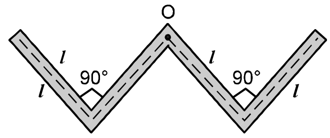
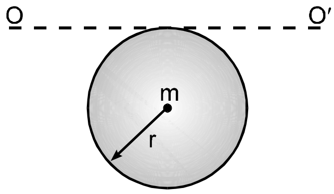
EXERCISE-1

1. A thin rod of length 4 l, mass 4m is bent at the points as shown in the fig. What is the moment of inertia of the rod about the axis passing point O & perpendicular to the plane of the paper.



(A) $\frac{M\ell^2}{3}$ (B) $\frac{10M\ell^2}{3}$ (C) $\frac{M\ell^2}{12}$ (D) $\frac{M\ell^2}{24}$

2. Moment of inertia of a disc about O O′ is:



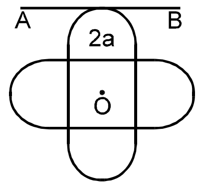
(A) $\frac{3mr^2}{2}$ (B) $\frac{mr^2}{2}$

(C) $\frac{5mr^2}{2}$ (D) $\frac{5mr^2}{4}$

3. The moment of inertia of a door of mass m, length 2 l and width l about its longer side is

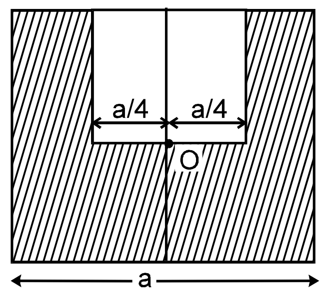
(A) $\frac{11m\ell^2}{24}$ (B) $\frac{5m\ell^2}{24}$ (C) $\frac{m\ell^2}{3}$ (D) none of these

4. A symmetric lamina of mass M consists of a square shape with a semicircular section over each of the edge of the square as in figure. The side of the square is 2 a. The moment of inertia of the lamina about an axis through its centre of mass and perpendicular to the plane is 1.6 Ma^2. The moment of inertia of the lamina about the tangent AB in the plane of lamina is _______________.



5. Three identical rods, each of length l, are joined to form a rigid equilateral triangle. Its radius of gyration about an axis passing through a corner and perpendicular to the plane of the triangle is _______.

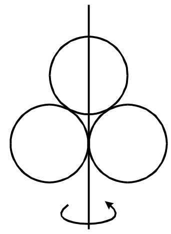
6. A square plate of edge a/2 is cut out from a uniform square plate of edge 'a' as shown in figure. The mass of the remaining portion is M. The moment of inertia of the shaded portion about an axis passing through 'O' (centre of the square of side a) and perpendicular to plane of the plate is :



(A) $\frac{9}{64}Ma^2$ (B) $\frac{3}{16}Ma^2$

(C) $\frac{5}{12}Ma^2$ (D) $\frac{Ma^2}{6}$

7. Three rings each of mass m and radius r are so placed that they touch each other. The radius of gyration of the system about the axis as shown in the figure is :



(A) $\sqrt{\frac{5}{3}}\,r$ (B) $\sqrt{\frac{5}{6}}\,r$

(C) $\sqrt{\frac{7}{3}}\,r$ (D) $\sqrt{\frac{7}{6}}\,r$

8. The moment of inertia of a hollow cubical box of mass M and side a about an axis passing through the centres of two opposite faces is equal to

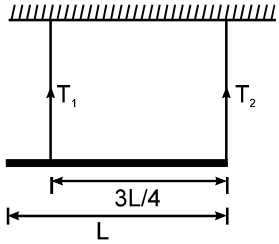
(A) $\frac{5Ma^2}{3}$ (B) $\frac{5Ma^2}{6}$ (C) $\frac{5Ma^2}{12}$ (D) $\frac{5Ma^2}{18}$

9. A hollow cylinder has mass M, outside radius R_2 and inside radius R_1. Its moment of inertia about an axis parallel to its symmetry axis and tangential to the outer surface is equal to :

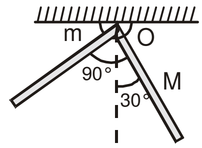
(A) $\frac{M}{2}(R_2^2 + R_1^2)$ (B) $\frac{M}{2}(R_2^2 - R_1^2)$ (C) $\frac{M}{4}(R_2 + R_1)^2$ (D) $\frac{M}{2}(3R_2^2 + R_1^2)$

10. Let I be the moment of inertia of a uniform square plate about an axis AB that passes through its centre and is parallel to two of its sides. CD is a line in the plane of the plate that passes through the centre of the plate and makes an angle θ with AB. The moment of inertia of the plate about the axis CD is then equal to :
(A) I (B) I $\sin^2\theta$ (C) I $\cos^2\theta$ (D) I $\cos^2(\theta/2)$

11. A uniform rod of mass m and length L is suspended with two massless strings as shown in the figure. If the rod is at rest in a horizontal position the ratio of tension in the two strings t_1/T_2 is:
(A) 1: 1
(B) 1: 2
(C) 2: 1
(D) 4: 3



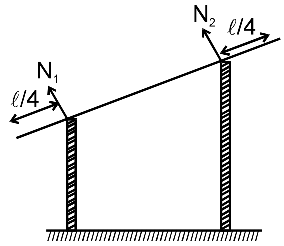
12. Two uniform rods of equal length but different masses are rigidly joined to form an L–shaped body, which is then pivoted about O as shown. If in equilibrium the body is in the shown configuration, ratio M/m will be:



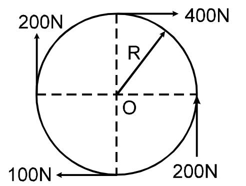
(A) 2 (B) 3 (C) $\sqrt{2}$ (D) $\sqrt{3}$

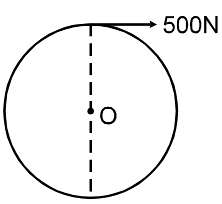
13. Two persons of equal height are carrying a long uniform wooden beam of length ℓ. They are at distance $\ell/4$ and $\ell/6$ from nearest ends of the rod. The ratio of normal reactions at their heads is:
(A) 2 : 3 (B) 1 : 3 (C) 4 : 3 (D) 1 : 2

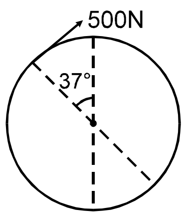
14. A uniform rod of length ℓ is placed symmetrically on two walls as shown in figure. The rod is in equilibrium. If N_1 and N_2 are the normal forces exerted by the walls on the rod then
(A) $N_1 > N_2$ (B) $N_1 > N_2$
(C) $N_1 = N_2$
(D) N_1 and N_2 would be in the vertical directions. N_1 N_2

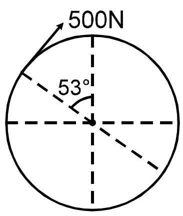


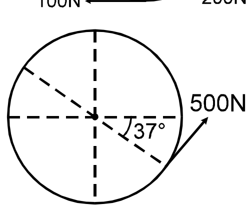
15. Four forces tangent to the circle of radius 'R' are acting on a wheel as shown in the figure. The resultant equivalent one force system will be :



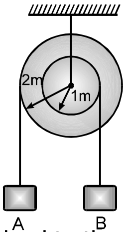
(A)


(B)


(C)


(D)


16. In the pulley system shown, if radii of the bigger and smaller pulley are 2 m and 1 m respectively and the acceleration of block A is 5 m/s² in the downward direction, then the acceleration of block B will be:



(A) 0 m/s² (B) 5 m/s² (C) 10 m/s² (D) 5/2 m/s²

17. A uniform thin rod of mass 'm' and length L is held horizontally by two vertical strings attached to the two ends. One of the string is cut. Find the angular acceleration soon after it is cut :

'm' L

(A) $\frac{g}{2L}$ (B) $\frac{g}{L}$ (C) $\frac{3g}{2L}$ (D) $\frac{2g}{L}$

18. A rigid body is in pure rotation.

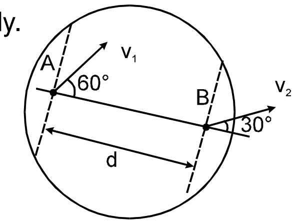
(A) You can find two points in the body in a plane perpendicular to the axis of rotation having same velocity.

(B) You can find two points in the body in a plane perpendicular to the axis of rotation having same acceleration.

(C) Speed of all the particles lying on the curved surface of a cylinder whose axis consides with the axis of rotation is same.

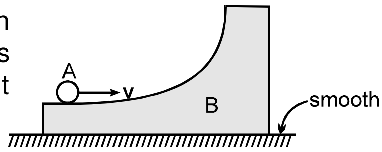
(D) Angular speed of the body is same as seen from any point in the body.

19. Two points A & B on a disc have velocities v_1 & v_2 at some moment. Their directions make angles 60° and 30° respectively with the line of separation as shown in figure. The angular velocity of disc is :



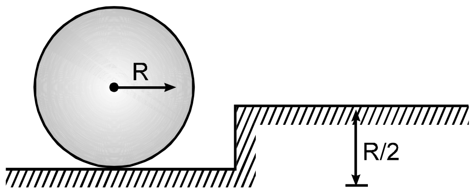
(A) $\frac{\sqrt{3}v_1}{d}$ (B) $\frac{v_2}{\sqrt{3}d}$ (C) $\frac{v_2 - v_1}{d}$ (D) $\frac{v_2}{d}$

20. In the figure shown a ring A is rolling without sliding with a velocity v on the horizontal surface of the body B (of same mass as A). All surfaces are smooth. B has no initial velocity. What will be the maximum height (from initial position) reached by A on B.



(A) $\frac{3v^2}{4g}$ (B) $\frac{v^2}{4g}$ (C) $\frac{v^2}{2g}$ (D) $\frac{v^2}{3g}$

21. A wheel (to be considered as a ring) of mass m and radius R rolls without sliding on a horizontal surface with constant velocity v. It encounters a step of height R/2 at which it ascends without sliding.



(A) the angular velocity of the ring just after it comes in contact with the step is 3v/4R

(B) the normal reaction due to the step on the wheel just after the impact is $\frac{mg}{2} + \frac{9mv^2}{16R}$

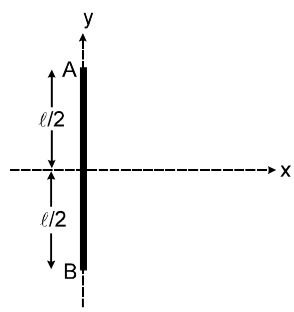
(C) the normal reaction due to the step on the wheel increases as the wheel ascends

(D) the friction will be absent during the ascent.

