THIS FILE CONTAINS

ALGEBRA

(COLLECTION # 2)

Very Important Guessing Questions For IIT JEE 2011 With Detail Solution

<u>Junior Students Can Keep It Safe For Future IIT-JEEs</u>

- ***Complex Number***
- ***Theory of Equation (Quadratic Equation)***
- ***Sequence and Series***
- ***Permutation and Combination***
- ***Determinants and Matrices***
- ***Logarithm and Their Properties***
- ***Probability***
- ***Binomial Theorem***

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For Collection # 2

- ***Same As Above***

TOPIC = ALGEBRA

SINGLE CORRECT TYPE

Q. 1 If $\frac{2}{1!13!}+\frac{2}{3!11!}+\frac{2}{5!9!}+\frac{2}{7!7!}=\frac{2^p}{q!}$ where *p* and *q* are relative prime then $(p+q)$ is: (codeV3T1PAQ8)

(A) 26 (B) 27 (C) 28 (D) 30

Q. 2 The expansion of $(1+x)^n$ has 3 consecutive terms with coefficients in the ratio 1:2:3 and can be written in the form ${}^nC_k : {}^nC_{k+1} : {}^nC_{k+2}$. The sum of all possible values of $(n+k)$ is(codeV3T4PAQ4)

(A) 18 (B) 21 (C) 28 (D) 32

Q. 3 Number of different terms in the sum $(1+x)^{2009}+(1+x^2)^{2008}+(1+x^3)^{2007}$, is (codeV3T10PAQ7)

(A) 3683 (B) 4007 (C) 4017 (D) 4352

Q. 4 Number of ordered pairs(s) (a,b) of real numbers such that $(a+ib)^{2008}=a-ib$ holds good, is

(A) 2008 (B) 2009 (C) 2010 (D) 1 (codeV3T5PAQ3)

Q. 5 If z is a complex number satisfying the equation $|z+i|+|z-i|=8$, on the complex plane then maximum value of $|z|$ is (codeV3T5PAQ7)

(A) 2 (B) 4 (C) 6 (D) 8

Q. 6 Given $f(z)=$ the real part of a complex number z. For example, $f(3-4i)=3$. If $a\in N$, $n\in N$ then the value of $\sum_{n=1}^{6a}\log_2\left|f\left(\left(1+i\sqrt{3}\right)^n\right)\right|$ has the value equal to (codeV3T5PAQ11)

(A) $18a^2+9a$ (B) $18a^2+7a$ (C) $18a^2-3a$ (D) $18a^2-a$

Q. 7 It is given that complex numbers z_1 and z_2 satisfy $|z_1|=2$ and $|z_2|=3$. If the included angle of their corresponding vectors is $60°$ then $\left|\frac{z_1+z_2}{z_1-z_2}\right|$ can be expressed as $\frac{\sqrt{N}}{7}$ where N is natural number then N equals (codeV3T5PAQ18)

(A) 126 (B) 119 (C) 133 (D) 19

Q. 8 The solution of the equation $z^4+4z^3i-6z^2-4zi-i=0$ are the vertices of a convex polygon in the complex plane. The area of the polygon is (codeV3T7PAQ3)

(A) $2^{3/4}$ (B) $2^{3/2}$ (C) $2^{5/4}$ (D) 2

Q. 9 Let z denotes a complex number and defined $S=\left\{\frac{1}{1-z}:|z|=1 \text{ and } z\neq 1\right\}$. Which of the following best describes the set S, when S is interpreted geometrically as a set of point in the complex plane. (codeV3T8PAQ5)

(A) S is a straight line parallel to the imaginary axis.
(B) S is a straight line parallel to the real axis.
(C) S is a circle with a single point missing.
(D) S is a branch of hyperbola

Q. 10 All complex number 'z' which satisfy the relation $\big|z-|z+1|\big|=\big|z+|z-1|\big|$ on the complex plane lie on the (codeV3T8PAQ6)

(A) line $y=0$ (B) line $x=0$
(C) circle $x^2+y^2=1$ (D) line $x=0$ or on a line segment joining $(-1,\ 0)$ to $(1,\ 0)$

Q. 11 If z is a complex number then the roots of the equation $(z+1)^4 = 16z^4$ lie on a circle with

(A) Centre $(0, 0)$ and radius $\frac{1}{3}$ (B) Centre $\left(\frac{1}{3}, 0\right)$ and radius $\frac{2}{3}$ (codeV3T10PAQ8)

(C) Centre $(0, 0)$ and radius $\frac{2}{3}$ (D) Centre $\left(\frac{1}{3}, 0\right)$ and radius 1

Q. 12 Let $S(x) = 1 + x - x^2 - x^3 + x^4 + x^5 - x^6 - x^7 + ...$; where $0 < x < 1$ If $S(x) = \frac{\sqrt{2}+1}{2}$ then the value of *x* equal (codeV3T1PAQ2)

(A) $2-\sqrt{2}$ (B) $\sqrt{2}-1$ (C) $\frac{1}{\sqrt{2}}$ (D) $\left(1-\frac{1}{\sqrt{2}}\right)$

Q. 13 Number of value of x satisfying the pair of quadratic equations $x^2 - px + 20 = 0$ and $x^2 - 20x + p = 0$ for some $p \in R$, is (codeV3T1PAQ3)

(A) 1 (B) 2 (C) 3 (D) 4

Q. 14 If quadratic polynomials defined on real coefficients $P(x) = a_1x^2 + 2b_1x + c_1$ and $Q(x) = a_2x^2 + 2b_2x + c_2$ take positive values $\forall\ x \in R$. What can we say for the trinomial $g(x) = a_1a_2x^2 + b_1b_2x + c_1c_2$? (codeV3T8PAQ8)

(A) $g(x)$ takes positive value only (B) $g(x)$ takes negative values only

(C) $g(x)$ can take positive as well as negative values

(D) nothing definite can be said about $g(x)$

Q. 15 Number of 4 digit positive integers if the product of their digits is divisible by 3, is (codeV3T8PAQ7)

(A) 2700 (B) 6628 (C) 7704 (D) 5464

Q. 16 The sum of all the numbers formed from the digit 1, 3, 5, 7, 9 which are smaller than 10,000 if repetition of digit is not allowed, is (codeV3T9PAQ7)

(A) $(28011)S$ (B) $(28041)S$ (C) $(28121)S$ (D) $(29152)S$ where $S = (1+3+5+7+9)$

Q. 17 Three people each flip two fair coins. The probability that exactly two of the people flipped one head and one tail, is (codeV3T4PAQ8)

(A) 1/2 (B) 3/8 (C) 5/8 (D) 3/4

Q. 18 Lot A consists of 1 defective and 5 good article, lot B consists of 2 defective and 4 good articles and lot C has 3 defective and 3 good articles. A mixed lot M is formed by taking 5 from lot A, 3 from lot B and 2 from C. The probability that an article randomly chosen from the mixed lot M is defective, is (codeV3T7PAQ4)

(A) $\frac{17}{60}$ (B) $\frac{15}{60}$ (C) $\frac{13}{60}$ (D) $\frac{19}{60}$

Q. 19 Suppose families always have one, two or three children, with probabilities $\frac{1}{4}, \frac{1}{2}$ and $\frac{1}{4}$ respectively. Assume everyone eventually gets married and has children, the probability of a couple having exactly four grandchildren is (codeV3T9PAQ6)

(A) $\frac{27}{128}$ (B) $\frac{37}{128}$ (C) $\frac{25}{128}$ (D) $\frac{20}{128}$

Q. 20 If $\tan\alpha, \tan\beta$ are the roots of the equation $x^2 + px + q = 0$, then the value of $\sin^2(\alpha+\beta) + p\sin(\alpha+\beta)\cos(\alpha+\beta) + q\cos^2(\alpha+\beta)$ is (codeV3T1PAQ1)

(A) independent of *p* but dependent on q (B) independent of *q* but dependent on *p*

(C) independent of both p and q (D) dependent on both p and q

Q. 21 If the first four terms of an arithmetic sequence are $a, 2a, b$ and $(a-6-b)$ for some numbers a and b, then the sum of the first 100 terms of the sequence is (codeV3T2PAQ3)

(A) -100 (B) 100 (C) 5050 (D) -5050

Q. 22 The value of the sum

$$S=\frac{1}{2}+\frac{1}{6}\left(1^2+2^2\right)+\frac{1}{12}\left(1^2+2^2+3^2\right)+\frac{1}{20}\left(1^2+2^2+3^2+4^2\right)+.....+\frac{1}{3660}\left(1^2+2^2+3^2+.......+60^2\right)$$

(A) 620 (B) 720 (C) 520 (D) 420 (codeV3T10PAQ5)

COMPREHENSION TYPE

Paragraph for question nos. 1 to 3

Consider $\left(1+x+x^2\right)^{2n}=\sum_{r=0}^{4n}a_r x^r$, where $a_0, a_1, ..., a_{4n}$ are real numbers and *n* is a positive integer. (codeV3T2PAQ12to14)

Q. 1 The value of $\sum_{r=0}^{n-1}a_{2r}$, is

(A) $\frac{9^n-2a_{2n}-1}{4}$ (B) $\frac{9^n+2a_{2n}+1}{4}$ (C) $\frac{9^n-2a_{2n}+1}{4}$ (D) $\frac{9^n+2a_{2n}-1}{4}$

