Rightarrow$ f = 40 cm

Ex.51 Figure shown a point object and a converging lens. Find the final image formed.

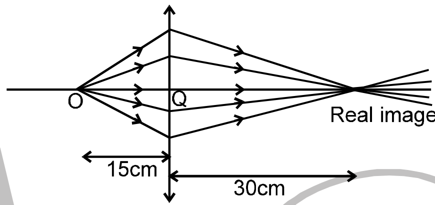


Sol. $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

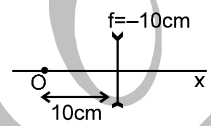
$\dfrac{1}{v} - \dfrac{1}{-15} = \dfrac{1}{10}$

$\dfrac{1}{v} = \dfrac{1}{10} - \dfrac{1}{15} = \dfrac{1}{30}$

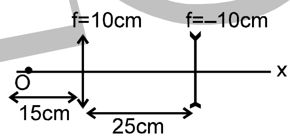
v = + 30 cm



Q.30 Figure shows a point object and a diverging lens. Find the final image formed.



Ex.52 See the figure
Find the position of final image formed.



Sol. For converging lens

u = –15 cm, f = 10 cm

$$v = \frac{fu}{f+u} = 30\ \text{cm}$$

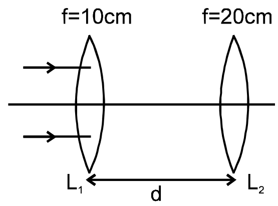
For diverging lens

u = 5 cm

f = –10 cm

$\therefore$ $v = \dfrac{fu}{f+u} = 10$ cm

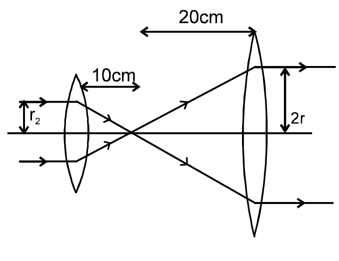
Ex.53 Figure shows two converging lenses. Incident rays are parallel to principal axis. What should be the value of d so that final rays are also parallel.



Sol. Final rays should be parallel. For this the II focus of L_1 must coincide with I focus of L_2.

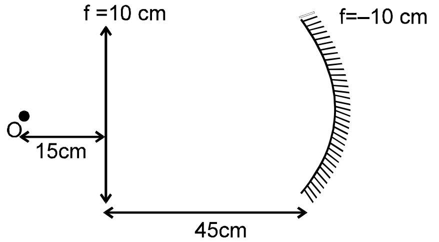
d = 10 + 20

= 30 cm



Here the diameter of ray beam becomes wider.

Ex.54 See the figure



Find the position of final image formed.

Sol. For lens,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{-15} = \frac{1}{10} \qquad \Rightarrow \qquad v = +30 \text{ cm}$$

Hence it is object for mirror

$u = -15$ cm

$$\frac{1}{v} + \frac{1}{-15} = \frac{1}{-10} \qquad \Rightarrow \qquad v = -30 \text{ cm}$$

Now for second time it again passes through lens

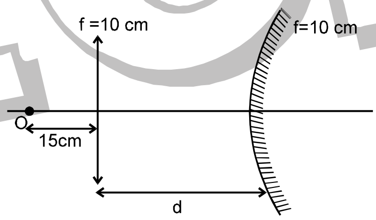
$u = -15$ cm

$v = ?$; $f = 10$ cm

$$\frac{1}{v} - \frac{1}{-15} = \frac{1}{10} \qquad \Rightarrow \qquad v = +30$$

Hence final image will form at a distance 30 cm from the lens towards left.

Ex.55 What should be the value of d so that image is formed on the object itself.

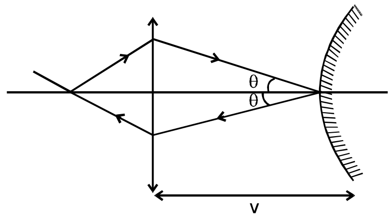


Sol. For lens :

$$\frac{1}{v} - \frac{1}{-15} = \frac{1}{10}$$

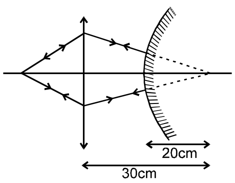
$v = +30$ cm

Case I :



If d = 30, the object for mirror will be at pole and its image will be formed there itself.