22. If $\vec{\tau} \times \vec{L} = 0$ for a rigid body, where $\vec{\tau}$ = resultant torque & $\vec{L}$ = angular momentum about a point and both are non - zero. Then:

(A) $\vec{L}$ = constant (B) $|\vec{L}|$ = constant (C) $|\vec{L}|$ will increase (D) $|\vec{L}|$ may increase.

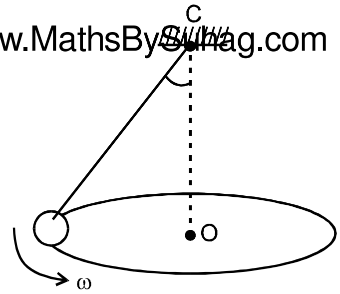
23. A uniform rod of mass m, length ℓ is placed over a smooth horizontal surface along y-axis and is at rest as shown in figure. An impulsive force F is applied for a small time Δt along x-direction at point A. The x–coordinate of end A of the rod when the rod becomes parallel to x–axis for the first time is (initially the coordinate of centre of mass of the rod is (0, 0)) :



(A) $\frac{\pi\ell}{12}$ (B) $\frac{\ell}{2}\left(1+\frac{\pi}{12}\right)$

(C) $\frac{\ell}{2}\left(1-\frac{\pi}{6}\right)$ (D) $\frac{\ell}{2}\left(1+\frac{\pi}{6}\right)$

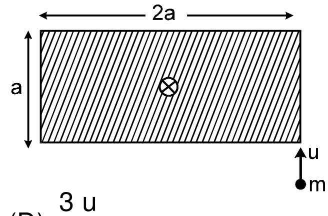
24. A conical pendulum consists of a simple pendulum moving in a horizontal circle as shown. C is the pivot, O the centre of the circle in which the pendulum bob moves and ω the constant angular velocity of the bob. If $\vec{L}$ is the angular momentum about point C, then



(A) $\vec{L}$ is constant (B) only direction of $\vec{L}$ is constant
(C) only magnitude of $\vec{L}$ is constant (D) none of the above.

25. In the above problem if $\vec{L}$ is the angular momentum about the axis CO, then,
(A) $\vec{L}$ is constant (B) only direction of $\vec{L}$ is constant
(C) only magnitude of $\vec{L}$ is constant (D) none of the above.

26. A uniform rectangular plate of mass m which is free to rotate about the smooth vertical hinge passing through the centre and perpendicular to the plate, is lying on a smooth horizontal surface. A particle of mass m moving with speed 'u' collides with the plate and sticks to it as shown in figure. The angular velocity of the plate after collision will be :

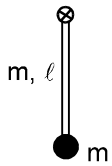


(A) $\frac{12}{5}\frac{u}{a}$ (B) $\frac{12}{19}\frac{u}{a}$ (C) $\frac{3}{2}\frac{u}{a}$ (D) $\frac{3}{5}\frac{u}{a}$

27. The angular momentum of a particle about origin is varying as L = 4t + 8 (SI units) when it moves along a straight line y = x – 4 (x,y in meters). The force acting on the particle would be :
(A) 1 N (B) 2 N (C) $\sqrt{2}$N (D) $\sqrt{3}$N

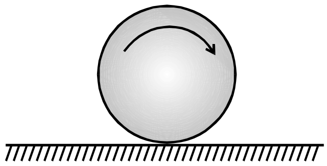
28. A rod can rotate about a fixed vertical axis. The mass is non-uniformly distributed along the length of the rod. A horizontal force of constant magnitude and always perpendicular to the rod is applied at the end. Which of the following quantity (after one rotation) will not depend on the information that through which end the axis passes ?
(A) angular momentum (B) kinetic energy
(C) angular velocity (D) none of these

29. A particle is attached to the lower end of a uniform rod which is hinged at its other end as shown in the figure. The minimum speed given to the particle so that the rod performs circular motion in a vertical plane will be:



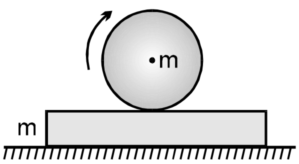
(A) $\sqrt{5g\ell}$ (B) $\sqrt{4g\ell}$ (C) $\sqrt{4.5g\ell}$ (D) none of these

30. A body of mass m and radius r is rotated with angular velocity ω as shown in the figure & kept on a surface that has sufficient friction then the body will move :



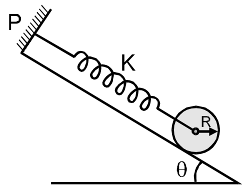
(A) backward first and then move forward
(B) forward first and then move backward
(C) will always move forward (D) none of these

31. A sphere of mass ' m ' is given some angular velocity about a horizontal axis through its centre and gently placed on a plank of mass ' m '. The co-efficient of friction between the two is μ. The plank rests on a smooth horizontal surface. The initial acceleration of the centre of sphere relative to the plank will be :



(A) zero (B) μg (C) (7/5) μg (D) 2 μg

32. A uniform cylinder of mass M and radius R rolls without slipping down a slope of angle θ to the horizontal. The cylinder is connected to a spring constant K while the other end of the spring is connected to a rigid support at P. The cylinder is released when the spring is unstretched. The maximum distance that the cylinder travels is



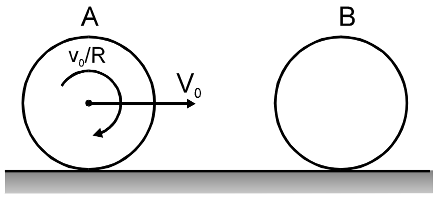
(A) $\frac{3}{4}\frac{Mg\sin\theta}{K}$ (B) $\frac{Mg\tan\theta}{K}$ (C) $\frac{2Mg\sin\theta}{K}$ (D) $\frac{4}{3}\frac{Mg\sin\theta}{K}$

33. When a person throws a meter stick it is found that the centre of the stick is moving with speed 10 m/

s & left end of stick with speed 20 m/s. Both points move vertically upwards at that moment. Then angular speed of the stick is:

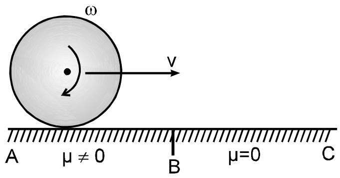
(A) 20 rad/ sec (B) 10 rad/sec (C) 30 rad/sec (D) none of these

34. A hollow smooth uniform sphere A of mass 'm' rolls without sliding on a smooth horizontal surface. It collides elastically and headon with another stationary smooth solid sphere B of the same mass m and same radius. The ratio of kinetic energy of 'B' to that of 'A' just after the collision is :



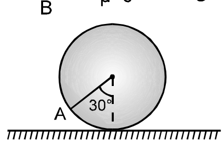
(A) 5 : 2 (B) 1 : 1 (C) 2 : 3 (D) 3 : 2

35. As shown in the figure, a disc of mass m is rolling without slipping a angular velocity ω. When it crosses point B disc will be in :



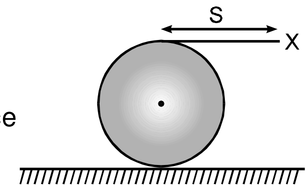
(A) translational motion only (B) pure rolling motion
(C) rotational motion only (D) none of these

36. A disc of radius R rolls on a rough horizontal surface. The linear distance covered by the point A in one complete revolution is:



(A) 8 R (B) $4\sqrt{3}$ R (C) $2\sqrt{3}$ R (D) none of these

37. A large spool of rope lies on the ground as shown in the fig. The end, labelled X is pulled a distance S in the horizontal direction. The spool rolls without slipping. The centre of mass of the spool moves a distance

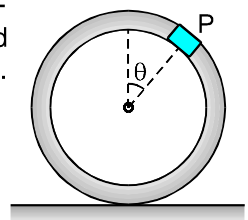


(A) 2 S (B) S (C) S/2 (D) S/4

38. A solid uniform disk of mass m rolls without slipping sown an inclined plane with an acceleration a. The frictional force on the disk due to surface of the plane is

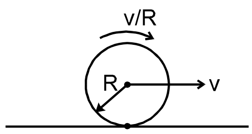
(A) 2 ma (B) $\frac{3}{2}$ ma (C) ma (D) $\frac{1}{2}$ ma

39. A small block of mass 'm' is rigidly attached at 'P' to a ring of mass '3m' and radius 'r'. The system is released from rest at $\theta = 90°$ and rolls without sliding. The angular acceleration of ring just after release is –



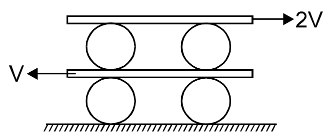
(A) $\frac{g}{4r}$ (B) $\frac{g}{8r}$ (C) $\frac{g}{3r}$ (D) $\frac{g}{2r}$

40 A disc is performing pure rolling on a smooth stationary surface with constant angular velocity as shown in figure. At any instant, for the lower most point of the disc



(A) Velocity is v, acceleration is zero
(B) Velocity is zero, acceleration is zero
(C) velocity is v, acceleration is $\frac{v^2}{R}$.
(D) velocity is zero, acceleration is $\frac{v^2}{R}$

41. A system of uniform cylinders and plates is shown in figure. All the cylinders are identical and there is no slipping at any contact. Velocity of lower & upper plate is V and 2V respectively as shown in figure. Then the ratio of angular speed of the upper cylinders to lower cylinders is



(A) 3 (B) 1/3 (C) 1 (D) none of these

42. A box of dimensions l and b is kept on a truck moving with an

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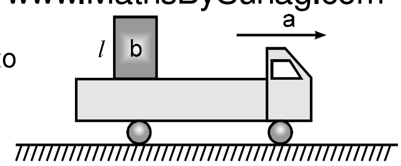
acceleration a. If box does not slide, maximum acceleration for it to remain in equilibrium (w.r.t.truck) is :



(A) $\frac{g\,\ell}{b}$ (B) $\frac{g\,b}{\ell}$ (C) g (D) none of these

43. If the positions of two like parallel forces are interchanged, their resultant shifts by one–fourth of the distance between them then the ratio of their magnitude is:
(A) 1: 2 (B) 2: 3 (C) 3: 4 (D) 3: 5

EXERCISE-2

SECTION (A) : KINEMATICS

A 1. A body rotating at 20 rad/s is acted upon by a constant torque providing it a deceleration of 2 rad/s^2. At what time will the body have kinetic energy same as the initial value if the torque continues to act?

