SHEET 20 TRIGONOMETRIY (COLLECTION # 1) Single Correct Type

Que. 1. If $A+B+C=180^o$ then $\frac{\cos A\cos C+\cos(A+B)\cos(B+C)}{\cos A\sin C-\sin(A+B)\cos(B+C)}$ simplifies to

(a) $-\cot C$ (b) 0 (c) tan C (d) cot C **(code-V1T2PAQ2)**

Que. 2 Let $3^a=4, 4^b=5, 5^c=6, 6^d=7, 7^e=8$ and $8^f=9$. The value of product (abcdef), is

(a) 1 (b) 2 (c) $\sqrt{6}$ (d) 3 **(code-V1T2PAQ3)**

Que. 3. Which of the following numbers is the largest ?

(a) $\cos 15^o$ (b) $\tan 60^o$ (c) $\sec 15^o$ (d) $\operatorname{cosec} 15^o$ **(code-V1T2PAQ4)**

Que. 4. If $\alpha+\gamma=2\beta$ then the expression $\frac{\sin\alpha-\sin\gamma}{\cos\gamma-\cos\alpha}$ simplifies to **(code-V1T2PAQ6)**

(a) $\tan\beta$ (b) $-\tan\beta$ (c) $\cot\beta$ (d) $-\cot\beta$

Que. 5. If $A=110^o$ then $\frac{1+\sqrt{1+\tan^2 2A}}{\tan 2A}$ equals

(a) tan A (b) – tan B (c) cot A (d) – cot A **(code-V1T4PAQ2)**

Que. 6. Minimum value of $y=256\sin^2 x+324\operatorname{cosec}^2 x\ \forall\ x\in R$ is

(a) 432 (b) 504 (c) 580 (d) 776 **(code-V1T4PAQ4)**

Que. 7. If $A=320^o$ then $\frac{-1+\sqrt{1+\tan^2 A}}{\tan A}$ is equal to **(code-V1T5PAQ1)**

(a) $\tan\frac{A}{2}$ (b) $-\tan\frac{A}{2}$ (c) $\cot\frac{A}{2}$ (d) $-\cot\frac{A}{2}$

Que. 8. The product $\left(\cos\frac{x}{2}\right).\left(\cos\frac{x}{4}\right).\left(\cos\frac{x}{8}\right)......\left(\cos\frac{x}{256}\right)$ is equal to **(code-V1T5PAQ3)**

(a) $\frac{\sin x}{128\sin\frac{x}{256}}$ (b) $\frac{\sin x}{256\sin\frac{x}{256}}$ (c) $\frac{\sin x}{128\sin\frac{x}{128}}$ (d) $\frac{\sin x}{512\sin\frac{x}{512}}$

Que. 9. In a triangle $ABC\angle A=60^o, \angle B=40^o$ and $\angle C=80^o$. If P is the centre of the circumcircle of triangle ABC with radius unity, then the radius of the circumcircle of triangle BPC is **(code-V1T5PAQ5)**

(a) 1 (b) $\sqrt{3}$ (c) 2 (d) $\sqrt{3}/2$

Que. 10. In a triangle ABC if angle C is 90^o and area of triangle is 30, then the minimum possible value of the hypotenuse c is equal to **(code-V1T5PAQ9)**

(a) $30\sqrt{2}$ (b) $60\sqrt{2}$ (c) $120\sqrt{2}$ (d) $2\sqrt{30}$

Que. 11. Which one of the following relations does not hold good ? **(code-V1T5PAQ13)**

(a) $\sin\frac{\pi}{10}\sin\frac{13\pi}{10}=-\frac{1}{4}$ (b) $\sin\frac{\pi}{10}+\sin\frac{13\pi}{10}=-\frac{1}{2}$

(c) $4\left(\sin 24^o+\cos 6^o\right)=\sqrt{15}+\sqrt{3}$ (d) $\sin^2 72^o-\sin^2 60^o=\frac{\sqrt{5}-1}{4}$

Que.12. With usual notations, in a triangle ABC, $a\cos(B-C)+b\cos(C-A)+c\cos(A-B)$ is equal to

(a) $\frac{abc}{R^2}$ (b) $\frac{abc}{4R^2}$ (c) $\frac{4abc}{R^2}$ (d) $\frac{abc}{2R^2}$ **(code-V1T5PAQ15)**

Que. 13. General solution of the equation, $2\sin^2 x+\sin^2 2x=2$, is **(code-V1T5PAQ16)**

(a) $(2n+1)\frac{\pi}{4}$ (b) $n\pi\pm\frac{\pi}{4}$ (c) $n\pi\pm\frac{\pi}{2}$ (d) $n\pi\pm\frac{\pi}{4}\cup n\pi\pm\frac{\pi}{2}$

Que. 14. In a triangle ABC, $\angle A=60^o$ and $b:c=\sqrt{3}+1:2$ then $(\angle B-\angle C)$ has the value equal to

(a) 15^o (b) 30^o (c) 22.5^o (d) 45^O **(code-V1T5PAQ17)**

Que. 15. If the two roots of the equation, $x^3-px^2+qx-r=0$ are equal in magnitude but opposite sign then

(a) $pr=q$ (b) $qr=p$ (c) $pq=r$ (d) $pq+r=0$ **(code-V1T5PAQ18)**

Que. 16. The equation, $\sin^2\theta-\frac{4}{\sin^2\theta-1}=1-\frac{4}{\sin^3\theta-1}$ has **(code-V1T5PAQ20)**

(a) no root (b) one root (c) two roots (d) infinite roots

Que. 17. If a,b,c are the sides of a triangle then the expression $\frac{a^2+b^2+c^2}{ab+bc+ca}$ lies in the interval

(a) (1,2) (b) [1,2] (c) [1,2) (d) (1,2] **(code-V1T5PAQ23)**

Que. 18. If α and β are the roots of the equation $a\cos 2\theta+b\sin 2\theta=c$ then $\cos^2\alpha+\cos^2\beta$ is equal to

(a) $\frac{a^2+ac+b^2}{a^2+b^2}$ (b) $\frac{a^2-ac+b^2}{a^2+b^2}$ (c) $\frac{2b^2}{a^2+c^2}$ (d) $\frac{2a^2}{b^2+c^2}$ **(code-V1T5PAQ24)**

Que. 19. If $\tan\theta=\frac{b}{a}$, then the value of $a\cos 2\theta+b\sin 2\theta$ is equal to **(code-V1T7PAQ5)**

(a) a (b) b (c) ab^2 (d) a^2b

Que. 20. Let $f_k(x)=\frac{\sin^k x+\cos^k x}{k}$ then $f_4(x)-f_6(x)$ is equal to **(code-V1T7PAQ6)**

(a) $\frac{1}{4}-\frac{1}{6}\cos^2 2x$ (b) $\frac{1}{12}+\frac{1}{4}\sin^2 2x$ (c) $\frac{1}{3}-\frac{1}{4}\cos^2 x$ (d) $\frac{1}{12}$

Que. 21. In a triangle ABC, $\cos^2\frac{A}{2}+\cos^2\frac{B}{2}+\cos^2\frac{C}{2}$ equal **(code-V1T7PAQ7)**

(a) $2+\frac{r}{2R}$ (b) $4-\frac{7r}{2R}$ (c) $2+\frac{r}{4R}$ (d) $\frac{3}{4}\left(\frac{4r}{R}+1\right)$

where r and R have their usual meaning.

Que. 22. A cricle is inscribed in a triangle ABC touches the side AB at D such that AD = 5 and BD = 5 and BD = 3. If $\angle A=60^o$ then the length of BC equals [Advise - Que. of properties of triangle but tru by 2D.]

(a) 9 (b) $\frac{120}{13}$ (c) 12 (d) 13 **(code-V1T7PAQ8)**

Que. 23. If in a trianlge ABC, $\sin A, \sin B, \sin C$ are in A.P., then **(code-V1T7PAQ10)**

(a) The altitudes are A.P. (b) The altitudes are in H.P

(c) The medians are in G.P. (d) The medians are in A.P.

Que. 24. Which of the following inequalitie(s) hold(s) true in any triangle ABC ? **(code-V1T7PAQ13)**

(a) $\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} \le \frac{1}{8}$ (b) $\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2} \le \frac{3\sqrt{3}}{8}$

(c) $\sin^2\frac{A}{2}\sin^2\frac{B}{2}\sin^2\frac{C}{2} < \frac{3}{4}$ (d) $\cos^2\frac{A}{2}+\cos^2\frac{B}{2}+\cos^2\frac{C}{2} \le \frac{9}{4}$

Que. 25. If $\alpha = \frac{\pi}{7}$ which of the following hold(s) good ? **(code-V1T7PAQ14)**

(a) $\tan\alpha.\tan 2\alpha.\tan 3\alpha = \tan 3\alpha - \tan 2\alpha - \tan\alpha$

(b) $\operatorname{cosec}\alpha = \operatorname{cosec} 2\alpha + \operatorname{cosec} 4\alpha$

(c) $\cos\alpha - \cos 2\alpha + \cos 3\alpha$ has the value equal to 1/2

(d) $8\cos\alpha.\cos 2\alpha.\cos 4\alpha$ has the value equal to 1.

Que. 26. Identify which of the following are correct ? **(code-V1T7PAQ16)**

(a) $(\tan x)^{\ell n(\sin x)} > (\cot x)^{\ell n(\sin x)}, \forall\ x \in \left(0, \frac{\pi}{4}\right)$

(b) $4^{\ell n \operatorname{cosec} x} < 5^{\ell n \operatorname{cosec} x}, \forall\ x \in \left(0, \frac{\pi}{2}\right)$

(c) $\left(\frac{1}{2}\right)^{\ell n(\cos x)} < \left(\frac{1}{3}\right)^{\ell n(\cos x)}, \forall\ x \in \left(0, \frac{\pi}{2}\right)$

(d) $2^{\ell n(\tan x)} < 2^{\ell n(\sin x)}, \forall\ x \in \left(0, \frac{\pi}{2}\right)$

Que. 27. Which of the following is always equal to $\cos^2 A - \sin^2 A$? **(code-V1T10PAQ4)**

(a) $\sin 2A$

(b) $\cos(A+B)\cos(A-B) - \sin(A+B)\sin(A-B)$

(c) $\sin(A+B)\cos(A-B) - \cos(A+B)\sin(A-B)$

(d) $\cos(A+B)\sin(A-B) - \sin(A+B)\cos(A-B)$

Que. 28. Number of values of $x \in [0, \pi]$ satisfying $\cos^2 5x - \cos^2 x + \sin 4x.\sin 6x = 0$, is **(code-V1T10PAQ5)**

(a) 2 (b) 3 (c) 5 (d) infinitely many

Que. 29. $\frac{\sin\alpha + \sin\beta + \sin(\alpha+\beta)}{\sin\alpha + \sin\beta - \sin(\alpha+\beta)}$ (wherever difined) simplifies to **(code-V1T10PAQ6)**

(a) $\cot\frac{\alpha}{2}\cot\frac{\beta}{2}$ (b) $\cot\frac{\alpha}{2}\tan\frac{\beta}{2}$ (c) $\tan\frac{\alpha}{2}\cot\frac{\beta}{2}$ (d) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2}$

Que. 30. Let A, B, C be three angles such that $A+B+C=\pi$. If $\tan A.\tan B=\csc\frac{\pi}{6}$ then the vlaue of $\frac{\cos A\cos B}{\cos C}$ is equal to **(code-V1T12PAQ5)**

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{3}$ (c) 1 (d) $\frac{1}{2}$

Que. 31. Which value of θ listed below leads to $2^{\sin\theta}>1$ and $3^{\cos\theta}<1$? **(code-V1T12PAQ7)**

(a) 70^o (b) 140^o (c) 210^o (d) 280^o

Que.32. In a trinalge ABC, a = 3, b = 4 and c = 5. The value of $\sin A+\sin 2B+\sin 3C$ equals

(a) $\frac{24}{25}$ (b) $\frac{14}{25}$ (c) $\frac{64}{25}$ (d) None. **(code-V1T13PAQ1)**

Que.33. Let $y=(\sin x+\operatorname{cosec} x)^2+(\cos x+\sec x)^2$, then the minimum value of y, $\forall\ x\in R$ is **(code-V1T13PAQ4)**

(a) 7 (b) 8 (c) 9 (d) 10

Que.34. If the equation $\cot^4 x-2\operatorname{cosec}^2 x+a^2=0$ has atleast one solution then, sum of all possible integral values of 'a' is equal to **(code-V1T13PAQ5)**

(a) 4 (b) 3 (c) 2 (d) 0

Que.35. If θ be an acute angle satistying the equation $8\cos 2\theta+8\sec 2\theta=65$, then value of $\cos\theta$ is equal to

(a) $\frac{1}{8}$ (b) $\frac{2}{3}$ (c) $\frac{2\sqrt{3}}{3}$ (d) $\frac{3}{4}$ **(code-V1T13PAQ7)**

Que.36. If $2\sin x+7\cos px=9$ has atleast one solution then p must be **(code-V1T13PAQ8)**

(a) an odd integer (b) an even integer
(c) a rational number (d) an irrational number

Que.37. If $\theta\in(\pi/4,\pi/2)$ and $\sum_{n=1}^{\infty}\frac{1}{\tan^n\theta}=\sin\theta+\cos\theta$ then the value of $\tan\theta$ is **(code-V1T13PAQ10)**

(a) $\sqrt{3}$ (b) $\sqrt{2}+1$ (c) $2+\sqrt{3}$ (d) $\sqrt{2}$

Que.38. If $\sin x+a\cos x=b$ then the value of $|a\sin x-\cos x|$ is equal to **(code-V1T13PAQ11)**

(a) $\sqrt{a^2+b^2+1}$ (b) $\sqrt{a^2+b^2-1}$ (c) $\sqrt{a^2-b^2-1}$ (d) $\sqrt{a^2-b^2+1}$

Que.39. The least value of x for $0<x<\pi/2$, such that $\cos(2x)=\sqrt{3}\sin(2x)$, is **(code-V1T13PAQ14)**

(a) $\frac{\pi}{12}$ (b) $\frac{2\pi}{12}$ (c) $\frac{3\pi}{12}$ (d) $\frac{4\pi}{12}$

Que.40. Let $\theta\in[0,4\pi]$ satisfying the equation $(\sin\theta+2)(\sin\theta+3)(\sin\theta+4)=6$. if the sum of all value of θ is of the form $k\pi$ then the value of 'k', is **(code-V1T13PAQ18)**

(a) 6 (b) 5 (c) 4 (d) 2

Que.41. Let $f(x)=a\sin x+c$, where a and c are real numbers and $a>0$. Then $f(x)<0\ \forall\ x\in R$ if

(a) $c<-a$ (b) $c>-a$ (c) $-a<c<a$ (d) $c<a$ **(code-V1T13PAQ19)**

Que.42. In which one of the following intervals the inequality, $\sin x < \cos x < \tan x < \cot x$ can hold good ?