Q. 2 The value of $\sum_{r=1}^{n}a_{2r-1}$, is

(A) $\left(\frac{9^n-1}{2}\right)$ (B) $\left(\frac{3^{2n}-1}{4}\right)$ (C) $\left(\frac{3^{2n}+1}{4}\right)$ (D) $\left(\frac{9^n+1}{2}\right)$

Q. 3 The value of a_2 is

(A) ${}^{4n+1}C_2$ (B) ${}^{3n+1}C_2$ (C) ${}^{2n+1}C_2$ (D) ${}^{n+1}C_2$

Paragraph for Question Nos. 4 to 6

Consider the binomial expansion $R=(1+2x)^n=1+f$, where I is the integral part of R and 'f' is the fractional part of $R, n\in N$. Also the sum of the coefficients of R is 6561. (codeV3T4PAQ14to16)

Q. 4 The value of $(n+R-Rt)$ for $x=\frac{1}{\sqrt{2}}$ equals

(A) 7 (B) 8 (C) 9 (D) 10

Q. 5 If i^{th} terms is the greatest term for $x=\frac{1}{2}$, then 'i' equals

(A) 4 (B) 5 (C) 6 (D) 7

Q. 6 If k^{th} terms is having greatest coefficient then sum of all possible value(s) of k is

(A) 6 (B) 7 (C) 11 (D) 13

Paragraph for question nos. 7 to 9

Let P be a point denoting a complex number z on the complex plane. Let Re(z) denotes the real part of z and Im(z) denotes the imaginary part of Z. (codeV3T1PAQ12to14)

Q. 7 If P moves such that $|\operatorname{Re} Z|+|\operatorname{Im} Z|=a\left(a\in R^+\right)$, then the locus of P is

(A) a parallelogram which is not a rhombus. (B) a rhombus which is not a square.
(C) a rectangle which is not a square (D) a square,

Q. 8 The area of the circle inscribed in the region denoted by $|\text{Re Z}|+|\text{Im Z}|=100$ equals

(A) 2500π (B) 5000π (C) 2550π (D) 5050

Q. 9 Number of integral solutions satisfying the inequality $|\text{Re z}|+|\text{Im Z}|<51$ is

(A) 5050 (B) 4901 (C) 5101 (D) 5100

Paragraph for question nos. 10 to 12

A multiple choice test question has five alternative answers, of which only one is correct. If a student has done his home work, then he is sure to identify the correct answer, otherwise, he choose an answer at random. (codeV3T3PAQ17to19)

Let E : denotes the event that a student does his home work with P(E) = P and
F : denotes the event that he answer the question correctly.

Q. 10 If $p=0.75$ the value of $P(E/F)$ equals

(A) $\frac{8}{16}$ (B) $\frac{10}{16}$ (C) $\frac{12}{16}$ (D) $\frac{15}{16}$

Q. 11 The relation $P(E/F)\ge P(E)$ holds good for

(A) all values of p in $[0, 1]$ (B) all values of p in $(0, 1)$ only

(C) all values of p in $[0.5, 1]$ only (D) no value of p

Q. 12 Suppose that each question has n alternative answers of which only one is correct, and p is fixed but not equal to 0 or 1 then $P(E/F)$

(A) decreases as *n* increases for all $p\in(0, 1)$

(B) increases as *n* increases for all $p\in(0, 1)$

(C) remains constant for all $p\in(0, 1)$

(D) decreases if $p\in(0,0.5)$ and increases if $p\in(0.5, 1)$ as n increases.

Paragraph for question nos. 13 to 15

Urn-I contains 5 Red balls and 1 Blue ball, Urn-II contains 2 Red balls and 4 Blue balls.
A fair die is tossed. If it results in even number, balls are repeatedly by drawn one at a time with replacement from urn-I. If it is an odd number, balls are repeatedly by drawn one at a time with replacement from urn-II. Given that the first two draws both have resulted in a blue ball.

Q. 13 Conditional probability that the first two drawns have resulted in blue balls given urn-II is used is (codeV3T6PAQ1)

(A) 1/2 (B) 4/9 (C) 1/3 (D) None

Q. 14 If the probability that the urn-I is being used is p, and q is the corresponding figure for urn-II then (codeV3T6PAQ2)

(A) $q=16p$ (B) $q=4p$ (C) $q=2p$ (D) $q=3p$

Q. 15 The probability of getting a red ball in the third draw, is (codeV3T6PAQ3)

(A) 1/3 (B) 1/2 (C) 37/102 (D) 41/102

Paragraph for question nos. 16 to 18

A bag contains 6 balls of 3 different colours namely white, Green and Red, atleast one ball of each different colour. Assume all possible probability distributions are equally likely. (codeV3T7PAQ14to16)

Q. 16 The probability that the bag contains 2 balls of each colour, is

(A) $\frac{1}{3}$ (B) $\frac{1}{5}$ (C) $\frac{1}{10}$ (D) $\frac{1}{4}$

Q. 17 Three balls are picked up at random from the bag and found to be one of each different colour. The probability that the bag contained 4 Red balls is

(A) $\frac{1}{14}$ (B) $\frac{2}{14}$ (C) $\frac{3}{14}$ (D) $\frac{4}{14}$

Q. 18 Three balls are picked at random from the bag and found to be one of each different colour. The probability that the bag contained equal number of White and Green balls, is

(A) $\frac{4}{14}$ (B) $\frac{3}{14}$ (C) $\frac{2}{14}$ (D) $\frac{5}{14}$

ASSERTION REASON TYPE

Q. 1 Consider four comlex numbers (codeV3T1PAQ17)

$z_1 = 1 + i$; $z_2 = 1 - i$; $z_3 = -1 - i$; and $z_4 = -1 + i$;

Statement-1: z_1, z_2, z_3 and z_4 constitute the vertices of a square on the complex plane.

because Statement-2 : The non zero complex numbers $z, \bar{z}, -z$, and $-\bar{z}$ always constitute the vertices of a square.

Q. 2 Consider the curves on the Argand plane as (codeV3T1PAQ18)

C_1 : amp $z = \frac{\pi}{4}$; C_2 : amp $z = \frac{3\pi}{4}$ and C_3 : amp $(z - 5 - 5i) = \pi$.

Statement-1: Area of the region bounded by the curves C_1, C_2 and C_3 is $\frac{25}{2}$.

because Statement-2 : The boundaries of C_1, C_2 and C_3 constitute a right isosceles triangle.

Q. 3 Consider z_1 and z_2 as two complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$. (codeV3T2PAQ15)

Statement-1: $\text{amp}.z_1 - \text{amp}.z_2 = 0$

because Statement-2 : The complex numbers z_1 and z_2 are collinear with origin.

Q. 4 Let z represent a variable point in complex plane such that $z - z_1$ is real, where z_1 is a fixed point in same plane, then (codeV3T7PAQ8)

Statement-1 : If $z_1 = -2 + i$, then there exits two values of z for which $|z| = 2$

because Statement - 2 : There always exist two values of z such that $|z| = \lambda, \lambda \in R, \lambda \geq |\text{Im}(z_1)|$.

Q. 5 Let z be a complex number such that $|z| = 1$ and both $\text{Re}(z)$ and $\text{Im}(z)$ are rational numbers

Statement-1 : for $n \in Z, |z^{2n} - 1|$ is always rational (codeV3T8PAQ12)

because Statement - 2 : $\sin\theta, \cos\theta \in Q \Rightarrow \sin n\theta, \cos n\theta \in Q \quad \forall n \in N$

Q. 6 Let A be any 3×2 matrix (codeV3T9PAQ18)

Statement-1 : Inverse of AA^T does not exist.

because Statement - 2 : AA^T is a singular matrix.

Q. 7 A and B be 3×3 matrices such that $AB + A + B = 0$ (codeV3T10PAQ15)

Statement-1 : $AB = BA$

because Statement - 2 : $PP^{-1} = I = P^{-1}P$ for every matrix P which is invertible.

Q. 8 Let $f(x) = x^3 + ax^2 + bx + c$ be a cubic polynomial with real coefficients and all real roots. Also $|f(i)| = 1$ where $i = \sqrt{-1}$ (codeV3T3PAQ10)

Statement-1: All 3 roots of $f(x) = 0$ are zero

because Statement-2 : $a + b + c = 0$

Q. 9 **Statement-1:** If a, b, c are not real complex and α, β are the roots of the equation $ax^2 + bx + c = 0$ then $\text{Im}(\alpha\beta) \neq 0$. (codeV3T4PAQ10)

because Statement-2 : A quadratic equation with non real complex coefficient do not have root which are conjugate of each other.