A 2. A solid body rotates about a stationary axis according to the law $\varphi = at - bt^3$, where a = 6.0rad/s and b = 2.0 rad/s^3. Find:
(a) the mean values of the angular velocity and angular acceleration averaged over the time interval between t = 0 and the complete stop;
(b) the angular acceleration at the moment when the body stops.

A 3. A solid body starts rotating about a stationary axis with an angular acceleration β = at , where a = 2.0 × 10^{-2} rad/s^3. How soon after the beginning of rotation will the total acceleration vector of an arbitrary point of the body form an angle α = 60º with its velocity vector.

SECTION (B) : MOMENT OF INERTIA

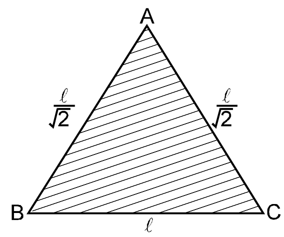
B 1. Find the M.I. of a rod about (i) an axis perpendicular to the rod and passing through left end. (ii) An axis through its centre of mass and perpendicular to the length whose linear density varies as λ = ax where a is a positive constant and 'x' is the position of an element of the rod relative to its left end. The length of the rod is ℓ.

B 2. Three particles, each of mass 200 g, are kept at the corners of an equilateral triangle of side 10 cm. Find the moment of inertia of the system about an axis
(a) joining two of the particles and
(b) passing through one of the particles and perpendicular to the plane of the particles.

B 3. Particles of masses 1 g, 2 g, 3 g,.........100 g are kept at the marks 1 cm, 2 cm, 3 cm,...... 100 cm respectively on a metre scale. Find the moment of inertia of the system of particles about a perpendicular bisector of the metre scale.

B 4. Find the moment of inertia of a uniform half-disc about an axis perpendicular to the plane and passing through its centre of mass. Mass of this disc is M and radius is R.

B 5. A uniform triangular plate of mass m whose vertices are ABC has lengths $\ell, \frac{\ell}{\sqrt{2}}$ and $\frac{\ell}{\sqrt{2}}$. Find the moment of inertia of this plate about an axis passing through point B and perpendicular to the plane of the plate.



B 6. Find the moment of inertia of a pair of spheres, each having a mass m and radius r, kept in contact about the tangent passing through the point of contact.

B 7. The moment of inertia of a uniform rod of mass m = 0.50 kg and length ℓ = 1 m is I = 0.10 kg -m^2 about a line perpendicular to the rod. Find the distance of this line from the middle point of the rod.

B 8. Calculate the moment of inertia.
(a) of a copper uniform disc relative to the symmetry axis perpendicular to the plane of the disc, if its thickness is equal to b = 2.0 mm and its radius to R = 100 mm;

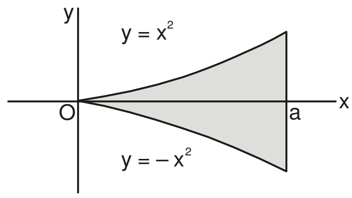
B 9. Find the moment of inertia of a thin uniform rectangular plate relative to the axis passing perpendicular to the plane of the plate through one of its vertices. If the sides of the plate are equal to a and b, and its mass is m.

B 10. Find the moment of inertia of a uniform square plate of mass m and edge a about one of its diagonals.

B 11. Find the radius of gyration of a circular ring of radius r about a line perpendicular to the plane of this line from the middle point of the rod. r

B 12. The radius of gyration of a uniform disc about a line perpendicular to the disc equals its radius. Find the distance of the line from the centre.

B 13. Find the moment of inertia about x–axis of uniform thin plate of density ρ kg/m^2 as shown in the Figure.



B 14. Calculate the moment of inertia of a uniform rod of mass m & length ℓ about an axis passing through one end & making angle $\theta = 45^\circ$ with its length.

B 15. The surface density (mass/area) of a circular disc of radius a depends on the distance from the centre of $\rho(r) = A + Br$. Find its moment of inertia about the line perpendicular to the plane of the disc through its centre.

B 16. Calculate the moment of inertia of a uniform solid cone relative to its symmetry axis, if the mass of the cone is equal to m and the radius of its base to R.

SECTION (C) : TORQUE

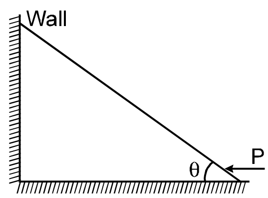
C 1. A force $F = A\hat{i} + B\hat{j}$ is applied to a point whose radius vector relative to the origin of coordinates O is equal to $r = a\hat{i} + b\hat{j}$, where a, b &, A, B are constants, and $\hat{i}$, $\hat{j}$ are the unit vectors of the x and y axes. Find the moment N and the arm ℓ of the force relative to the point O.

C 2. A force $\vec{F}_1 = A\hat{j}$ is applied to a point whose radius vector $\vec{r}_1 = a\hat{i}$, while a force $\vec{F}_2 = B\hat{i}$ is applied to the point whose radius vector $\vec{r}_2 = b\hat{j}$. Both radius vectors are determined relative to the origin of coordinates O, $\hat{i}$ and $\hat{j}$ are the unit vectors of the x and y axes, a, b, A. B are constants. Find the arm ℓ of the resultant force relative to the point O.

C 3. When a sphere of radius $r_1 = 1.2$ mm moves in glycerin the laminar flow is observed if the velocity of the sphere does not exceed $\upsilon_1 = 23$ cm/s. At what minimum velocity υ_2 of a sphere of radius $r_2 = 5.5$ cm will the flow in water become turbulent? The vicosities of glycerin and water are equal to $\eta_1 = 13.9$ P and $\eta_2 = 0.011$ P respectively.

SECTION (D) : ROTATIONAL EQUILIBRIUM

D 1. Assuming frictionless contacts, determine the magnitude of external horizontal force P applied at the lower end for equilibrium of the rod. The rod is uniform and its mass is 'm'.

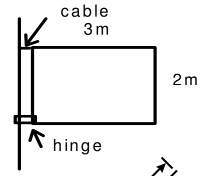


D 2. A uniform ladder of length 10.0 m and mass 16.0 kg is resting against a vertical wall making an angle of 37° with it. The vertical wall is frictionless but the ground is rough. An electrician weighing 60.0 kg climbs up the ladder. If he stays on the ladder at a point 8.00 m from the lower end, what will be the normal force and the force of friction on the ladder by the ground? What should be the minimum of friction for the electrician to work safely?

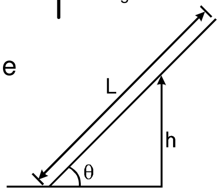
D 3. A uniform metre stick of mass 200 g is suspended from the ceiling through two vertical strings of equal lengths fixed at the ends. A small object of mass 20 g is placed on the stick at a distance of 70 cm from the left end. Find the tensions in the two strings.

D 4. The door of an almirah is 6 ft high, 1.5 ft wide and weighs 8 kg. The door is supported by two hinges situated at a distance of 1 ft from the ends. If the magnitudes of the forces exerted by the hinges on the door are equal, find this magnitude.

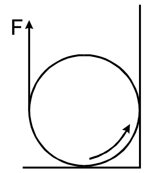
D 5. In figure the uniform gate weighs 300 N and is 3 m wide and 2 m high. It is supported by a hinge at the bottom left corner and a horizontal cable at the top left corner, as shown. Find :
(a) The tension in the cable
(b) The force that the hinge exerts on the gate (magnitude and direction).



D 6. A uniform rod of length L rests against a smooth wall as shown in figure. Find the friction coefficient between the ground and the lower end if the minimum angle that the rod can make with the horizontal is θ.



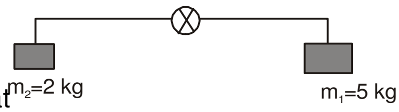
D 7. Figure shows a vertical force F that is applied tangentially to a uniform cylinder of weight W. The coefficient of static friction between the cylinder and all surfaces is 0.5. Find in terms of W, the maximum force that can be applied without causing the cylinder to rotate.



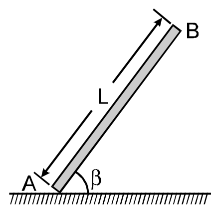
SECTION (E) : ROTATION ABOUT FIXED AXIS ($\tau_H = I_H\alpha$)

E 1. A rod of mass m and length L, lying horizontally , is free to rotate about a vertical axis through its centre. A horizontal force of constant magnitude F acts on the rod at a distance of L/4 from the centre. The force is always perpendicular to the rod. Find the angle rotated by the rod during the time t after the motion starts.

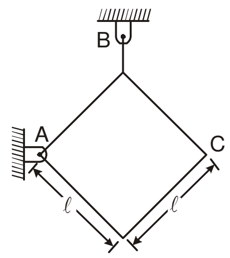
E 2. A light rod of length 1 m is pivoted at its centre and two masses of 5 kg and 2 kg are hung from the ends as shown in figure.
(a) Find the initial angular acceleration of the rod assuming that it was horizontal in the beginning.
(b) If the rod has a mass of 1 kg distributed uniformly over its length.
(i) Find the initial angular acceleration of the rod.
(ii) Find the tension in the supports to the blocks of mass 2 kg and 5 kg.



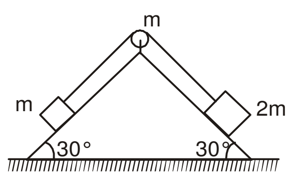
E 3. The uniform rod AB of mass m is released from rest when β = 60º. Assuming that the friction force between end A and the surface is large enough to prevent sliding, determine (for the instant just after release)
(a) The angular acceleration of the rod
(b) The normal reaction and the friction force at A.
(c) The minimum value of μ compatible with the described motion.
(a)
(b) A
(c) μ



E 4. A uniform square plate of mass m is supported as shown. If the cable suddenly breaks, determine just after that momentum;
(a) The angular acceleration of the plate.
(b) The acceleration of corner C.
(c) The reaction at A.
(a)
(b) C
(c) A

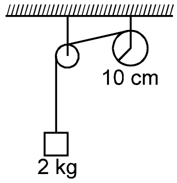


E 5. The pulley (uniform disc) shown in figure has mass m.
(a) Assuming the inclined planes to be frictionless, calculate the acceleration of the mass 2m.
(b) If the friction coefficient between the block A and the plane



below it is μ = 0.5 and the plane below the 2m block is frictionless.

