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## THINGS TO REMEMBER :

1. 
2. If a body is thrown vertically up with a velocity u in the uniform gravitational field then (neglecting air resistance) :
(i) Maximum height attained $\mathrm{H}=\frac{\mathrm{u}^{2}}{2 \mathrm{~g}}$
(ii) Time of ascent $=$ time of descent $=\frac{u}{g}$
(iii) Total time of flight $=\frac{2 \mathrm{u}}{\mathrm{g}}$
(iv) Velocity of fall at the point of projection $=u$ downwards
3. KINEMATIC GRAPH :

Slope of the displacement time graph at any particular time gives the magnitude of the instantaneous velocity at that particular time.
Slope of the $v$ - $t$ graph will give the magnitude of the instantaneous acceleration.
The area between the $v-t$ graph, the time axis and the ordinates erected at the beginning \& end of time interval considered will represent the total displacement of the body.
5. RELATIVE VELOCITY:
(a) Velocity of ' $A$ ' relative to ' $B$ ' is given by $\vec{V}_{A B}=\vec{V}_{A}-\vec{V}_{B}$.
(b) Angular velocity of A relative to B i.e. $\omega_{A B}$ is given by
$\omega_{A B}=\frac{\text { velocityof ArelativetoBinadirectionperpendiculartoAB }}{\mathrm{AB}}$
6. LEVEL GROUND PROJECTILE MOTION :
(i) $\mathrm{v}=\mathrm{u}+\mathrm{at}$
(ii) $\mathrm{s}=\left(\frac{\mathrm{u}+\mathrm{v}}{2}\right) \mathrm{t}=\mathrm{ut}+\frac{\mathrm{at}{ }^{2}}{2}=v t-\frac{a t^{2}}{2}$
(iv) $\mathrm{s}_{\mathrm{n}}=\mathrm{u}+\frac{1}{2} \mathrm{a}(2 \mathrm{n}-1)$
(v) $S=\left(\frac{\mathrm{v}+\mathrm{u}}{2}\right) \mathrm{t}$
(iii) $\mathrm{v}^{2}=\mathrm{u}^{2}+2$ as
$\dot{8}$
$\infty$
$\infty$
0
$\overrightarrow{\mathrm{V}}_{\mathrm{AB}}$ refers to the velocity which ' $A$ ' appears to have as seen by $B$. The above idea of 1
dimensional relative motion can be extended to motion in 2 dimensions.

When a body is thrown obliquely (in a vertical plane) into the uniform gravitational field then the trajectory (actual path of motion) is a parabola. The horizontal component of velocity ucos $\alpha$ remains unchanged where as vertical component decreases up to the maximum height and then increases .
(a) Time taken to reach the height point $\mathrm{t}_{\mathrm{H}}=\frac{\mathrm{u} \sin \alpha}{\mathrm{g}}$
(b) Maximum height $\mathrm{H}=\frac{\mathrm{u}^{2} \sin ^{2} \alpha}{2 \mathrm{~g}}$
(c) Total time of flight $=\frac{2 \mathrm{u} \sin \alpha}{\mathrm{g}}=2 \mathrm{t}_{\mathrm{H}}$
(d) Horizontal range $=(\mathrm{u} \cos \alpha) \cdot \mathrm{T}=\frac{2}{\mathrm{~g}}(\mathrm{u} \cos \alpha)(\mathrm{u} \sin \alpha)=\frac{\mathrm{u}^{2} \sin 2 \alpha}{\mathrm{~g}}$

[Figure 1]
(e) $\mathrm{R}_{\max }=\frac{\mathrm{u}^{2}}{\mathrm{~g}}$ if $\alpha=45^{\circ}$

Note that for a given velocity of projection \& a given horizontal range there are in general two directions of projection which are complement of each other and are equally inclined to the direction of the maximum range.

