

SHORT REVISION

THINGS TO REMEMBER :

$$v = \frac{ds}{dt}$$
; $a = \frac{dv}{dt} = v \frac{dv}{ds}$; $s = \int v dt$; $v = \int a dt$; $\frac{v^2}{2} = \int a ds$

where the symbols have their usual meaning.

The equations of motion for a body moving in straight line with uniform acceleration, are

(i)
$$v = u + at$$
 (ii) $s = \left(\frac{u+v}{2}\right)t = ut + \frac{at^2}{2} = vt - \frac{at^2}{2}$ (iii) $v^2 = u^2 + 2as$
(iv) $s_n = u + \frac{1}{2}a(2n-1)$ (v) $S = \left(\frac{v+u}{2}\right)t$

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(iv)
$$s_n = u + \frac{1}{2} a (2n-1)$$
 (v) $S = \left(\frac{v+1}{2}\right)^n$

0 98930 58881. If a body is thrown vertically up with a velocity u in the uniform gravitational field then (neglecting air resistance):

(i) Maximum height attained
$$H = \frac{u^2}{2g}$$
 (ii) Time of ascent = time of descent = $\frac{u}{g}$
(iii) Total time of flight = $\frac{2u}{g}$ (iv) Valacity of fall at the point of projection

(a) Velocity of 'A' relative to 'B' is given by $\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$.

(b) Angular velocity of A relative to B i.e. ω_{AB} is given by

The second at more defined in the second of the instantaneous acceleration. (i) Maximum height attained $H = \frac{u^2}{2g}$ (ii) Velocity of fall at the point of projection = u downwards **KINEMATIC GRAPH :** Slope of the displacement time graph at any particular time gives the magnitude of the instantaneous velocity at that particular time. Slope of the displacement time graph at any particular time gives the magnitude of the instantaneous velocity at that particular time. Slope of the v t graph will give the magnitude of the instantaneous acceleration. The area between the v - t graph, the time axis and the ordinates erected at the beginning & end of time interval considered will represent the total displacement of the body. **RELATIVE VELOCITY:** Velocity of 'A' relative to 'B' is given by $\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$. Vab refers to the velocity which 'A' appears to have as seen by B. The above idea of 1 dimensional relative motion can be extended to motion in 2 dimensions. Angular velocity of A relative to B i.e. ω_{AB} is given by $\omega_{AB} = \frac{velocity of Arelative to Bi a extended to motion in 2 dimensions.$ $Angular velocity of A relative to B i.e. <math>\omega_{AB}$ is given by $\omega_{AB} = \frac{velocity of Arelative to Bi a vertical plane) into the uniform gravitational field then the trajectory (actual path of motion) is a parabola. The horizontal component of velocity ucos <math>\alpha$ remains unchanged where as vertical component decreases up to the maximum height and then increases. Time taken to reach the height point $t_{H} = \frac{u \sin \alpha}{g}$ Maximum height $H = \frac{u^2 \sin^2 \alpha}{2g}$ Maximum height $H = \frac{u^2 \sin^2 \alpha}{2g}$ Horizontal range = (ucos α). $T = \frac{2}{g}$ (ucos α) (usin α) = $\frac{u^2 \sin 2\alpha}{g}$ $Horizontal range = (ucos \alpha)$. $T = \frac{2}{g}$ (ucos α) (usin α) = $\frac{u^2 \sin 2\alpha}{g}$ $Horizontal range = (ucos \alpha)$. $T = \frac{2}{g}$ if $\alpha = 45^\circ$

(a) Time taken to reach the height point
$$t_{\rm H} = \frac{u \sin \alpha}{\sigma}$$

(b) Maximum height
$$H = \frac{u^2 \sin^2 r}{2 \sigma}$$

(c) Total time of flight =
$$\frac{2 u \sin \alpha}{\alpha} = 2 t_{\rm H}$$

(d) Horizontal range =
$$(u\cos\alpha) \cdot T = \frac{2}{g} (u\cos\alpha) (u\sin\alpha) = \frac{u^2 \sin 2\alpha}{g}$$

(e)
$$R_{max} = \frac{u^2}{g}$$
 if $\alpha = 45^\circ$

Note that for a given velocity of projection & a given horizontal range there are in general two directions of projection which are complement of each other and are equally inclined to the direction of the maximum range.

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(f) VELOCITY & DIRECTION OF MOTION AT A GIVEN TIME :

 $V\cos\theta = u\cos\alpha$ $V\sin\theta = u\sin\alpha - gt$ Squaring & adding these 2 equations we will get the velocity of the projectile. Dividing the velocities in y and x directions gives the direction of motion.