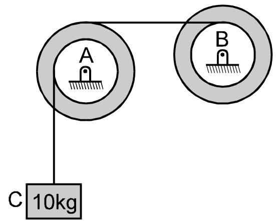
E 6. A string is wrapped on a wheel of moment of inertia 0.20 kg -m^2 and radius 10 cm and goes through a light pulley to support a block of mass 2.0 kg as shown in figure. 0.20 kg -m^2 10 cm 2.0 kg



(a) If the smaller pulley is assume to be light, find the acceleration of the block.
(b) The smaller pulley has its radius 5.0 cm and moment of inertia 0.10 kg-m^2. Find the tension in the part of the string joining the pulleys.

E 7. The ends of thin threads tightly wound on the axle of radius r of the Maxwell disc are attached to a horizontal bar. When the disc unwinds, the bar is raised to keep the disc at the same height. The mass of the disc at the axle is equal to m, the moment of inertia of the arrangement relative to its axis is I. Find the tension of each thread and the acceleration of the bar.

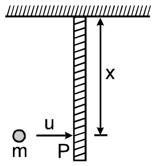
E 8. Each of the double pulleys shown has a centroidal mass moment of inertial of 0.25 kg m^2, an inner radius of 100 mm and an outer radius of 150 mm. Assuming that the bearing friction at A and at B is equivalent to torque of magnitude 0.45 N. m, determine ;
(a) The velocity of the cylinder 3 sec after the system is released from rest.
(b) The tension in the cord connecting the pulleys.



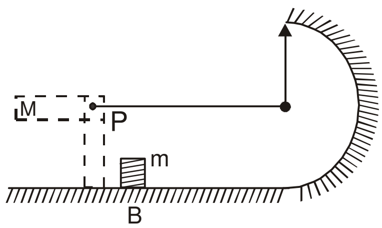
E 9. A uniform disc of radius R is spinned to the angular velocity ω and then carefully placed on a horizontal surface such that its axis is vertical. How long will the disc be rotating on the surface if the friction coefficient is equal to k? The pressure exerted by the disc on the surface can be regarded as uniform

E 10. A flywheel with the initial angular velocity ω_0 decelerates due to the forces whose moment relative to the axis is proportional ,to the square root of its angular velocity .Find the mean angular velocity of the flywheel averaged over the total deceleration time.

E 11. One end of a uniform rod of mass M and length L is hinged to the ceiling such that it can rotate freely about the hinge in vertical plane. The arrangement is shown in the figure. A ball of mass m moving horizontally with speed u hits the rod at point P separated by a distance x from hinge. For what value of x reaction in horizontal direction of the hinge on the rod is minimum and what is it? xx



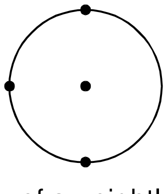
E 12. A rod of length R and mass M is free to rotate about a horizontal axis passing through hinge P as in figure. First it is taken aside such that it becomes horizontal and then released. At the lowest point the rod hits the block B of mass m and stops. Find the ratio of masses such that the block B completes the circular track of radius R. Neglect any friction.



SECTION (F) : ROTATION ABOUT FIXED AXIS (ENERGY CONSERVATION)

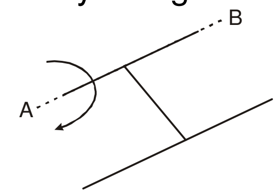
F 1. A metre stick is held vertically with one end on a rough horizontal floor. It is gently allowed to fall on the floor. Assuming that the end at the floor does not slip, find the angular speed of the rod when it hits the floor.

F 2. A solid cylinder of mass M = 1kg & radius R = 0.5m is pivoted at its centre & has three particles of mass m = 0.1kg mounted at its perimeter as shown in the figure. The system is originally at rest. Find the angular speed of the cylinder, when it has swung through 90^0 in anticlockwise direction.

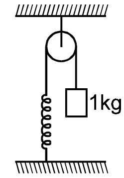


F 3. Two particles of masses 7 m and 3 m are fastened to the ends, A, B respectively of a weightless rigid rod, 15 feet long, which is freely hinged at a point O, 5 feet from A. If the rod is just disturbed from its position of unstable equilibrium. Find the velocity with which A will pass through its position of stable equilibrium.

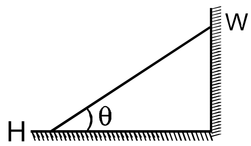
F 4. A rigid body is made of three identical thin rods each of length L fastened together in the form of letter H. The body is free to rotate about a fixed horizontal axis AB that passes through one of the legs of the H. The body is allowed to fall from rest from a position in which the plane of H is horizontal. What is the angular speed of the body, when the plane of H is vertical.



F 5. The pulleys shown in figure has a radius of 20 cm and moment of inertia 0.2 kg-m^2. The string going over it attached at one end to a vertical spring of spring constant 50 N/m fixed from below, and supports a 1 kg mass at the other end. The system is released from rest with the spring at its natural length. Find the speed of the block when it has descended through 10 cm. Take g = 10 m/s^2.



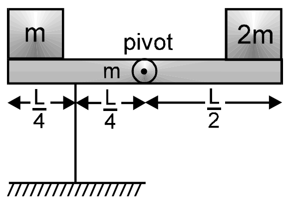
F 6. A uniform rod of length 'ℓ' is kept as shown in the figure. H is a horizontal smooth surface and W is a vertical smooth wall. The rod is released from this position. Find the angular acceleration of the rod just after the release.



SECTION (G) : ROTATION ABOUT FIXED AXIS ($\tau_H = I_H\alpha$ & ENERGY CONSERVATION)

G 1. A uniform rod pivoted at its upper end hangs vertically. It is displaced through an angle of 60°C and then released. Find the magnitude of the force acting on a particle of mass dm at the tip of the rod when the rod makes an angle of 37° with the vertical.

G 2. A bar of length L and mass m has a frictionless pivot through its mid point. There is an additional point mass 2m on the right end of the bar and an additional point mass m on the left end of the bar. The bar is held in horizontal position by a vertical cord attached at L/4 from the left end as shown in the Figure. The additional masses m & 2m remain fixed on the rod. The rod is initially horizontal.



(a) Find the tension in the cord.
(b) Find the force that the pivot exerts on the bar.
(c) If the cord is cut, what is the angular acceleration of the bar immediately after the cord is cut.
(d) When the bar has rotated through 90^0 and is vertical, what is the linear velocity of the mass 2m.

SECTION (H) : ANGULAR MOMENTUM

H 1. A particle having mass 2 kg is moving along straight line 3x + 4y = 5 with speed 8m/s. Find angular momentum of the particle about origin. x and y are in meters.

H 2. A particle having mass 2 kg is moving with velocity $(2\hat{i} + 3\hat{j})$m/s. Find angular momentum of the particle about origin when it is at (1, 1, 0).

H 3. A uniform square plate of mass 2.0 kg and edge 10 cm rotates about one of its diagonals under the action of a constant torque of 0.10 N.m. Calculate the angular momentum and the kinetic energy of the plate at the end of the fifth second after the start.

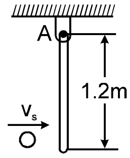
H 4. A wheel of moment of inertia 0.500 kg-m^2 and radius 20.0 cm is rotating about its axis at an angular speed of 20.0 rad/s. It picks up a stationary particle of mass 200 g at its edge. Find the new angular speed of the wheel.

H 5. A diver having a moment of inertia of 6.0 kg-m^2 about an axis through its centre of mass rotates at an angular speed of 2 rad/s about this axis. If he folds his hands and feet to decrease the moment of inertia to 5.0 kg-m^2, what will be the new angular speed ?

H 6. A wheel of moment of inertia 0.10 kg-m^2 is rotating about a shaft at an angular speed of 160 rev/minute. A second wheel is set into rotation at 300 rev/minute and is coupled to the same shaft so that both the wheels finally rotate with a common angular speed of 20 rev/minute. Find the moment of inertia of the second wheel. Show that the kinetic energy of the combined system is less than the sum of initial kinetic energeis of the two disc. How do you account for this loss?

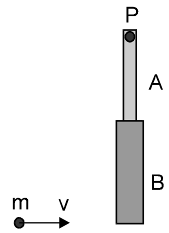
H 7. A dumb-bell consists of two identical small balls of mass 1/2 kg each connected to the two ends of a 50 cm long light rod. The dumb-bell is rotating about a fixed axis through the centre of the rod and perpendicular to it at an angular speed of 10 rad/s. An impulsive force of average magnitude 5.0 N acts on one of the masses in the direction of its velocity for 0.10 s. Find the new angular velocity of the system.

H 8. A 2 kg sphere moving horizontally to the right with an initial velocity of 5 m/s strikes the lower end of an 8 kg rigid rod AB. The rod is suspended from a hinge at A and is initially at rest. Knowing that the coefficient of restitution between the rod and sphere is 0.80, determine the angular velocity of the rod and the velocity of the sphere immediately after the impact.

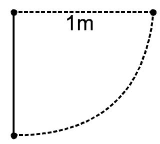


H 9. A round board of mass M and radius R is placed on a fixed smooth horizontal plane and is free to rotate about an axis which passes through its centre. A man is standing on the point marked A on the circumference of the board. At first the board & the man are at rest. The man starts moving along the rim of the board at constant speed v_o relative to the board. Find the angle of board's rotation when the man passes his starting point on the disc first time.

H 10. Two uniform thin rods A & B of length 0.6 m each and of masses 0.01 kg & 0.02 kg respectively are rigidly joined, end to end. The combination is pivoted at the lighter end P as shown in figure such that it can freely rotate about the point P in a vertical plane. A small object of mass 0.05 kg moving horizontally hits the lower end of the combination and sticks to it. What should be the velocity of the object so that the system could just be raised to the horizontal position ? [JEE - 94]



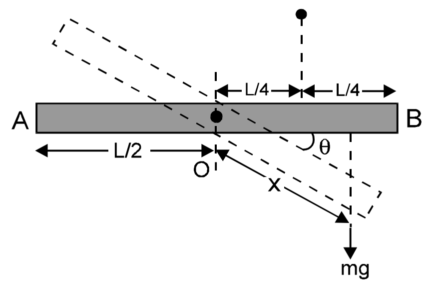
H 11. A metre stick weighing 240 g is pivoted at its upper end in such a way that it can freely rotate in a vertical plane through this end (figure). A particle of mass 100 g is attached to the upper end of the stick through a light string of length 1 m. Initially, the rod is kept vertical and the spring horizontal when the system is released from rest. The particle collides with the lower end of the stick and sticks there. Find the maximum angle through which the stick will rise.