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(f) Velocity \& Direction Of Motion At A Given Time :
(g) Velocity \& Direction Of Motion At A Given Height h:
$\left.\begin{array}{cc}\mathrm{V}^{2} \cos ^{2} \theta & =\mathrm{u}^{2} \cos ^{2} \alpha \\ \mathrm{~V}^{2} \sin ^{2} \theta & =\mathrm{u}^{2} \sin ^{2} \alpha-2 \mathrm{gh}\end{array}\right]$ on adding $\mathrm{V}^{2}=\mathrm{u}^{2}-2 \mathrm{gh}$
(h) Equations Of Motion In Vector Notation :
(i) $\overrightarrow{\mathrm{V}}=\overrightarrow{\mathrm{u}}+\overrightarrow{\mathrm{g}} \mathrm{t}$
(ii) $\overrightarrow{\mathrm{S}}=\overrightarrow{\mathrm{ut}}+\frac{1}{2} \overrightarrow{\mathrm{~g}} \mathrm{t}^{2}$
(iii) $\overrightarrow{\mathrm{V}}_{\mathrm{av}}=\frac{\overrightarrow{\mathrm{S}}}{\mathrm{t}}=\overrightarrow{\mathrm{u}}+\frac{1}{2} \overrightarrow{\mathrm{~g}} \mathrm{t} \quad\left(\overrightarrow{\mathrm{V}}_{\mathrm{av}}=\right.$ average velocity vector $)$
(i) Equation Of Trajectory :

Oblique Projection (refer fig-1) $y=x \tan \alpha-\frac{\mathrm{gx}^{2}}{2 \mathrm{u}^{2} \cos ^{2} \alpha}=x \tan \alpha\left(1-\frac{x}{R}\right)$
Note that $\frac{d y}{d x}$ represent the direction of motion .

## 7. PROJECTILE UP AN INCLINED PLANE :

(a) Total time of flight on the inclined plane

$$
T=\frac{2 \mathrm{u}}{\mathrm{~g}} \frac{\sin (\alpha-\beta)}{\cos \beta}
$$

(b) Range PQ on the inclined plane

(c) For Maxmimum range $2 \alpha-\beta=\frac{\pi}{2} \Rightarrow \alpha=\frac{\pi}{4}+\frac{\beta}{2}$

Hence the direction for maximum range bisects the angle between the vertical and the inclined $\propto^{\dot{\circ}}$ plane.
8. PROJECTILE DOWN AN INCLINED PLANE:
(a) Time of flight $=\frac{2 \mathrm{u} \sin (\alpha+\beta)}{\mathrm{g} \cos \beta}$
(b) $\quad$ Range $\mathrm{OP}=\frac{2 \mathrm{u}^{2} \sin (\alpha+\beta) \cdot \cos \alpha}{\mathrm{g} \cos ^{2} \beta}$
(c) Maximum range $=\frac{\mathrm{u}^{2}}{\mathrm{~g}(1-\sin \beta)}$
 $S=\frac{u^{2} \sin ^{2}(\alpha-\beta)}{2 \mathrm{~g} \cos \beta}$ when the projectile is at $H$, its velocity perpendicular to the plane is zero
(d) Angle of projection $\alpha$ for maximum range $=\frac{\pi}{4}-\frac{\beta}{2}$

# Get Solution of These Packages \& Learn by Video Tutorials on www.MathsBySuhag.com EXERCISE - I 

A butterfly is flying with velocity $10 \hat{\mathrm{i}}+12 \hat{\mathrm{j}} \mathrm{m} / \mathrm{s}$ and wind is blowing along x axis
with velocity u. If butterfly starts motion from A and after some time reaches
point B , find the value of u .
Find the change in velocity of the tip of the minute hand (radius $=10 \mathrm{~cm}$ ) of a clock in 45 minutes.

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Q. 12 A boat starts from rest from one end of a bank of a river of width $d$ flowing with velocity $u$. The boat is
Q. 13 A ball is thrown horizontally from a cliff such that it strikes ground after 5 sec . The line of sight from the point of projection to the point of hitting makes an angle of $37^{\circ}$ with the horizontal. What is the initial velocity of projection.
Q. 14 A ball is projected on smooth inclined plane in direction perpendicular to line of greatest slope with velocity of $8 \mathrm{~m} / \mathrm{s}$. Find it's speed after 1 sec .
Q. 15 A particle is projected from point P with velocity $5 \sqrt{2} \mathrm{~m} / \mathrm{s}$ perpendicular to the surface of a hollow right angle cone whose axis is vertical. It collides at Q normally. Find the time of the flight of the particle.
Q. 16 Find range of projectile on the inclined plane which is projected perpendicular to the incline plane with velocity $20 \mathrm{~m} / \mathrm{s}$ as shown in figure.