MORE THAN ONE MAY CORRECT TYPE

Q. 1 Equation of a straight line on the complex plane passing through a point P denoting the complex number α and perpendicular to the vector $\overline{OP}$ where 'O' in the origin can be written as (codeV3T3PAQ22)

(A) $\text{lm}\left(\frac{z-\alpha}{\alpha}\right)=0$ (B) $\text{Re}\left(\frac{z-\alpha}{\alpha}\right)=0$ (C) $\text{Re}(\alpha Z)=0$ (D) $\alpha z+\alpha z-2|\alpha|^2=0$

Q. 2 Let A and B be two distinct points denoting the complex numbers α and β respectively. A complex number z lies between A and B where $z\neq\alpha, z\neq\beta$. Which of the following relation(s) hold good? (codeV3T7PAQ22)

(A) $|\alpha-z|+|z-\beta|=|\alpha-\beta|$ (B) $\exists$ a positive real number 't' such that $z=(1-t)\alpha+t\beta$

(C) $\begin{vmatrix} z-\alpha & \bar{z}-\bar{\alpha} \\ \beta-\alpha & \bar{\beta}-\bar{\alpha} \end{vmatrix}=0$ (D) $\begin{vmatrix} z & \bar{z} & 1 \\ \alpha & \bar{\alpha} & 1 \\ \beta & \bar{\beta} & 1 \end{vmatrix}=0$

Q. 3 If $\alpha_1, \alpha_2, \alpha_3, ..., \alpha_{n-1}$ are the imaginary n^{th} roots of unity then the product $\prod_{r=1}^{n-1}(i-\alpha_r)$ (where $i=\sqrt{-1}$) can take the value equal to (codeV3T10PAQ20)

(A) 0 (B) 1 (C) i (D) $(1+i)$

Q. 4 Let $P=\begin{bmatrix} 3 & -5 \\ 7 & -12 \end{bmatrix}$ and $Q=\begin{bmatrix} 12 & -5 \\ 7 & -3 \end{bmatrix}$ then the matrix $(PQ)^{-1}$ is (codeV3T3PAQ21)

(A) nilpotent (B) idempotent (C) involutory (D) symmetric

Q. 5 The p^{th}, $(2p)^{th}$ and $(4p)^{th}$ terms of an A.P. are in G.P. the common ratio of G.P. is(codeV3T2PAQ20)

(A) 2 (B) 1 (C) 4 (D) 1/2

Q. 6 Solution of the inequality $(\log_2 x)^4-\left(\log_{1/2}\frac{x^3}{8}\right)+9\log_2\left(\frac{32}{x^2}\right)<4(\log_{1/2} x)^2$ is $(a, b)\cup(c, d)$ then the correct statement is (codeV3T2PAQ19)

(A) $a=2b$ and $d=2c$ (B) $b=2a$ and $d=2c$

(C) $\log_e d=\log_b a$ (D) there are 4 integers in (c, d)

Q. 7 If the equation $x^2+4+3\cos(ax+b)=2x$ has atleast one solution where $a, b\in[0, 5]$ then the value of $(a+b)$ equal to (codeV3T9PAQ19)

(A) 5π (B) 3π (C) 2π (D) π

Q. 8 Let $S=1+10+10^2+10^3+10^4+10^5$ which of the following number(s) can be divide the sum S?

(A) 37 (B) 13 (C) 11 (D) 21 (codeV3T7PAQ21)

Q. 9 Consider the word D = F R E E W H E E L. Which of the following statement(s) is/are correct (codeV3T6PAQ11)

(A) Number of other ways in which the letters of the word D can be arranged is ${}^9P_5$.

(B) Number of ways is which the letters of the word D can be an arranged in a circle is ${}^8P_4$ distisguishing between clockwise and anticlockwise.

(C) Number of ways is which the letters of the word D can be arranged if vowels and consonts both are in alphabetical order is $\frac{1}{2}\,{}^{10}C_5$

(D) If as many more words are formed as possible using the letters of the word D the number of words which contain the word FEEL is ${}^6P_4$.

Q. 10 A boy has a collection of blue and green marbles. The number of blue marbles belong to the sets $\{2, 3, 4, ...13\}$. If two marbles are chosen simultaneously and at random from his collection, then the probability than they have different colour is ½. Possible number of blue marbles is : (codeV3T9PAQ20)

(A) 2 (B) 3 (C) 6 (D) 10

Q. 11 The p^{th} term T_p of H.P. is $q(p+q)$ and q^{th} term T_q is $p(p+q)$ when $p>2,\ q>2,$ then

(A) $T_{pq} > T_{p+q}$ (B) $T_{p+q} = pq$ (C) $T_{pq} = p+q$ (D) $T_{p+q} = T_{pq}$ (codeV3T1PAQ21)

MATCH THE COLUMNS

Q. 1 Match the equation in z, in **Column-I** with the corresponding values of arg(z) in **Column-II**.

Column-I (equations in z)	**Column-II** (principal value of arg(z)) (codeV3T4PBQ1)
(A) $z^2 - z + 1 = 0$	(P) $-\frac{2\pi}{3}$
(B) $z^2 + z + 1 = 0$	(Q) $-\frac{\pi}{3}$
(C) $2z^2 + 1 + i\sqrt{3} = 0$	(R) $\frac{\pi}{3}$
(D) $2z^2 + 1 - i\sqrt{3} = 0$	(S) $\frac{2\pi}{3}$

Q. 2 6 married couples are present in a birthday party. Number of ways in which four persons are selected if (codeV3T1PBQ1)

Column-I	**Column-II**
(A) they form no pair	(P) 220
(B) they form atleast one pair	(Q) 240
(C) they form fewer than two pairs	(R) 255
	(S) 480

Q. 3 Let $f(x) = \ln x$ and $g(x) = x^2 - 1$ Column-I contains composite functions and column-II contains their domain. Match the entries of column-I with their corresponding answer is column-II. (codeV3T1PBQ2)

Column-I	**Column-II**
(A) fog	(P) $(1, \infty)$
(B) gof	(Q) $(-\infty, \infty)$
(C) fof	(R) $(-\infty, -1) \cup (1, \infty)$
(D) gog	(S) $(0, \infty)$

SOLUTION
SINGLE CORRECT TYPE

Q. 1 (B)

Sol $\frac{1}{14!}\left(2^{14}C_1+2^{14}C_3+2^{14}C_5+2^{14}C_7\right)=\frac{1}{14!}\left({}^{14}C_1+{}^{14}C_3+{}^{14}C_5+{}^{14}C_7+{}^{14}C_9+{}^{14}C_{11}+{}^{14}C_{13}\right)=\frac{1}{14!}.2^{14-1}=\frac{2^{13}}{14!}$]

Q. 2 (A) Sol $\frac{{}^nC_k}{{}^nC_{k+1}}=\frac{1}{2}$ $\Rightarrow$ $\frac{n!}{k!(n-k)!}.\frac{(k+1)!(n-k-1)!}{n!}=\frac{1}{2}$ or $\frac{k+1}{n-k}=\frac{1}{2}$

$2k+2=n-k$ $\Rightarrow$ $n-3k=2$ …(1)

|||ly $\frac{{}^nC_{k+1}}{{}^nC_{k+2}}=\frac{2}{3}$ $\Rightarrow$ $\frac{n!}{(k+1)!(n-k-1)!}.\frac{(k+2)!(n-k-2)!}{n!}=\frac{2}{3}$

$\frac{k+2}{n-k-1}=\frac{2}{3}$ $\Rightarrow$ $3k+6=2n-2k-2$ $\Rightarrow$ $2n-5k=8$ …(2)

From (1) and (2) $n=14$ and $k=4$ $\Rightarrow$ $\therefore$ $n+k=18$ Ans.]

Q. 3 (C) Sol Number of terms in $(1+x)^{2009}=2010$ …..(1)

+ addition terms in $\left(1+x^2\right)^{2008}=x^{2010}+x^{2012}+.........+x^{4016}=1004$ ……(2)

+ addition terms in $\left(1+x^2\right)^{2007}=x^{2010}+x^{2013}+.........+x^{4014}+......+x^{6021}=1338$ ……(3)

- (common to 2 and 3) $=x^{2010}+x^{2016}+.........+x^{4014})=335$

Hence total $=2010+1004+1338-335=4352-335=4017$ **Ans.**

Alternatively :

$$n(A\cup B\cup C)=n(A)+n(B)+n(C)-\left(n(A\cap B)+n(B\cap C)+n(C\cap A)+n(A\cap B\cap C)\right)$$

$$(2010+2009+2008)-\underbrace{1005}_{A\cap B}+\underbrace{670}_{B\cap C}+\underbrace{670}_{C\cap A}+335=4017$$ Ans.]

Q. 4 (C) Sol Let $z=a+ib$ $\Rightarrow$ $\bar{z}=a-ib$

Hence we have $\therefore$ $|z|^{2008}=|\bar{z}|=|z|$

$|z|\left[|z|^{2007}-1\right]=0 \Rightarrow |z|=0$ or $|z|=1$; if $|z|=0$ $\Rightarrow$ $z=0$ $\Rightarrow$ $(0, 0)$

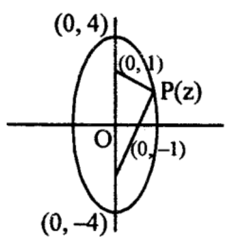
if $|z|=1$ $z^{2009}=z\bar{z}=|z|^2=1\Rightarrow$ 2009 value of z $\Rightarrow$ Total = 2010 Ans.]