(a) $\left(0, \frac{\pi}{4}\right)$ (b) $\left(\frac{3\pi}{4}, \pi\right)$ (c) $\left(\frac{5\pi}{4}, \frac{3\pi}{2}\right)$ (d) $\left(\frac{7\pi}{4}, 2\pi\right)$ **(code-V1T13PAQ21)**

Que.43. If $\sin x + \sin y + \sin z = 0 = \cos x + \cos y + \cos z$ then the expression, $\cos(\theta - x) + \cos(\theta - y)$ $+ \cos(\theta - z)$, for $\theta = \in R$ is **(code-V1T15PAQ1)**

(a) independent of θ but dependent on x, y, z

(b) dependent on θ but independent of x, y, z

(c) dependent on x, y, z and θ

(d) independent of x, y, z and θ

Que.44. In $\Delta ABC, AB = 1, BC = 1$ and $AC = 1/\sqrt{2}$. In $\Delta MNP, MN = 1$, and $\angle MNP - 2\angle ABC$. The side MP equals

(a) $3\sqrt{2}$ (b) $7/4$ (c) $2\sqrt{2}$ (d) $\sqrt{7}/2$ **(code-V1T15PAQ5)**

Que.45. The sum $\sum_{n=1}^{9} \sin^2 \frac{n\pi}{18}$ equals **(code-V1T15PAQ6)**

(a) 5 (b) 4 (c) $\left(\sqrt{5} + 1\right)$ (d) $3\sqrt{5}$

Que.46. Let area of a triangle ABC is $\frac{\sqrt{3} - 1}{2}$, $b = 2$ and $c = \left(\sqrt{3} - 1\right)$ and $\angle A$ is acute. The measure of the angle C is **(code-V1T15PAQ10)**

(a) 15^o (b) 30^o (c) 60^o (d) 75^o

Que.47. If the inequality $\sin^2 x + a \cos x + a^2 > 1 + \cos x$ holds for any $x \in R$ then the largest negative intergral value of 'a' is **(code-V1T18PAQ2)**

(a) – 4 (b) – 3 (c) – 2 (d) – 1

Que. 48. The value of the expression $\sin\left(\frac{a}{2}\right) \cdot \left(\frac{1}{2} + \sum_{k=1}^{n} \cos(ka)\right)$, is **(code-V1T18PAQ3)**

(a) $\frac{1}{2}\sin\left(\left(n + \frac{1}{2}\right)a\right)$ (b) $\frac{1}{2}\sin\left(\left(n - \frac{1}{2}\right)a\right)$

(c) $\sin\left((n+1)a\right)$ (b) $\cos^n(a)$

Que. 49. Suppose ABC is a triangle with 3 acute angle A,B and C. The point whose coordinates are $(\cos B - \sin A, \sin B - \cos A)$ can be in the **(code-V1T18PAQ4)**

(a) first and 2nd quadrant (b) second the 3rd quadrant

(c) third and 4th quadrant (d) second quadrant only

Que. 50. Number of solution of the equation, $\sin^4 x - \cos^2 x \sin x + 2\sin^2 x + \sin x = 0$ is $0 \le x \le 3\pi$, is

(a) 3 (b) 4 (c) 5 (d) 6 **(code-V1T19PAQ3)**

Que. 51. Let α, β and γ be the angles of a triangle with $0<\alpha<\frac{\pi}{2}$ and $0<\beta<\frac{\pi}{2}$, satisfying $\sin^2\alpha+\sin^2\beta=\sin\gamma$, then **(code-V1T20PAQ3)**

(a) $\gamma\in(0,30^o)$ (b) $\gamma\in(30^o,60^o)$ (c) $\gamma\in(60^o,90^o)$ (d) $\gamma=90^o$

Que. 52. Let L and M be the respective intersections of the internal and external angle bisectors of the triangle ABC at C and the side AB produced. If CL = CM, then the value of (a^2+b^2) is (where a & b have their usual meanings) **(code-V2T1PAQ4)**

(a) $2R^2$ (b) $2\sqrt{2}\,R^2$ (c) $4R^2$ (d) $4\sqrt{2}\,R^2$

Que. 53. In a triangle ABC, if $A-B=120^o$ and $R=8r$ where R and r have their usual meaning then cos C equals

(a) 3/4 (b) 2/3 (c) 5/6 (d) 7/8 **(code-V2T3PAQ9)**

Que. 54. The system of equations **(code-V2T5PAQ13)**

$$x-y\cos\theta+z\cos 2\theta=0$$
$$-x\cos\theta+y-z\cos\theta=0$$
$$x\cos 2\theta-y\cos\theta+z=0$$

has non trivial solution for θ equals

(a) $n\pi$ only, $n\in I$. (b) $n\pi+\frac{\pi}{4}$ only, $n\in I$.

(c) $(2n-1)\frac{\pi}{2}$ only, $n\in I$. (d) all value of θ

Que. 55. If $\tan\left(\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{1}{5}\right)$ is expressed as a rational $\frac{a}{b}$ in lowest form $(a+b)$ has the value equal to **(code-V2T8PAQ1)**

(a) 19 (b) 27 (c) 38 (d) 45

Que. 56. A sector OABO of central angle θ is constructed in a circle with centre O and of radius 6. The radius of the circle that is circumscribed about the triangle OAB, is **(code-V2T8PAQ7)**

(a) $6\cos\frac{\theta}{2}$ (b) $6\sec\frac{\theta}{2}$ (c) $3\left(\cos\frac{\theta}{2}+2\right)$ (d) $3\sec\frac{\theta}{2}$

Que. 57. Let s, r, R respectively specify the semiperimeter, inradius and circumradius of a triangle ABC. Then $(ab+bc+ca)$ in terms of s, r and R given by **(code-V2T8PAQ8)**

(a) $sr+rR+Rs$ (b) R^2+r^2+rs (c) s^2+R^2+2rs (d) r^2+s^2+4Rr

Que. 58. The value of the expression $(1+\tan A)(1+\tan B)$ when $A=20^o$ and $B=25^o$ reduces to

(a) prime number **(b)** composite number **(code-V2T13PAQ5)**

(c) irrational number **(d)** rational which is not an integer.

Que. 59. The least value of the expression $\dfrac{\cot 2x - \tan 2x}{1+\sin\left(\dfrac{5\pi}{2}-8x\right)}$ in $\left(0, \dfrac{\pi}{8}\right)$ equals **(code-V2T13PAQ11)**

(a) 1 (b) 2 (c) 4 (d) none.

Que. 60. The value of x satisfying the equation $\sin\left(\tan^{-1} x\right) = \cos\left(\cos^{-1}(x+1)\right)$ is **(code-V2T13PAQ22)**

(a) $\dfrac{1}{2}$ (b) $-\dfrac{1}{2}$ (c) $\sqrt{2}-1$ (d) No finite value

Que. 61. Which one of the following quantities is negative ? **(code-V2T13PAQ23)**

(a) $\cos\left(\tan^{-1}(\tan 4)\right)$ (b) $\sin\left(\cot^{-1}(\cot 4)\right)$ (c) $\tan\left(\cos^{-1}(\cos 5)\right)$ (d) $\cot\left(\sin^{-1}(\sin 4)\right)$

Que. 62. The product of all real values of x satisfying the equation **(code-V2T13PAQ29)**

$$\sin^{-1}\cos\left(\frac{2x^2+10|x|+4}{x^2+5|x|+3}\right) = \cot\left(\cot^{-1}\left(\frac{2-81|x|}{9|x|}\right)\right)+\frac{\pi}{2} \text{ is}$$

(a) 9 (b) – 9 (c) – 3 (d) – 1

Que. 63. Product of all the solution of the equation $\tan^{-1}\left(\dfrac{2x}{x^2-1}\right)+\cot^{-1}\left(\dfrac{x^2-1}{2x}\right)=\dfrac{2\pi}{3}$, is **(code-V2T14PAQ4)**

(a) 1 (b) –1 (c) 3 (d) $-\sqrt{3}$

Que. 64. If the value of $\tan\left(37.5^o\right)$ can be expressed as $\sqrt{a}-\sqrt{b}+\sqrt{c}-\sqrt{d}$ where $a, b, c, d \in N$ and $(a > b > c > d)$ then the value of $\dfrac{ad}{bc}$ is equal to **(code-V2T14PAQ13)**

(a) 1 (b) 2 (c) 3 (d) 4

Que. 65. If the minimum value of expression $y = (27)^{\cos x} + (81)^{\sin x}$ can be expressed in the form $\sqrt{a/b}$ where $a, b \in N$ and are in their lowest term then the value of $(a+b)$ equals **(code-V2T17PAQ10)**

Que. 66. Let $f(x) = \sin x + \cos x + \tan x + \text{arc}\sin x + \text{arc}\cos x + \text{arc}\tan x$. If M and m are maximum and minimum values of f(x) then their arithmetic mean is equal to **(code-V2T18PAQ3)**

(a) $\dfrac{\pi}{2}+\cos 1$ (b) $\dfrac{\pi}{2}+\sin 1$ (c) $\dfrac{\pi}{4}+\tan 1+\cos 1$ (d) $\dfrac{\pi}{4}+\tan 1+\sin 1$

Que. 67. The sum $\displaystyle\sum_{r=1}^{2009}\cos\left(\frac{r\pi}{6}\right)$ equals **(code-V2T19PAQ1)**

(a) 0 (b) 1 (c) $\dfrac{\sqrt{3}+1}{2\sqrt{2}}$ (d) $\dfrac{\sqrt{2}+\sqrt{3}}{2}$

Que. 68. $\left(\tan\dfrac{3\pi}{8}\right)^{2009}+\left(-\cot\dfrac{3\pi}{8}\right)^{2009}$ is **(code-V2T19PAQ3)**

(a) even integer (b) odd integer
(c) rational which is not an integer (d) irraional

Comprehesion Type

1 Paragraph for Q. 1 to Q. 3

Let r and R denote the radii of the incircle and the circumcircle of the triangle ABC with sides a, b, c and $a+b+c=2s$. Also Δ denotes the area of the trianlge. **(code-V1T16PAQ7,8,9)**

1. The value of the product $\prod_{\substack{\text{cyclic}\\ ABC}} \cot\frac{A}{2}$ equals

(a) $\frac{r^2}{\Delta}$ (b) $\frac{\Delta}{s^2}$ (c) $\frac{R(a+b+c)^2}{abc}$ (d) $\frac{r}{s}$

2. The vlaue of the sum $\prod_{\substack{\text{cyclic}\\ A,B,C}} \cot\frac{A}{2}\cot\frac{B}{2}$ equals

(a) $\frac{R+4r}{r}$ (b) $\frac{4R+r}{r}$ (c) $\frac{4R+r}{R}$ (d) $\frac{4R}{r}$

3. Let $f(x)=0$ denotes a cubic whose roots are $\cot\frac{A}{2},\cot\frac{B}{2},\cot\frac{C}{2}$. If the triangle ABC is such that one of its anlge is 90° then which one of the following holds good ?

(a) $r+2R=s$ (b) $3r+2R=s+2$ (c) $1+r+4R=2s$ (d) $4r+R=s$

2 Paragraph for Q. 4 to Q. 6

Let ABC be an acute triangle with orthocenter H.D,E,F are the feet of the perpendiculars from A,B,and C on the opposite sides. Also R is the circumradius of the triangle ABC. **(code-V1T18PAQ6,7,8)**

Given $(AH)(BH)(CH)=3$ and $(AH)^2+(BH)^2+(CH)^2=7$

4. The ratio $\frac{\prod \cos A}{\sum \cos^2 A}$ has the value equal to

(a) $\frac{3}{14R}$ (b) $\frac{3}{7R}$ (c) $\frac{7}{3R}$ (d) $\frac{14}{3R}$

5. The product $(HD)(HE)(HF)$ has the value equal to

(a) $\frac{9}{64R^3}$ (b) $\frac{9}{8R^3}$ (c) $\frac{8}{9R^3}$ (d) $\frac{9}{32R^3}$

6. The value of R is

(a) 1 (b) $3^{1/3}$ (c) $\frac{3}{2}$ (d) $\sqrt{\frac{5}{2}}$

Assertion & Reason Type

In this section each que. contains STATEMENT-1 (Assertion) & STATEMENT-2(Reason).Each question has 4 choices (A), (B), (C) and (D), out of which **only one is correct.**

Bubble (A) STATEMENT-1 is true, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1.

Bubble (B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1.

Bubble (C) STATEMENT-1 is True, STATEMENT-2 is False.

Bubble (D) STATEMENT-1 is False, STATEMENT-2 is True.

Que. 1. Statement - 1 : **(code-V1T4PAQ8)**

In a triangle ABC if A is obtuse then tan B tan C > 1

because

Statement - 2 :

In any triangle ABC, $\tan A = \frac{\tan B + \tan C}{\tan B \tan C - 1}$

Que. 2. Statement - 1 : **(code-V1T4PAQ9)**

$\cos(10)^c$ and $\cos(-10)^c$ both are negative and have the same value.

because

Statement - 2 :

$\cos\theta = \cos(-\theta)$ and the real numbers $(10)^c$ and $(-10)^c$ both lie in the third quadrant.

Que. 3. Let α, β and γ satisfy $0 < \alpha < \beta < \gamma < 2\pi$ and $\cos(x+\alpha) + \cos(x+\beta) + \cos(x+\gamma) = 0\ \forall x \in R$

Statement - 1 : $\gamma - \alpha = \frac{2\pi}{3}$. **(code-V1T6PAQ1)**

because

Statement - 2 : $\cos\alpha + \cos\beta + \cos\gamma = 0$ and $\sin\alpha + \sin\beta + \sin\gamma = 0$.

Que. 4. If $A + B + C = \pi$ then **(code-V1T6PAQ3)**

Statement - 1 : $\cos^2 A + \cos^2 B + \cos^2 C$ has its minimum value $\frac{3}{4}$.

because

Statement - 2 : Maximum value of $\cos A \cos B \cos C = \frac{1}{8}$.

Que. 5. Let $f(x) = 3\sin^2 x + 4\sin x \cos x + 4\cos^2 x, x \in R$. **(code-V1T8PAQ11)**

Statement - 1 : Greatest and least values of $f(x)\ \forall\ x \in R$ are $\frac{7+\sqrt{17}}{2}$ and $\frac{7-\sqrt{17}}{2}$ respectively.

because

Statemtnt 2 : $\frac{a+b-\sqrt{a^2+b^2+c^2-2ab}}{2} \le a\sin^2 x + b\sin x\cos x + c\cos^2 x \le \frac{a+b+\sqrt{a^2+b^2+c^2-2ab}}{2}$

where $a, b, c \in R$

Que. 6. Statement - 1 : $\tan\frac{6\pi}{7}-\tan\frac{5\pi}{7}-\tan\frac{\pi}{7}=\tan\frac{6\pi}{7}.\tan\frac{5\pi}{7}.\tan\frac{\pi}{7}$ **(code-V1T10PAQ7)**

because

Statement - 2 : If $\theta=\alpha+\beta$, then $\tan\theta-\tan\alpha-\tan\beta=\tan\theta.\tan\alpha.\tan\beta$.

Que. 7. Consider the following statements **(code-V1T12PAQ8)**

Statement - 1 : In any right angled triangle ABC, $\sin^2 A+\sin^2 B+\sin^2 C=2$

because

Statement - 2 : In any triangle ABC, $\sin^2 A+\sin^2 B+\sin^2 C=2-2\cos A\cos B\cos C$]

Que. 8. Statement-1: In any triangle ABC $\ell n\left(\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}\right)$ $=\ell n\cot\frac{A}{2}+\ell n\cot\frac{B}{2}+\ell n\cot\frac{C}{2}$

because **(code-V1T14PAQ5)**

Statement - 2 : $\ell n\left(1+\sqrt{3}+\left(2+\sqrt{3}\right)\right)=\ell n 1+\ell n\sqrt{3}+\ell n\left(2+\sqrt{3}\right)$

Que. 9. Statement - 1 : General solution of $\frac{\tan 4x-\tan 2x}{1+\tan 4x\tan 2x}=1$ is $x=\frac{n\pi}{2}+\frac{\pi}{8}, n\in I$

because **(code-V1T18PAQ11)**

Statement - 2 : General solution of $\tan\alpha=1$ is $\alpha=n\pi+\frac{\pi}{4}, n\in I$.

Que. 10. Statement - 1 : The equation $\sin(\cos x)=\cos(\sin x)$ has no real solution **(code-V1T19PAQ10)**

because

Statement - 2 : $\sin x\pm\cos x$ is bounded in $\left[-\sqrt{2},\sqrt{2}\right]$

Que. 11. Statement 1: In any triangle ABC, $\cot A+\cot B+\cot C>0$ **(code-V2T7PAQ10)**

because

Statement 2: Minimum value of $\cot A+\cot B+\cot C$ in any triangle ABC is 1.

Que. 12. Let P be the point lying inside the acute triangle ABC such that angles sutended by each side at P is 120°. Equilateral triangles AFB, BDC,CEA are constructed outwardly on sides AB, BC, CA of ΔABC

Statement 1: Lines AD, BE, CF are concurrent at P. **(code-V2T18PAQ8)**

because

Statement 2: P is the radical centre of circumcircles of triangles ABF, BDE, CEA

Que. 13. Let ABC be an acute triangle whose orthocentre is at H. Altitude from A is produced to meet the circumcircle of the triangle ABC at D. **(code-V2T19PAQ7)**

Statement 1: The distange HD = 4R $\cos B\cos C$ where R is the circumradius of the triangle ABC.

because

Statement 2: Image of orthocentre H in any side of an acute triangle lies on its circumcircle.