Q. 17 Initial acceleration of a particle moving in a straight line is $a_{0}$ and initial velocity is zero. The acceleration reduces continuously to half in every $\mathrm{t}_{0}$ seconds $\mathrm{as} \mathrm{a}=\frac{\mathrm{a}_{0}}{\frac{t}{t}}$. Find the terminal velocity of the particle.
Q. 18 Find the acceleration of movable pulley P and block B if

Q. 19 The velocities of A and B are marked in the figure. Find the velocity of block C (assume that the pulleys are ideal and string inextensible).

Q. 20 Aparticle is moving in $x-y$ plane such that $x=t+\sin (t)$ meter, $y=\cos (t)$ meter. $t$ is the time in sec. Find the length of the path taken by the particle from $t=0$ to $t=2 \pi \mathrm{sec}$.
Q. 21 The speed of a particle when it is at its greatest height $\sqrt{2 / 5}$ is of its speed when it is at its half the maximum height. The angle of projection is $\qquad$ and the velocity vector angle at half the maximum height is $\qquad$ .

Q. 23 The horizontal range of a projectiles is R and the maximum height attained by it is H. A strong wind now begins to blow in the direction of motion of the projectile, giving it a constant horizontal
Q. 24 A rocket is launched at an angle $53^{\circ}$ to the horizontal with an initial speed of $100 \mathrm{~ms}^{-1}$. It moves along its initial line of motion with an acceleration of $30 \mathrm{~ms}^{-2}$ for 3 seconds. At this time its engine falls \& the rocket proceeds like a free body. Find :
(i) the maximum altitude reached by the rocket
(ii) total time of flight.
(iii) the horizontal range. $\quad\left[\sin 53^{\circ}=4 / 5\right]$
Q. 25 A particle is thrown horizontally with relative velocity $10 \mathrm{~m} / \mathrm{s}$ from an inclined plane, which is also moving with acceleration $10 \mathrm{~m} / \mathrm{s}^{2}$ vertically upward. Find the time after which it lands on the plane ( $\mathrm{g}=10 \mathrm{~m} / \mathrm{s}^{2}$ )


List of recommended questions from I.E. Irodov. 1.1, 1.4 to 1.8, 1.10, 1.11, 1.14, 1.15, 1.17, 1.18, 1.19, 1.21 , $1.24,1.26,1.27,1.31,1.32,1.33,1.34(a)$


Q. 10 Ahunter is riding an elephant of height 4 m moving in straight line with uniform speed of $2 \mathrm{~m} / \mathrm{sec}$. Adeer running with a speed V in front at a distance of $4 \sqrt{5} \mathrm{~m}$ moving perpendicular to the direction of motion of the elephant. If hunter can throw his spear with a speed of $10 \mathrm{~m} / \mathrm{sec}$. relative to the elephant, thenat what angle $\theta$ to it's direction of motion must he throw his spear horizontally for a successful hit. Find also the speed ' $V$ ' of the deer.
Q. 11 A projectile is to be thrown horizontally from the top of a wall of height 1.7 m . Calculate the initial velocity of projection if it hits perpendicularly an incline of angle $37^{\circ}$ which starts from the ground at the bottom of the wall. The line of greatest slope of incline lies in the plane of motion of projectile.
Q. 12 Two inclined planes OA and OB having inclination (with horizontal) $30^{\circ}$ and $60^{\circ}$ respectively, intersect each other at O as shown in fig. A particle is projected from point P with velocity $\mathrm{u}=10 \sqrt{3} \mathrm{~m} \mathrm{~s}^{-1}$ along a direction perpendicular to plane OA . If the particle strikes plane OB perpendicularly