Q. 5 (B) Sol

If $|z+i|+|z-i|=8$,

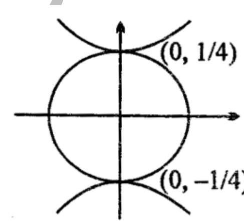
$PF_1+PF_2=8$

$\therefore$ $|z|_{max}=4$ $\Rightarrow$ (B)



]

Q. 6 (D) Sol



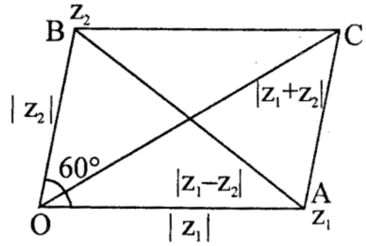
$$\left(1+i\sqrt{3}\right)^n=\left[2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\right]^n=2^n\left(\cos\frac{n\pi}{3}+i\sin\frac{n\pi}{3}\right)$$

$f\left(1+i\sqrt{3}\right)^n=$ real part of $z=2^n\cos\frac{n\pi}{3}$]

$$\therefore \sum_{n=1}^{6a}\log_2\left|2^n\cos\frac{n\pi}{3}\right|=\sum\left(n+\log_2\left|\cos\frac{n\pi}{3}\right|\right)=\frac{6a(6a+1)}{2}+\underbrace{(-1-1+0-1-1+0)}_{\text{a such term}}$$

$=3a(6a+1)-4a=18a^2-a$ **Ans.]**

Q. 7 (C) Sol Using consine rule



$$|z_1+z_2| = \sqrt{|z_1|^2+|z_2|^2-2|z_1||z_2|\cos 120^\circ}$$

$$\sqrt{4+9+2.3} = \sqrt{19}$$

and $|z_1-z_2| = \sqrt{|z_1|^2+|z_2|^2-2|z_1||z_2|\cos 60^\circ}$

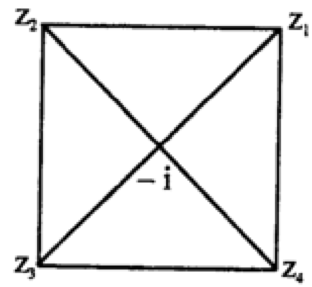
$$= \sqrt{4+9-6} = \sqrt{7}$$

$\therefore \quad \left|\dfrac{z_1+z_2}{z_1+z_2}\right| = \sqrt{\dfrac{19}{7}} = \dfrac{\sqrt{133}}{7} \Rightarrow N = 133$ **Ans.**]

Q. 8 (C) Sol $z^4+4z^3i+6z^2i^2+4zi^3+i^4 = 1+i$

$(z+i)^4 = 1+i \quad \Rightarrow \quad |z+1|^4 = \sqrt{2} \Rightarrow |z+i| = 2^{1/8}$

$|z+i| = 2^{1/8}$



$\text{Area} = \dfrac{d^2}{2} = \dfrac{4|z_1+i|^2}{2}$

$= 2.2^{1/8}.2^{1/8} = 2^{5/4}$ **Ans]**

Q. 9 (A) Sol $W = \dfrac{1}{1-z} = \dfrac{1}{(1-\cos\theta)-i\sin\theta} = \dfrac{1}{2\cos^2(\theta/2)-2i\sin(\theta/2)\cos(\theta/2)}$

$$= \frac{1}{-2i\sin(\theta/2)\left[\cos(\theta/2)+i\sin(\theta/2)\right]} = \frac{\cos(\theta/2)-i\sin(\theta/2)}{-2i\sin(\theta/2)} = \frac{1}{2}+\frac{1}{2}\cot\frac{\theta}{2}i$$

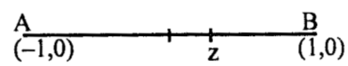
Hence $\operatorname{Re}(w) = \dfrac{1}{2} \Rightarrow \therefore$ w moved on the line $2x-1=0$ parallel to y-axis.]

Q. 10 (D) Sol Given $\left|z-|z+1|\right|^2 = \left|z+|z-1|\right|^2$

$\therefore \quad (z-|z+1|)(\bar{z}-|z+1|) = (z+|z-1|)(\bar{z}+|z-1|)$

$z\bar{z}-z|z+1|-\bar{z}|z+1|+|z+1|^2 = z\bar{z}+z|z-1|+\bar{z}|z-1|+|z-1|^2$

$|z+1|^2-|z-1|^2 = (z+\bar{z})\left[|z-1||z+1|\right]$



$(z+1)(\bar{z}+1)-(z-1)(\bar{z}-1) = (z+\bar{z})\left[|z-1|+|z+1|\right]$

$(z\bar{z}+z+\bar{z}+1)-(z\bar{z}-z-\bar{z}+1) = (z+\bar{z})\left[|z-1|+|z+1|\right]$

$2(z+\bar{z}) = (z+\bar{z})\left[|z+1|+|z-1|\right]$

$(z+\bar{z})\left[|z+1|+|z-1|-2\right] = 0$

$\Rightarrow$ either $z+\bar{z} = \ \Rightarrow$ z ix purely imaginary

$\Rightarrow$ z lies on $y-$axis $\Rightarrow$ $x=0$

or $|z+1|+|z-1| = 2$

$\Rightarrow$ z lie on the segment joining $(-1, 0)$ and $(1, 0)$ $\Rightarrow$ (D)]

Q. 11 (B) Sol $(z+1)^4 = 16z^4$

$|z+1| = 2|z| \Rightarrow |z+1|^2 = 4|z|^2 \Rightarrow (z+1)(\bar{z}+1) = 4z\bar{z}$

$3z\bar{z}-z-\bar{z}-1 = 0$ or $z\bar{z}-\dfrac{1}{3}z-\dfrac{1}{3}\bar{z}-\dfrac{1}{3} = 0$

Center = - coefficient of $\bar{z} = \left(\dfrac{1}{3}, 0\right) \Rightarrow$ Radius $= \sqrt{\alpha\bar{\alpha}-r} = \sqrt{\dfrac{1}{9}+\dfrac{1}{3}} = \sqrt{\dfrac{4}{9}} = \dfrac{2}{3}$

Hence centre $\left(\dfrac{1}{3}, 0\right)$ & radius $= \dfrac{2}{3}$ $\Rightarrow$ (B)]

Q. 12 (B) Sol $\frac{\sqrt{2}+1}{2}=\left(1-x^2+x^4-x^6+x^8..........\right)+\left(x-x^3+x^5-x^7.......\right)$

$\frac{\sqrt{2}+1}{2}=\frac{1}{1+x^2}+\frac{x}{1+x^2}=\frac{1+x}{1+x^2}$

or $\left(\sqrt{2}+1\right)x^2+\left(\sqrt{2}+1\right)=2+2x$

$\left(\sqrt{2}+1\right)x^2-2x+\left(\sqrt{2}-1\right)=0$ (divide by $\sqrt{2}+1$)

$x^2-2\left(\sqrt{2}-1\right)x+\left(\sqrt{2}-1\right)^2=0 \quad\Rightarrow\quad \left[x-\left(\sqrt{2}-1\right)\right]^2=0 \quad\Rightarrow\quad x=\sqrt{2}-1$ Ans.]

Q. 13 (C) Sol $x^2-px+20=0$

$x^2-20x+p=0$

If $p\neq 20$ then

$x^2-px+20=x^2-20x+p \quad\Rightarrow\quad (20-p)x+(20-p)=0 \quad\Rightarrow x=-1$ and $p=-21$

Hence there are 3 values of *x* i.e. $\left\{10+4\sqrt{5},\ 10-4\sqrt{5},\ -1\right\}$]

Q. 14 (A) Sol $D_1=4b_1^2-4a_1c_1<0$

i.e. $a_1c_1>b_1^2$ …..(1)

$D_2=4b_2^2-4a_2c_2<0$

hence $a_2c_2>b_2^2$ ….(2)

multiplying (1) and (2)

$a_1a_2c_1c_2>b_1^2b_2^2 \quad\Rightarrow$ Now consider for $f(x)$

$D=b_1^2b_2^2-4a_1a_2c_1c_2 \quad\Rightarrow\quad <b_1^2b_2^2-4b_1^2b_2^2 \quad\Rightarrow\quad =-3b_1^2b_2^2$

$\therefore \quad D<0 \quad\Rightarrow\quad g(x)>0 \quad \forall\ x\in R \quad\Rightarrow\quad (A)$]

Q. 15 (C) Sol Product will be divisible by 3 if atleast one digit is 0, 3, 6, 9

Hence total 4 digit numbers = 9.10^3

Number of 4 digit numbers without

0, 3, 6 or 9 = 6^4 = 1296

$\therefore$ Number of numbers $=9000-1296=7704$ Ans.]

Q. 16 (B) Sol Sum of single digit number $1+3+5+7+9=25=S$

Sum of two digit number $4S(1+10)=4(S+10S)=44S$

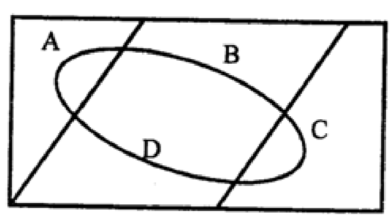
Sum of three digit number $12S\left(1+10+10^2\right)+(12)(111)S=1332S$

Sum of four digit number $24S\left(1+10+10^2+10^3\right)=24(1111)S=26664S$

Total $=28041S$]

Q. 17 (B) Sol $n=3;$ P (success)

$=P(HT \text{ or } TH)=\frac{1}{2} \quad\Rightarrow\quad p=q=\frac{1}{2}$ and $r=2$

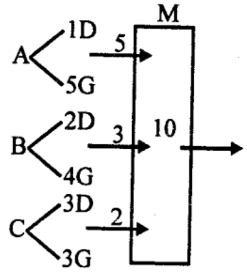


$P(r=2)={}^3C_2\left(\frac{1}{2}\right)^2.\frac{1}{2}=\frac{3}{8}$ Ans.]