More than One May Correct Type

Que. 1. If $2\cos\theta + 2\sqrt{2} = 3\sec\theta$ where $\theta \in (0, 2\pi)$ then which of the following can be correct ?

(a) $\cos\theta = \frac{1}{\sqrt{2}}$ (b) $\tan\theta = 1$ (c) $\sin\theta = -\frac{1}{\sqrt{2}}$ (d) $\cot\theta = -1$ **(code-V1T2PAQ9)**

Que. 2 Which of the following real numbers when simplified are neither terminating nor repeating decimal ?

(a) $\sin 75^\circ . \cos 75^\circ$ (b) $\log_2 28$ (c) $\log_3 5 . \log_5 6$ (d) $8^{-(\log_{27} 3)}$ **(code-V1T2PAQ10)**

Que. 3. Suppose ABCD (in order) is a quadrilateral inscribed in a circle. Which of the following is/are always True ? **(code-V1T2PAQ11)**

(a) $\sec B = \sec D$ (b) $\cot A + \cos C = 0$

(c) $\text{cosec} A = \text{cosec} C$ (d) $\tan B + \tan D = 0$

Que. 4. which of the following quantities are rational ? **(code-V1T4PAQ15)**

(a) $\sin\left(\frac{11\pi}{12}\right)\sin\left(\frac{5\pi}{12}\right)$ (b) $\text{cosec}\left(\frac{9\pi}{10}\right)\sec\left(\frac{4\pi}{5}\right)$

(c) $\sin^4\left(\frac{\pi}{8}\right) + \cos^4\left(\frac{\pi}{8}\right)$ (d) $\left(1+\cos\frac{2\pi}{9}\right)\left(1+\cos\frac{4\pi}{9}\right)\left(1+\cos\frac{8\pi}{9}\right)$

Que. 5. In a triangle ABC, a semicircle is inscribed, whose diameter lies on the side c. If x is the length of the angle bisector through angle C then the radius of the semicircle is **(code-V1T6PAQ6)**

(a) $\frac{abc}{4R^2(\sin A + \sin B)}$ (b) $\frac{\Delta}{x}$

(c) $x \sin\frac{C}{2}$ (d) $\frac{2\sqrt{s(s-a)(s-b)(s-c)}}{s}$

Que. 6. The possible value(s) of x satisfying the equation $\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$, is/are. **(code-V1T6PAQ9)**

(a) $-\frac{7\pi}{8}$ (b) $-\frac{\pi}{8}$ (c) $\frac{\pi}{8}$ (d) $\frac{\pi}{4}$

Que. 7. In which of the following sets the inequality $\sin^6 x + \cos^6 x > \frac{5}{8}$ holds good ? **(code-V1T6PAQ12)**

(a) $\left(-\frac{\pi}{8}, \frac{\pi}{8}\right)$ (b) $\left(\frac{3\pi}{8}, \frac{5\pi}{8}\right)$ (c) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ (d) $\left(\frac{7\pi}{8}, \frac{9\pi}{8}\right)$

Que. 8. The value of x satisfying the equation $\cos(\ell n\, x) = 0$, is **(code-V1T10PAQ9)**

(a) $e^{\pi/2}$ (b) $e^{-\pi/2}$ (c) e^{π} (d) $e^{3\pi/2}$

Que. 9. Which of the following do/does not reduce to unity ? **(code-V1T10PAQ11)**

(a) $\dfrac{\sin(180^\circ+A)}{\tan(180^\circ+A)}.\dfrac{\cot(90^\circ+A)}{\tan(90^\circ+A)}.\dfrac{\cos(360^\circ-A)\operatorname{cosec}A}{\sin(-A)}$

(b) $\dfrac{\sin(-A)}{\tan(180^\circ+A)}-\dfrac{\tan(90^\circ+A)}{\cot A}+\dfrac{\cos A}{\sin(90^\circ+A)}$

(c) $\dfrac{\sin 24^\circ\cos 6^\circ-\sin 6^\circ\sin 66^\circ}{\sin 21^\circ\cos 39^\circ-\cos 51^\circ\sin 69^\circ}$

(d) $\dfrac{\cos(90^\circ+A)\sec(-A)\tan(180^\circ-A)}{\sec(360^\circ+A)\sin(180^\circ+A)\cot(90^\circ-A)}$

Que. 10. Which of the following identities wherever difined hold(s) good ? **(code-V1T10PAQ12)**

(a) $\cot\alpha-\tan\alpha=2\cot 2\alpha$ (b) $\tan(45^\circ+\alpha)-\tan(45^\circ-\alpha)=2\operatorname{cosec}2\alpha$

(c) $\tan(45^\circ+\alpha)+\tan(45^\circ-\alpha)=2\sec 2\alpha$ (d) $\tan\alpha+\cot\alpha=2\tan 2\alpha.$

Que. 11. Let α,β and γ are some angles in the 1^{st} quadrant satisfying $\tan(\alpha+\beta)=\dfrac{15}{8}$ and $\operatorname{cosec}\gamma=\dfrac{17}{8}$ then which of the following holds good ? **(code-V1T14PAQ7)**

(a) $\alpha+\beta+\gamma=\pi$

(b) $\cot\alpha.\cot\beta.\cot\gamma=\cot\alpha+\cot\beta+\cot\gamma$

(c) $\tan\alpha+\tan\beta+\tan\gamma=\tan\alpha.\tan\beta.\tan\gamma$

(d) $\tan\alpha.\tan\beta+\tan\beta.\tan\gamma+\tan\gamma.\tan\alpha=1$

Que. 12. Which of the following statements are always correct ? (where Q denotes the set of rationals)

(a) $\cos 2\theta\in Q$ and $\sin 2\theta\in Q \Rightarrow \tan\theta\in Q$ (in difinde) **(code-V1T14PAQ8)**

(b) $\tan\theta\in Q \Rightarrow \sin 2\theta, \cos 2\theta$ and $\tan 2\theta\in Q$ (if defined)

(c) If $\sin\theta\in Q$ and $\cos\theta\in Q \Rightarrow \tan 3\theta\in Q$ (if defined)

(d) if $\sin\theta\in Q \Rightarrow \cos 3\theta\in Q$

Que. 13. Given that $\sin 3\theta=\sin 3\alpha$, then which of the following angles will be equal to $\cos\theta$?

(a) $\cos\left(\dfrac{\pi}{3}+\alpha\right)$ (b) $\cos\left(\dfrac{\pi}{3}-\alpha\right)$ (c) $\cos\left(\dfrac{2\pi}{3}+\alpha\right)$ (d) $\cos\left(\dfrac{2\pi}{3}-\alpha\right)$ **(code-V1T14PAQ9)**

Que. 14. If the quadratic equation $(\operatorname{cosec}^2\theta-4)x^2+(\cot\theta+\sqrt{3})x+\cos^2\dfrac{3\pi}{2}=0$ holds true for all real x then the most general values of θ can be given by **(code-V1T15PAQ12)**

(a) $2n\pi+\dfrac{11\pi}{6}$ (b) $2n\pi+\dfrac{5\pi}{6}$ (c) $2n\pi\pm\dfrac{7\pi}{6}$ (d) $n\pi\pm\dfrac{11\pi}{6}$

Que. 15. If α and β are two different solution of $a\cos\theta+b\sin\theta=c$, then which of the following hold(s) good ? **(code-V1T15PAQ13)**

(a) $\sin\alpha+\sin\beta=\dfrac{2bc}{a^2+b^2}$ (b) $\sin\alpha\sin\beta=\dfrac{c^2-a^2}{a^2+b^2}$

(c) $\cos\alpha\cos\beta=\dfrac{2ac}{a^2+b^2}$ (d) $\cos\alpha\cos\beta=\dfrac{c^2+a^2}{a^2+b^2}$

Que. 16. If in a triangle ABC, $\cos 3A+\cos 3B+\cos 3C=1$, then **(code-V1T15PAQ14)**

(a) ΔABC is a right angled triangle.

(b) ΔABC is an obtuse angle triangle

(c) ΔABC is an acute angled triangle

(d) Inradius 'r' of the triangle ABC is either $\sqrt{3}(s-a)$ or $\sqrt{3}(s-b)$ or $\sqrt{3}(s-c)$.

Que. 17. The value of x in $(0,\pi/2)$ satisfying the equation, $\dfrac{\sqrt{3}-1}{\sin x}+\dfrac{\sqrt{3}+1}{\cos x}=4\sqrt{2}$ is **(code-V2T2PAQ11)**

(a) $\dfrac{\pi}{12}$ (b) $\dfrac{5\pi}{12}$ (c) $\dfrac{7\pi}{24}$ (d) $\dfrac{11\pi}{36}$

Que. 18. In a triangle ABC, $3\sin A+4\cos B=6$ and $3\cos A+4\sin B=1$ then $\angle C$ can be **(code-V2T8PAQ12)**

(a) 30^o (b) 60^o (c) 90^o (d) 150^o

Que. 19. If $\cos 3\theta=\cos 3\alpha$ then the value of $\sin\theta$ can be given by **(code-V2T15PAQ9)**

(a) $\pm\sin\alpha$ (b) $\sin\left(\dfrac{\pi}{3}\pm\alpha\right)$ (c) $\sin\left(\dfrac{2\pi}{3}+\alpha\right)$ (d) $\sin\left(\dfrac{2\pi}{3}-\alpha\right)$

Que. 20. Let ABC be a triangle with AA_1, BB_1, CC_1 as their medians and G be the centriod. If the points A, C_1, G, B_1 be concyclic then which one of the following relations do/does not hold good ? **(code-V2T19PAQ10)**

(a) $2a^2=b^2+c^2$ (b) $3b^2+c^2+a^2$ (c) $4c^2+a^2+b^2$ (d) $3a^2+b^2+c^2$

Match Matrix Type

Que. 1. Match the general solution of the trigonometric equation given in **Column - I** with their corresponding entries given **Column - II.** **(code-V1T14PBQ1)**

	Column - I		**Column - II**
A.	$\cos^2 2x+\cos^2 x=1$	**P.**	$x=n\pi+\dfrac{\pi}{4}\cup n\pi+\dfrac{\pi}{6}, n\in I.$
B.	$\cos x=\sqrt{3}(1-\sin x)$	**Q.**	$x=\dfrac{n\pi}{3}, n\in I$
C.	$1+\sqrt{3}\tan^2 x=(1+\sqrt{3})\tan x$	**R.**	$x=(2n-1)\dfrac{\pi}{6}, n\in I.$
D.	$\tan 3x-\tan 2x-\tan x=0$	**S.**	$x=2n\pi+\dfrac{\pi}{2}\cup 2n\pi+\dfrac{\pi}{6}, n\in I.$

Subjective Type (Up to 4 digit)

Que. 1. The expression $2\cos\frac{\pi}{17}.\cos\frac{9\pi}{17}+\cos\frac{7\pi}{17}+\cos\frac{9\pi}{17}$ simplifies to an integer P. Find the value of P. **(code-V1T1PAQ1)**

Que. 2. Show that the expression $\frac{\sin(\alpha+\beta)-2\sin\alpha+\sin(\alpha-\beta)}{\cos(\alpha+\beta)-2\cos\alpha+\cos(\alpha-\beta)}$ is independent of β. **(code-V1T1PAQ2)**

Que. 3. If the expression $\frac{\sin\theta.\sin 2\theta+\sin 3\theta\sin 6\theta+\sin 4\theta\sin 13\theta}{\sin\theta\cos 2\theta+\sin 3\theta\cos 6\theta+\sin 4\theta\cos 13\theta}=\tan k\theta$, where $k\in N$. Find the value of k. **(code-V1T1PAQ4)**

Que. 4. If $y=\cos^8\frac{x}{2}-\sin^8\frac{x}{2}$. Find the value of y when $x=\frac{\pi}{4}$ and also when $x=\frac{\pi}{6}$. **(code-V1T3PAQ4)**

Que. 5. In a trianlge ABC, given $\sin A:\sin B:\sin C=4:5:6$ and $\cos A:\cos B:\cos C=x:y:z$. If the ordered pair (x, y) satisfies this, then compute the value of $\left(x^2+y^2+z^2\right)$ where $x, y, z\in N$ and are in their lowest form. **(code-V1T7PBQ2)**

Que. 6. Let $A=\cos 360^o.\sin^2 270^o-2\cos 180^o.\tan 225^o$, $B=3\sin 540^o.\sec 720^o+2\operatorname{cosec}450^o-\cos 3600^o$

$C=2\sec^2 2\pi.\cos 0^c+3\sin^3\frac{3\pi}{2}-\operatorname{cosec}\frac{5\pi}{2}$ and $D=\tan\pi.\cos\frac{3\pi}{2}+\sec 2\pi-\operatorname{cosec}\frac{3\pi}{2}$. Find the value of $A+B-C\div D$. **(code-V1T9PAQ1)**

Que. 7. If the value of the expression $E=\cos^4 x-k^2\cos^2 2x+\sin^4 x$, is independent of x then find the set of values of k. **(code-V1T9PAQ3)**

Que. 8. If $\cot\frac{\pi}{24}=\sqrt{p}+\sqrt{q}+\sqrt{r}+\sqrt{s}$ where $p, q, r, s\in N$, find the value of $(p+q+r+s)$. **(code-V1T9PAQ5)**

Que. 9. Let L denotes the value of the expression, $\frac{\sin 2\theta-\sin 6\theta+\cos 2\theta-\cos 6\theta}{\sin 4\theta-\cos 4\theta}$, when $\theta=27^o$

and M denotes the value of $\frac{\tan x\tan 2x}{\tan 2x-\tan x}$ when $x=9^o$. **(code-V1T9PAQ7)**

and N denotes the numerical value of the expression (wherever difined) $\frac{1-\cos 4\alpha}{\sec^2 2\alpha-1}+\frac{1+\cos 4\alpha}{\operatorname{cosec}^2 2\alpha-1}$ when it is simplified.

Find the value of the product (LMN).

Que. 10. If $\cos(\alpha+\beta)=\frac{4}{5};\sin(\alpha-\beta)=\frac{5}{13}$ and α,β lie between 0 and $\frac{\pi}{4}$, then find the value of $\tan 2\beta$. **(code-V1T11PAQ1)**

Que. 11. Find the range of values of k for which the equation $2\cos^4 x-\sin^4 x+k=0$, has atleast one solution. **(code-V1T11PAQ3)**

Que. 12. Prove that, $\sin^3\theta+\sin^3\left(\theta+\frac{2\pi}{3}\right)+\sin^3\left(\theta+\frac{4\pi}{3}\right)-=-\frac{3}{4}\sin 3\theta$. **(code-V1T11PAQ5)**

Que. 13. Compute the value of the sum $\sum_{r=1}^{n}\left(\frac{\tan 2^{r-1}}{\cos 2^r}\right)$ **(code-V1T11PAQ6)**

Que. 14. If $15\sin^4\alpha + 10\cos^4\alpha = 6$ then find the value of $8\operatorname{cosec}^6\alpha + 27\sec^6\alpha$. **(code-V1T15PDQ1)**

Que. 15. Given that $x + \sin y = 2008$ and $x + 2008\cos y = 2007$ where $0 \le y \le \pi/2$. Find the value $[x+y]$. (Here [x] denotes greatest interger function) **(code-V2T1PDQ2)**

Que. 16. The sum $\sum_{x=2}^{44} 2\sin x.\sin 1[1+\sec(x-1).\sec(x+1)]$ can be written in the form as $\sum_{n=1}^{4}(-1)^n \frac{\phi^2(\theta_n)}{\psi(\theta_n)}$ where ϕ and ψ are trigonometric functions and $\theta_1, \theta_2, \theta_3, \theta_4$ are in degrees $\in [0, 45]$. Find $(\theta_1+\theta_2+\theta_3+\theta_4)$.
(code-V2T17PDQ1)

Que. 17. If the total between the curves $f(x) = \cos^{-1}(\sin x)$ and $g(x) = \sin^{-1}(\cos x)$ on the interval $[-7\pi, 7\pi]$ is A, find the value of 49A. (Take $\pi = 22/7$) **(code-V2T17PDQ3)**

[SOLUTION]

Single Correct Type

Que. 1. (D)

$$\frac{\cos A\cos C + (-\cos C)(-\cos A)}{\cos A\sin C - (\sin C)(-\cos A)} \Rightarrow \frac{2\cos A\cos C}{2\cos A\sin C} = +\cos C$$

Que. 2 (B) $3^{a;} = 4; a = \log_3 4;$ |||ly $b = \log_4$ etc.