## at Q , calculate

(a) velocity with which particle strikes the plane OB ,
(b) time of flight,
(c) vertical height h of P from O ,
(d) maximum height fromO attained by the particle and
(e) distance PQ
Q. 13 A particle is projected with a velocity $2 \sqrt{\mathrm{ag}}$ so that it just clears two walls of equal height 'a which are at a distance ' 2 a ' apart. Show that the time of passing between the walls is $2 \sqrt{\mathrm{a} / \mathrm{g}}$.
Q. 14 A stone is projected from the point of a ground in such a direction so as to hit a bird on the top of a telegraph post of height $h$ and then attain the maximum height 2 h above the ground. If at the instant of $\frac{\bar{\sigma}}{\bar{\infty}}$ projection, the bird were to fly away horizontally with a uniform speed, find the ratio between the horizontal velocities of the bird and the stone, if the stone still hits the bird while descending.
Q. 15 Two persons Ram and Shyam are throwing ball at each other as shown
 in the figure. The maximum horizontal distance from the building where Ram can stand and still throw a ball at Shyam is $\mathrm{d}_{1}$. The maximum horizontal distance of Ram from the building where Shyam can throw a ball is $d_{2}$. If both of them can throw ball with a velocity of $\sqrt{2 g k}$, find the ratio of $d_{1} / d_{2}$. Neglect the height of each person.

## EXERCISE \# III

Q. $1 \quad$ The motion of a body is given by the equation $\frac{d v(t)}{d t}=6.0-3 v(t)$; where $v(t)$ is the speed in $\mathrm{m} / \mathrm{s} \& \mathrm{t}$ in sec., if the body has $\mathrm{v}=0$ at $\mathrm{t}=0$ then
(A) the terminal speed is $2.0 \mathrm{~m} / \mathrm{s}$
(B) the magnitude of the initial acceleration is $6.0 \mathrm{~m} / \mathrm{s}^{2}$
(C) the speed varies with time as $v(t)=2\left(1-\mathrm{e}^{-3 \mathrm{t}}\right) \mathrm{m} / \mathrm{s}$
(D) the speed is $1.0 \mathrm{~m} / \mathrm{s}$ when the acceleration is half the initial value.
[JEE' 1995]
Q. 2 Two guns, situated at the top of a hill of height 10 m , fire one shot each with the same speed $5 \sqrt{3} \mathrm{~m} / \mathrm{s}$ at some interval of time. One gun fires horizontally and other fires upwards at an angle of $60^{\circ}$ with the horizontal. The shots collide in air at a point $P$. Find
(a) the time interval between the firings, and
(b) the coordinates of the point P. Take origin of the coordinates system at the foot of the hill right below the muzzle and trajectories in X-Y plane.
[JEE'1996]
Q. 3 The trajectory of a projectile in a vertical plane is $y=a x-b x^{2}$, where $a, b$ are constants \& $x$ and $y$ are respectively the horizontal \& vertical distances of the projectile from the point of projection. The maximum height attained is $\qquad$ \& the angle of projection from the horizontal is $\qquad$ . [JEE '1997]
Q. 4 A large heavy box is sliding without friction down a smooth plane of inclination $\theta$. From a point P on the bottom of a box, a particle is projected inside the box. The initial speed of the particle with respect to box is $u$ and the direction of projection makes an angle $\alpha$ with the bottom as shown in figure.

(a) Find the distance along the bottom of the box between the point of projection P and the point Q where the particle lands. (Assume that the particle does not hit any other surface of the box. Neglect air resistance).
(b) If the horizontal displacement of the particle as seen by an observer on the ground is zero, find the speed of the box with respect to the ground at the instant when the particle was projected. [JEE'1998]
Q. 5 A particle of mass $10^{-2} \mathrm{~kg}$ is moving slong the positive x -axis under the influence of a force $F(x)=\frac{-K}{2 x^{2}}$ where $K=10^{-2} \mathrm{Nm}^{2}$. At time $\mathrm{t}=0$ it is at $\mathrm{x}=1.0 \mathrm{~m} \&$ its velocity is $\mathrm{v}=0$. Find :
(i) its velocity when it reaches $x=0.50 \mathrm{~m}$
(ii) the time at which it reaches $x=0.25 \mathrm{~m}$.
[JEE '1998]
Q. 6 In 1.0 sec . a particle goes from point A to point B moving in a semicircle of radius 1.0 m . The magnitude of average velocity is :
[JEE '99]
(A) $3.14 \mathrm{~m} / \mathrm{sec}$
(B) $2.0 \mathrm{~m} / \mathrm{sec}$
(C) $1.0 \mathrm{~m} / \mathrm{sec}$
(D) zero
Q. $7 \quad$ The co-ordinates of a particle moving in a plane are given by $x(t)=a \cos (\pi t)$ and $y(t)=b \sin (\pi t)$ where $a, b(<a) \& \pi$ are positive constants of appropriate dimensions.
(A) the path of the particle is an ellipse