Q. 18 (A) Sol $P(A)=\frac{5}{10},\ P(B)=\frac{3}{10};\ P(C)=\frac{2}{10}$

$P(D)=P(A).P(D/A)+P(B).P(D/B)+P(C).P(D/C)$

$=\frac{5}{10}.\frac{1}{6}+\frac{3}{10}.\frac{2}{6}+\frac{2}{10}.\frac{3}{6} \quad\Rightarrow\quad =\frac{5+6+6}{60}=\frac{17}{60}$



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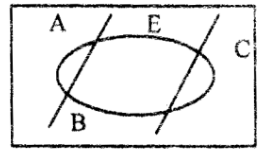
Q. 19 (A) Sol **A: exactly one child**

B: exactly two children

C: exactly 3 children



$P(A)=\frac{1}{4};\ P(B)=\frac{1}{2};\ P(C)=\frac{1}{4}$

E: couple has exactly 4 grandchildren

$P(E)=P(A).P(E/A)+P(B).P(E/B)+P(C).P(E/C)$

$$=\frac{1}{4}.0+\frac{1}{2}\left[\underbrace{\left(\frac{1}{2}\right)^2}_{2/2}+\underbrace{\frac{1}{4}.\frac{1}{4}.2}_{(1,3)}\right]+\frac{1}{4}\left[3\underbrace{\left(\frac{1}{4}.\frac{1}{4}.\frac{1}{2}\right)}_{1\quad 1\quad 2}\right] \Rightarrow =\frac{1}{8}+\frac{1}{16}+\frac{3}{128}=\frac{27}{128} \quad \text{Ans.}$$

IIIly 2/2 denotes each child having two children

$2.\frac{1}{4}.\frac{1}{4}$ denotes each child having 1 and 3 or 3 and 1 children

$=\frac{16}{128}+\frac{8}{128}+\frac{3}{128}=\frac{27}{128}$ **Ans.]**

Q. 20 (A) Sol $\tan\alpha+\tan\beta=-p$

$\tan\alpha\tan\beta=q \quad \Rightarrow \quad \tan(\alpha+\beta)=\frac{-p}{1-q}=\frac{p}{q-1}$

$$\frac{1}{1+\tan^2(\alpha+\beta)}\left[\tan^2(\alpha+\beta)+p\tan(\alpha+\beta)+q\right] \Rightarrow \frac{1}{1+\frac{p^2}{(q-1)^2}}\left[\frac{p^2}{(q-1)^2}+\frac{p^2}{(q-1)}+q\right]$$

$$\frac{1}{(q-1)^2+p^2}\left[p^2+p^2(q-1)+q(q-1)^2\right] \Rightarrow \frac{1}{p^2+(q-1)^2}\left[p^2q+q(q-1)^2\right]$$

$$q\left[\frac{p^2+(q-1)^2}{p^2+(q-1)^2}\right]=q\,]$$

Q. 21 (D) Sol a, 2a, b, $(a-b-6)$ in A.P.

$a+a-b-6=2a+b \Rightarrow \quad b=-3 \quad \Rightarrow \quad 2a-a=b-2a \quad \Rightarrow \quad 3a=b;\ a=-1$

Hence the series is $\Rightarrow$ $-1, -2, -3, -4, -5, \ldots\ldots\ldots$

$\therefore \quad s_{100}=-[1+2+3+\ldots\ldots+100]=-5050$ Ans.]

Q. 22 (A) Sol $T_r=\frac{1^2+2^2+3^2+\ldots\ldots+r^2}{r(r+1)}$

$$\therefore \quad S=\sum_{r=1}^{60}T_r=\sum_{r=1}^{60}\frac{r(r+1)(2r+1)}{6r(r+1)}=\frac{1}{6}\sum_{r=1}^{60}(2r+1)=\frac{1}{6}\underbrace{[3+5+7+\ldots\ldots+121]}_{\text{A.P. with a=3, d=2, n=60}}$$

$$=\frac{60}{2.6}\left[6+(60-1)2\right]=5[6+59\times 2]=5[6+118]=620 \quad \text{Ans.]}$$

COMPREHENSION TYPE

Q. 1 C 2.B 3. C

[Sol. $\therefore \quad (1+x+x^2)^{2n}=\sum_{r=0}^{4n}a_r x^r$ .....(1)

Replacing x by $\frac{1}{x}$ in equation (1) then

$\left(1+\frac{1}{x}+\frac{1}{x^2}\right)^{2n}=\sum_{r=0}^{4n}a_r\left(\frac{1}{x}\right)^r$ or $\left(1+x+x^2\right)^{2n}=\sum_{r=0}^{4n}a_r x^{4n-r}$ ….(2)

From equation (1) and (2), we get

$$\sum_{r=0}^{4n}a_r x^r=\sum_{r=0}^{4n}a_r x^{4n-r}$$

Comparing coefficient of x^{4n-r} on both sides, then we get

$a_r=a_{4n-r}$ ….(3)

(10) Put $x=1$ and $x=-1$ in equation (1), then

$9^n=a_0+a_1+a_2+a_3+a_4+....+a_{2n}+....+a_{4n}$

and $1=a_0-a_1+a_2-a_3+a_4-....+a_{2n}+....+a_{4n}$

adding and subtracting, then we get

$\frac{9^n+1}{2}=a_0+a_2+a_4+.........+a_{2n}+........+a_{4n-2}+a_{4n}$ ….(4)

and $\frac{9^n-1}{2}=a_1+a_3+a_5+.........+a_{2n-1}+........+a_{4n-1}$ ….(5)

Now, $\therefore$ $a_r=a_{4n-r}$

Put $r=0,2,4,6,\ ....,a_{2n-2},a_{2n}$

$\therefore$ $a_0=a_{4n}$ $\Rightarrow$ $a_2=a_{4n-2}$ $\Rightarrow$ $a_4=a_{4n-2}$

$\vdots \quad \vdots$

$a_{2n-2}=a_{2n+2}$

$\therefore$ $a_0+a_2+a_4+.....+a_{2n-2}=a_{2n-2}+......+a_{2n-4}+a_{4n-2}+a_{4n}$

Now from equation (4)

$\frac{9^n+1}{2}=2\left(a_0+a_2+a_4+.....+a_{2n-2}\right)+a_{2n}\Rightarrow$ $\frac{9^n+1-2a_{2n}}{4}=a_0+a_2+a_4+.....+a_{2n-2}$

$\therefore$ $\sum_{r=0}^{n-1}a_{2r}=\frac{9^n+1-2a_{2n}}{4}$ **Ans.**

(13) $\therefore$ $a_r=a_{4n-r}$

Put $r=1,\ 3,5,\ 7,\ ....,\ 2n-3,\ 2n-1$

$a_1=a_{4n-1}$ $\Rightarrow$ $a_3=a_{4n-3}$ $\Rightarrow$ $a_5=a_{4n-5}$ $\Rightarrow$ $a_{2n-3}=a_{2n+3}$

$a_{2n-1}=a_{2n+1}$ $\Rightarrow$ $\therefore$ $a_1+a_3+a_5+.......+a_{2n-1}=a_{2n+1}+a_{2n+3}+......+a_{4n-3}+a_{4n-1}$

Now from equation (5)

$\frac{9^n-1}{2}=2\left(a_1+a_3+a_5+.....+a_{2n-1}\right)\Rightarrow\therefore$ $\sum_{r=1}^{n}a_{2r-1}=\left(\frac{9^n-1}{4}\right)=\left(\frac{3^{2n}-1}{4}\right)$ **Ans.**

(14) $a_2=$ coefficient of x^2 in $\left(1+x+x^2\right)^{2n}$

$=$ coefficient of x^2 in $\left\{1+{}^{2n}C_1\left(x+x^2\right)+{}^{2n}C_2\left(x+x^2\right)^2+......\right\}$

$={}^{2n}C_1+{}^{2n}C_2$ $\Rightarrow$ $={}^{2n+1}C_2$ **Ans.]**

Q. **4.** C **Q. 5** B **Q. 6** D

[Sol. $R=\left(1+2x\right)^n$

put $x=1$ to get sum of all the coefficients

$\therefore$ $3^n=6561=3^8\Rightarrow$ $n=8$

(i) for $x=\frac{1}{\sqrt{2}};R=\left(\sqrt{2}+1\right)^8$

consider $\dfrac{\left(\sqrt{2}+1\right)^8+\left(\sqrt{2}-1\right)^8}{1+f+f'}=2\left[{}^8C_0\left(\sqrt{2}\right)^8+........\right]=$ even integer

since I is integer $\Rightarrow$ $f+f'$ must be an integer

but $0<f+f'<2$ $\Rightarrow$ $f+f'=1$ $\Rightarrow$ $f'=1-f$

now $n+R-Rf$

$$n+R\left(1-f\right)=8+\left(\sqrt{2}+1\right)^n.\left(\sqrt{2}-1\right)^n=8+1=9 \qquad \text{Ans.}$$