Case II :



If the rays strike the mirror normally, they will retrace and the image will be formed on the object itself

$\therefore$ d = 30 – 20 = 10 cm

13.2 Transverse magnification (m)

Transverse magnification (m) of (of dimension perpendicular to principal axis) is given by

$$m = \frac{v}{u}$$

If the lens is thick or/and the medium on both sides is different, then we have to apply the formula given for refraction at spherical surfaces step by step.

Ex.56 An extended real object of size 2 cm is placed perpendicular to the principal axis of a converging lens of focal length 20 cm. The distance between the object and the lens is 30 cm.
(i) Find the lateral magnification produced by the lens.
(ii) Find the height of the image.
(iii) Find the change in lateral magnification, if the object is brought closer to the lens by 1 mm along the principal axis.

Sol. Using

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

and $m = \frac{v}{u}$

we get $m = \frac{f}{f+u}$ ....(A)

$\therefore$ $m = \frac{+20}{+20+(-30)}$

$$= \frac{+20}{-10} = -2$$

–ve sign implies that the image is inverted.

(ii) $\frac{h_2}{h_1} = m$

$\therefore$ $h_2 = mh_1 = (-2)\ (2) = -4$ cm

(iii) Differentiating (A) we get

$$dm = \frac{-f}{(f+u)^2}\ du$$

$$= \frac{-(20)}{(-10)^2}\ (0.1) = \frac{-2}{100} = -.02$$

Note that the method of differential is valid only when changes are small.

Aliter u (after displacing the object)
= –(30 + 0.1) = – 29.9 cm

Applying the formula

$$m = \frac{f}{f+u}$$

$$m = \frac{20}{20+(-29.9)} = -2.02$$

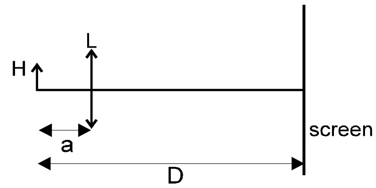
$\therefore$ change in 'm' = – 0.02.

Since in this method differential is not used, this method can be used for any changes, small or large.

Q.31 An extended real object is placed perpendicular to the principal axis of a concave lens of focal length –10 cm, such that the image found is half the size of object.
(a) Find the object distance from the lens
(b) Find the image distance from the lens and draw the ray diagram
(c) Find the lateral magnification if object in moved by 1 mm along the principal axis towards the lens.

13.3 Displacement Method to find Focal length of Converging Lens :

Fix an object of small height H and a screen at a distance D from object (as shown in figure). Move a converging lens from the object towards the screen. Let a sharp image forms on the screen when the distance between the object and the lens is 'a'. From lens formula we have



$$\frac{1}{D-a} - \frac{1}{-a} = \frac{1}{f}$$

or $a^2 - D\,a + f\,D = 0$ ...(A)

This is quadratic equation and hence two values of 'a' are possible. Call them a_1 and a_2. Thus a, and a_2 are the roots of the equation. From the properties of roots of a quadratic equation,

$\therefore$ $a_1 + a_2 = D$

$a_1 a_2 = f\,D$

Also $(a_1 - a_2) = \sqrt{(a_1 + a_2)^2 - 4a_1a_2} = \sqrt{D^2 - 4fD} = d$ (suppose).

'd' physically means the separation between the two position of lens.

Q.32 Find the relation for the focal length of lens is terms of D and d.

Q.33 For what condition, d = 0, i.e. the two position coincide

Q.34 Roots of the eq (A) become imaginary if ....

Q.35 What type of image is formed on screen .... (Real/virtual)

Q.36 Prove that minimum distance between real point object and its image in case of converging lens is 4f.

Q.37 If m_1 and m_2 are the lateral magnifications in the two position of lens then show that $m_1m_2=1$.

Q.38 If image length are h_1 and h_2 in the two cases, prove that $h_1h_2 = H^2$.

14. COMBINATION OF LENSES:

The equivalent focal length of thin lenses in contact is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} \ldots$$

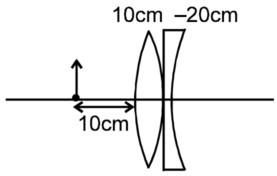
where f_1, f_2, f_3 are focal lengths of individual lenses.

If two converging lenses are separated by a distance d and the incident light rays are parallel to the common principal axis ,then the combination behaves like a single lens of focal length given by the relation

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1f_2}$$

and the position of equivalent lens is $\frac{-d\ F}{f_1}$ with respect to 2^{nd} lens

Ex.57 Find the lateral magnification produced by the combination of lenses shown in the figure.