(B) the velocity \& acceleration of the particle are normal to each other at $\mathrm{t}=\pi /(2 \pi)$
(C) the acceleration of the particle is always directed towards a focus
(D) the distance travelled by the particle in time interval $\mathrm{t}=0 \mathrm{ot}=\pi /(2 \pi)$ is a .
[JEE '1999]

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Q. 8 A ball is dropped vertically from a height $d$ above the ground it hits the ground and bounces up vertically to a height $d / 2$. Neglecting subsequent motion and air resistances, its velocity $v$ varies with the height $h$ above the ground as
[JEE'2000 (Scr)]
(A)

(B)

(C)

(D)

Q. $9 \quad$ An object A is kept fixed at the point $\mathrm{x}=3 \mathrm{~m}$ and $\mathrm{y}=1.25 \mathrm{~m}$ on a plank $P$ raised above the ground. At time $t=0$ the plank starts moving along the $+x$ direction with an acceleration $1.5 \mathrm{~m} / \mathrm{s}^{2}$. At the same instant a stone is projected from the origin with a velocity $u$ as shown. A stationary person on the ground observes the stone hitting the object during its downward motion at an angle of $45^{\circ}$ to the horizontal. All the motions are in $x-y$ plane. Find $u$ and the time after which the stone hits the object. Take $\mathrm{g}=10 \mathrm{~m} / \mathrm{s}^{2}$. [JEE 2000]

Q. 10 On a frictionless horizontal surface, assumed to be the $x$ - $y$ plane, a small trolley $A$ is moving along a straight line parallel to the $y$-axis (see figure) with a constant velocity of $(\sqrt{3}-1) \mathrm{m} / \mathrm{s}$. At a particular instant, when the line OA makes an angle of $45^{\circ}$ with the x -axis, a ball is thrown along the surface from the origin O. Its velocity makes an angle $\phi$ with the x -axis and it hits the trolley.

(a) The motion of the ball is observed from the frame of trolley. Calculate the angle $\theta$ made by the velocity yector of the ball with the x -axis in this frame.
(b) Find the speed of the ball with respect to the surface, if $\phi=\frac{4 \theta}{3}$.
[JEE 2002]
Q. 11 A particle starts from rest. Its acceleration (a) versus time ( t ) is as shown in the figure. The maximum speed of the particle will be
[JEE' 2004 (Scr)]
(A) $110 \mathrm{~m} / \mathrm{s}$
(B) $55 \mathrm{~m} / \mathrm{s}$
(C) $550 \mathrm{~m} / \mathrm{s}$
(D) $660 \mathrm{~m} / \mathrm{s}$

Q. 12 A small block slides without friction down an inclined plane starting from rest. Let Sn be the distance travelled from time $\mathrm{t}=\mathrm{n}-1$ to $\mathrm{t}=\mathrm{n}$. Then $\frac{\mathrm{S}_{\mathrm{n}}}{\mathrm{S}_{\mathrm{n}+1}}$ is $\quad$ [JEE' 2004 (Scr)]
(A) $\frac{2 n-1}{2 n}$
(B) $\frac{2 \mathrm{n}+1}{2 \mathrm{n}-1}$
(C) $\frac{2 n-1}{2 n+1}$
(D) $\frac{2 \mathrm{n}}{2 \mathrm{n}+1}$
Q. 13 The velocity displacement graph of a particle moving along a straight line is shown.