Hence abcdef $= \log_3 4; \log_4 5; \log_5 6.\log_6 7.\log_7 8.\log_8 9 = \log_3 9 = 2 \quad \Rightarrow \quad 2.$

Que. 3. (D)

$\cos 15^o = 2+\sqrt{3} \cong 3.732;$ $\tan 60^o = \sqrt{3} \cong 1.732;$ $\sec 15^o = \frac{4}{\sqrt{6}+\sqrt{2}} = \sqrt{6}-\sqrt{2} = 1.035;$

$\operatorname{cosec} 15^o = \frac{4}{\sqrt{6}-\sqrt{2}} = \sqrt{6}+\sqrt{2} = 3.86$ which is largest

Que. 4. (C)

$$\frac{\sin\alpha - \sin\gamma}{\cos\gamma - \cos\alpha} = \frac{2\sin\left(\frac{\alpha-\gamma}{2}\right)\cos\left(\frac{\alpha+\gamma}{2}\right)}{2\sin\left(\frac{\alpha-\gamma}{2}\right)\sin\left(\frac{\alpha+\gamma}{2}\right)} = \cot\left(\frac{\alpha+\gamma}{2}\right) = \cot\beta$$

Que. 5. (B) $\frac{1+\sqrt{1+\tan^2 2A}}{\tan 2A} = \frac{1+|\sec 2A|}{\tan 2A}$ $(2A = 220^o)$ $= \frac{1-\sec 2A}{\tan 2A} = -\left(\frac{1-\cos 2A}{\sin 2A}\right) = -\tan A.$

Que. 6. (C) $y = 256(\sin^2 x + \operatorname{cosec}^2 x) + 68\operatorname{cosec}^2 x, 256((\sin x - \operatorname{cosec} x)^2 + 2) + 68\operatorname{cosec}^2 x$

Minimum when $x = \frac{\pi}{2}$ or $-\frac{\pi}{2}$ and minimum vlue $= 512 + 68 = 580$

Que. 7. **(a)** $E = \frac{-1+|\sec A|}{\tan A} = \frac{1-\cos A}{\sin A} = \tan\frac{A}{2}$

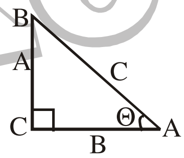
Que. 8. (B).

Que. 9. (A) Let R_1 be the radius of the circumcircle of triangle ABC using sine law in trianlge BPC

$\frac{a}{\sin 120^o} = 2R_1$ .... (1) also $\frac{a}{\sin 60^o} = 2R$ (in ΔABC)

$a = 2R\sin 60^o$ $(R = 1$ given$)$ $a = \sqrt{3}$; form (1) $R_1 = \frac{2\sqrt{3}}{\sqrt{3}}.\frac{1}{2} = 1.$

Que. 10. (D) $\Delta = \frac{1}{2}ab \Rightarrow ab = 60$ $c = \sqrt{a^2 + b^2}$ also $a^2 + b^2 \geq 2ab$



$\therefore$ $\sqrt{a^2 + b^2} \geq \sqrt{2ab}$

$\therefore$ equality occurs when a = b

minimum value of $\sqrt{a^2 + b^2} = \sqrt{2}\sqrt{ab} = \sqrt{120} = 2\sqrt{30}$

Alternatively: $b = c\cos\theta$; $a = c\sin\theta$ $\Delta = \frac{1}{2}c^2\sin\theta\cos\theta = \frac{c^2\sin 2\theta}{4} = 3\theta$ $c^2 = 120\,\text{cosec}\,2\theta$

$c^2|_{min} = 120 \Rightarrow c = 2\sqrt{30}$

Que. 11. (D) In (D) it should be $\frac{\sqrt{5} - 1}{8}$.

Que 12. (A) put $a = 2R\sin A$ etc.

$T_1 = 2R\sin(B + C)\cos(B - C) = R[\sin 2B + \sin 2C]$ etc.

$E = R[\sin 2B + \sin 2C + \sin 2C + \sin 2A + \sin 2A + \sin 2B] = 2R(\sin 2A + \sin 2B + \sin 2C)$

$= 8R.\frac{a}{2R}.\frac{b}{2R}.\frac{c}{2R} = \frac{abc}{R^2}.$

Que. 13. (D) where $n \in I$.

$\sin^2 2x = 2\cos^2 x$ $4\sin^2 x\cos^2 x = 2\cos^2 x$ $\cos^2 x[1 - 2\sin^2 x] = 0$

$\cos^2 x = 0$ or $\sin^2 x = \frac{1}{2}$ $\therefore$ $x = n\pi \pm \frac{\pi}{2}$ or $x = n\pi \pm \frac{\pi}{4}$

Que. 14. (B) $\frac{b}{c} = \frac{\sqrt{3}+1}{2}; \frac{b-c}{b+c} = \frac{\sqrt{3}+1-2}{\sqrt{3}+1+2} = \frac{\sqrt{3}-1}{(\sqrt{3}+1)}.\frac{1}{\sqrt{3}}$

now using $\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2} = \frac{\sqrt{3}-1}{(\sqrt{3}+1)}.\frac{\sqrt{3}}{\sqrt{3}} = 2 - \sqrt{3} \Rightarrow \frac{B-C}{2} = 15^o$ $\therefore$ $B - C = 30^o$

Que. 15. (C)

Que. 16. (D) $\sin^2\theta = 1 \quad [\sin\theta \neq +1] \Rightarrow \sin\theta = -1 \Rightarrow \theta = 2n\pi - \pi/2 \quad \Rightarrow$ infinite roots

Que. 17. (C) In a triangle $b + c > a \quad \Rightarrow \quad b + c - a > 0 \quad \therefore \quad a(b + c - a) > 0$ |||1y $\quad b(c + a - b) > 0$

and $\quad c(a + b - c) > 0$

$a(b + c) + b(c + a) + c(a + b) > a^2 + b^2 + c^2 \qquad 2(ab + bc + ca) > a^2 + b^2 + c^2$

$\therefore \quad \dfrac{a^2+b^2+c^2}{ab+bc+ca} < 2$ ....(1) also forany $a,b,c \in R$ $\quad a^2 + b^2 + c^2 \geq ab + bc + ca$

$\therefore \quad \dfrac{a^2+b^2+c^2}{ab+bc+ca} \geq 1$ ....(2) (equality holds if $a = b = c$) form (1) and (2) $\quad 1 \leq \dfrac{\sum a^2}{\sum ab} < 2$

Que. 18. (A) Make a quadratic in $\cos 2\theta$ to get $\cos 2\alpha + \cos 2\beta = \dfrac{2ac}{a^2 + b^2}$

$$\Rightarrow \qquad 2\left(\cos^2\alpha + \cos^2\beta\right) = \frac{2ac}{a^2 + b^2} + 2; \qquad \cos^2\alpha + \cos^2\beta = \frac{a^2 + ac + b^2}{a^2 + b^2}$$

Que. 19. (A) $\dfrac{a(1 - \tan^2\theta)}{1 + \tan^2\theta} + \dfrac{2b.\tan\theta}{1 + \tan^2\theta} = \dfrac{a\left(1 - \frac{b^2}{a^2}\right) + \frac{2b^2}{a}}{1 + \frac{b^2}{a^2}} = \dfrac{a(a^2 - b^2) + 2ab^2}{a^2 + b^2} = \dfrac{a(a^2 + b^2)}{a^2 + b^2} = a.$

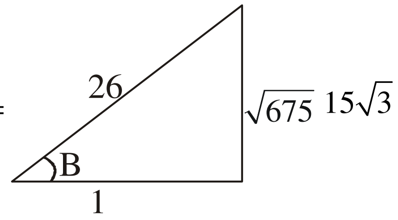
Que. 20. (D) $f_4(x) - f_6(x) = \dfrac{1}{4}(\sin^4 x + \cos^2 x) - \dfrac{1}{6}(\sin^6 x + \cos^6 x) = \dfrac{1}{4}(1 - 2\sin^2\cos^2 x) - \dfrac{1}{6}(1 - 3\sin^2\cos^2 x)$

$$= \frac{1}{4}\left[1 - \frac{1}{2}\sin^2 2x\right] - \frac{1}{6}\left[1 - \frac{3}{4}\sin^2 2x\right] = \frac{1}{4} - \frac{1}{6} = \frac{1}{12}.$$

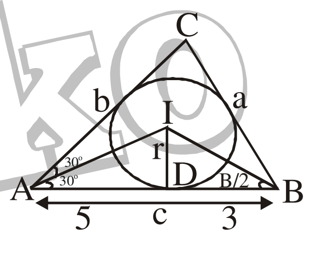
Que. 21. (A)

Que. 22. (D) Given $A = 60^\circ$; $\tan 30^\circ = \dfrac{r}{5} \Rightarrow r = \dfrac{5}{\sqrt{3}}$ now $\tan\dfrac{B}{2} = \dfrac{r}{3} = \dfrac{5}{3\sqrt{3}} \quad (a = ?)$

$\cos B = \dfrac{1 - \tan^2(B/2)}{1 + \tan^2(B/2)} = \dfrac{1 - (25/27)}{1 + (25/27)} = \dfrac{2}{521} = \dfrac{1}{26} =$



Hence $\quad \sin B = \dfrac{15\sqrt{3}}{26} \sin C = \sin(A + B)$



$= \sin A \cos B + \cos A \sin B =$

$\dfrac{\sqrt{3}}{2}.\dfrac{1}{26} + \dfrac{1}{2}.\dfrac{15\sqrt{3}}{26} = \dfrac{1}{52}\left[16\sqrt{3}\right] = \dfrac{4\sqrt{3}}{13} = \sin C \Rightarrow \dfrac{c}{\sin C} = \dfrac{a}{\sin A}; \qquad a = \dfrac{8.\sqrt{3}}{2}.\dfrac{13}{4\sqrt{3}} = 13.$

Que. 23. (B)

Que. 24. (A,B,D) (c) $\sum \sin^2\frac{A}{2} = \frac{1}{2}\left[3-(\cos A+\cos B+\cos C)\right] = \frac{3}{2}-\frac{1}{2}(\cos A+\cos B+\cos C)$

but $[\cos A+\cos B+\cos C]_{max} = \frac{3}{2}$ $\therefore$ $\left.\sum \sin^2\frac{A}{2}\right]_{min} = \frac{3}{2}-\frac{3}{4} = \frac{3}{4}$ $\therefore$ $\sum\sin^2\frac{A}{2} \ge \frac{3}{4} \Rightarrow$ (c) si wrong.

a,b,c are correct and hold good in an equilateral triangle as their maximumum values.

Que. 25. (A,B,C) (b) $\text{RHS} = \frac{\sin 4\alpha+\sin 2\alpha}{\sin 2\alpha.\sin \alpha 4} = \frac{2\sin 3\alpha.\cos\alpha}{\sin 2\alpha.\sin 4\alpha} = \frac{1}{\sin\alpha} = \text{cosec}\,\alpha\,(\text{using } \pi = 7\alpha) \Rightarrow$ (b).

(c) $\cos\alpha+\cos 3\alpha+\cos 5\alpha$ sum of a series with constant $d = 2\alpha$ sum $= \frac{1}{2} \Rightarrow$ (c) is wrong.

(d) continued product = 1 $\Rightarrow$ (d) is also wrong.

Que. 26. (A,B,C) (a) $x = \pi/8, \Rightarrow (\tan x)^{\ln(\sin x)} > 1$ and $(\cot x)^{\ln(\sin x)} < 1 \Rightarrow$ True.

(b) $x = \pi/6, \Rightarrow 4^{\ln 2} < 5^{\ln 2} \Rightarrow$ True.

(c) $x = \pi/2, 2^{\ln 2} < 3^{\ln 2} \Rightarrow$ True.

(d) $x = \pi/4, 2^0 > 2^{-\ln 2} \Rightarrow 1 < \frac{1}{2^{\ln 2}}$ is not correct $\Rightarrow$ False.

Que. 27. (B) $(\cos^2 A-\sin^2 B)-(\sin^2 A-\sin^2 B) = \cos^2 A-\sin^2 A = \cos 2A$

(C) $\sin 2B$; (D) $-\sin 2B$

Que. 28. (D) $(1-\sin^2 5x)-(1-\sin^2 x)+\sin 4x\sin 6x = 0 \Rightarrow \sin^2 x-\sin^2 5x+\sin 4x.\sin 6x = 0$

$-\sin 6x.\sin 4x+\sin 4x.\sin 6x = 0$ which is true for all $x\in[0,\pi] \Rightarrow$ it is indentity

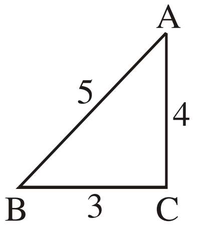
Que. 29. (A)
$$\frac{2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)+2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)}{2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)-2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)} = \frac{\cos\left(\frac{\alpha-\beta}{2}\right)+\cos\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)-\cos\left(\frac{\alpha+\beta}{2}\right)}$$

$$= \frac{2\cos\frac{\alpha}{2}\cos\frac{\beta}{2}}{2\sin\frac{\alpha}{2}\sin\frac{\beta}{2}} = \cot\frac{\alpha}{2}\cot\frac{\beta}{2}.$$

Que. 30. C. Given $\tan A.\tan B = 2$

Let $y = \frac{\cos A\cos B}{\cos C} = -\frac{\cos A.\cos B}{\cos(A+B)} = \frac{\cos A.\cos B}{\sin A\sin B-\cos A\cos B} = \frac{1}{\tan A\tan B-1} = \frac{1}{2-1} = 1$

Que. 31. B. $2^{\sin\theta} > 1 \Rightarrow \sin\theta > 0 \Rightarrow \theta\in 1^{st}$ or 2^{nd} quadrant , $3^{\sin\theta} < 1 \Rightarrow \cos\theta < 0 \Rightarrow \theta\in 2^{nd}$ or 3^{rd} quadrant hence $\theta\in 2^{nd} \Rightarrow$ possible answer is (B).



Que. 32. B. $E = \sin A+\sin 2B+\sin 3C \Rightarrow E = \frac{3}{5}+2.\frac{4}{5}.\frac{3}{5}-1 = \frac{15}{25}+\frac{24}{25}-1 = \frac{39-25}{25} = \frac{14}{25}.$

Que. 33. C. $y=\left(\sin^2 x+\cos^2 x\right)+2\left(\sin x\operatorname{cosec}x+\cos x\sec x\right)+\sec^2 x+\operatorname{cosec}^2 x$

$=5+2+\tan^2 x+\cot^2 x=7+\left(\tan x-\cot x\right)^2+2\ \therefore\ y_{min}=9.$

Que.34. D. $\cot^4 x-2\left(1+\cot^2 x\right)+a^2=0\Rightarrow\cot^4 x-2\cot^2 x+a^2-2=0\Rightarrow\left(\cot^2 x-1\right)^2=3-a^2$ to have atleast one solution $3-a^2\ge 0\Rightarrow a^2-3\le 0\Rightarrow a\in\left[-\sqrt{3},\sqrt{3}\right]$ integral values $-1,0,1\ \therefore$ sum $=0.$

Que.35. D. Let $\cos 2\theta=t\ \therefore 8t+\frac{8}{t}=65\Rightarrow 8t^2-65t+8=0\Rightarrow 8t-64t-t+8\Rightarrow 8t\left(t-8\right)-\left(t-8\right)=0$

$\Rightarrow t=8$ or $t=\frac{1}{8}$ ($t=8$ is rejected, think !) $\therefore\ \cos 2\theta=\frac{1}{8};\ 2\cos^2\theta-1=\frac{1}{8}\Rightarrow\cos^2\theta=\frac{9}{16}\Rightarrow\cos\theta=\frac{3}{4}.$

Que.36. C. $2\sin x+7\cos px=9$ is possible only if $\sin x=1$ $\cos px=1$

$x=(4n+1)\frac{\pi}{2}$ and $px=2m\pi\Rightarrow x=\frac{2m\pi}{p}\left(m,n,\in I\right)\ \therefore (4n+1)\frac{\pi}{2}=\frac{2m\pi}{p}\Rightarrow p=\frac{4m}{4n+1}\ \therefore\ p\in$ rational.