(g) VELOCITY & DIRECTION OF MOTION AT A GIVEN HEIGHT h:

$$\begin{bmatrix} V^2 \cos^2 \theta & = u^2 \cos^2 \alpha \\ V^2 \sin^2 \theta & = u^2 \sin^2 \alpha - 2gh \end{bmatrix}$$
 on adding $V^2 = u^2 - 2gh$

(h) Equations Of Motion In Vector Notation :

(i)
$$\vec{V} = \vec{u} + \vec{g}t$$
 (ii) $\vec{S} = \vec{u}t + \frac{1}{2}\vec{g}t^2$ (iii) $\vec{V}_{av} = \frac{S}{t} = \vec{u} + \frac{1}{2}\vec{g}t$ (\vec{V}_{av} = average velocity vector)

(i) EQUATION OF TRAJECTORY :

Oblique Projection (refer fig-1) $y = x \tan \alpha - \frac{gx^2}{2u^2 \cos^2 \alpha} = x \tan \alpha \left(1 - \frac{x}{R}\right)$

Note that $\frac{dy}{dx}$ represent the direction of motion .

7. PROJECTILE UP AN INCLINED PLANE:

(a) Total time of flight on the inclined plane

$$T = \frac{2u}{g} \frac{\sin(\alpha - \beta)}{\cos\beta}$$

(b) Range PQ on the inclined plane

$$PQ = \frac{2u^2}{g} \frac{\cos\alpha \cdot \sin(\alpha - \beta)}{\cos^2 \beta} = \frac{u^2}{g\cos^2 \beta} [\sin(2\alpha - \beta) - \sin\beta]$$

(c) For Maxmimum range
$$2\alpha - \beta = \frac{\pi}{2} \Rightarrow \alpha = \frac{\pi}{4} + \frac{\beta}{2}$$

Hence the direction for maximum range bisects the angle between the vertical and the inclined plane.

(d)
$$R_{max} = \frac{u^2}{g(1+\sin\beta)}$$

S =

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(e) Greatest distance of the projectile from the inclined plane ;

 $\frac{u^2 \sin^2 (\alpha - \beta)}{2g \cos \beta}$ when the projectile is at H, its velocity perpendicular to the plane is zero

PROJECTILE DOWN AN INCLINED PLANE:

(a) Time of flight =
$$\frac{2 u \sin (\alpha + \beta)}{g \cos \beta}$$

(b) Range OP = $\frac{2 u^2 \sin (\alpha + \beta) \cdot \cos \alpha}{g \cos^2 \beta}$
(c) Maximum range = $\frac{u^2}{g (1 - \sin \beta)}$
(d) Angle of projection α for maximum range = $\frac{\pi}{4} - \frac{\beta}{2}$

- A butterfly is flying with velocity $10\hat{i} + 12\hat{j}$ m/s and wind is blowing along x axis Q.1 with velocity u. If butterfly starts motion from A and after some time reaches point B, find the value of u.
- Q.2 Find the change in velocity of the tip of the minute hand (radius = 10 cm) of a clock in 45 minutes.
- Q.3 A, B & C are three objects each moving with constant velocity. A's speed is 10 m/sec in a direction \overrightarrow{PQ} . The velocity of B relative to A is 6 m/sec at an angle of, $\cos^{-1}(15/24)$ to PQ. The velocity of C relative
- The velocity of B relative to A is 6 m/sec at an angle of, $\cos^{-1}(15/24)$ to PQ. The velocity of C relative to B is 12 m/sec in a direction \overrightarrow{QP} , then find the magnitude of the velocity of C. Rain is falling vertically with a speed of 20 ms⁻¹ relative to air. A person is running in the rain with a velocity of 5 ms⁻¹ and a wind is also blowing with a speed of 15 ms⁻¹ (both towards east). Find the angle with the vertical at which the person should hold his umbrella so that he may not get drenched. Q.4
- Q.5 The velocity-time graph of the particle moving along a straight line is shown. The rate of acceleration and deceleration is constant and it is equal to 5 ms⁻². If the average velocity during the motion is 20 ms⁻¹, then find the value of t.
- Q.6 The fig. shows the v-t graph of a particle moving in straight line. Find the time when particle returns to the starting point.

20

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Q.7 A particle is projected in the X-Y plane. 2 sec after projection the velocity of the particle makes an 9 angle 45° with the X - axis. 4 sec after projection, it moves horizontally. Find the velocity of \underline{g} projection (use g = 10 ms⁻²).