H 12. A homogeneous rod AB of length L = 1.8 m and mass M is pivoted at the centre O in such a way that it can rotate freely in the vertical plane as shown (fig). The rod is initially in the horizontal position. An insect S of the same mass M falls vertically with speed V on the point C, midway between the point O & B. Immediately after falling, the insect moves towards the end B such that the rod rotates with constant angular velocity ω.

(a) Determine the velocity ω in terms of V & L .

(b) If the insect reaches the end B when the rod has turned through an angle of 90º, determine V.



H 13. A boy is standing on a platform which is free to rotate about its axis. The boy holds an open umbrella in his hand. The axis of the umbrella coincides with that of the platform. The moment of inertia of the "the platform plus the boy system" is 3.0×10^{-3} kg -m^2 and that of the umbrella is 2.0×10^{-3} kg-m^2. The boy starts spinning the umbrella about the axis at an angular speed of 2.0 rev/s with respect to himself. Find the angular velocity imparted to the platform.

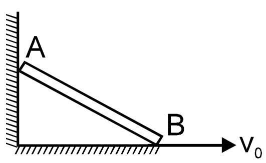
SECTION (I) : COMBINED TRANSLATIONAL + ROTATION MOTION (KINEMATICS)

I 1. The end B of rod AB which makes angle θ with the floor is being pulled with a velocity v_0 as shown. Taking the length of the rod as ℓ, calculate the following at the instant when θ = 37º

(a) The velocity of end A

(b) The angular velocity of rod

(c) Velocity of CM of the rod.



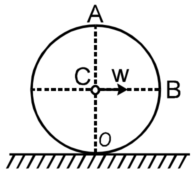
I 2. A point A is located on the rim of a wheel of radius R = 0.50 m which rolls without slipping along a horizontal surface with velocity v = 1.00 m/s. Find:

(a) the modulus and the direction of the acceleration vector of the point A ;

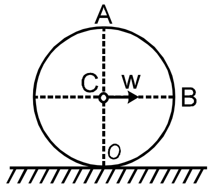
(b) the total distance s traversed by the point A between the two successive moments at which it touches the surface.

I 3. A sphere of mass m rolls on a plane surface. Find its kinetic energy at an instant when its centre moves with speed v.

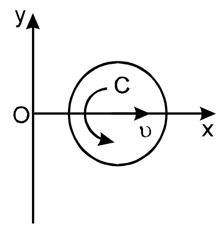
I 4. A ball of radius R =10.0 cm rolls without slipping down an inclined plane so that its centre moves with constant acceleration ω = 2.50 cm/s^2; t = 2.00 s after the beginning of motion its position corresponds to that shown in Fig. Find :
(a) the velocities of the points A, B and O ;
(b) the accelerations of these points.



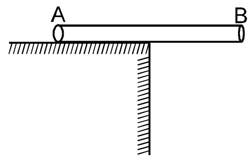
I 5. A cylinder rolls without slipping over a horizontal plane. The radius of the cylinder is equal to r. Find the curvature radii of trajectories traced out by the points A and B in Fig



I 6. A rotating disc (figure) moves in the positive direction of the x axis. Find the equation y (x) describing the position of the instantaneous axis of rotation, if at the initial moment the axis C of the disc was located at the point O after which it moved
(a) With a constant velocity v, while the disc started rotating counter clockwise with a constant angular acceleration β (the initial angular velocity is equal to zero) ;
(b) With a constnat acceleration ω (and the zero initial velocity), while the disc rotates counterclockwise with a constnat angular velocity ω.



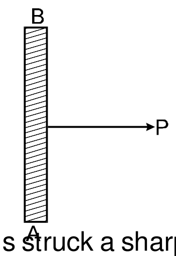
I 7. A drinking straw of mass 2m is placed on a table orthogonally to the edge such that half of it extends beyond the table. A fly with mass m lands on the inner end of the straw and walks along the straw untill it reaches the outer end. It does not tip even when another fly lands on the top of the first one. Find the largest mass that the second fly can have. (Neglect the friction between straw and table). 2m (drinking straw) m ()



SECTION (J) : COMBINED TRANSLATIONAL & ROTATIONAL MOTION (DYNAMICS)

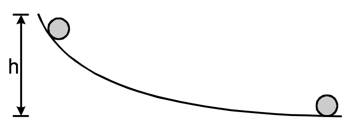
J 1. A cylinder rolls on a horizontal plane surface. If the speed of the centre is 25 m/s, what is the speed of the highest point ?

J 2. Determine the point of the rod AB of length 0.9 m at which the force P should be applied if the acceleration of point B is to be zero. Knowing that the magnitude of P is 2.70 N, determine the corresponding angular acceleration of the rod and the acceleration of the center of the rod. (Mass of the rod is 1.5 kg. Force P is to be applied perpendicular to the rod and the rod is in horizontal smooth plane).



J 3. A uniform rod 1.0m long is initially standing vertically on a smooth horizontal surface. It is struck a sharp horizontal blow at the top end, with the blow directed at right angles to the rod axis. As a result, the rod acquired an angular velocity of 3 rad/sec.
(a) What is the translational velocity of the centre of mass of the rod after the blow.
(b) Which point on the rod is stationary on the rod just after the blow.

J 4. A small spherical ball is released from a point at a height h on a rough track shown in figure. Assuming that it does not slip anywhere, find its linear speed when it rolls on the horizontal part of the track.

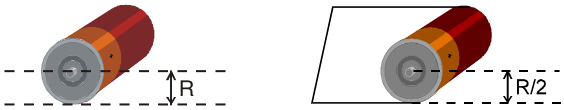


J 5. A small disc is set rolling with a speed v on the horizontal part of the track of the previous problem from right to left. Two what height will it climb up the curved part ?

J 6. A sphere starts rolling down an incline of inclination θ. Find the speed of its centre when it has covered a distance ℓ.

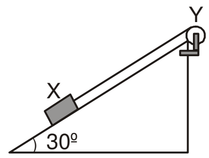
J 7. A solid sphere of mass m is released from rest from the rim of a hemispherical cup so that it rolls along the surface. If the rim of the hemisphere is kept horizontal, find the normal force exerted by the cup on the ball when the ball reaches the bottom of the cup.

J 8. A carpet of mass 'M' made of inextensible material is rolled along its length in the form of a cylinder of radius 'R' and is kept on a rough floor. The carpet starts unrolling without sliding on the floor when a negligibly small push is given to it. Calculate the horizontal velocity of the axis of the cylindrical part of the carpet when its radius reduces to R/2.



[JEE - 90]

J 9. A block X of mass 0.5 kg is held by a long massless string on a frictionless inclined plane inclined at 30º to the horizontal. The string is wound on a uniform solid cylindrical drum Y of mass 2 kg and radius 0.2 m as shown in figure. The drum is given an initial angular velocity such that block X starts moving up the plane.



(a) Find the tension in the string during motion.

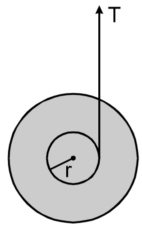
(b) At a certain instant of time the magnitude of the angular velocity of Y is 10 rad s^{-1}. Calculate the distance travelled by X from that instant of time until it comes to rest.

[JEE - 94]

J 10. By pulling on the cord of a yo-yo just fast enough, a person manages to make the yo-yo spin counterclockwise, while remaining at a constant height above the floor. Denoting the weight of the yo-yo by W, the radius of the inner drum on which the cord is wound by r, and the radius of gyration of the yo-yo byk, determine ;

(a) The tension in the cord

(b) The angular acceleration of the yo-yo.



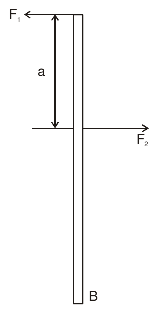
J 11. A string is wrapped over the edge of a uniform disc and the free end is fixed with the ceiling. The disc moves down, unwinding the string. Find the downward acceleration of the disc.

J 12. A hollow sphere is released from the top of an inclined plane of inclination θ. (a) What should be the minimum coefficient of friction between the sphere and the plane to prevent sliding. (b) Find the kinetic energy of the ball as it moves down a length l on the incline if the friction coefficient is half the value calculated in part (a).

J 13. A solid sphere of mass 0.50 kg is kept on a horizontal surface. The coefficient of static friction between the surfaces in contact is 2/7. What maximum force can be applied at the highest point in the horizontal direction so that the sphere does not slip on the surface?

J 14. The 6 kg carriage is supported as shown by two uniform disks, each having a mass of 4 kg and a radius of 75 mm. Knowing that the carriage is initially at rest, determine the velocity of the carriage 2.5 s after the 10 N force has been applied. Assume that the disks roll without sliding.

J 15. A thin uniform rod AB of mass m = 1.0 kg moves transtionally with acceleration w = 2.0 m/s^2 due to two antiparallel forces F_1 and F_2 (Fig.) . The distance between the points at which these forces are applied is equal to a = 20 cm. Besides, it is known that F_2 = 5.0.N. Find the length of the rod.



J 16. A uniform slender bar AB of mass m is suspended from two springs as shown.

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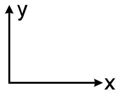
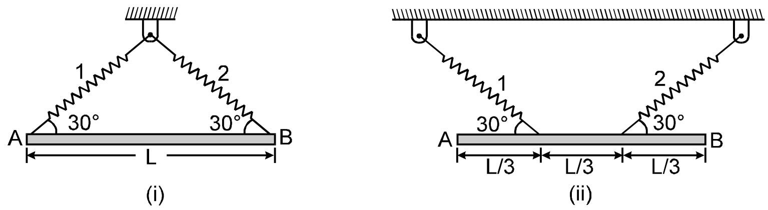
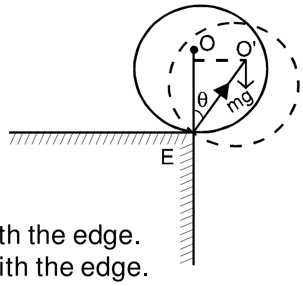


If spring 2 breaks, determine at that instant ;



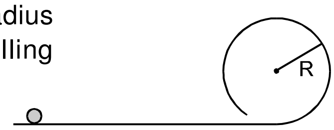
(a) The angular acceleration of the bar.
(b) The acceleration of point A.
(c) The acceleration of point B.