Que.37. A. $\tan\theta>1\Rightarrow 0<\frac{1}{\tan\theta}<1$ now, $\frac{1}{\tan\theta}+\frac{1}{\tan^2\theta}+\frac{1}{\tan^2\theta}+........\infty=\sin\theta+\cos\theta$

$\Rightarrow\ \frac{\frac{1}{\tan\theta}}{1-\frac{1}{\tan\theta}}=\sin\theta+\cos\theta\Rightarrow\frac{1}{\tan\theta-1}=\sin\theta+\cos\theta\Rightarrow\frac{\cos\theta}{\sin\theta-\cos\theta}=\sin\theta+\cos\theta$

$\Rightarrow\cos\theta=\sin^2\theta-\cos^2\theta=1-2\cos^2\theta\qquad\Rightarrow 2\cos^2\theta+\cos\theta-1=0\Rightarrow\left(2\cos\theta-1\right)\left(\cos\theta+1\right)=0$

$\cos\theta=\frac{1}{2}$ or $\cos\theta=-1$ (rejected) $\Rightarrow\ =\frac{\pi}{3}\Rightarrow\ \tan\theta=\sqrt{3}.$

Que.38. D. Let $\left|a\sin x-\cos x\right|=k,\ k\ge 0$ .... (1) also $\sin x+a\cos x=b$ .... (2) Square and add (1) and (2) $a^2+1=k^2+b^2\Rightarrow k^2=a^2-b^2+1\Rightarrow k=\sqrt{a^2-b^2+1}.$

Que.39. A. $\frac{\cos 2x}{2}-\frac{\sqrt{3}}{2}\sin 2x=0$ or $\cos 2x.\cos 2x.\cos\frac{\pi}{3}-\sin 2x.\text{son}\frac{\pi}{3}\Rightarrow\cos\left(2x+\frac{\pi}{3}\right)=0$

$\Rightarrow 2x+\frac{\pi}{3}=\frac{\pi}{2};2x=\frac{\pi}{6}\Rightarrow x=\frac{\pi}{12}.$

Que.40. B. $\left(\sin\theta+2\right)\left(\sin\theta+3\right)\left(\sin\theta+4\right)=6$ LHS ≥ 6 and RHS $=6\ \Rightarrow$ equality only can hold if $\sin\theta=-1.\ \Rightarrow\sin\theta=-1\Rightarrow\theta=\frac{3\pi}{2},\frac{7\pi}{2}\ \therefore$ sum $=5\pi\ \Rightarrow\ 5.$

Que.41. A. $a\sin x+c<0\Rightarrow\sin x<-\frac{c}{a};\ -\frac{c}{a}>\sin x;\ -\frac{c}{a}>1;\ -c>a\Rightarrow a+c<0\Rightarrow$ (A)

Que.42. A. In 2nd quadrant $\sin x < \cos x$ is False (think !)

In 4th quadrant $\cos x < \tan x$ is False (think !)

in 3rd quadrant, i.e. $\left(\frac{5\pi}{4}, \frac{3\pi}{2}\right)$ if $\tan x < \cot x \Rightarrow \tan^2 x < 1$ which is not correct hence A can be correct

now $\sin x < \cos x$ is true in $\left(0, \frac{\pi}{4}\right)$ and $\tan x < \cot x$ is also true

$\therefore$ only the value of x for which $\cos x < \tan x$ is be determined

$\therefore$ now $\cos x = \tan x$ i.e. $\cos^2 x = \sin x$ or $1 - \sin^2 x = \sin x \Rightarrow \sin^2 x + \sin x - 1 > 0$

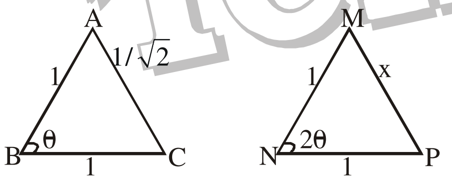
$\Rightarrow \sin x = \frac{-1 \pm \sqrt{5}}{2};\ \sin x = \frac{\sqrt{5}-1}{2} \Rightarrow x = \sin^{-1}\left(\frac{\sqrt{5}-1}{2}\right)$

$\therefore \cos x < \tan x$ in $\left(\sin^{-1}\frac{\sqrt{5}-1}{2}, \frac{\pi}{4}\right)$ and $\cos x > \tan x$ in $\left(0, \sin^{-1}\frac{\sqrt{5}-1}{4}\right)$

Que.43. D. $\cos\theta\left(\sum \cos x\right) + \sin\theta\left(\sum \sin x\right) = 0$

Que.44. D. $\cos\theta = \frac{1+1-\frac{1}{2}}{2} = \frac{3}{4}$ $\therefore$ $\cos 32\theta = 2\cos^2\theta - 1 = 2.\frac{9}{16} - 1 = \frac{2}{16} = \frac{1}{8}$

agin $x^2 = 1 + 1 - 2\cos 2\theta = 2(1 - \cos 2\theta) = 2\left(1 - \frac{1}{8}\right) = \frac{7.2}{8} = \frac{7}{4} \Rightarrow x = \frac{\sqrt{7}}{2}$



Que.45. A. $\text{sum} = \sin^2\frac{\pi}{18} + \sin^2\frac{2\pi}{18} + ... + \sin^2\frac{8\pi}{18} + 1$ now $\sin^2\frac{\pi}{18} + \sin^2\frac{8\pi}{18} = 1$ etc. $\Rightarrow \text{sum} = 5.$

Que.46. A. Using $\Delta = \frac{1}{2}bc\sin A \therefore \frac{1}{2}.2\left(\sqrt{3}-1\right)\sin A = \frac{\sqrt{3}-1}{2} \therefore \sin A = \frac{1}{2} \Rightarrow A = 30^o$

$\Rightarrow \tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2} = \frac{3-\sqrt{3}}{\sqrt{3}+1}.\frac{\sqrt{3}+1}{\sqrt{3}-1} = \sqrt{3} \Rightarrow B - C = 120^o$ also $B + C = 150^o \Rightarrow C = 15^o.$

Que.47. B. $\sin^2 x + a\cos x + a^2 > 1 + \cos x$ put $x = 0 \Rightarrow a + a^2 > 2 \Rightarrow a^2 + a = 2 > 0 \Rightarrow (a+2)(a-1) > 0$

$\therefore$ 

$\therefore$ largest negative integral value of 'a' = – 3.

Que. 48. A. $\sin\left(\frac{a}{2}\right).\left(\frac{1}{2}+\sum_{k=1}^{n}\cos(ka)\right) \Rightarrow \sin\left(\frac{a}{2}\right).\left(\frac{1}{2}+\cos a+\cos 2a+\cos 3a+......+\cos na\right)$

$\frac{1}{2}\sin\frac{a}{2}+\frac{1}{2}\left[\left(\sin\frac{3a}{2}-\sin\frac{a}{2}\right)+\left(\sin\frac{5a}{2}-\sin\frac{3a}{2}\right)+........+\left(\sin\left(n+\frac{1}{2}\right)a-\sin\left(n-\frac{1}{2}\right)a\right)\right]$

$\frac{1}{2}\sin\frac{a}{2}+\frac{1}{2}\left[\sin\left(n+\frac{1}{2}\right)a-\sin\frac{a}{2}\right]=\frac{1}{2}\sin\left(n+\frac{1}{2}\right)a.$

Que. 49. D. Since ABC are acute angle

$\therefore \quad A+B>\pi/2 \Rightarrow A>\frac{\pi}{2}-B \Rightarrow \sin A-\cos B>0 \Rightarrow \cos B-\sin A<0$ .... (1)

Again, $B>\frac{\pi}{2}-A \Rightarrow \sin B>\cos A \Rightarrow \sin B-\cos A>0$ .... (2)

Form (1) and (2) x-coordinate is – ve and y-coordinate is +ve

$\Rightarrow$ line in 2nd quadrant only.

Que. 50. B. $\sin^4 x-\cos^2\sin x+2\sin^2 x+\sin x=0 \qquad \Rightarrow \sin x\left[\sin^3 x-\cos^2 x+2\sin x+1\right]=0$

$\Rightarrow \sin x\left[\sin^3 x-1+\sin^2 x+2\sin x+1\right]=0 \quad \Rightarrow \quad \sin x\left[\sin^3 x+\sin^2 x+2\sin x\right]=0$

$\Rightarrow \quad \sin^2 x=0$ or $\sin^2 x+\sin x+2=0 \quad \Rightarrow$ not possible for real x. $\sin x=0$

$\Rightarrow \quad x=0,\ \pi,\ 2\pi,\ 3\pi, \quad \Rightarrow$ 4 solution.

Que. 51. D.

Que. 52. C. $a^2+b^2=4R^2\left[\sin^2\left(45^o-\theta\right)+\sin^2\left(135^o-\theta\right)\right]=4R^2\left[\sin^2\left(45^o-\theta\right)+\cos^2\left(45^o-\theta\right)\right]=4R^2.$

Que. 53. D. $R=8r=8\left(4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right) \quad \therefore \quad 2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}=\frac{1}{16}$

$\Rightarrow \quad \left(\cos\frac{A-B}{2}-\cos\frac{A+B}{2}\right)\sin\frac{C}{2}=\frac{1}{16} \qquad \Rightarrow \qquad \sin\frac{C}{2}.\left(\frac{1}{2}-\sin\frac{C}{2}\right)=\frac{1}{16};$

$\sin^2\frac{C}{2}-\frac{1}{2}\sin\frac{C}{2}+\frac{1}{16}=0 \qquad \Rightarrow \qquad \left(\frac{1}{4}-\sin\frac{C}{2}\right)^2=0 \qquad \Rightarrow \qquad \sin\frac{C}{2}=\frac{1}{4}$

$\cos C=1-2\sin^2\frac{C}{2}=1-\frac{1}{8}=\frac{7}{8}.$

Que. 54. D. For non trivial solution $\begin{vmatrix} 1 & -\cos\theta & \cos 2\theta \\ -\cos\theta & 1 & -\cos\theta \\ \cos 2\theta & -\cos\theta & 1 \end{vmatrix}=0$ using $C_1 \to C_1 \to C_3$

$\begin{vmatrix} 2\sin^2\theta & -\cos\theta & \cos 2\theta \\ 0 & 1 & -\cos\theta \\ -2\sin^2\theta & -\cos\theta & 1 \end{vmatrix}=0 \Rightarrow 2\sin^2\theta\begin{vmatrix} 1 & -\cos\theta & \cos 2\theta \\ 0 & 1 & -\cos\theta \\ -1 & -\cos\theta & 1 \end{vmatrix}=0 \Rightarrow \sin^2\theta=0$

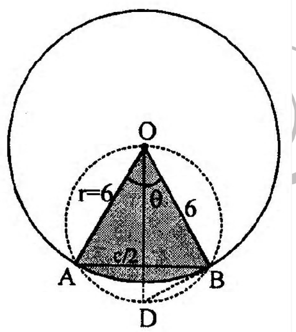
or $1\left[1-\cos^2\theta\right]-1\left[\cos^2\theta-\cos 2\theta\right] \Rightarrow \sin^2\theta-\left[\cos^2\theta-(\cos^2\theta-\sin^2\theta\right] \Rightarrow \sin^2\theta-\sin^2\theta=0$

hence $D=0 \ \forall\theta\in R \Rightarrow$ **D.**

Que. 55. A. $\tan\left(\frac{\pi}{4}+\alpha\right)$ when $\alpha = \tan^{-1}\left(\frac{\frac{1}{4}+\frac{1}{5}}{1-\frac{1}{20}}\right)$; $\alpha = \tan^{-1}\left(\frac{9}{19}\right) = \frac{1+\frac{9}{19}}{1-\frac{9}{19}} = \frac{28}{10} = \frac{14}{5} = \frac{a}{b} \Rightarrow 14+5=19.$

Que. 56. D. $R = \frac{abc}{4\Delta}$; $\Delta = \frac{1}{2}.6.6\sin\theta = 18\sin\theta \Rightarrow a=b=6 \Rightarrow \sin\frac{\theta}{2} = \frac{c}{12} \Rightarrow c = 12\sin\frac{\theta}{2}.$

Alternatively :



$OD^2 = 6^2 + x^2 \Rightarrow \frac{x}{6} = \tan(\theta/2) \Rightarrow OD^2 = 6^2.\sec^2(\theta/2) \Rightarrow 4r^2 = 36\sec^2(\theta/2) \Rightarrow r = 3\sec(\theta/2).$

Que. 57. D. $\Delta^2 = s(s-a)(s-b)(s-c) \Rightarrow r^2 s = (s-a)(s-b)(s-c) \quad (\because \quad r = \Delta/s)$

$= s^3 - s^2(a+b+c) + (ab+bc+ca)s - abc \;\; \therefore \;\; r^2 s = s^3 - 2s^3 + (ab+bc+ca)s - abc$

using $\frac{abc}{4R} = \Delta \Rightarrow abc = 4Rrs \Rightarrow r^2 = (ab+bc+ca) - s^2 - 4Rr \;\therefore\; \sum ab = r^2 + s^2 + 4Rr.$

Que. 58. A. $E = (1+\tan A)(1+\tan B) = 1 + \tan A + \tan B + \tan A \tan B$

Now $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = 1 \Rightarrow A = 20^o$ and $B = 25^o \Rightarrow 1 - \tan A \tan B = \tan A + \tan B$

$\Rightarrow \tan A + \tan B + \tan A \tan B = 1 \quad \therefore \quad E = 2. \Rightarrow$ **A.**

Que. 59. B. Expression reduces to 2 corosec 8x

Que. 60. D.

$\frac{x}{\sqrt{1+x^2}} = \frac{x+1}{\sqrt{(x+1)^2+1}} \Rightarrow x^2\left[(x+1)^2+1\right] = (x+1)^2\left[(x^2+1)\right] \Rightarrow x^2(x+1)^2 + x^2 = x^2(x+1)^2 + (x+1)^2$

$x^2 = (x+1)^2 \Rightarrow x+1 = x$ not possible as $x \to \infty$ or $x+1 = -x \Rightarrow x = -1/2$ not possible (think!).

Que. 61. D. (A) $\cos\left(\text{tra}^{-1}\left(\tan(4-\pi)\right)\right) = \cos(4-\pi) = \cos(\pi-4) = -\cos 4 > 0$

(B) $\sin\left(\cot^{-1}\left(\cot(4-\pi)\right)\right) = \sin(4-\pi) = -\sin 4 > 0$ (as $\sin 4 < 0$)

(C) $\tan\left(\cos^{-1}\left(\cos(2\pi-5)\right)\right) = \tan(2\pi-5) = -\tan 5 > 0$ (as $\tan 5 < 0$)

(D) $\cot\left(\sin^{-1}\left(\sin(\pi-4)\right)\right) = \cot(\pi-4) = -\cot 4 < 0 \Rightarrow$ **(D)** si correct.