The most suitable acceleration-displacement graph will be
(A)

(B)

(C)

(D)

[JEE' 2005 (Scr)]
Q. $35 \mathrm{~m} / \mathrm{sec}$
Q. $4 \tan ^{-1}(1 / 2)$
Q. $9 \quad 20$ sec
Q. $10 \quad 50 \mathrm{~m}$
Q. $11 \frac{\pi \mathrm{v}^{4}}{\mathrm{~g}^{2}}$
Q. $12 y=\frac{a x^{2}}{2 u^{2}}$
Q. $13 \quad 100 / 3 \mathrm{~m} / \mathrm{s}$
Q. $14 \quad 10 \mathrm{~m} / \mathrm{s}$
Q. $15 \quad 1 \mathrm{sec}$
Q. 1675 m
Q. $17 \frac{\mathrm{a}_{0} \mathrm{t}_{0}}{\ln (2)}$
Q. $18 \quad a_{P}=1 \mathrm{~m} / \mathrm{s}^{2} \downarrow, \mathrm{a}_{\mathrm{B}}=2 \mathrm{~m} / \mathrm{s}^{2} \uparrow$
Q. $195 \mathrm{~m} / \mathrm{s}$
Q. $20 \quad 8 \mathrm{~m}$
Q. $2160^{\circ}, \tan ^{-1}(\sqrt{3 / 2})$
Q. $22 \operatorname{2asin}(\alpha / 2)$
Q. $23 \quad \mathrm{R}+2 \mathrm{H}$
Q. 24 (i) 1503.2 m (ii) 35.54 sec (iii) 3970.56 m
Q. $25 \frac{1}{\sqrt{3}} \mathrm{sec}$
$\begin{array}{llllll}\text { Q. } 1 & 160 \mathrm{sec} & \text { Q. } 2 & 122.7 \mathrm{~km} / \mathrm{hr} & \text { Q. } 3 & \frac{\mathrm{~S}_{0} \pi}{4}\end{array}$
Q. $4 \quad 2 \tan ^{-1}(1 / 3) \quad$ Q. $5 \quad \theta=\tan ^{-1} 2, v=134.16 \mathrm{~km} / \mathrm{h}$

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(b)

Q. $10 \quad \theta=37^{0}, v=6 \mathrm{~m} / \mathrm{s}$
Q. 7
$\frac{d}{2}$
Q. $8 \quad 3$
Q. $7 \quad 20 \sqrt{5}$
Q. 8

Q. 4
Q. 1 A, B, C, D
Q. 2
(a) 1 sec ,
(b) $(5 \sqrt{3} \mathrm{~m}, 5 \mathrm{~m})$
Q. $3 \quad \frac{\mathrm{a}^{2}}{4 \mathrm{~b}}, \tan ^{-1} \mathrm{a}$
(a) $\frac{\mathrm{u}^{2} \sin 2 \alpha}{\mathrm{~g} \cos \theta}$, (b) $\mathrm{v}=\frac{\mathrm{u} \cos (\alpha+\theta)}{\cos \theta}$
Q. 5
(i) $\overrightarrow{\mathrm{V}}=-1 \hat{\mathrm{i}} \mathrm{m} / \mathrm{s}$
(ii) $t=\frac{\pi}{3}+\frac{\sqrt{3}}{4}$
Q. 7

A, B
Q. 11 B
Q. 12 C
Q. 13 B
(a) $45^{\circ}$, (b) $2 \mathrm{~m} / \mathrm{sec}$
Q. $8 \quad \mathrm{~A}$
Q. $9 \quad u=7.29 \mathrm{~m} / \mathrm{s}, \mathrm{t}=1 \mathrm{sec}$
(a) $10 \mathrm{~ms}^{-1}$, (b)
(b) 2 sec , (c) 5 m , (d
(d) 16.25 m , (e)
(e) 20 m

## EXERCISE \# III

(a) $5 \mathrm{~d} / 4 \tan \alpha$, (b) 9 , (c) point O
Q. 9 (a) $5 \mathrm{~d} / 4 \tan \alpha$, (b)


## Q. 12

$$
\text { Q. } 14 \frac{2}{\sqrt{2}+1} \quad \text { Q. } 15 \sqrt{\frac{\mathrm{k}-\mathrm{h}}{\mathrm{k}+\mathrm{h}}}
$$

Q. $11 u=3 \mathrm{~m} / \mathrm{s}$
(a) $10 \mathrm{~ms}^{-1}$

2 sec ,

> m