(ii) T_{r+1} in $\left(1+2x\right)^8={}^8C_r\left(2x\right)^r={}^8C_r$ when $x=\dfrac{1}{2}$

now $T_{r+1}\ge T_r$

$\dfrac{T_{r+1}}{T_r}\gtrless 1$ $\Rightarrow$ $\dfrac{{}^8C_r}{{}^8C_{r-1}}\gtrless 1$ $\Rightarrow T_{r+1}\gtrless T_r$ $\dfrac{8!}{r!(8-r)!}.\dfrac{(r-1)!;(9-r)!}{8!}\ge 1$

$(9-r)\ge r$ $\Rightarrow$ $9\ge 2r$ $\Rightarrow$ for $r=1,\ 2,\ 3,\ 4$ this is true

i.e. $T_5>T_4$ $\Rightarrow$ but for $r=5$ $T_6<T_5$

$\Rightarrow$ T_5 is the greatest term $\Rightarrow$ (B)

(iii) again $T_{k+1}={}^8C_k.2^k.x^k$; $T_k={}^8C_{k-1}.2^{k-1}.x^{k-1}$

$T_{k-1}={}^8C_{k-2}.2^{k-2}.x^{k-2}.x^{k-2}$

we want to find the term having the greatest coefficient

$\therefore$ $2^{k-1}.{}^8C_{k-1}>2^k.{}^8C_k$ …(1) and $2^{k-1}.{}^8C_{k-1}>2^{k-2}.{}^8C_{k-2}$ …(2)

from (1)

$\dfrac{8!.2^{k-1}}{(k-1)!(9-k)!}>\dfrac{2^k.8!}{k!(8-k)!}$ $\Rightarrow$ $\dfrac{1}{(9-k)}>\dfrac{2}{k}$ $\Rightarrow$ $k>18-3k$ $\Rightarrow$ $k>6$

Again $2^{k-1}.{}^8C_{k-1}>2^{k-2}.{}^8C_{k-2}$

$\dfrac{8!.2^{k-1}}{(k-1)!(9-k)!}>\dfrac{2^{k-2}.8!}{(k-2)!(10-k)!}$ $\Rightarrow$ $\dfrac{2}{k-1}>\dfrac{1}{10-k}$

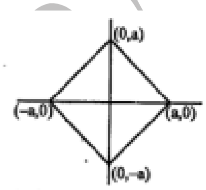
$\Rightarrow$ $20-2k>k-1$ $\Rightarrow 21>3k$ $\Rightarrow$ $k<7$

$\Rightarrow$ $6<k<7$ $\Rightarrow$ T_6 and T_7 term has the greatest coefficient

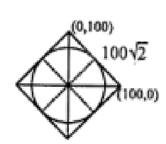
$\Rightarrow$ $k=6$ or 7 $\Rightarrow$ $\text{sum}=6+7=13$ Ans.]

Q. 7 D **Q. 8** B **Q. 9** C

[Sol.



(i) $|x|+|y|=a$ Figure is a square Ans.



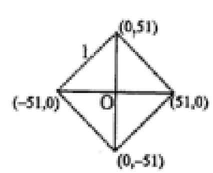
(ii) Area of the circle $=\dfrac{\pi d^2}{4}$

(where $d=$ diameter of circle = side of the square)

$=\dfrac{\pi(100)^2.2}{4}$ $\Rightarrow$ $=5000\pi$ Ans.

(iii) $x+y<51$ $x\ge 0,\ y\ge 0$

or $x+y\le 50$



give one each to *x* and *y*

$x+y\le 48$

$x + y + z = 48 \quad \Rightarrow \quad$ number of solutions $= {}^{50}C_2$

$\dfrac{50\times 49}{2}$

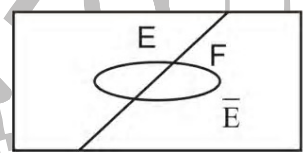
Number of solutions in all the four quadrants $= 100.49 = 4900$

Number of solutions except $(0, 0)$ on x and y axis from $(-51, 0)$ to $(51, 0)$ and $(0, 51)$ to $(0, -51)$ are 200

Total solutions $= 4900 + 200 + 1 = 5101$ Ans.]

Q. 10 D **Q. 11** A **Q. 12** B

[Sol. $P(E) = P$



$P(F) = P(E\cap F) + P(\overline{E}\cap F)$

$P(F) = P(E)P(F/E) + P(\overline{E})P(F/E)$

$$= p.1 + (1-p).\frac{1}{5} = \frac{4p}{5} + \frac{1}{5}$$

(i) if $p = 0.75$

$p(F) = \frac{1}{5}(4p+1) = \frac{1}{5}(4) = 0.8$

$\therefore \quad P(E/F) = \dfrac{P(E\cap F)}{P(F)} = \dfrac{0.75}{0.80} = \dfrac{15}{16}$ Ans.

(ii) now $P(E/F) = \dfrac{5p}{(4p+1)} \ge p$

Equality holds for $p = 0$ or $p = 1$

For all others value of $p \in (0, 1)$, LHS > RHS, hence **(A)**

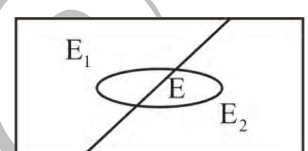
(iii) If each question has n alternatives than

$P(F) = p + (1-p)\dfrac{1}{n} = p\left(1 - \dfrac{1}{n}\right) + \dfrac{1}{n} = \dfrac{(n-1)p+1}{n}$

$\therefore \quad P(E/F) = \dfrac{np}{(n-1)p+1}$ which increases as n increases for a fixed p $\Rightarrow$ **(B)]**

Q. 13 B **Q. 14** A **Q. 15** C

[Sol. Urn – I $<^{5R}_{1B}$



Urn – I $<^{2R}_{4B}$

A : first two draws resulted in a blue ball.

B_1 : urn-I is used $\quad P(B_1) = \dfrac{1}{2} \quad \Rightarrow \quad B_2$: urn-II is used $\quad P(B_2) = \dfrac{1}{2}$

$P(A/B_1) = \dfrac{1}{6}.\dfrac{1}{6} = \dfrac{1}{36} \quad \Rightarrow \quad P(A/B_2) = \dfrac{4}{6}.\dfrac{4}{6} = \dfrac{16}{36} = \dfrac{4}{9}$ **Ans.(i)**

$$\left.\begin{aligned} \underbrace{P(B_1/A)}_{E_1} &= \frac{\frac{1}{2}.\frac{1}{36}}{\frac{1}{2}.\frac{1}{36}+\frac{1}{2}.\frac{16}{36}} = \frac{1}{17} \\ \underbrace{P(B_2/A)}_{E_2} &= \frac{\frac{1}{2}.\frac{16}{36}}{\frac{1}{2}.\frac{16}{36}+\frac{1}{2}.\frac{16}{36}} = \frac{16}{17} \end{aligned}\right] \Rightarrow \textbf{Ans. (ii)}$$

E: third ball drawn is red

$$P(E) = P(E \cap E_1) + P(E \cap E_2)$$

$$= \frac{1}{17}.\frac{5}{6} + \frac{16}{17}.\frac{2}{6} = \frac{5}{102} + \frac{32}{102} = \frac{37}{102}$$ **Ans. (iii)**

Q. 16 [C] **Q. 17** [A] **Q. 18** [B]

[Sol.

(1) A: 3 balls drawn found to be one each of different colours.

B_1: $1(W)+1(G)+4(R)$ are drawn; $P(B_1) = \frac{1}{10}$

B_2: $1(W)+4(G)+1(R)$ are drawn; $P(B_2) = \frac{1}{10}$

B_3: $4(W)+1(G)+1(R)$ are drawn; $P(B_3) = \frac{1}{10}$

B_4: They are drawn in groups of 1, 2, 3 (WGR) - (6 cases); $P(B_4) = \frac{6}{10}$

B_5: $2(W)+2(G)+1(R)$; $P(B_5) = \frac{1}{10}$ **Ans.**

$P(A/B_1) = \frac{{}^4C_1}{{}^6C_3} = \frac{4}{20}$ W G R R R R

$P(A/B_2) = \frac{{}^4C_1}{{}^6C_3} = \frac{4}{20}$ W G G G G R

$P(A/B_3) = \frac{{}^4C_1}{{}^6C_3} = \frac{4}{20}$ W W W W G R

$P(A/B_4) = 6.\frac{{}^1C_1.{}^2C_1.{}^3C_1}{{}^6C_3} = \frac{36}{20}$ W G G R R R,

$P(A/B_5) = \frac{{}^2C_1.{}^2C_1.{}^2C_1}{{}^6C_3} = \frac{8}{20}$ W W G G R R

$$\sum_{r=1}^{5} P(B_i).P(A/B_i) = \frac{1}{10}.\frac{4}{20} + \frac{1}{10}.\frac{4}{20} + \frac{1}{10}.\frac{4}{20} + \frac{1}{10}.\frac{36}{20} + \frac{1}{10}.\frac{8}{20} = \frac{56}{200}$$

(2) $P(B_1/A) = \frac{\frac{1}{10}.\frac{4}{20}}{\frac{56}{200}} = \frac{4}{56} = \frac{1}{14}$ **Ans.**

(3) $P(B_5/A) = \frac{\frac{1}{10}.\frac{8}{20}}{\frac{56}{200}} = \frac{8}{56} = \frac{2}{14}$

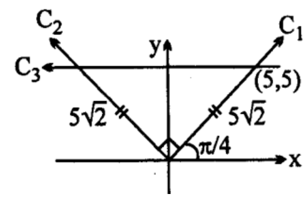
Hence P (bag had equals number of W and G balls/A)

$= P(B_1/A) = P(B_5/A) = \frac{1}{14} + \frac{2}{14} = \frac{3}{14}$ **Ans.**]

ASSERTION REASON TYPE

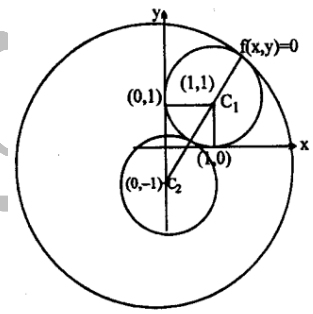
Q. 1 (C) Sol Statement-2 is False $\Rightarrow$ Take eg. $z = 2+3i$ $\Rightarrow$
$-z = -2-3i$
$-\bar{z} = -2+3i$ then figure is rectangle]

Q. 2 (D) Sol Area $= \dfrac{5\sqrt{2}.5\sqrt{2}}{2} = 25$

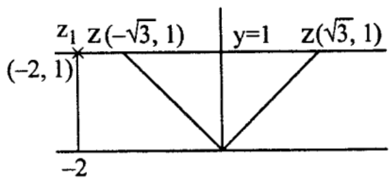


Hence S-1 is false and S-2 is true.]