Que. 62. A.

$$\frac{\pi}{2}-\cos^{-1}\cos\left(\frac{2\left(x^2+5|x|+3\right)-2}{\underbrace{x^2+5|x|+3}_{0<\downarrow<2}}\right)=\cot\cot^{-1}\left(\frac{2}{9|x+}-2\right)+\frac{\pi}{2}\Rightarrow\frac{\pi}{2}-2+\frac{2}{x^2+5|x|+3}=\frac{2}{9|x|}-2+\frac{\pi}{2}$$

$$\Rightarrow \quad |x|^2-4|x|+3=0 \quad \Rightarrow \quad |x|=1,3 \quad \Rightarrow \quad x=\pm1,\pm3.$$

Que. 63. A. Solution are $\sqrt{3},-\frac{1}{\sqrt{3}},2-\sqrt{3},-\left(2+\sqrt{3}\right) \Rightarrow$ Product $=1.$

Que. 64. A.

$$\tan 37.5^o=\tan\left(\frac{75}{2}\right)^o=\frac{1-\cos 75^o}{\sin 75^o}=\frac{1-\frac{\sqrt{3}-1}{2\sqrt{2}}}{\frac{\sqrt{3}+1}{2\sqrt{2}}}=\frac{2\sqrt{2}-\sqrt{3}+1}{\sqrt{3}+1}=\frac{\left(2\sqrt{2}-\sqrt{3}+1\right)\left(\sqrt{3}-1\right)}{2}=2\sqrt{6}-4-2\sqrt{2}$$

$$\frac{2\left(\sqrt{6}-3+\sqrt{3}\right)-\left(2\sqrt{2}-\sqrt{3}+1\right)}{2}=\frac{2\sqrt{6}-4+2\sqrt{3}-2\sqrt{2}}{2}=\sqrt{6}-\sqrt{4}+\sqrt{3}-\sqrt{2}$$

$$\therefore \quad a=6;\ b=4,\ c=3,\ d=2 \quad \Rightarrow \quad \frac{ad}{bc}=\frac{12}{12}=1.$$

Que. 65. C. $y=(3)^{3\cos x}+(3)^{4\sin x}$ now using $AM\ge GM$ $\frac{3^{3\cos x}+3^{4\sin x}}{2}\ge\left(3^{3\cos x}.3^{4\sin x}\right)^{1/2}$

$$\Rightarrow 3^{3\cos x}+3^{4\sin x}\ge 2\sqrt{3^{3\cos x+4\sin x}}\ge 2\sqrt{3^{-5}} \quad \text{but} \quad -5\le 3\cos x+4\sin x\le 5 \quad \therefore \quad 3^{3\cos x}+3^{4\sin x}\ge 2\sqrt{3^{-5}}$$

$$=\frac{2}{3^{5/2}}=\frac{2}{3.3.\sqrt{3}}=\frac{2}{\sqrt{243}}=\sqrt{\frac{4}{243}} \Rightarrow a+b=247.$$

Que. 66. A Domain of f is $[-1,1]$; $f(x)=\sin x+\cos x+\tan x+\sin^{-1}x+\cos^{-1}x+\tan^{-1}x$

$$f'(x)=\cos x-\sin x+\underbrace{\sec^2 x}_{>1}+0+\frac{1}{\underbrace{1+x^2}_{[1/2,\,1]}}$$

Hence $f'(x)>0$ $\Rightarrow$ f is increasing $\Rightarrow$ range is $\left[f(-1),\,f(1)\right]$

$$\therefore \quad f(x)|_{min}=f(-1)=-\sin 1+\cos 1-\tan 1-\frac{\pi}{2}+\pi-\frac{\pi}{4}=\frac{\pi}{4}+\cos 1-\sin 1-\tan 1$$

$$\Rightarrow \quad \frac{M+m}{2}=\frac{\pi}{2}+\cos 1 \quad \Rightarrow \quad (A)]$$

Que. 67. A. $S=\cos\theta+\cos 2\theta+.......+\cos n\theta$ where $\theta=\pi/6$ and $n=2009$

$$S=\frac{\sin\frac{n\theta}{2}}{\sin\frac{\theta}{2}}\cos(n+1)\frac{\theta}{2} \quad \Rightarrow \text{now } (n+1)\frac{\theta}{2}=\left(\frac{2010}{2}\right)\frac{\pi}{6}=(335)\frac{\pi}{2}$$

Hence $\cos(n+1)\frac{\theta}{2}=0$ $\Rightarrow$ $S=0$ **Ans.**

Que. 68. A. $=\left(\sqrt{2}+1\right)^{2009}-\left(\sqrt{2}-1\right)^{2009}$

$$\Rightarrow 2\left[{}^{2009}C_1\left(\sqrt{2}\right)^{2008}+{}^{2009}C_3\left(\sqrt{2}\right)^{2006}+{}^{2009}C_5\left(\sqrt{2}\right)^{2004}+........+{}^{2009}C_{2009}\left(\sqrt{2}\right)^{0}\right]$$

= which is an even integer $\Rightarrow$ (A)

Comprehesion Type

1 Paragraph for Q. 1 to Q. 3

1. **C.** **2.** **B.** **3.** **A.**

(i) $\tan\frac{A}{2}=\frac{\Delta}{s(s-a)}=\frac{r}{s-a}\left(r=\frac{\Delta}{s}\right)\ \therefore\ \cot\frac{A}{2}=\frac{s-a}{r}$

in any triangle, $\sum\cot\frac{A}{2}=\prod\cot\frac{A}{2}=\frac{s-a+s-b+s-c}{r}=\frac{s}{r}=\frac{s^2}{\Delta}=\frac{4s^2}{4\Delta}=\frac{(a+b+c)^2}{abc}.R\left(\Delta\frac{abc}{4R}\right).$

(ii) $$\sum\cot\frac{A}{2}\cot\frac{B}{2}=\frac{\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}+\cos\frac{B}{2}\cos\frac{C}{2}\sin\frac{A}{2}+\cos\frac{C}{2}\cos\frac{A}{2}\sin\frac{B}{2}}{\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}=\prod\sin\frac{A}{2}}\qquad ....(1)$$

Now consider $\sin\left(\overline{\frac{A}{2}+\frac{B}{2}}+\frac{C}{2}\right)=1\Rightarrow\sin\left(\frac{A}{2}+\frac{B}{2}\right)\cos\frac{C}{2}+\cos\left(\frac{A}{2}+\frac{B}{2}\right)\sin\frac{C}{2}$

$$\left(\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}+\sin\frac{B}{2}\cos\frac{C}{2}\cos\frac{C}{2}\cos\frac{A}{2}+\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}\right)-\left(\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right)=1$$

$$\therefore\ \cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}+\cos\frac{B}{2}\cos\frac{C}{2}\sin\frac{A}{2}+\cos\frac{C}{2}\cos\frac{A}{2}\sin\frac{B}{2}=1+\prod\sin\frac{A}{2}$$

$$\therefore\ \sum\cot\frac{A}{2}\cot\frac{B}{2}=\frac{1+\prod\sin\frac{A}{2}}{\prod\sin\frac{A}{2}}\quad\left(\text{using } r=4R\prod\sin\frac{A}{2}\right)=\frac{1+\frac{r}{4R}}{\frac{r}{4R}}=\frac{4R+r}{r}.$$

(iii) We have $\sum\cot\frac{A}{2}=\prod\cot\frac{A}{2}=\frac{s}{r}$ and $\sum\cot\frac{A}{2}\cot\frac{B}{2}=\frac{4R+r}{r}$ hence an equation whose roots are not $\cot\frac{A}{2},\cot\frac{B}{2}$ and $\cot\frac{C}{2}$ is $x^3-\left(\sum\cot\frac{A}{2}\right)x^2+\left(\sum\cot\frac{A}{2}\cot\frac{B}{2}\right)x-\prod\cot\frac{A}{2}=0$

$f(x)=x^3-\frac{s}{r}x^2+\left(\frac{4R+r}{r}\right)x-\frac{s}{r}=0$ as A or B $C\in\left\{\frac{\pi}{2}\right\}$.

$\therefore\ x=1$ must a root $\left(\text{as }\cot\frac{A}{2}\text{ or }\cot\frac{B}{2}\text{ or }\cot\frac{C}{2}=1\right)\ \therefore\ f(1)=0\ \Rightarrow\ 1-\frac{s}{R}+\frac{4R+r}{r}-\frac{s}{r}=0$

$\Rightarrow\quad r-2s+4R+r=0\quad\Rightarrow\quad 2R+r=s.$

 # 2 Paragraph for Q. 4 to Q. 6

4. A. 5. B 6. C.

$(AH)(BH)(CH) = 3$ i.e. $(2R\cos A)(2R\cos B)(2R\cos C) = 3 \Rightarrow \prod \cos A = \frac{3}{8R^3}$ ....(1)

$(HD)(HE)(HF) = (2R\cos B\cos C)(2R\cos C\cos A)(2R\cos A\cos B) = 8R^3\left(\cos^2 A\cos^2 B\cos^2 C\right)$ ....(2)

From (1) and (2) $\prod(HD) = 8R^3.\frac{9}{64R^6} = \frac{9}{8R^3}$ also $(AH)^2 + (BH)^2 + (CH)^2 = 7$ $\therefore$ $(1)\div(3)$

$\frac{\prod \cos A}{\sum \cos^2 A} = \frac{3}{8R^3}.\frac{4R^2}{7} = \frac{3}{14R}$ Now we know that, in a triangle ABC $\cos^2 A + \cos^2 B + \cos^2 C$

$= 1 - 2\cos A\cos B\cos C$ $\Rightarrow \frac{7}{4R^2} = 1 - 2.\frac{3}{8R^3} \Rightarrow \frac{7}{4R^2} = 1 - \frac{3}{4R^3}$

$\therefore$ $4R^3 - 7R - 3 = 0$ $\Rightarrow$ $(R+1)(2R+1)(2R-3) = 0$ $\therefore$ $R = \frac{3}{2}$.

Assertion & Reason Type

Que. 1. (D) $A = \pi - (B+C)$ $\tan A = -\tan(B+C) = \frac{\tan B + \tan C}{\tan B\tan C - 1}$ $\Rightarrow$ S − 2 is True

hence if A is acute then tan B tan C > 1 if A is obtuse then thanB tan C < 1
$\Rightarrow$ S − 1 is False $\Rightarrow$ answer is (D)

Que. 2. (C)

Que. 3. (C) $f(x) = ax^2 + bx + c$ given $f(0) + f(1) = 2 \Rightarrow f(x) > 0\,\forall x \in R \Rightarrow$ S − 1 is true.

Let $f(x) = x^2 - x + 1$ $\Rightarrow$ $a + b = 0$ $\Rightarrow$ S − 2 is False

Que. 4. (A) $\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2\cos A\cos B\cos C$

$\therefore$ $\sum \cos^2 A|_{min} = 1 - 2.\frac{1}{8} = 1 - \frac{1}{4} = \frac{3}{4}$.

Que. 5. B.

Que.6. (a) $\tan\theta - \tan\alpha - \tan\beta = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} - (\tan\alpha + \tan\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}(\tan\alpha.\tan\beta) \Rightarrow \tan\theta.\tan\alpha.\tan\beta$

Que. 7. C.

Que. 8. B.

Que. 9. D. Givne $\tan 2x = 1$ $\therefore$ $2x = n\pi + \frac{\pi}{4}$ (note that $\tan 4x$ is not defined)

Hence given equation has no solution $\therefore$ Statement - 1 is false and Statement - 2 is true.

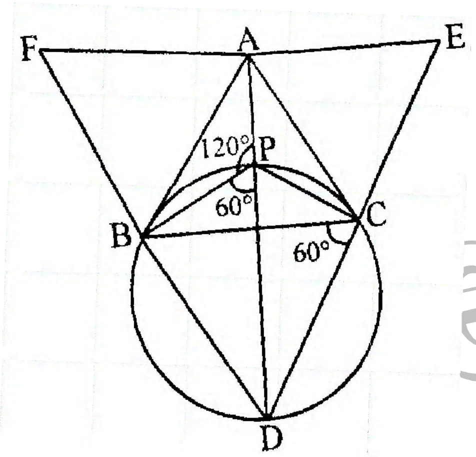
Que. 10. A.

Que. 11. C. If it is acute triangle then statement -1 is abviously true

Let A be obtuse say $A = 150^o$ $\therefore$ $B + C = 30^o$ both angles $< 30^o$ and if C $= 30^o$

Now $\cot A$ and $\cot(B+C)$ will be of equal magnitke but opposite sign, As $\cot\theta$ is decreasing hence, $\cos B+\cos A$ alone is +ve $\quad\therefore\quad \cos A+\cot B+\cot C>0$.

Que. 12. B $\quad \angle BPD=\angle BCD=60°\quad$ ($\therefore$ chord BD subtends equal angle in same segment)



$\therefore\quad \angle APB+\angle DPB=180°$

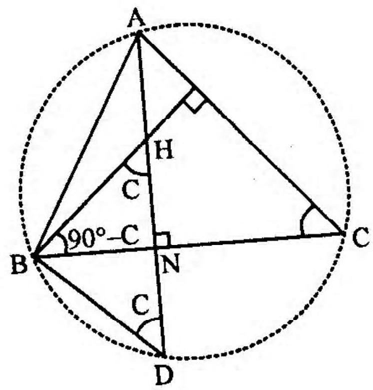
A, P, D are collinear

|||ly B, P, E and C, P, F are also collinear

hence AD, BE, CF are concurrent at P.

Que. 13. A. $\quad \Delta BHN$ and BDN are congruent

$\therefore\quad HN=ND=2R\cos B\cos C \quad\Rightarrow\quad HD=4R\cos B\cos C$



More than One May Correct Type

Que. 1. (A,B,C,D)

$2\cos\theta+2\sqrt{2}=3\sec\theta$

$\therefore\quad 2\cos^2\theta+2\sqrt{2}\cos\theta-3=0$

$$\cos\theta=\frac{-2\sqrt{2}\pm\sqrt{32}}{4}=\frac{-2\sqrt{2}\pm4\sqrt{2}}{4}$$

$\therefore\quad \cos\theta=\frac{1}{\sqrt{2}}\quad$ or $\quad\cos\theta=-\frac{3}{\sqrt{2}}$ (rejected)

$\therefore\quad \theta=\frac{\pi}{4}$ or $-\frac{\pi}{4}$

$\Rightarrow\quad \sin\theta=-\frac{1}{\sqrt{2}};\cot\theta=-1;\quad \tan\theta=1\quad$ and $\quad\cos\theta=-\frac{1}{\sqrt{2}}$

Que. 2 (B,C)

(A) $=\frac{1}{2}\sin 150^{o}=\frac{1}{4} \Rightarrow$ rational

(B) $2+\log_2 7 \Rightarrow$ irrational

(C) $\log_3 6 = 1+\log_3 2 \Rightarrow$ irrational

(D) $8^{-1/3}=2^{-1}=\frac{1}{2} \Rightarrow$ irrational

Que. 3. (B,C,D)

Opposite angles of a cyclic quadrilateral are supplementary

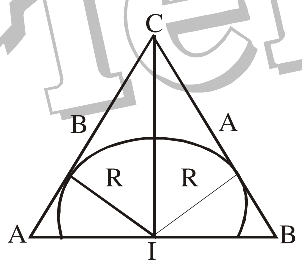
Que. 4. (A,B,C,D) (A) $\sin\left(\frac{11\pi}{12}\right)\sin\left(\frac{5\pi}{12}\right)=\sin\left(\frac{\pi}{12}\right)\cos\left(\frac{\pi}{12}\right)=\frac{1}{2}\sin\left(\frac{\pi}{6}\right)=\frac{1}{4}\in Q$

(B). $\operatorname{cosec}\left(\frac{9\pi}{19}\right)\sec\left(\frac{4\pi}{5}\right)=-\operatorname{cosec}\left(\frac{\pi}{10}\right)\sec\left(\frac{\pi}{5}\right)=\frac{1}{\sin 18^{o}\cos 36^{o}}=-\frac{16}{\left(\sqrt{5}-1\right)\left(\sqrt{5}+1\right)}=-4\in Q$

(C). $\sin^4\left(\frac{\pi}{8}\right)+\cos^4\left(\frac{\pi}{8}\right)=1-2\sin^2\left(\frac{\pi}{8}\right)\cos^2\left(\frac{\pi}{8}\right)=1-\frac{1}{2}\sin^2\left(\frac{\pi}{4}\right)=1-\frac{1}{4}=\frac{3}{4}\in Q$

(D). $2\cos^2\frac{\pi}{9}.2\cos^2\frac{2\pi}{9}.2\cos^2\frac{4\pi}{9}=8\left(\cos 20^{o}.\cos 40^{o}.\cos 80^{o}\right)=\frac{1}{8}\in Q$

Que. 5. (A,C) $\frac{1}{2}ra+\frac{1}{2}rb=\frac{1}{2}ab\sin C \Rightarrow r(a+b)=2\Delta \Rightarrow r=\frac{2\Delta}{a+b}$ ......(1)



$\therefore\ r=\frac{2abc}{4R\left(2R\sin A+2R\sin B\right)}=\frac{abc}{4R^2\left(\sin A+\sin B\right)}\Rightarrow (A)$ also $x=\frac{2ab}{a+b}\cos\frac{C}{2}$

form (1) $r=\frac{2.\frac{1}{2}ab\sin C}{a+b}=\frac{2ab\sin\frac{C}{2}\cos\frac{C}{2}}{a+b}=\frac{2ab\cos\frac{C}{2}}{a+b}.\sin\frac{C}{2}=x\sin\frac{C}{2}$

Que. 6. (A,C) $\sin x-3\sin 2x+\sin 3x=\cos x-3\cos 2x+\cos 3x$ or $2\sin 2x\cos x-3\sin 2x=2\cos x-3\cos 2x$

$\sin 2x\left[\left(2\cos x\right)-3\right]=\cos 2x\left[2\cos x-3\right]\Rightarrow\left(\sin 2x-\cos 2x\right)\left(2\cos x-3\right)=0$ but $2\cos x-3\neq 0$

as $\cos x\leq 1$ hence, $\sin 2x-\cos 2x=0\Rightarrow\tan 2x=1\Rightarrow 2x=n\pi+\pi/4$ or $x=\frac{n\pi}{2}+\frac{\pi}{8}\Rightarrow a,b,c$

Que. 7. (A,B,D) $\left(\sin^2x+\cos^2x\right)^3 - 3\sin^2x\cos^2x\left(\sin^2+\cos^2x\right) > \frac{5}{8}$

$= 1-3\sin^2 x\cos^2 x > \frac{5}{8} \Rightarrow 1-\frac{5}{8} > 3\sin^2 x\cos^2 x$

$\Rightarrow \frac{3}{8} > 3\sin^2 x\cos^2 x \Rightarrow 1-2\sin^2 2x > 0 \Rightarrow \cos 4x > 0 \Rightarrow 4x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \Rightarrow 4x \in \left(2\pi r-\frac{\pi}{2}, 2\pi r+\frac{\pi}{2}\right)$

$x \in \left(\frac{n\pi}{2}-\frac{\pi}{8}, \frac{n\pi}{2}+\frac{\pi}{8}\right) n \in I$ now verify.