10

25

20

Q.8

- projection (use $g = 10 \text{ ms}^{-2}$). A stone is dropped from a height h . Simultaneously another stone is thrown up from the ground with such a velocity that it can reach a height of 4h. Find the time when two stones cross each other. A particle is projected upwards with a velocity of 100 m/sec at an angle of 60° with the vertical. Find the time when the particle will move perpendicular to its initial direction, taking $g = 10 \text{ m/sec}^2$. A balloon is ascending vertically with an acceleration of 0.2m/s^2 . Two stones are dropped from it at an interval of 2 sec. Find the distance between them 1.5 sec after the second stone is released.(use $g=9.8 \text{m/s}^2$) A large number of bullets are fired in all direction with the same speed v. What is the maximum area on ground on which these bullets can spread? Q.9
- Q.10
- Q.11

B

page

- A boat starts from rest from one end of a bank of a river of width d flowing with velocity u. The boat is Q.12 steered with constant acceleration a in a direction perpendicular to the bank. If point of start is origin, direction of bank is x axis and perpendicular to bank is y axis. Find the equation of trajectory of the boat.
- A ball is thrown horizontally from a cliff such that it strikes ground after 5 sec. Q.13 The line of sight from the point of projection to the point of hitting makes an angle of 37° with the horizontal. What is the initial velocity of projection.
- Q.14 A ball is projected on smooth inclined plane in direction perpendicular
- Q.15
- Q.16
- Q.17
- Q.18

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- Q.19
- Q.20
- Q.21
- Q.22 horizontal. One end of the rope is fixed to the wall at point A. A small load is attached to the rope at point B. The wedge starts moving to the right with a constant acceleration. Determine the acceleration a, of the load when it is still on the wedge.



8 m/s

- Q.23 The horizontal range of a projectiles is R and the maximum height attained by it is H. A strong wind now begins to blow in the direction of motion of the projectile, giving it a constant horizontal acceleration = g/2. Under the same conditions of projection, find the horizontal range of the projectile.
- Q.24 A rocket is launched at an angle 53° to the horizontal with an initial speed of 100 ms⁻¹. It moves along its initial line of motion with an acceleration of 30 ms⁻² for 3 seconds. At this time its engine falls & the rocket proceeds like a free body. Find :
- (i) the maximum altitude reached by the rocket
- (ii) total time of flight.

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- (iii) the horizontal range . [$\sin 53^\circ = 4/5$]
- Q.25 A particle is thrown horizontally with relative velocity 10 m/s from an inclined plane, which is also moving with acceleration 10 m/s² vertically upward. Find the time after which it lands on the plane ($g = 10 \text{ m/s}^2$)

 10m/s^2

List of recommended questions from I.E. Irodov. 1.1, 1.4 to 1.8, 1.10, 1.11, 1.14, 1.15, 1.17, 1.18, 1.19, 1.21, 1.24, 1.26, 1.27, 1.31, 1.32, 1.33, 1.34(a)

- Q.1 A train takes 2 minutes to acquire its full speed 60kmph from rest and 1 minute to come to rest from the full speed. If somewhere in between two stations 1 km of the track be under repair and the limited speed on this part be fixed to 20kmph, find the late running of the train on account of this repair work, assuming otherwise normal at running of the train between the stations.
- page Q.2 A speeder in an automobile passes a stationary policeman who is hiding behind a bill board with a motorcycle. After a 2.0 sec delay (reaction time) the policeman accelerates to his maximum speed of 150 km/hr in 12 sec and catches the speeder 1.5 km beyond the billboard. Find the speed of speeder in km/hr.
- Q.3 A particle is moving on a straight line. Its displacement from the initial position is plotted against time in the graph shown. What will be the velocity of the particle at $2/3 \sec$? Assume the graph to be a sine curve.