Sol. $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{10} - \frac{1}{20} = \frac{1}{20}$

f = + 20

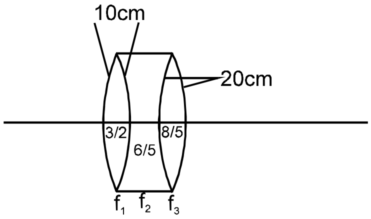
$\therefore$ $\frac{1}{v} - \frac{1}{-10} = \frac{1}{20}$

$\frac{1}{v} = \frac{1}{20} - \frac{1}{10}$

$= \frac{-1}{20} = -20$ cm

$\therefore \quad m = \frac{-20}{-10} = 2$

Ex.58 Find the focal length of equivalent system.



Sol. $\frac{1}{f_1} = \left(\frac{3}{2}-1\right)\left(\frac{1}{10}+\frac{1}{10}\right)$

$= \frac{1}{2} \times \frac{2}{10} = \frac{1}{10}$

$\frac{1}{f_2} = \left(\frac{6}{5}-1\right)\left(\frac{-1}{10}-\frac{1}{20}\right)$

$= \frac{1}{5} \times \left(\frac{-30}{10\times 20}\right)$

$= \frac{-3}{100}$

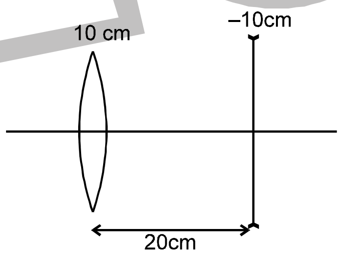
$\frac{1}{f_3} = \left(\frac{8}{5}-1\right)\left(\frac{1}{20}+\frac{1}{20}\right)$

$= \frac{3}{50}$

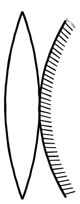
$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} = \frac{1}{10} + \frac{-3}{100} + \frac{3}{50}$

$f = \frac{100}{13}$

Q.39 Find the equivalent focal length of the system for paraxial rays parallel to axis.



15. COMBINATION OF LENS AND MIRROR :

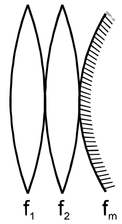


The combination of lens and mirror behaves like a mirror of focal length ‘f’ given by

$$\frac{1}{f} = \frac{1}{F_m} - \frac{2}{F_\ell}$$

If lenses are more then one, ‘f’ is given by

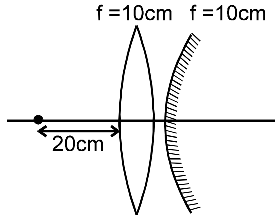
$$\frac{1}{f} = \frac{1}{F_m} - 2\left(\sum \frac{1}{f_\ell}\right)$$



For the following figure
'f' is given by

$$\frac{1}{f} = \frac{1}{F_m} - 2\left(\frac{1}{f_1} + \frac{1}{f_2}\right)$$

Ex.59 Find the position of final image formed.
(The gap shown in figure is of negligible width)



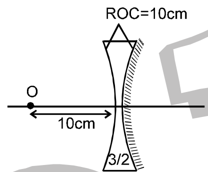
Sol. $\frac{1}{f_{eq}} = \frac{1}{10} - \frac{2}{10} = \frac{-1}{10}$

$f_{eq} = -10$ cm

$\frac{1}{v} + \frac{1}{-20} = \frac{1}{-10}$ $\Rightarrow$ $v = -20$ cm

Hence image will be formed on the object itself

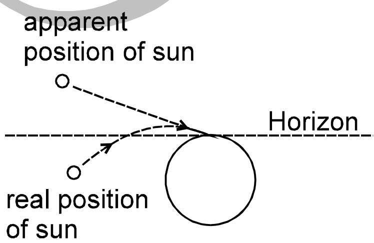
Q.40 See the figure



Find the equivalent focal length of the combination shown in the figure and position of image.

SOME INTERESTING FACTS ABOUT LIGHT :

(1) The Sun Rises Before It Actually Rises And Sets After It Actually Sets :



The atmosphere is less and less dense as its height increase, and it is also known that the index of refraction decrease with a decrease in density. So, there is a decrease of the index of refraction with height.Due to this the light rays bend as they move in the earth's atmosphere

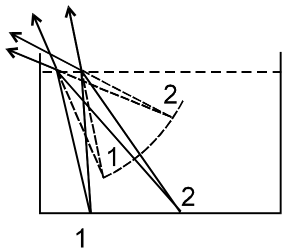
(2) The Sun Is Oval Shaped At The Time Of Its Rise And Set :
The rays diverging from the lower edge of the sun have to cover a greater thickness of air than the rays from the upper edge. Hence the former are refracted more than the latter, and so the vertical diameter of the sun appears to be a little shorter than the horizontal diameter which remains unchanged.