Q. 3 (B) Sol z_1, z_2 and '0' are on the same side then only $S-2$ is the reason of $S-1$]



Q. 4 (C) Sol For $\lambda = |\mathrm{Im}(z_1),|$ then number of values of $z = 1$
For $\lambda > |\mathrm{Im}(z_1)|$, then number values of $z = 2$



]

Q. 5 (A)Sol **Let** $z = \cos\theta + i\sin\theta$ where $\cos\theta, \sin\theta \in Q \Rightarrow z^{2n} - 1 = -1 + \cos 2n\theta + i\sin 2n\theta$
$= -2\sin^2 n\theta + 2i\sin n\theta\cos n\theta \Rightarrow \quad = -2\sin n\theta(\sin n\theta - i\cos n\theta) \Rightarrow |z^{2n} - 1| = 2|\sin n\theta|$
Now $P(n): \sin n\theta, \cos n\theta \in Q \quad \forall\, n \in N$ can be provided by induction if $\sin\theta, \cos\theta \in Q$]

Q. 6 (A) Sol **Let** $A = \begin{bmatrix} a & p \\ b & q \\ c & e \end{bmatrix} A^T = \begin{bmatrix} a & b & c \\ p & q & r \end{bmatrix}$

$$AA^T = \begin{bmatrix} a^2+p^2 & ab+pq & ac+pr \\ ab+pq & b^2+q^2 & bc+qr \\ ac+pr & bc+qr & c^2+r^2 \end{bmatrix} \Rightarrow |AA^T| = \begin{vmatrix} a & p & 0 \\ b & q & 0 \\ c & r & 0 \end{vmatrix}\begin{vmatrix} a & b & c \\ p & q & r \\ 0 & 0 & 0 \end{vmatrix} = 0 \quad \Rightarrow \quad AA^T \text{ is singular.}]$$

Q. 7 (A) Sol **Given** $AB + A + B = 0$
$AB + A + B + I = I \quad \Rightarrow \quad A(B+I) + (B+I) = I$
$(A+I)(B+I) = I \quad \Rightarrow$
$\Rightarrow$ $(A+I)$ and $(B+I)$ are inverse of each other $\Rightarrow$ $(A+I)(B+I) = (B+I)(A+I)$
$\Rightarrow$ $AB = BA$]

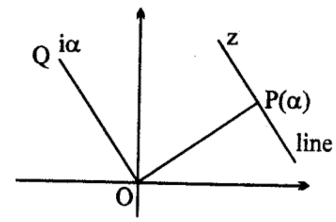
Q. 8 (B) Sol Let $x_1, x_2, x_3 \in R$ be the roots of $f(x) = 0$
$\therefore \quad f(x) = (x-x_1)(x-x_2)(x-x_3)$
$f(i) = (i-x_1)(i-x_2)(i-x_3)$
$|f(i)| = |x_1 - i||x_2 - i||x_3 - i| = 1$
$\therefore \quad \sqrt{x_1^2+1}\sqrt{x_2^2+1}\sqrt{x_3^2+1} = 1$
This is possible only if $x_1 = x_2 = x_3 = 0$
$\Rightarrow \quad f(x) = x^3 \quad \Rightarrow \quad a = 0 = b = c \quad \Rightarrow \quad a+b+c = 0*$

Q. 9 (D) Sol $ix^2 + (1+i)x + i = 0 \quad \Rightarrow \quad \alpha\beta = 1 \Rightarrow \quad \mathrm{Im}(\alpha\beta) = 0.]$

MORE THAN ONE MAY CORRECT TYPE

Q. 1 B, D

[Sol. Required line is passing through $P(\alpha)$ and parallel to the vector $\overrightarrow{OQ}$



Hence $z = \alpha + i\lambda a,\ \lambda \in R$

$\dfrac{z-\alpha}{\alpha} =$ purely imaginary

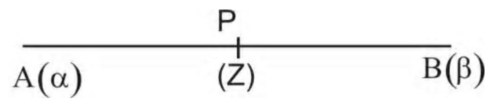
$\Rightarrow \quad \text{Re}\left(\dfrac{z-\alpha}{\alpha}\right) = 0 \Rightarrow \quad$ (**B**)

$\Rightarrow \quad \text{Re}\left((z-\alpha)\bar{\alpha}\right) = 0 \quad \Rightarrow \quad \text{Re}\left(z\bar{\alpha} - |\bar{\alpha}|\right) = 0$

Also $\quad \dfrac{z-\alpha}{\alpha} + \dfrac{\bar{z}-\bar{\alpha}}{\bar{\alpha}} = 0 \quad \Rightarrow \bar{\alpha}(z-\alpha) + \alpha\left(\bar{z}-\bar{\alpha}\right) = 0$

$\bar{\alpha}z + \alpha\bar{z} - 2|\alpha|^2 = 0 \quad \Rightarrow \quad$ (**D**)]

Q. 2 [A, B, C, D]



[Sol. $AP + PB = AB$

$|z-\alpha| + |\beta - z| = |\beta - \alpha| \qquad \Rightarrow \qquad$ A is true

Now $z = \alpha + t(\beta - \alpha)$

$= (1-t)\alpha + t\beta$ where $t \in (0, 1) \qquad \Rightarrow \qquad$ B is true

Again $\dfrac{z-\alpha}{\beta-\alpha}$ is real $\Rightarrow \quad \dfrac{z-\alpha}{\beta-\alpha} = \dfrac{\bar{z}-\bar{\alpha}}{\bar{\beta}-\bar{\alpha}}$

$$\Rightarrow \quad \begin{vmatrix} z-\alpha & \bar{z}-\bar{\alpha} \\ \beta-\alpha & \bar{\beta}-\bar{\alpha} \end{vmatrix} = 0 \qquad \text{Ans.}$$

$$\text{Again } \begin{vmatrix} z & \bar{z} & 1 \\ \alpha & \bar{\alpha} & 1 \\ \beta & \bar{\beta} & 1 \end{vmatrix} = 0 \text{ if and only if } \begin{vmatrix} z-\alpha & \bar{z}-\bar{\alpha} & 0 \\ \alpha & \bar{\alpha} & 1 \\ \beta-\alpha & \bar{\beta}-\bar{\alpha} & 0 \end{vmatrix} = 0$$

$$\Rightarrow \quad \begin{vmatrix} (z-\alpha) & \bar{z}-\bar{\alpha} \\ \beta-\bar{\alpha} & \bar{\beta}-\bar{\alpha} \end{vmatrix} = 0 \qquad \textbf{Ans.}$$

Q. 3 A, B, C, D

[Sol. $\dfrac{z^n-1}{z-1} = (z-\alpha_1)(z-\alpha_2)............(z-\alpha_{n-1})$

put $z = i$

$$\prod_{r=1}^{n-1}(i-\alpha_r) = \frac{i^n-1}{i-1} = \begin{bmatrix} 0 & \text{if } n = 4k \\ 1 & \text{if } n = 4k+1 \\ 1+i & \text{if } n = 4k+2 \\ i & \text{if } n = 4k+3 \end{bmatrix}$$

Q. 4 B, C, D

[Hint. $PQ = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \Rightarrow \quad$ **B,C,D**]

Q. 5 A, B

[Sol. $\therefore \quad \dfrac{t_{2p}}{t_p} = \dfrac{t_{4p}}{t_{2p}} = r$ (say)

If we start from t_p, then t_{2p} is the $(p+1)^{th}$ term and if we start from t_{2p}, then t_{4p} is the $(2p+1)^{th}$ term

$\therefore \quad t_{2p} = t_p + pd \qquad \ldots (1)$

and $\quad t_{4p} = t_{2p} + 2pd \;(d = c.d)$

$\Rightarrow \quad t_{4p} = t_{2p} + 2(t_{2p} - t_p) \qquad$ (from equation (1))

$\Rightarrow \quad t_{4p} = 3t_{2p} - 2t_p \Rightarrow \quad \dfrac{t_{4p}}{t_{2p}} = 3 - \dfrac{2t_p}{t_{2p}} \quad \Rightarrow \quad r = 3 - \dfrac{2}{r}$