- Q.4 A glass wind screen whose inclination with the vertical can be changed, is mounted on a cart as shown in figure. The cart moves uniformly along the horizontal path with a speed of 6 m/s. At what maximum angle α to the vertical can the wind screen be placed so that the rain drops falling vertically downwards with velocity 2 m/s, do not enter the cart?
 - 903 903 7779, An aeroplane is observed by two persons travelling at 60 km/hr in two vehicles moving in opposite directions on a straight road. To an observer in one vehicle the plane appears to cross the road track at right angles while to the observer in the other vehicle the angle appears to be 45°. At what angle does the plane actually cross the road track and what is its speed relative to the ground. plane actually cross the road track and what is its speed relative to the ground.
- Bhopal How long will a plane take to fly around a square with side a with the wind blowing at a velocity u, in the two cases Q.6 the direction of the wind coincides with one of the sides
 - the direction of the wind coincides with one diagonal of the square. The velocity of the plane in still air is v > u.
- Ŀ. Q.7 Two ships A and B originally at a distance d from each other depart at the same time from a straight $\vec{\alpha}$ coastline. Ship A moves along a straight line perpendicular to the shore while ship B constantly heads for \dot{o} ship A, having at each moment the same speed as the latter. After a sufficiently great interval of time the second ship will obviously follow the first one at a certain distance. Find the distance.
- The slopes of the wind-screen of two motorcars are $\beta_1 = 30^\circ$ and $\beta_2 = 15^\circ$ respectively. The first car is travelling with a velocity of v_1 horizontally. The second car is travelling with a velocity v_2 in the same direction. The hail stones are falling vertically. Both the drivers observe that the hail stones rebound σ Q.8 Teko Classes, Maths : vertically after elastic collision with the wind-screen. Find the ratio of v_1/v_2 .
- Q.9 A small ball is thrown between two vertical walls such that in the absence of the wall its range would have been 5d. The angle of projection is α . Given that all the collisions are perfectly elastic, find
 - Maximum height attained by the ball.
 - Total number of collisions before the ball comes back to the ground, and
 - Point at which the ball falls finally. The walls are supposed to be very tall.

d/2

- Q.10 A hunter is riding an elephant of height 4m moving in straight line with uniform speed of 2m/sec. Adeer running with a speed V in front at a distance of $4\sqrt{5}$ m moving perpendicular to the direction of motion of the elephant. If hunter can throw his spear with a speed of 10m/sec. relative to the elephant, then at what angle θ to it's direction of motion must he throw his spear horizontally for a successful hit. Find also the speed 'V' of the deer.
- Q.11 A projectile is to be thrown horizontally from the top of a wall of height 1.7 m. Calculate the initial $\overset{\infty}{\underline{o}}$ velocity of projection if it hits perpendicularly an incline of angle 37° which starts from the ground at the bottom of the wall. The line of greatest slope of incline lies in the plane of motion of projectile.
- Q.12 Two inclined planes OA and OB having inclination (with horizontal) 30° and 60° respectively, intersect each other at O as shown in fig. A particle is projected from point P with velocity $u = 10\sqrt{3}$ m s⁻¹ along a direction perpendicular to plane OA. If the particle strikes plane OB perpendicularly at Q, calculate



- (a) velocity with which particle strikes the plane OB,
- (b) time of flight,
- (c) vertical height h of P from O,
- (d) maximum height from O attained by the particle and
- (e) distance PQ

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- Q.13 A particle is projected with a velocity $2\sqrt{ag}$ so that it just clears two walls of equal height 'a' which are at a distance '2a' apart. Show that the time of passing between the walls is $2\sqrt{a/g}$.
- Q.14 A stone is projected from the point of a ground in such a direction so as to hit a bird on the top of a telegraph post of height h and then attain the maximum height 2h above the ground. If at the instant of more projection, the bird were to fly away horizontally with a uniform speed, find the ratio between the horizontal velocities of the bird and the stone, if the stone still hits the bird while descending.
- Q.15 Two persons Ram and Shyam are throwing ball at each other as shown in the figure. The maximum horizontal distance from the building where Ram can stand and still throw a ball at Shyam is d₁. The maximum horizontal distance of Ram from the building where Shyam can throw a ball is d₂. If both of them can throw ball with a velocity of $\sqrt{2gk}$, find the ratio of d₁/d₂. Neglect the height of each person.