(3) The Stars Twinkle But Not The Planets. The refractive index of atmosphere fluctuates by a small amount due to various reasons. This causes slight variation in bending of light due to which the apparent position of star also changes, producing the effect of twinkling.

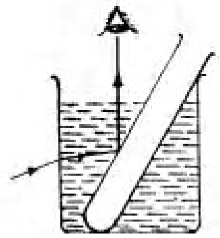
(4) Glass Is Transparent, But Its Powder Is White :
When powerded, light is reflected from the surface of innumerable small pieces of glass and so the poweder appears white. Glass transmits most of the incident light and reflects very little hence it appears transparent.

(5) Greased Or Oiled Paper Is Transprent, But Paper Is White :
The rough surface of paper diffusely reflects incident light and so it appears white. When oiled or greased very little reflection takes place and most of the light is allowed to pass and hence it appears transparent.

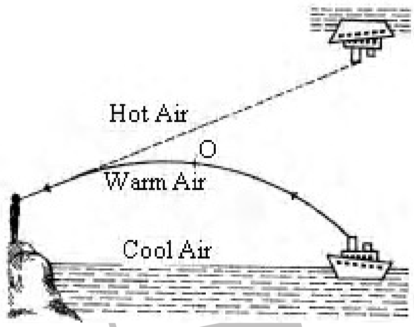
(6) An Extended Water Tank Appears Shallow At The Far End :

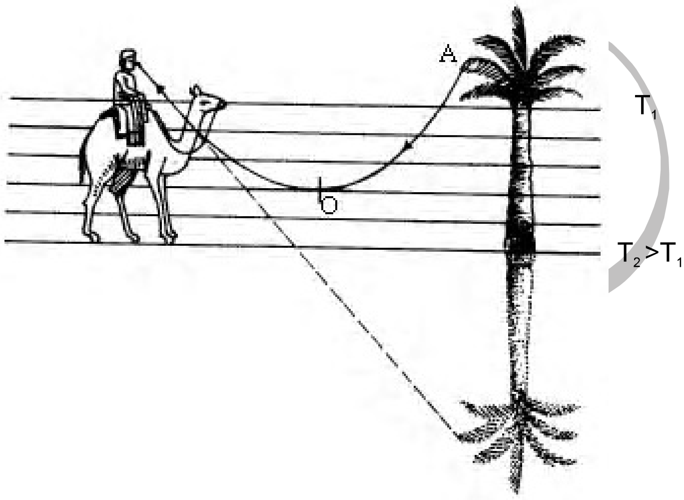


(7) A Test Tube Or A Smoked Ball Immersed In Water Appears Silvery White When Viewed From The Top : This is due to Total internal reflection



(8) Ships Hang Inverted In The Air In Cold Countries And Trees Hang Inverted Underground In Deserts:
This is due to Total internal reflection





SUMMARY

- Light is said to move rectilinearly if, $a >> \sqrt{D\lambda}$.
- **Mirror formula :**

$$\frac{1}{v}+\frac{1}{u}=\frac{2}{R}=\frac{1}{f}$$

- **Newton's Formula**: $XY = f^2$
- Optical power of a mirror (in Diopters) = $\frac{1}{f}$
- **Laws of refraction :**

$$\frac{\text{Sin } i}{\text{Sin } r} = \frac{n_2}{n_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$$

- Refractive index of the medium relative to vacuum $= \sqrt{\mu_r \in_r}$
- Deviation (δ) of ray incident at $\angle$ i and refracted at $\angle$ r is given by $\delta = |i - r|$
- **Principle of Reversibility of Light Rays:**

$${}_1n_2 = \frac{1}{{}_2n_1}$$

- **Refraction through a Parallel Slab**

$$d = \frac{t \sin(i-r)}{\cos r}$$

- **Apparent Depth and shift of Submerged Object:**

Apparent shift = $d\left(1-\frac{1}{n_{rel}}\right)$

- **Refraction through a Composite Slab**

Apparent shift =

$$t_1\left[1-\frac{1}{n_{1rel}}\right] + t_2\left[1-\frac{1}{n_{2rel}}\right] + \ldots\ldots\ldots + \left[1-\frac{n}{n_{n\,rel}}\right]t_n$$