Que. 8. (A,B,D)

Que. 9. (B,C,D) **(A)** 1 **(B)** 3 **(C)** $\frac{\sin 24^\circ\cos 60^\circ - \cos 24^\circ \sin 6^\circ}{\sin 21^\circ \cos 39^\circ - \sin 39^\circ \cos 21^\circ} = \frac{\sin\left(18^\circ\right)}{\sin\left(-18^\circ\right)} = -1$ **(D)** -1

Que. 10. (A,C) **(A)** $\frac{\cos^2\alpha - \sin\alpha}{\sin\alpha\cos\alpha} = 2\cot 2\alpha$

Que. 11. B,D. $\tan(\alpha+\beta) = \frac{15}{8}$ and $\tan\gamma = \frac{8}{15}$ $\therefore$ $\alpha+\beta+\gamma = \frac{\pi}{2}$ $\Rightarrow$ (B) and (D)

Que. 12. A,B,C. **(A).** $\tan\alpha = \frac{1-\cos 2\theta}{\sin 2\theta} \Rightarrow$ (A) is correct.

(B). $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}; \cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}; \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} \Rightarrow$ (B) is correct.

(C). $\tan 3\theta = \frac{\sin\theta}{\cos 3\theta} \Rightarrow$ (C) is correct.

(D). $\sin\theta = \frac{1}{3}$ which is rational but $\cos 3\theta = \cos\theta\left(4\cos^2\theta - 3\right)$ which is irrational $\Rightarrow$ (D) is correct.

Que. 13. A,B,C,D. $3\theta = n\pi + (-1)^n(3\alpha)$ $\therefore$ $3\theta = 3\alpha$; $3\theta = \pi - 3\alpha$; $3\theta = -\pi - 3\alpha$ or $3\theta = 2\pi + 3\alpha$; $3\theta = -2\pi + 3\alpha$

Hence $\theta = \alpha; \theta = \frac{\pi}{3} - \alpha;$ $\theta = -\left(\frac{\pi}{3}+\alpha\right);$ $\theta = \left(\frac{2\pi}{3}+\alpha\right); \theta = -\frac{2\pi}{3} + 3\alpha$

$\Rightarrow$ $\cos\theta = \cos\left(\frac{\pi}{3} \pm \alpha\right)$ or $\cos\theta = \cos\left(\frac{2\pi}{3} \pm \alpha\right) \Rightarrow$ (A), (B), (C) and (D) all are correct.

Que. 14. A,B. Given quadratic equaton is an identity $\therefore \operatorname{cosec}^2\theta = 4$ and $\cot\theta = -\sqrt{3}$ $\Rightarrow$ $\operatorname{cosec}\theta = 2$ or -2 and $\tan\theta = -\frac{1}{\sqrt{3}}$ $\Rightarrow$ $\theta = \frac{5\pi}{6}$ or $\frac{11\pi}{6}$

Que. 15. A,B,C. Making quadrtic in sine from $a\cos\theta + b\sin\theta + c$, we get

$\left(a^2+b^2\right)\sin^2\theta - 2bc\sin + c^2 - a^2 = 0 <^{\alpha}_{\beta}$ .... (1)

$\Rightarrow \sin\alpha+\sin\beta=\dfrac{2bc}{a^2+b^2}\Rightarrow$ (A) is correct $\Rightarrow \sin\alpha+\sin\beta=\dfrac{c^2-a^2}{a^2+b^2}\Rightarrow$ (B) is correct

Making quadratic equation in cos, we get (changing a and b)

$$\left(a^2+b^2\right)\cos^2\theta-2ac\cos\theta+c^2-b^2= \begin{cases}\alpha\\ \beta\end{cases}$$

$\Rightarrow \cos\alpha+\cos\beta=\dfrac{2bc}{a^2+b^2}\Rightarrow$ (C) is correct $\Rightarrow \cos\alpha+\cos\beta=\dfrac{c^2-b^2}{a^2+b^2}\Rightarrow$ (D) is correct

Que. 16. B,D. $\sum\cos 3A=1\Rightarrow \sin\dfrac{3A}{2}.\sin\dfrac{3B}{2}.\sin\dfrac{3C}{2}=0\ \therefore\ A=\dfrac{2\pi}{3}$ or $B=\dfrac{2\pi}{3}$ or $C=\dfrac{2\pi}{3}\Rightarrow$ (B)

alos $r=(s-a)\tan\dfrac{A}{2}$ or $(s-b)\tan\dfrac{B}{2}$ or $(s-c)\tan\dfrac{C}{2}$

$r=\sqrt{3}(s-a)$ or $\sqrt{3}(s-b)$ or $\sqrt{3}(s-c)\ \Rightarrow$ (D)

Que. 17. A,D. $\dfrac{\sqrt{3}-1}{2\sqrt{2}\sin x}+\dfrac{\sqrt{3}+1}{2\sqrt{2}\cos x}=2\Rightarrow \sin\dfrac{\pi}{12}\cos x+\cos\dfrac{\pi}{12}\sin x=\sin 2x\Rightarrow \sin 2x=\sin\left(x+\dfrac{\pi}{12}\right)$

$\therefore\ 2x=x+\dfrac{\pi}{12}$ or $2x=\pi-x-\dfrac{\pi}{12}\ \Rightarrow x=\dfrac{\pi}{12}$ or $3x=\dfrac{11\pi}{12}\ \ \therefore\ x=\dfrac{\pi}{12}$ or $\dfrac{11\pi}{36}\Rightarrow$ A,C.

Que. 18. A. Square and adding $9+16+24\sin(A+B)=37\Rightarrow 24\sin(A+B)=12\Rightarrow \sin(A+B)=\dfrac{1}{2}$

$\Rightarrow \sin C=\dfrac{1}{2};\ C=30^o$ Or 150^o if $C=150^o$ then even of B = 0 and $A=30^o$ the quantity $3\sin A+4\cos B\ \Rightarrow 3.\dfrac{1}{2}+4=5\dfrac{1}{2}<6$ hence $C=150^o$ is not possible $\Rightarrow \angle C=30^o$ only

Que. 19. A,B,C,D. $\cos 3\theta=\cos 3\alpha$ put $n=0,1\Rightarrow 3\theta=2n\pi\pm 3\alpha$

$\therefore\ 3\theta=3\alpha$ or -3α or $2\pi+3\alpha$ or $2\pi-3\alpha$

$\theta=\alpha$ or $-\alpha$ or $\dfrac{2\pi}{3}+\alpha$ or $\dfrac{2\pi}{3}-\alpha\ \Rightarrow$ **(A), (C), (D)** are correct.

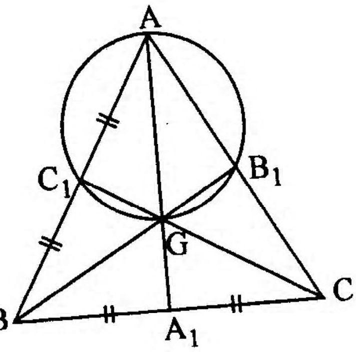
if $n=-1\Rightarrow 3\theta=-2\pi\pm 3\alpha\Rightarrow \theta=-\dfrac{2\pi}{3}\pm\alpha$

$$\sin\theta=\sin\left(-\frac{2\pi}{3}\pm\alpha\right)=-\sin\left(\frac{2\pi}{3}\pm\alpha\right)=-\sin\left(\pi-\frac{\pi}{3}\pm\alpha\right)=-\sin\left(\pi-\left(\frac{\pi}{3}\pm\alpha\right)\right)=-\sin\left(\frac{\pi}{3}\pm\alpha\right)$$

hence (B) is not correct.

Que. 20. B, C, D use power of point B w.r.t. the circle passing through AC_1GB_1

i.e. $BC_1\times BA=BG\times BB_1\Rightarrow \dfrac{c}{2}\times c=\dfrac{2}{3}BB_1\times BB_1\Rightarrow \dfrac{c^2}{3}\times\dfrac{2}{3}(m_b)^2\Rightarrow \dfrac{c^2}{3}=\dfrac{2}{3}\left(\dfrac{2c^2+2a^2-b^2}{4}\right)$



$\Rightarrow \quad 2a^2 = b^2 + c^2$ Ans. $\quad \Rightarrow \quad B, C, D$ are the answers.

Match Matrix Type

Que. 1. A - R. B - S. C- P. D - Q.

A. $\cos^2 2x - \sin^2 x = 0 \Rightarrow \cos 3x.\cos x = 0 \Rightarrow \cos 3x = 0$ or $\cos x = 0 \Rightarrow 3x = (2n-1)\frac{\pi}{2}$

$x = (2n-1)\frac{\pi}{6}$ or $x = (2n-1)\frac{\pi}{2}$ hence general solution is $(2n-1)\frac{\pi}{6}$ as $(2n-1)\frac{\pi}{2}$ is contained in $(2n-1)\frac{\pi}{6} \Rightarrow (R)$

B. $\cos x + \sqrt{3}\sin x = \sqrt{3} \Rightarrow \frac{\cos x}{2} + \frac{\sqrt{3}}{2}\sin x = \frac{\sqrt{3}}{2} \Rightarrow \cos\left(x - \frac{\pi}{3}\right) = \cos\frac{\pi}{6} \Rightarrow x - \frac{\pi}{3} = 2n\pi \pm \frac{\pi}{6}$

$\therefore \quad x - \frac{\pi}{3} = 2n\pi + \frac{\pi}{6}$ or $2n\pi - \frac{\pi}{6} \Rightarrow x = 2n\pi + \frac{\pi}{2}$ or $x = 2n\pi + \frac{\pi}{6} \Rightarrow (S)$

C. $\sqrt{3}\tan^2 x - \left(\sqrt{3}+1\right)\tan x + 1 = 0 \Rightarrow \sqrt{3}\tan x\left(\tan x - 1\right) - \left(\tan x - 1\right) = 0 \Rightarrow \left(\tan x - 1\right)\left(\sqrt{3}\tan x - 1\right) = 0$

$$\left.\begin{array}{lll} \therefore \quad \tan x - 1 & \Rightarrow & x = n\pi + \frac{\pi}{4} \\ \text{or} \quad \tan x = \frac{1}{\sqrt{3}} & \Rightarrow & x = n\pi + \frac{\pi}{6} \end{array}\right] \Rightarrow (P)$$

D. $\tan 3x - \tan 2x - \tan x = 0$ or $\tan x.\tan 2x.\tan 3x = 0 \Rightarrow x = n\pi$ or $\frac{n\pi}{2}$(rejected) or $\frac{n\pi}{3}$

$\Rightarrow$ general solution $\frac{n\pi}{3}$ are $n\pi$ is contained in $\frac{n\pi}{3} \quad \Rightarrow \quad (Q)$

Subjective Type (Up to 4 digit)

Que. 1. $\frac{\pi}{17} = \theta; 17\theta = \pi \Rightarrow 2\cos\theta.\cos 9\theta + \cos 7\theta + \cos 9\theta \Rightarrow \cos(100)\theta + \cos 8\theta + \cos 7\theta + \cos 9\theta = 0$

Sum of cosines of supplymentary angles is zero.

Que. 2 $\tan\alpha$

$\frac{2\sin\alpha\cos\beta - 2\sin\alpha}{2\cos\alpha\cos\beta - 2\cos\alpha} = \frac{2\sin\alpha(\cos\beta - 1)}{2\cos\alpha(\cos\beta - 1)} = \tan\alpha$ Hence proved.

Que. 3. Multiple numerator and denominator by 2

$$\therefore \quad \frac{\cos\theta-\cos3\theta+\cos3\theta-\cos9\theta+\cos9\theta-\cos17\theta}{\sin3\theta-\sin\theta+\sin9\theta-\sin3\theta+\sin17\theta-\sin9\theta}$$

$$=\frac{\cos\theta-\cos17\theta}{\sin17\theta--\sin\theta}=\frac{2\sin9\theta\sin4\theta}{2\sin4\theta\cos9\theta}=\tan9\theta=\tan k\theta \Rightarrow \quad k=9.$$

Que. 4. $\frac{3\sqrt{2}}{8};\frac{7\sqrt{3}}{16}$

$$y=\left(\cos\frac{x}{2}\right)^8-\left(\sin\frac{x}{2}\right)^8=\left(\cos^4\frac{x}{2}-\sin^4\frac{x}{2}\right)\left(\cos^4\frac{x}{2}+\sin^4\frac{x}{2}\right)=\cos x\left(1-2\sin^2\frac{x}{2}\cos^2\frac{x}{2}\right)$$

$$=\cos x\left(1-\frac{1}{2}\sin^2 x\right)$$

(i) $$y\left(\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}\left(1-\frac{1}{2}.\frac{1}{2}\right)=\frac{1}{\sqrt{2}}.\frac{3}{4}=\frac{3}{4\sqrt{2}}=\frac{3\sqrt{2}}{8}.$$

(ii) $$y\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}\left(1-\frac{1}{2};\frac{1}{4}\right)=\frac{7\sqrt{3}}{16}$$

Que. 5. (229) $$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}\Rightarrow a=4k, b=5k, c=6k \ \therefore\ \cos A=\frac{5^2+6^2-4^2}{2.5.6}=\frac{3}{4};$$

$$\cos B\frac{4^2+6^2-5^2}{2.5.6}=\frac{9}{16};\cos C=\frac{4^2+5^2-6^2}{2.4.5}=\frac{1}{8}\text{ hence }\frac{\cos A}{3/4}=\frac{\cos B}{9/16}=\frac{\cos C}{1/8}$$

deividing by 16 $\frac{\cos A}{12}=\frac{\cos B}{9}=\frac{\cos C}{2}\ \therefore\ x=12, y=9$ and $z=2.$

Que. 6. $A=3; B=1; C=-2; D=2\Rightarrow 3+1-(-2)\div2=5.$

Que. 7. $E=\left(\cos^2x+\sin^2x\right)^2-2\sin^2x\cos^2x-k^2\left(\cos^2x-\sin^2x\right)^2$

$=1-2-\sin^2x\cos^2-k^2\left[\left(\cos^2x+\sin^2x\right)-4\sin^2x\cos^2x\right]=\left(1-k^2\right)-2\sin^2x\cos^2x\left(1-2k^2\right)$ for this to be independent of x, $1-2k^2=0\Rightarrow k=\frac{1}{\sqrt{2}}$ or $-\frac{1}{\sqrt{2}}$ **Note :** The value of expression for this value of k is $\frac{1}{2}$.