com	Q.1	The motion of a body is given by the equation $\frac{d v(t)}{dt} = 6.0 - 3 v(t)$; where v(t) is the sp			
FREE Download Study Package from website: www.TekoClasses.com & www.MathsBySuhag.		m/s & t in sec., if the body has $v = 0$ at $t = 0$ th (A) the terminal speed is 2.0 m/s (B) the magnitude of the initial acceleration is 6 (C) the speed varies with time as $v(t) = 2(1-e)$ (D) the speed is 1.0 m/s when the acceleration	nen 5.0 m/s² e ^{-3t}) m/s is half the initial value.	[IEE '1995]	page 9
	0.2	Two guns situated at the top of a hill of height 10 m fire one shot each with the same speed			81.
	Q.2	5 $\sqrt{3}$ m/s at some interval of time. One gun fires horizontally and other fires upwards at an angle of 60° with the horizontal. The shots collide in air at a point P. Find			3930 588
	(a) (b)	the time interval between the firings, and the coordinates of the point P. Take origin of the muzzle and trajectories in X-Y plane.	e coordinates system at the foot of the hill	right below the [JEE'1996]	79, 098
	Q.3	The trajectory of a projectile in a vertical plan respectively the horizontal & vertical distances of height attained is & the angle of projec	e is $y = ax - bx^2$, where a, b are constant The projectile from the point of projection tion from the horizontal is	s & x and y are . The maximum [JEE '1997]	03 903 903 77
	Q.4	A large heavy box is sliding without friction d inclination θ . From a point P on the bottom of a bo inside the box. The initial speed of the particle and the direction of projection makes an angle	own a smooth plane of ox, a particle is projected with respect to box is u α with the bottom as	pla o	pal Phone:(
	(a)	Find the distance along the bottom of the box between the point of projection P and the point Q where the particle lands. (Assume that the particle does not hit any other surface of the box. Neglect air resistance)			
	(b)	If the horizontal displacement of the particle as a of the box with respect to the ground at the inst	seen by an observer on the ground is zero tant when the particle was projected.	, find the speed [JEE'1998]	a (S. R. K
	Q.5	A particle of mass 10^{-2} kg is moving slong the positive x-axis under the influence of a force			Sariya
		$F(x) = \frac{-K}{2x^2}$ where $K = 10^{-2}$ N m ² . At time t = 0 it is at x = 1.0 m & its velocity is v = 0. Find :			g R.
	(i) (ii)	its velocity when it reaches $x = 0.50 \text{ m}$ the time at which it reaches $x = 0.25 \text{ m}$.		[JEE '1998]	Suha
	Q.6	In 1.0 sec. a particle goes from point A to point I The magnitude of average velocity is : (A) 3.14 m/sec (C) 1.0 m/sec	B moving in a semicircle of radius 1.0 m. [JEE '99] (B) 2.0 m/sec (D) zero	A 1m B	sses, Maths :
	Q.7	The co-ordinates of a particle moving in a pla where a, b (<a) &="" <math="">\pi are positive constants of (A) the path of the particle is an ellipse (B) the velocity & acceleration of the particle a (C) the acceleration of the particle is always dif (D) the distance travelled by the particle in tim</a)>	ane are given by $x(t) = a \cos(\pi t)$ and y appropriate dimensions. are normal to each other at $t = \pi/(2\pi)$ rected towards a focus me interval $t = 0$ o $t = \pi/(2\pi)$ is a.	$f(t) = b \sin(\pi t)$ [JEE '1999]	Teko Cla

Q.8 A ball is dropped vertically from a height d above the ground it hits the ground and bounces up vertically to a height d/2. Neglecting subsequent motion and air resistances, its velocity v varies with the height h above the ground as [JEE'2000 (Scr)]



Q.9 An object A is kept fixed at the point x = 3 m and y = 1.25 m on aplank P raised above the ground. At time t = 0 the plank starts moving along the + x direction with an acceleration 1.5 m/s^2 . At the same instant a stone is projected from the origin with a velocity u as shown. A stationary person on the ground observes the stone hitting the object during its downward motion at an angle of 45° to the horizontal. All the motions are in x-y plane. Find u and the time after which the stone hits the object. Take $g = 10 \text{ m/s}^2$. [JEE 2000]



(a) The motion of the ball is observed from the frame of trolley. Calculate the angle θ made by the velocity vector of the ball with the x-axis in this frame.

1.25m

Find the speed of the ball with respect to the surface, if ϕ = (b)

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- Q.11 A particle starts from rest. Its acceleration (a) versus time (t) is as shown in the figure. The maximum speed of the particle will be 10m/s² [JEE' 2004 (Scr)] (A) 110 m/s (B) 55 m/s(C) 550 m/s (D) 660 m/s
- A small block slides without friction down an inclined plane starting from rest. Let Sn be the distance Q.12
 - travelled from time t = n 1 to t = n. Then $\frac{S_n}{S_{n+1}}$ is [JEE' 2004 (Scr)]

(A)
$$\frac{2n-1}{2n}$$
 (B) $\frac{2n+1}{2n-1}$ (C) $\frac{2n-1}{2n+1}$ (D) $\frac{2n}{2n+1}$

Q.13 The velocity displacement graph of a particle moving along a straight line is shown. The most suitable acceleration-displacement graph will be





a

2n