$\Rightarrow \quad (r-1)(r-2) = 0 \qquad \Rightarrow \qquad r = 1, 2$ **Ans.**

Alternative solution : $R = \dfrac{A + (2p-1)D}{A + (p-1)D} = \dfrac{A(4p-1)D}{A + (2p-1)D}$

$R = \dfrac{(\ )-(\ )}{(\ )-(\ )}; \; R = \dfrac{2PD}{PD} = 2$

Also if $PD = 0 \qquad \Rightarrow \qquad D = 0 \Rightarrow T_P = T_{2p} = T_{4p} \Rightarrow R = 1$]

Q. 6 B, C

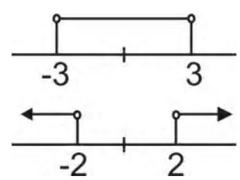
[Sol. $(\log_2 x)^4 - \left(\log_2 \left(\dfrac{x}{2}\right)^3\right)^2 + 9\left[\log_2 32 - \log_2 x^2\right] < 4(\log_2 x)^2$

$(\log_2 x)^4 - (3\log_2 x - 3)^2 + 45 - 15\log_2 x < 4(\log_2 x)^2$

Let $\quad \log_2 x = t$

$t^4 - (3t-3)^2 + 45 - 18t < 4t^2 \Rightarrow \qquad t^4 - (9t^2 + 9 - 18t) - 18t + 45 < 4t^2$

$\Rightarrow \quad t^4 - 13t^2 + 36 < 0 \qquad \Rightarrow \qquad (t^2 - 4)(t^2 - 9) < 0$



$\Rightarrow \quad 4 < t^2 < 9$

$\quad t^2 < 9 \quad \Rightarrow \quad -3 < t < 3$

and $\quad t^2 > 4 \quad \Rightarrow \quad t > 2$ or $t < -2$

hence, $t \in (-3, -2) \cup (2, 3)$

$x \in \left(\dfrac{1}{8}, \dfrac{1}{4}\right) \cup (4, 8) \qquad \Rightarrow \qquad$ B,C

Q. 7 B, D

[Sol. $x^2 - 2x + 4 = -3\cos(ax + b)$

$(x-1)^2 + 3 = -3\cos(ax+b)$

for above equation to have atleast one solution

let $f(x) = (x-1)^2 + 3$ and $f(x) = -3\cos(ax+b)$

if $x = 1$ then L.H.S. = 3 and R.H.S. $= -3\cos(a+b)$

hence, $\cos(a+b) = -1 \qquad \Rightarrow \qquad \therefore \qquad a + b = \pi, 2\pi, 5\pi$

but $\quad 0 \le a + b \le 10 \quad \Rightarrow \quad a + b = \pi$ or $3\pi \qquad \Rightarrow \qquad$ B, D]

Q. 8 [A, B, C, D]

[Sol. $S = 111111 = 3.7.11.13.37 \qquad \Rightarrow \qquad$ [ABCD]

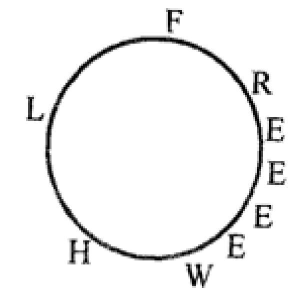
Q. 9 B, C, D

[Sol. **(A)** False is should be ${}^9P_5 - 1$

(B) $x.4! = 8!$

$\therefore \quad x = \frac{8!}{4!} = {}^8C_4$

(C) Vowels E E E E select 4 places in ${}^9C_4$ ways arrange consonant alphabetically only us one ways.



$\therefore {}^9C_4 = 126 = \frac{1}{2}.256 = \frac{1}{2}.{}^{10}C_5$

(D) True

$\therefore$ correct answer are (B), (C) and (D)

Q. 10 B, C, D

[Sol. Let number of blue marbles is b and number of green marbles is g

Hence $\frac{bg}{{}^{b+g}C_2} = \frac{1}{2} \quad \Rightarrow \quad (b+g)(g+b-1) = 4bg \Rightarrow (b+g)^2 - (b+g) = 4bg$

$b^2 + g^2 + 2bg - b - g = 4bg \quad \Rightarrow \quad g^2 - 2bg - g + b^2 - b = 0 \quad \Rightarrow \quad D = (2b+1)^2 - 4(b^2 - b)$

$= 8b + 1$ must a perfect square. Hence3 possible values of b are 3, 6, 10 $\Rightarrow [B, C, D]$]

Q. 11 B, C, D

[Sol. Let the H.P. be $\frac{1}{A} + \frac{1}{A+D} + \frac{1}{A+2D} + ...$

Corresponding A.P. $A + (A+D) + (A+2D) + ...$

T_p of $AP = \frac{1}{q(p+q)} = A + (p-1)D \quad \quad ....(1)$

T_q of $AP = \frac{1}{p(p+q)} = A + (q-1)D \quad \quad ....(2)$

T_{p+q} of $AP = A + (P+q-1)D$

Now solving equation (1) and (2), we get

$A = D = \frac{1}{pq(p+q)} \Rightarrow \therefore \quad T_{p+q}$ of $AP = A + (p+q-1)D = (p+q)D = \frac{1}{pq}$

And T_{pq} of $AP = A + (pq-1)D = pqD = \frac{1}{p+q}$

$\Rightarrow \quad T_{p+q}$ of $HP = pq$ and T_{pq} of $HP = p+q \Rightarrow$ also $\because p > 2, q > 2$

$\Rightarrow \therefore \quad pq > p+q \quad$ i.e. $\quad T_{p+q} > T_{pq}$]

MATCH THE COLUMNS

Q. 1 (A) Q, R; (B) P, S; (C) Q, S; (D), P, R

[Sol. (A) $z = \frac{1 \pm \sqrt{-3i}}{2} = \frac{1+\sqrt{-3i}}{2}$ or $\frac{1-\sqrt{-3i}}{2}$

amp $z = \frac{\pi}{3}$ or ampz $= -\frac{\pi}{3} \Rightarrow$ **Q, R**

(B) $z = \frac{-1 \pm \sqrt{3}i}{2} = \frac{-1+\sqrt{3}i}{2}$ or $\frac{-1-\sqrt{3}i}{2}$

amp $z = \frac{2\pi}{3}$ or $-\frac{2\pi}{3} \Rightarrow$ **P, S**

(C) $2z^2 = -1 - i\sqrt{3}$ $\Rightarrow$ $z^2 = \frac{-1-\sqrt{3}i}{2} = \cos\left(-\frac{2\pi}{3}\right) + i\sin\left(-\frac{2\pi}{3}\right)$

$z = \cos\left(\frac{2m\pi - (2\pi/3)}{2}\right) + i\sin\left(\frac{2m\pi - (2\pi/3)}{2}\right)$

$m = 0,\ z = \cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)$

$m = 1,\ z = \cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)$ $\Rightarrow$ $\text{amp}\, z = -\frac{\pi}{3}$ or $\frac{2\pi}{3} \Rightarrow$ **Q,S**

(D) $2z^2 + 1 - i\sqrt{3} = 0$

$z^2 = \frac{-1+i\sqrt{3}}{2} = \cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)$

$z = \cos\left(\frac{2m\pi + (2\pi/3)}{2}\right) + i\sin\left(\frac{2m\pi + (2\pi/3)}{2}\right)$

$m = 0,$ $z = \cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)$

$m = 1,$ $\cos\left(\frac{4\pi}{3}\right) + i\sin\left(\frac{4\pi}{3}\right)$ or $\cos\left(-\frac{2\pi}{3}\right) + i\sin\left(-\frac{2\pi}{3}\right) \Rightarrow$ **P, R]**

Q. 2 (A) Q: (B) R; (C) S

[Sol: (A) No pair $= {}^6C_4.2^4 = 15.16 = 240$ Ans. $\Rightarrow$ (Q)

(B) atleast one pair = exactly one + both pair

$= {}^6C_1.{}^5C_2.2^2 + {}^6C_2$

$= 240 + 15 = 255$ Ans. $\Rightarrow$ (R)

(C) fewer than 2 pairs = no pair + exactly one pair

$= {}^6C_4.2^4 + {}^6C_1.{}^5C_2.2^2$

$= 240 + 240 = 480$ Ans $\Rightarrow$ (S)]

Q. 3 (A)-R (B)-S (C)-P (D)-Q

[Sol: (A) $fog : f[g(x)]$

$= \ln[g(x)] = \ln(x^2 - 1)$

$\therefore$ $x^2 - 1 > 0$ $\Rightarrow$ $(-\infty, -1) \cup (1, \infty)$ $\Rightarrow$ R

(B) $gof : g[f(x)] = \ln^2 x - 1$ $\Rightarrow$ $(0, \infty)$ $\Rightarrow$ S

(C) $fof : f[f(x)] = \ln[\ln(x)]$ $\Rightarrow \ln x > 0$

$x > 1$

$\therefore$ $(1, \infty)$ $\Rightarrow$ P

(D) $gog : g[g(x)] = g^2(x) - 1$

$(x^2 - 1)^2 - 1 \Rightarrow x \in (-\infty, \infty)$ $\Rightarrow$ Q]