Que. 8. $$\cot\frac{\pi}{24}=\cos7.5^\circ=\frac{1+\cos15^\circ}{\sin15^\circ}=\frac{1+\frac{\sqrt{6}+\sqrt{2}}{4}}{\frac{\sqrt{6}-\sqrt{2}}{4}}=\frac{4+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}=\frac{\left(4+\sqrt{6}+\sqrt{2}\right)\left(\sqrt{6}+\sqrt{2}\right)}{4}$$

$$=\frac{4\left(\sqrt{6}+\sqrt{2}\right)+\left(8+4\sqrt{3}\right)}{4}=\sqrt{6}+\sqrt{2}+2+\sqrt{3}=\sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}\quad\therefore\ p+q+r+s=15.$$

Que. 9. $$L=\frac{-2\cos4\theta\sin2\theta+2\sin4\theta\sin2\theta}{\sin4\theta-\cos4\theta}=\frac{2\sin2\theta\left(\sin4\theta-\cos4\theta\right)}{\sin4\theta-\cos4\theta}=2\sin2\theta$$

If $\theta = 27^o$, $L = 2\sin 54^o == 2\cos 36^o \Rightarrow L = \dfrac{\sqrt{5}+1}{2}$

$$M = \frac{\tan x \tan 2x}{\tan 2x - \tan x} = \frac{\tan x.\dfrac{2\tan x}{1-\tan^2 x}}{\dfrac{2\tan x}{1-\tan^2 x} - \tan x} = \frac{2\tan x}{2-\left(1-\tan^2 x\right)} = \frac{2\tan x}{1+\tan^2 x} = \sin 2x$$

when $x = 9^o, M = \sin 18^o \Rightarrow M = \dfrac{\sqrt{5}-1}{4}.$

$$N = \frac{1-\cos 4\alpha}{\sec^2 2\alpha - 1} + \frac{1+\cos 4\alpha}{\operatorname{cosec}^2 2\alpha - 1} = \frac{2\sin^2 2\alpha.\cos^2 2\alpha}{\left(1-\cos^2 2\alpha\right)} + \frac{2\cos^2 2\alpha.\sin^2 2\alpha}{\left(1-\sin^2 2\alpha\right)} = 2\left(\cos^2 2\alpha + \sin^2 2\alpha\right) \Rightarrow N = 2.$$

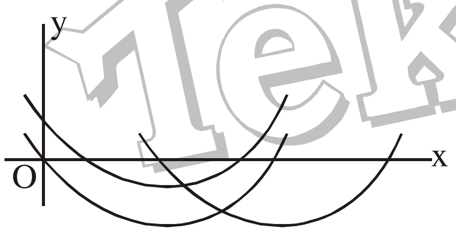
$$\therefore\ \ LMN = \left(\frac{\sqrt{5}+1}{2}\right)\left(\frac{\sqrt{5}-1}{4}\right)(2) = 1.$$

Que. 10. $\cos(\alpha+\beta) = \dfrac{4}{5} \Rightarrow \tan(\alpha+\beta) = \dfrac{3}{4} \Rightarrow \sin(\alpha-\beta) = \dfrac{5}{13} \Rightarrow \tan(\alpha-\beta) = \dfrac{5}{12}$ now $2\beta = (\alpha+\beta) - (\alpha-\beta)$

$$\tan 2\beta = \frac{\tan(\alpha+\beta) - \tan(\alpha-\beta)}{1+\tan(\alpha+\beta).\tan(\alpha-\beta)} = \frac{\left(\dfrac{3}{4} - \dfrac{5}{12}\right)}{1+\dfrac{3}{4}.\dfrac{5}{12}} = \frac{16}{63}.$$

Que. 11. $2\left(1-\sin^2 x\right)^2 - \sin^4 x + k = 0$ put $\sin^2 x = t$, $t \in [0,1] \Rightarrow 2(1-t)^2 - t^2 + k = 0 \Rightarrow t^2 - 4t + k + 2 = 0$

since sum of the roots is 4 $\Rightarrow$ one root in (0,1) and other greater then 1 as shown



now $\quad f(0) \ge 0$ and $f(1) \le 0 \Rightarrow k+2 \le 0$ and $k-1 \ge 0 \Rightarrow k \in [-2,1].$

<u>Alternatively :</u> $2\cos^4 x - \sin^4 x + k = 0 \Rightarrow \cos^4 x + \left(\cos^4 x - \sin^4 x\right) + k = 0 \Rightarrow \cos^4 x + \cos 2x + k = 0$

$$\left(\frac{1+\cos 2x}{2}\right)^2 + \cos 2x + k = 0 \Rightarrow 1+\cos^2 2x + 6\cos 2x + 4k = 0 \quad .....(A) \Rightarrow \cos 2x = t \Rightarrow t^2 + 6t + 1 + 4k = 0$$

$$\Rightarrow (t+3)^2 = 8-4k \Rightarrow (t+3)^2_{max.} = 16 \Rightarrow (t+3)^2_{min.} = 4 \therefore 4 \le 8-4k \le 16 \Rightarrow -4 \le -4k \le 18 \Rightarrow 1 \ge k \ge -2$$

<u>Alternatively :</u> After step (A) $\quad \cos 2x = \dfrac{-6 \pm \sqrt{36-16k-4}}{2} = \dfrac{-6 \pm \sqrt{32-16k}}{2}$

$\cos 2x = -3 + 2\sqrt{2-k}$ or $-3 - 2\sqrt{2-k}$ (rejected, think !)

$$\Rightarrow -1 \le -3 + 2\sqrt{2-k} \le 1 \Rightarrow 2 \le 2\sqrt{2-k} \le 4 \Rightarrow 1 \le \sqrt{2-k} \le 2 \Rightarrow 1 \le (2-k) \le 4 \Rightarrow -1 \le -k \le 2 \Rightarrow 1 \ge k \ge -2$$

Que. 12. Let $a = \sin\theta; b = \sin\left(\theta + \dfrac{2\pi}{3}\right); c = \sin\left(\theta + \dfrac{4\pi}{3}\right)$ Hence, $a+b+c = \sin\theta + \sin\left(\theta + \dfrac{2\pi}{3}\right) + \sin\left(\theta + \dfrac{4\pi}{3}\right)$

$= \sin\theta + \sin\left(\frac{\pi}{3}-\theta\right) - \sin\left(\frac{\pi}{3}+\theta\right)$ use : $\left[\sin\left(\pi+\frac{2\pi}{3}\right) = \sin\left(\pi - \left(\theta+\frac{2\pi}{3}\right)\right) = \sin\left(\frac{\pi}{3}-\theta\right)\right]$ (using C-D)

$= \sin\theta - 2\cos\frac{\pi}{3}\sin\theta = \sin\theta - \sin\theta = 0$ since a + b + c = 0 hence $a^3+b^3+c^3 = abc$

$\therefore\ \sin^3\theta + \sin^3\left(\theta+\frac{2\pi}{3}\right) + \sin^3\left(\theta+\frac{4\pi}{3}\right) = -3\sin\theta\sin\left(\frac{\pi}{3}-\theta\right)\sin\left(\frac{\pi}{3}+\theta\right) = -3\sin\theta\left(\sin^2\frac{\pi}{3} - \sin^2\theta\right)$

$= -\frac{3}{4}\left(3\sin\theta - 4\sin^3\theta\right) = -\frac{3}{4}\sin 3\theta.$ H.P.

Que. 13. $T_r = \frac{\sin 2^{r-1}}{\cos 2^{r-1}.\cos 2^r} = \frac{\sin\left(2^r - 2^{r-1}\right)}{\cos 2^{r-1}.\cos 2^r} = \frac{\sin 2^r \cos^{r-1} - \cos 2^r \sin 2^{r-1}}{\cos 2^{r-1}.\cos 2^r} = \tan 2^r - \tan 2^{r-1}$

$\therefore$ $\text{Sum} = \sum_{r=1}^{n}\left(\tan 2^r - \tan 2^{r-1}\right) = \tan 2 - \tan 1 + \tan 2^2 - \tan 2 + \tan 2^3 - \tan 2^2 + ... + \tan 2^n - \tan 2^{n-1}$

$\text{Sum} = \tan 2^n - \tan(1)$

Que. 14. (0250.00) Dividing by $\cos^4\alpha$ $15\tan^4\alpha + 10 = 6\sec^4\alpha \Rightarrow 15\tan^4\alpha + 10 = 6(1+\tan^2\alpha)^2$

$\Rightarrow 9\tan^4\alpha - 12\tan^2\alpha + 4 = 0 \Rightarrow \left(3\tan^2\alpha - 2\right)^2 = 0 \Rightarrow \tan^2\alpha = \frac{2}{3}$

Now $8\operatorname{cosec}^6\alpha + 27\sec^6\alpha \Rightarrow 8\left(1+\cot^2\alpha\right)^3 + 27\left(1+\tan^2\alpha\right)^3 \Rightarrow 8\left(1+\frac{3}{2}\right)^3 + 27\left(1+\frac{2}{3}\right) \Rightarrow 125+125 = 250.$

Que. 15. (2008). Subtract
$$\begin{array}{l} x+\sin y = 2008 \\ x + 2008\cos y = 2007 \\ \hline \sin y - 2008\cos y = 1 \end{array}$$
$\Rightarrow \sin y = 1 + 2008\cos y$ This is possible only if $\cos y = 0$

$\therefore\ y = \frac{\pi}{2}$ and $x = 2007 \Rightarrow x + y = 2007 + \frac{\pi}{2} \Rightarrow [x+y] = 2008.$

Que. 16. (92) $2\sin x\sin 1 = \cos(x-1) - \cos(x+1)$

$\therefore\ S\sum_{x=2}^{44}\left[\cos(x-=1) - \cos(x+1)\right]\left[1+\sec(x-1).\sec(x+1)\right]$

$= \sum_{x=2}^{44}\left(\cos(x-1) + \frac{1}{\cos(x+1)} - \cos(x-1) - \frac{1}{\cos(x-1)}\right)$ $= \sum_{x=2}^{44}\left(\frac{1-\cos^2(x+1)}{\cos(x+1)} - \frac{1-\cos^2(x-1)}{\cos(x-1)}\right)$

$= \sum_{x=2}^{44}\left(\frac{\sin^2(x+1)}{\cos(x+1)} - \frac{\sin^2(x-1)}{\cos(x-1)}\right) \therefore$ $S = \frac{\sin^2 3}{\cos 3} - \frac{\sin^2 1}{\cos 1} + \frac{\sin^2 4}{\cos 4} - \frac{\sin^2 2}{\cos 2} + \frac{\sin^2 4}{\cos 5} - \frac{\sin^2 3}{\cos 3} .... + \frac{\sin^2 44}{\cos 44} - \frac{\sin^2 42}{\cos 42}$

$+ \frac{\sin^2 44}{\cos 44} - \frac{\sin^2 42}{\cos 42} + \frac{\sin^2 45}{\cos 45} - \frac{\sin^2 43}{\cos 43} \Rightarrow S = \frac{\sin^2 44}{\cos 44} + \frac{\sin^2 45}{\cos 45} - \frac{\sin^2 1}{\cos 1} - \frac{\sin^2 2}{\cos 2}$

$S = -\frac{\sin^2 1}{\cos 1} + \frac{\sin^2 44}{\cos 44} - \frac{\sin^2 2}{\cos 2} + \frac{\sin^2 45}{\cos 45}$ which resembles 4 term of $\sum_{n=1}^{4}(-1)^n\frac{\phi^2(\theta_n)}{\psi(\theta_n)}$

$\therefore$ $\theta_1 + \theta_2 + \theta_3 + \theta_4 = 1 + 2 + 44 + 45 = 92.$

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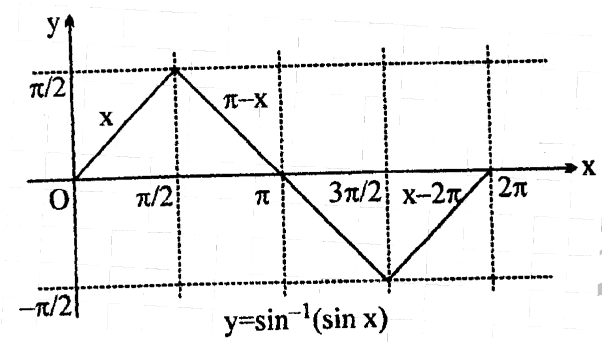
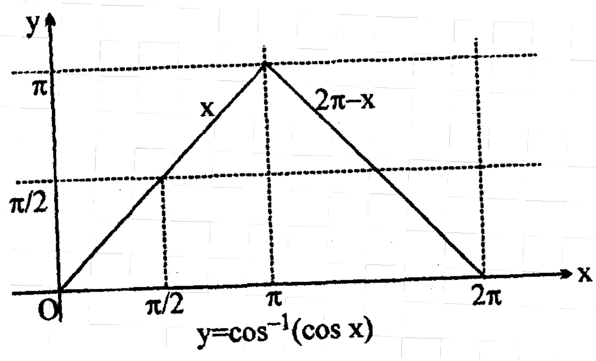
Que. 17. (3388) $f(x) = \cos^{-1}(\sin x) = \frac{\pi}{2} - \sin^{-1}(\sin x)$ ....(1) and

$g(x) = \sin^{-1}(\cos x) = \frac{\pi}{2} - \cos^{-1}(\cos x)$ .... (2) both f(x) and g(x) are periodic with period

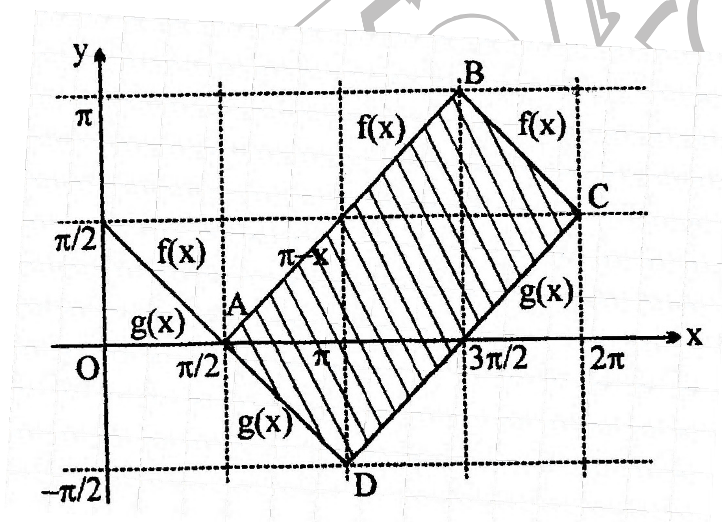
2π. The graphs of $\sin^{-1}(\sin x)$ and $\cos^{-1}(\cos x)$ as follows





hence $f(x) = \begin{cases} \frac{\pi}{2} - x & \text{if } x \in [0, \pi/2] \\ x - \frac{\pi}{2} & \text{if } \frac{\pi}{2} < x \le \frac{3\pi}{2} \\ \frac{5\pi}{2} - x & \text{if } \frac{3\pi}{2} < x \le 2\pi \end{cases}$; $g(x) \begin{cases} \frac{\pi}{2} - x & \text{if } x \in [0, \pi] \\ x - \frac{3\pi}{2} & \text{if } x \in [\pi, 2\pi] \end{cases}$

Now



Area enclosed between the two curves is the area of the rectangle ABCD in one pepriod.

Now now $AD = \sqrt{\frac{\pi^2}{4} + \frac{\pi^2}{4}} = \sqrt{\frac{\pi^2}{2}} = \frac{\pi}{\sqrt{2}}$ $\Rightarrow$ $\frac{\pi}{\sqrt{2}}.\sqrt{2}[= \pi^2$

and $DC = \sqrt{2}\pi$

and $DC = \sqrt{2}\pi$ $\therefore$ $A = 7.\pi^2 = 7\pi^2$ $(\text{in}[-7\pi, 7\pi])$ $49A = 49.7\pi^2 = 7.22.22 = 7.484 = 3388.$