J 17. A rectangular rigid fixed block has a long horizontal edge. A solid homogeneous cylinder of radius r is placed horizontally at rest with its length parallel to the edge such that the axis of the cylinder and the edge of the block are in the same vertical plane. There is sufficient friction present at the edge so that a very small displacement causes the cylinder to roll of the edge without slipping. Determine : :

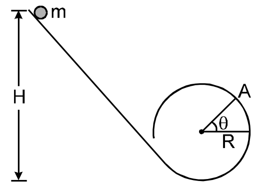


(a) The angle θ_c through which the cylinder rotates before it leave contact with the edge.
(b) The speed of the centre of mass of the cylinder before leaving contact with the edge.
(c) The ratio of translational to rotational kinetic energies of the cylinder when its centre of mass is in horizontal line with the edge. **[JEE - 95]**

J 18. Figure shows a rough track, a portion of which is in from of a cylinder of radius R. With what minimum linear speed should a sphere of radius r be set rolling on the horizontal part so that it completely goes round the circle on the cylindrical part.

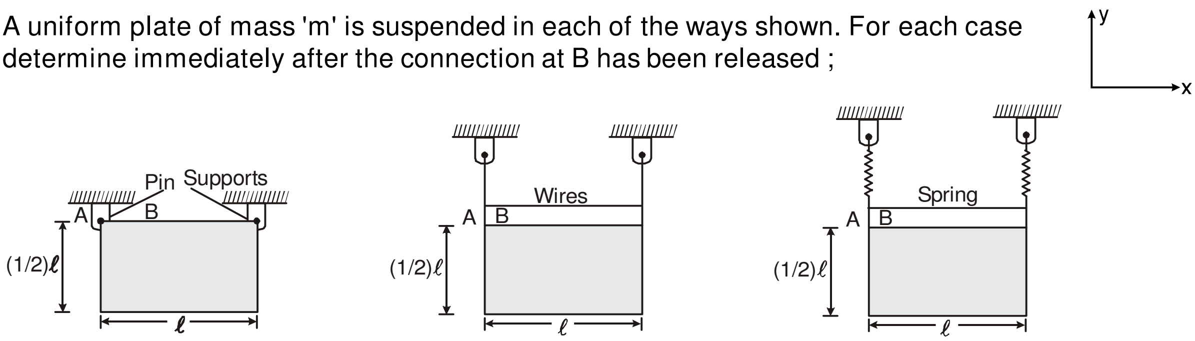


J 19. Figure shows a small spherical ball of mass m rolling down the loop track. The ball is released on the linear portion at a vertical height H from the lowest point. The circular part shown has a radius R.



(a) Find the kinetic energy of the ball when it is at a point A where the radius make an angle θ with the horizontal
(b) Find the radial and the tangential accelerations of the centre when the ball is at A.
(c) Find the normal force and the frictional force acting on the ball if H = 60 cm, R = 10 cm, θ = 0 and m = 70 g.

J 20. A uniform plate of mass 'm' is suspended in each of the ways shown. For each case determine immediately after the connection at B has been released ;



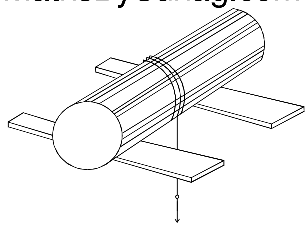
(i) (ii) (iii)

(a) The angular acceleration of the plate.
(b) The acceleration of its center of mass.

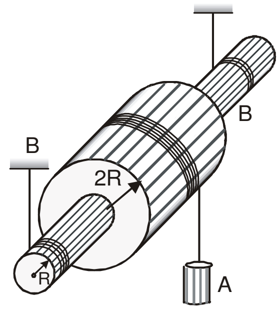
J 21. A uniform disc rolls without sliding on a horizontal surface. Find the ratio of total kinetic energy of upper half part to the total kinetic energy of the disc.

J 22. A uniform solid cylinder of mass m rests on two horizontal planks. A thread is

wound on the cylinder. The hanging end of the thread is pulled vertically down with a constant force F.
Find the maximum magnitude of the force F which still does not bring about any sliding of the cylinder, if the coefficient of friction between the cylinder and the planks is equal to k. What is the acceleration w_{max} of the axis of the cylinder rolling down the inclined plane.

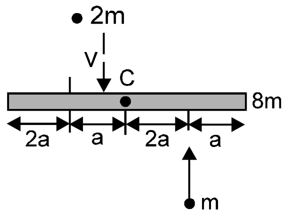


J 23. In the arrangement shown in the figure weight A possesses mass m, a pulley B possesses mass M. Also known are the moment of inertia I of the pulley relative to its axis and the radii of the pulley are R and 2R respectively. Consider the mass of the threads is negligible. Find the acceleration of weight A after the system is set free. (Assume no slipping takes place anywhere)



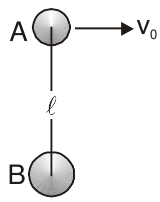
SECTION (K) : CONSERVATION OF ANGULAR MOMENTUM (COMBINED TRANSLATION & ROTATIONAL MOTION)

K 1. A uniform bar of length 6 a & mass 8 m lies on a smooth horizontal table. Two point masses 'm' & 2 m moving in the same horizontal plane with speeds 2v & 'v' respectively strike the bar as shown & stick to the bar after collision. Calculate ;
(a) Velocity of the centre of mass.
(b) Angular velocity about centre of mass
(c) Total kinetic energy, just after collision.



[JEE - 91]

K 2. Two small spheres A & B respectively of mass m & 2m are connected by a rigid rod of length ℓ & negligible mass. The two spheres are resting on a horizontal, frictionless surface. When A is suddenly given the velocity v_0 as shown. Find velocities of A & B after the rod has rotated through 180º.



K 3. A uniform rod of mass m and length ℓ is struck at an end by a force F perpendicular to the rod for a short time interval t. Calculate
(a) the speed of the centre of mass, (b) the angular speed of the rod about the centre of mass, (c) the kinetic energy of the rod and (d) the angular momentum of the rod about the centre of mass after the force has stopped to act. Assume that t is so small that the rod does not appreciably change its direction while the force acts.

K 4. A uniform rod of length L lies on a smooth horizontal table. A particle moving on the table strikes the rod perpendicularly at an end and stops. Find the distance travelled by the centre of the rod by the time it turns through a right angle. Show that if the mass of the rod is four times that of the particle the collision is elastic.

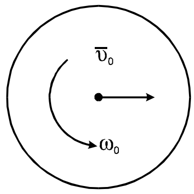
K 5. A ring rolls on a horizontal surface without sliding. The velocity of the centre is v. It encounters a step of height 0.3 R where R is the radius of the ring. Calculate the angular velocity of the ring just after the impact. Assume that the ring does not return back. (and there is sufficient friction to avoid slipping) . Find the minimum value of 'v' so that the ring ascends the step ..

K 6. A thin spherical shell of radius R lying on a rough horizontal surface is hit sharply and horizontally by a cue. Where should it be hit so that the shell does not slip on the surface?

K 7. A uniform wheel of radius R is set into rotation about its axis at an angular speed ω. This rotating wheel is now placed on a rough horizontal surface with its axis horizontal. Because of friction at the contact, the wheel accelerates forward and its rotation decelerates till the wheel starts pure rolling on the surface. Find the linear speed of the wheel after it starts pure rolling.

K 8. A thin spherical shell lying on a rough horizontal surface is hit by a cue in such a way that the line of action passes through the centre of the shell. As a result, the shell starts moving with a linear speed v without any initial angular velocity. Find the linear speed of the shell after it starts pure rolling on the surface.

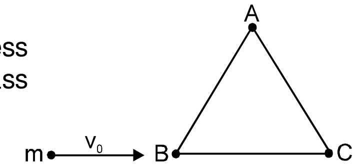
K 9. A sphere of mass m and radius r is projected along a rough horizontal surface with the initial velocities indicated. If the final velocity of the sphere is to be zero, express ;



(a) The required ω_0 in terms of $\overline{\upsilon}_0$ and r.

(b) The time required for the sphere to come to rest in terms of $\overline{\upsilon}_0$ & the coefficient of friction μ_k.

K 10. Three particles A, B, C of mass m each are joined to each other by massless rigid rods to form an equilateral triangle of side a. Another particle of mass m hits B with a velocity v_0 directed along BC as shown. The colliding particle stops immediately after impact. A, B, C m 'a' 'm' v_0 BC 'B' -



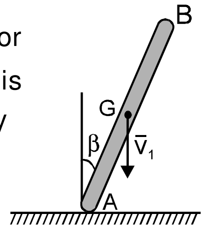
(a) Calculate the time required by the triangle ABC to complete half revolution in its subsequent motion.

(b) What is the net displacement of point B during this interval ?

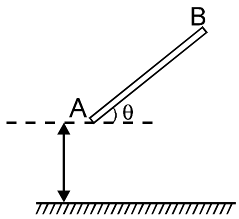
K 11. Suppose the rod with the balls A and B of the previous problem is clamped at the centre in such a way that is can rotate freely about a horizontal axis through the clamp. The system is kept at rest in the horizontal position. A particle P of the same mass m is dropped from a height h on the ball B. The particle collides with B and sticks to it. (a) Find the angular momentum and the angular speed of the system just after the collision. (b) What should be the minimum value of h so that the system makes a full rotation after the collision.

K 12. A solid sphere rolling on a rough horizontal surface with a linear speed v collides elastically with a fixed, smooth, vertical wall. Find the speed of the sphere after it has started pure rolling in the backward direction.

K 13. A slender rod of length L forming an angle β with the vertical strikes a frictionless floor at A with a vertical velocity $\overline{v}_1$ and no angular velocity. Assuming that the impact at A is perfectly elastic derive an expression for the angular velocity of the rod immediately after the impact.



K 14. A rod AB of length ℓ is released from rest with AB inclined at angle θ with horizontal. It collides elastically with ground after falling through a height h What is the height upto which the centre of mass the rod rebounds after impact? Take $\theta = 60^\circ$. Find the height *h* so that the rod is horizontal first time when its centre of mass is at the maximum height.

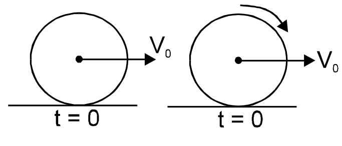


SECTION (L) : TOPPLING.