- **Critical Angle and Total Internal Reflection (T. I. R.):**

$$C = \sin^{-1}\frac{n_r}{n_d}$$

- **Angle of deviation :**

$\delta = i + e - A.$

- **Angle of dispersion.**

$\theta = \delta_v - \delta_r$

- **Cauchy's formula**

$$n(\lambda) = a + \frac{b}{\lambda^2}$$

- **Dispersive power**

$$\omega = \frac{n_v - n_r}{n_y - 1}$$

- **Dispersion without deviation**

The condition for direct vision combination is :

$$[n_y - 1]\ A = [n'_y - 1]\ A' \Leftrightarrow \left[\frac{n_v + n_r}{2} - 1\right]$$

$$A = \left[\frac{n'_v + n'_r}{2} - 1\right] A'$$

- **Deviation without dispersion**

Condition for achromatic combination is:

$(n_v - n_r)\ A = (n'_v - n'_r)\ A'$

- **Spherical Surfaces**

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

- **Transverse magnification**

$$m = \frac{v-R}{u-R} = \left(\frac{v/n_2}{u/n_1}\right)$$

- **Refraction at plane Surfaces**

$$v = \frac{n_2 u}{n_1}$$

For a spherical, thin lens *having the same medium on both sides*:

$$\frac{1}{v} - \frac{1}{u} = (n_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \ldots\ldots\ldots(a),$$

where $n_{rel} = \frac{n_{lens}}{n_{medium}}$

- **Transverse magnification (m)**

$$m = \frac{v}{u}$$

- **Lens Maker's Formula**

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

- The equivalent focal length of thin lenses in contact is given by

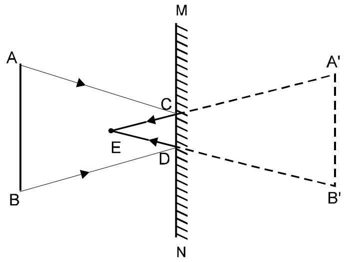
$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} \ldots$$

- The combination of lens and mirror behaves like a mirror of focal length 'f' given by

$$\frac{1}{f} = \frac{1}{F_m} - \frac{2}{F_\ell}$$

ANSWERS OF THE UNSOLVED PROBLEMS

1. O_3 .

2. 

Part CD is used to view the image.

3. 20 m/s in the direction of motion of mirror.

4. **Hint:** Join $I\,I_1$, $I\,I_2$ and IO and show them to be of equal length.

5. At the centre of curvature of curved mirror.

6. Final image will be formed at distance 30 cm infront of m_1.

7. 40 cm. **8.** 20/3 cm, concave

9. $\mu = \frac{2}{\sqrt{3}}$ **10.** $\left(1+\frac{2}{\sqrt{3}}\right)$. **11.** d = 0.2 m **13.** 120 cm.

14. 7.5 cm above the water surface **16.** $\frac{t_1}{\mu_1/\mu_2}$. **17.** $t_2 + \frac{t_1}{\mu_1/\mu_2}$.

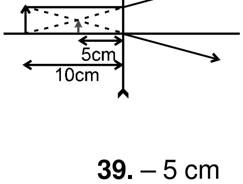
18. $$\frac{t_2 + \dfrac{t_1}{\mu_1/\mu_2}}{\mu_2/\mu_3} = \frac{t_2}{\mu_2/\mu_3} + \frac{t_1}{\mu_1/\mu_3} = \sum \frac{t}{\mu_{rel}} \text{ where } \mu_{rel} = \frac{\mu \text{ of corresponding medium}}{\mu \text{ of observer medium}}$$

19. r = h tan C $= \frac{\mu_r h}{\sqrt{\mu_d^2 - \mu_r^2}}$. **20.** $\sin^{-1}\frac{\mu_a}{\mu_w}$.

21. r = 30º ; r´ = 45º ; e = 90º ; δ = 60º **23.** 60º **24.** θ = 0º, δ = 0.6º

25. Ac = 4.8^0 , Af = 2.4^0 . **26.** (1) at O (2) 10 cm right of O.

27. 0.188 m **28.** 0.30 m **29.** 0.75 m

30. – 5 cm. **31.** (a) 10 cm, (b) –5 cm 

(c) 0.5025 **32.** f = $\frac{D^2 - d^2}{4D}$.

33. D = 4f. **34.** D < 4f **35.** Real **39.** – 5 cm **40.** 2.5 cm.

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