L 1. A cubical block of mass m and edge a slides down a rough inclined plane of inclination θ with a uniform speed. Find the torque of the normal force acting on the block about its centre.

EXERCISE-3

1. A uniform disc of mass m and radius R is projected horizontally with velocity v_0 on a rough horizontal floor so that it starts off with a purely sliding motion at t = 0. After t_0 seconds, it acquires a purely rolling motion as shown in figure.

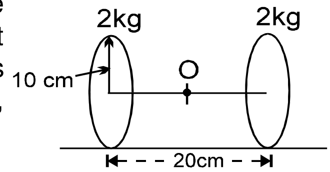


(a) Calculate the velocity of the centre of mass of the disc at t_0 .

(b) Assuming the coefficient of friction to be μ, calculate t_0 . Also calculate the work done by the frictional force as a function of time & the total work done by it over a time t much longer than t_0 . **[JEE - 97]**

2. A uniform disc of mass m and radius R is rolling up a rough inclined plane, which makes an angle of 30º with the horizontal . If the coefficients of static and kinetic friction are each equal to μ and only the forces acting are gravitational and frictional, than the magnitude of the frictional force acting on the disc is ______ and its direction is _______ (write 'up' or 'down') the inclined plane. **[JEE - 97]**

3. Two thin circular disks of mass 2 kg and radius 10 cm each are joined by a rigid massless rod of length 20 cm. The axis of the rod is along the perpendicular to the planes of the disk through their centres. This object is kept on a truck in such a way that the axis of the object is horizontal and perpendicular to the direction of motion of the truck. The friction with the floor of the truck is large enough, so that object can roll on the truck without slipping. Take x-axis as the direction of motion of the truck and z-axis as the vertically upward direction. If the truck has an acceleration of 9 m/s^2, calculate

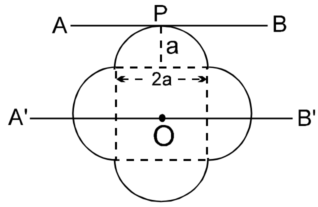


[JEE - 97]

(a) the force of friction on each disk. (b) the magnitude & direction of frictional torque acting on each disk about the centre of mass O of the object. Express the torque in the vector form in terms of unit vectors $\hat{i}$, $\hat{j}$ & $\hat{k}$ along x, y & z direction. **[JEE - 97' 5/100]**

4. A mass m is moving with a constant velocity along a line parallel to the, x-axis, away from the origin. Its angular momentum with respect to the origin : **[JEE- 97]**
(A) Is zero (B) Remains constant
(C) Goes on increasing (D) Goes on decreasing

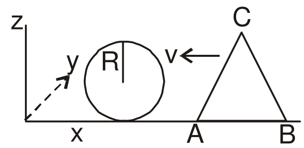
5. A symmetric lamina of mass M consists of a square shape with a semicircular section over each of the edge of the square as in figure . The side of the square is 2 a. The moment of inertia of the lamina about an axis through its centre of mass and perpendicular to the plane is 1.6 Ma^2 . The moment of inertia of the lamina about the tangent AB in the plane of lamina is _______ .



[JEE - 97' 2/100]

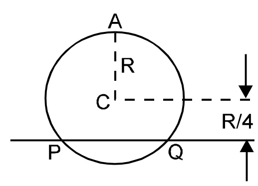
6. A rod of weight 'w' is supported by two parallel knife edges 'A' and 'B' is in equilibrium in a horizontal position. The knives are at a distance 'd' from each other. The centre of mass of the rod is at a distance 'x' from 'A'. The normal reaction on 'A' is __ and on 'B' is __ **[JEE - 97' 2/100] [JEE - 97' 2/100]**

7. A wedge of mass 'm' and triangular cross section (AB = BC = CA =2R) is moving with a constant velocity-v $\hat{i}$ towards a sphere of radius R fixed z on a smooth horizontal table as shown in the figure. The wedge makes an elastic collision with the fixed sphere and return along the same path without any rotation. Neglect all friction and suppose that the wedge remains in contact with the sphere for a very short time At, during which the sphere exerts a constant force $\vec{F}$ on the wedge.

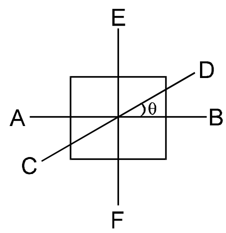


(a) Find the force $\vec{F}$ and also the normal force $\vec{N}$ exerted by the table on the wedge during the time Δt.
(b) Let 'h' denote the perpendicular distance between the centre of mass of the wedge and the line of action of force $\vec{F}$. Find the magnitude of the torque due to the normal force $\vec{N}$ about the centre of the wedge, during the time Δt. **[JEE - 98]**

8. A uniform circular disc has radius R & mass m. A particle also of mass m is fixed at a point A on the edge of the disc as shown in the figure. The disc can rotate freely about a fixed horizontal chord PQ that is at a distance R/4 from the centre C of the disc. The line AC is perpendicular to PQ. Initially the disc is held vertical with the point A at its highest position. It is then allowed to fall so that it starts rotating about PQ. Find the linear speed of the particle as it reaches its lowest position. **[JEE - 98' 8/200]**



9. Let I be the moment of inertia of a uniform square plate about an axis AB that passes through its centre and is parallel to two of its sides. CD is a line in the plane of the plate that passes through the centre of the plate and makes an angle θ with AB. The moment of inertia of the plate about the axis CD is then equal to **[JEE - 98' 2/200]**



(A) I (B) I $\sin^2\theta$ (C) I $\cos^2\theta$ (D) I $\cos^2(\theta/2)$

10. The $\vec{\tau}$ torque on a body about a given point is found to be equal to $\vec{A} \times \vec{J}$ where $\vec{A}$ is a constant vector $\vec{J}$ and is the angular momentum of the body about that point. From this it follows that
[JEE - 98' 2/200]

(A) $\frac{d\vec{J}}{dt}$ is perpendicular to $\vec{J}$ at all instants of time

(B) the components of $\vec{J}$ in the direction of $\vec{A}$ does not change with time

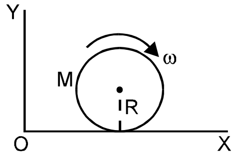
(C) the magnitude of $\vec{J}$ does not change with time

(D) $\vec{J}$ does not change with time

11. A smooth sphere 'A' is moving on a frictionless horizontal plane with angular speed ω and centre of mass velocity 'v'. It collides elastically and head on with an identical sphere 'B' at rest. Neglect friction everywhere. After the collision, their angular speeds are ω_A and ω_B, respectively. Then **[JEE - 99' 2/200]**

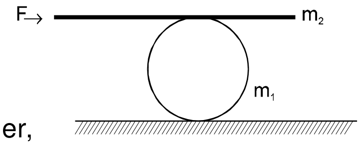
(A) $\omega_A < \omega_B$ (B) $\omega_A = \omega_B$ (C) $\omega_A = \omega$ (D) $\omega_B = \omega$

12. A disc of mass M and radius R is rolling with angular speed ω on a horizontal plane as shown. The magnitude of angular momentum of the disc about the origin O is **[JEE - 99' 2/200]**



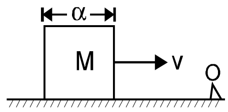
(A) $(1/2)MR^2\omega$ (B) $MR^2\omega$ (C) $(3/2)MR^2\omega$ (D) $2MR^2\omega$

13. A man pushes a cylinder of mass m_1 with help of a plank of mass m_2 as shown. There is no slipping at any contact. The horizontal component of the force applied by the man is F. Find : **[JEE 99' 6+4/200]**



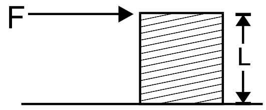
(a) The accelerations of the plank and the centre of mass of the cylinder, and (b) The magnitudes and directions of frictional forces at contact points.)

14. A cubical block of side a is moving with velocity v on a horizontal smooth plane, as shown It hits a ridge at point O. The angular speed of the block after it hits O is . **[JEE - 99' 2/200]**



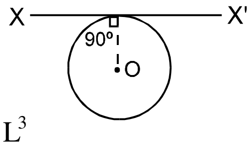
(A) $\frac{3v}{4a}$ (B) $\frac{3v}{2a}$ (C) $\frac{\sqrt{3}v}{\sqrt{2}a}$ (D) zero

15. A cubical block of side L rests on a rough horizontal surface with coefficient of friction μ . A horizontal force F is applied on the block as shown If the coefficient of friction is sufficiently high so that the block does not slide before toppling, the minimum force required to topple the block is L μ F (toppling) - **[JEE - 2000 (Scr' 1/35]**



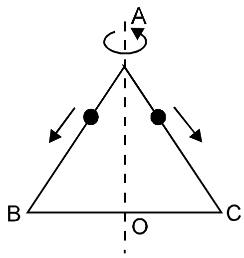
(A) Infinitesimal (B) $\frac{mg}{4}$ (C) mg/2 (D) $mg(1-\mu)$

16. A thin wire of length L and uniform density ρ is bent into a circular loop with centre at O as shown. The moment of inertia of the loop about the axis XX' is **[JEE - 2000 Scr' 1/35]**



(A) $\frac{\rho L^3}{8\pi^2}$ (B) $\frac{\rho L^3}{16\pi^2}$ (C) $\frac{5\rho L^3}{16\pi^2}$ (D) $\frac{3\rho L^3}{8\pi^2}$

17. An equilateral triangle ABC formed from a uniform wire has two small identical beads initially located at A. The triangle is set rotating about the vertical axis AO. Then the beads are released from rest simultaneously and allowed to slide down, one along AB and the other along AC as shown. Neglecting frictional effects, the quantities that are conserved as the beads slide down, are **[JEE - 2000 Scr.' 1/35]**



(A) Angular velocity and total energy (kinetic energy and potential energy)
(B) Total angular momentum and total energy
(C) Angular velocity and moment of inertia about the axis of rotation
(D) Total angular momentum and moment of inertia about

18. A rod AB of mass M and length L is lying on a horizontal frictionless surface. A particle of mass 'm' travelling along the surface hits the end A of the rod with a velocity v_0 in the direction perpendicular to AB. The collision is completely elastic. After the collision the particle comes to rest.
(a) Find the ratio m/M. **[JEE - 2000 (Mains) 3+3+3/100]**

(b) A point P on the rod is at rest immediately after the collision. Find the distance AP.
(c) Find the linear speed of the point P at a time $\pi L/(3v_0)$ after the collision.

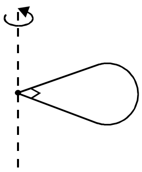
19. A particle is placed at a corner P of a cube of side 1 meter. Forces of magnitudes 2,3 and 5 kg. wt. act on the particle long the diagonals of the faces passing through the point P. Find the moment of these forces about the corner opposite to P. **[REE - 2000]**

20. A uniform rod AB of length 10 meters and weight 6kg.wt. is resting with its end A on a smooth horizontal plane AD and end B on a smooth plane DB inclined at angle 60° with the horizontal. The rod is kept in equilibrium by typing a string DP to a point P of the rod. If the length of the string is equal to AP and AD = BD, find the tension in the string. **[REE - 2000]**

21. One end of a uniform rod of mass M and length L is supported by a frictionless hinge which can withstand a tension of 1.75 Mg. The rod is free to rotate in a vertical plane. To what maximum angle should the rod be rotated from the vertical position so that when left, the hinge does not break. **[REE - 2000]**

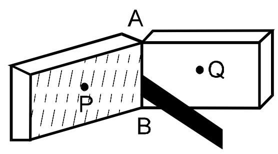
22. Two discs of same thickness but of different radii are made of two different materials such that their masses are same. The densities of the materials are in the ratio 1 : 3. The moments of inertia of these discs about the respectively axes passing through their centres and perpendicular to their planes will be in the ratio **[REE - 2000]**
(A) 1 : 3 (B) 3 : 1 (C) 1 : 9 (D) 9 : 1

23. One quarter sector is cut from a uniform circular disc of radius R. This sector has mass M . It is made to rotate about a line perpendicular to its plane and passing through the centre of the original disc. Its moment of inertial about the axis of rotation is :
[JEE - 01 (Scr.)' 3/105]



(A) $\frac{1}{2}MR^2$ (B) $\frac{1}{4}MR^2$ (C) $\frac{1}{8}MR^2$ (D) $\sqrt{2}MR^2$

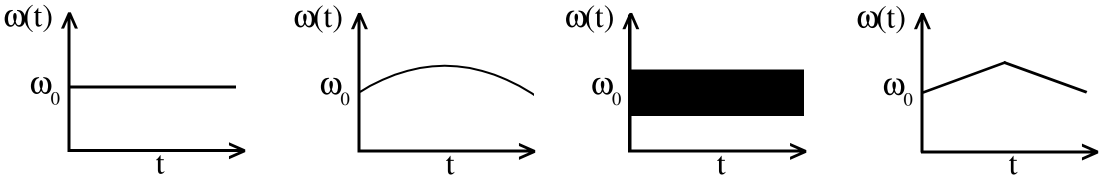
24. Two heavy metallic plates are joined together at 90º to each other. A laminar sheet of mass 30 kg is hinged at the line AB joining the two heavy metallic plates. The hinges are frictionless. The moment of inertia of the laminar sheet about an axis parallel to AB and passing through its centre of mass is 1.2 kg–m². Two rubber obstacles P and Q are fixed one on each metallic plate at a distance 0.5m from the line AB. This distance is chosen so that the reaction due to the hinges on the laminar sheet is zero during the impact. Initially the laminar sheet hits one of the obstacles with an angular velocity 1rad/sec and turns back. If the impulse on the sheet due to each obstacle is 6 N/s,



(a) Find the location of the centre of mass of the laminar sheet from AB
(b) At what angular velocity does the laminar sheet come back after the first impact
(c) After how many impacts, does the laminar sheet come to rest. **[JEE - 01(Mains) 8+1+1/100]**

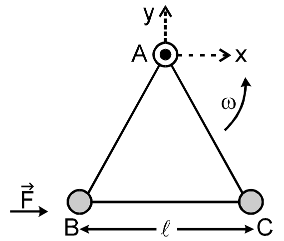
25. A cylinder rolls up an inclined plane, reaches some height and then rolls down (without slipping throughout these motions). The directions of the frictional force acting on the cylinder are : **[JEE - 02 (Scr.) 3/90]**
(A) Up the incline while ascending and down the incline while descending
(B) Up the incline while ascending as well as descending
(C) Down the incline while ascending and up the incline while descending
(D) Down the incline while ascending as well as descending.

26. A circular platform is free to rotate in a horizontal plane about a vertical axis passing through its centre. A tortoise is sitting at the edge of the platform . Now the platform is given an angular velocity ω_0 . When the tortoise moves along a chord of the platform with a constant velocity (with respect to the platform) the angular velocity of the platform $\omega(t)$ will vary with time t as : **[JEE - 02 (Scr.) 3/90]**



(A) (B) (C) (D)

27. Three particles A , B and C each of mass m are connected to each other by three massless rigid rods to form a rigid , equilateral triangular body of side ℓ . This body is placed on a horizontal frictionless table (x - y plane) and is



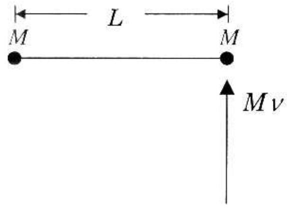
hinged to it at the point A so that it can move without friction about the vertical axis through A as shown in figure. The body is set into rotational motion on the table about A with a constant angular velocity ω .

(a) Find the magnitude of the horizontal force exerted by the hinge on the body . (hinge)

(b) At time T , when the side BC is parallel to the x – axis , a force F is applied on B along BC as shown. Obtain the x – component and the y – component of the force exerted by the hinge on the body, immediately after time T . **[JEE Mains 02, (1+4)/60]**

28. A particle is in uniform circular motion in a horizontal plane. Its angular momentum is constant when the origin is taken at :
(A) centre of the circle (B) any point on the circumference of the circle
(C) any point inside the circle (D) any point outside the circle **[JEE Sc. 2003' 3/84]**

29. Two particles, each of mass M, are connected a rod of negligible mass and length L. The system is lying on a horizontal frictionless surface. An impulse Mv, perpendicular to the rod, is given at one end of the rod as shown in the figure. The angular velocity acquired by the rod is **[JEE Sc. 2003' 3/84]**

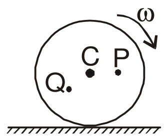


(A) $\frac{4v}{L}$ (B) $\frac{2v}{L}$ (C) $\frac{v}{L}$ (D) $\frac{v}{4L}$

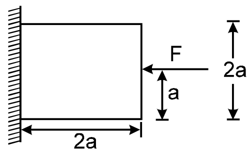
30. A platform is revolving in horizontal plane about a fixed axis and a boy is sitting at centre. The initial kinetic energy of system is K. If the boy stretches his arms then moment of inertia of system becomes double. Final kinetic energy of system is : **[JEE Sc. 2004' 3/84]**

(A) K (B) $\frac{K}{2}$ (C) $\frac{K}{4}$ (D) 2K

31. A disc is moving without slipping on ground then the relation between magnitude of velocity of points P, C and Q is [distance CP = CQ] **[JEE Sc. 2004' 3/84]**
(A) Q > C > P (B) P > C > Q
(C) C > Q > P (D) All will be same

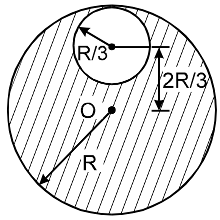


32. A block of mass m is held fixed against a wall by a applying a horizontal force F. Which of the following option is incorrect. **[JEE Scr. 2005 , 3/54]**



(A) friction force = mg (B) normal will not produce torque
(C) F will not produce torque (D) normal reaction = F

33. A disc has mass 9m. A hole of radius $\frac{R}{3}$ is cut from it as shown in the figure. The moment of inertia of the remaining part about an axis passing through the centre 'O' of the disc and perpendicular to the plane of the disc is : **[JEE Scr. 2005 , 3/54]**

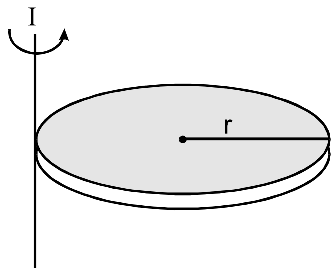


(A) 8 mR^2 (B) 4 mR^2
(C) $\frac{40}{9}mR^2$ (D) $\frac{37}{9}mR^2$

34. A particle moves in circular path with decreasing speed. Which of the following is correct
(A) $\vec{L}$ is constant **[JEE Scr. 2005 , 3/54]**
(B) only direction of $\vec{L}$ is constant
(C) acceleration $\vec{a}$ is towards the centre
(D) it will move in a spiral and finally reach the centre

35. A rod of mass M, length L hinged at its one end is in vertical equilibrium position. A bullet of mass m, moving with velocity v strikes the lower end of the rod and gets embedded into it. Find the angular velocity of the rod just after the collision. **[JEE 2005 , 2/60]**

36. A solid sphere of radius R has moment of inertia I about its geometrical axis. If it is melted into a disc of radius r and thickness t. If it's moment of inertia about the tangential axis (which is perpendicular to plane of the disc), is also equal to I, then the value of r is equal to : **[JEE 2006 , 3/184]**

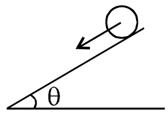


(A) $\frac{2}{\sqrt{15}}R$ (B) $\frac{2}{\sqrt{5}}R$

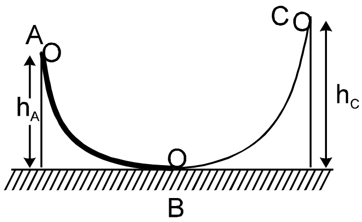
(C) $\frac{3}{\sqrt{15}}R$ (D) $\frac{\sqrt{3}}{\sqrt{15}}R$

37. A solid sphere is in pure rolling motion on an inclined surface having inclination θ. θ - **[JEE 2006 , 5/184]**

(A) frictional force acting on sphere is $f = \mu\, mg \cos\theta$.
(B) f is dissipative force.
(C) friction will increase its angular velocity and decreases its linear velocity.
(D) If θ decreases, friction will decrease.



38. A ball moves over a fixed track as shown in the figure. From A to B the ball rolls without slipping. If surface BC is frictionless and K_A, K_B and K_C are kinetic energies of the ball at A,B and C respectively, then : A B BC K_A, K_B K_C A,B C **[JEE 2006 , 5/184]**



(A) $h_A > h_C$; $K_B > K_C$ (B) $h_A > h_C$; $K_C > K_A$
(C) $h_A = h_C$; $K_B = K_C$
(D) $h_A < h_C$; $K_B > K_C$