THIS FILE CONTAINS

DIFFERENTIAL CALCULUS

(COLLECTION # 2)

Very Important Guessing Questions For IIT JEE 2011 With Detail Solution

Junior Students Can Keep It Safe For Future IIT-JEEs

- ***Function***
- ***Limit, Continuity and Differentiability of Function***
- ***Differentiation & L Hospital Rule***
- ***Application of Derivatives***

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For Collection # 1 Question (Next File)

- ***Single Correct Answer Type Question***
- ***Comprehension Type Questions***
- ***Assertion Reason Type Question***
- ***More Correct Answers Type Questions***
- ***Subjective (Up to 4 Digits)***
- ***Detiail Solution By Genuine Method (But In) Classroom I Will Give Short Tricks)***

For Collection # 2

- ***Same As Above***

TOPIC = DIFFERENTIAL CALCULUS

SINGLE CORRECT TYPE

Q. 1 Range of the function $f(x)=|x-1|+|x-2|+|x-8|+|x-9|$

(codeV3T1PAQ5)

(A) $[14,\infty)$ (B) $[13,\infty)$ (C) $[3,\infty)$ (D) $[0,\infty)$

Q. 2 Given $f(x)=\begin{cases}\sqrt{10-x^2} & \text{if } -3<x<3\\ 2-e^{x-3} & \text{if } x\geq 3\end{cases}$

(codeV3T4PAQ5)

The graph of $f(x)$ is

(A) continuous and differentiable at $x=3$
(B) continuous but not differentiable at $x=3$
(C) differentiable but not continuous at $x=3$
(D) neither differentiable nor continuous at $x=3$

Q. 3 Let $f(k)=\dfrac{k}{2009}$ and $g(k)=\dfrac{f^4(k)}{(1-f(k))^4+(f(k))^4}$ then the sum $\sum_{k=0}^{2009} g(k)$ is equal :

(A) 2009 (B) 2008 (C) 1005 (D) 1004

(codeV3T8PAQ4)

Q. 4 Suppose that $f(0)=0$ and $f'(0)=2,$ and let $g(x)=f(-x+f(f(x)))$. The value of $g'(0)$ is equal to

(A) 0 (B) 1 (C) 6 (D) 8

(codeV3T9PAQ1)

Q. 5 A line is tangent to the curve $f(x)=\dfrac{41x^3}{3}$ at the point P in the first quadrant, and has a slope of 2009. This line intersects the y-axis at $(0, b)$. The value of 'b' equals **(codeV3T9PAQ4)**

(A) $-\dfrac{7^3}{3}$ (B) $-\dfrac{2.7^3}{3}$ (C) $-\dfrac{82.7^3}{3}$ (D) $-\dfrac{82.7^2}{3}$

Q. 6 If the primitive of the function $f(x)=\dfrac{x^{2009}}{(1+x^2)^{1006}}$ w.r.t. x is equal to $\dfrac{1}{n}\left(\dfrac{x^2}{1+x^2}\right)+C$ then $(m+n)$ (where $m, n\in N$) is equal to

(codeV3T10PAQ3)

(A) 3012 (B) 3014 (C) 3016 (D) None

Q. 7 $\lim\limits_{x\to\infty} n^{-2009}\left(1+2^{2008}+3^{2008}+.......+n^{2008}\right)$ equals

(codeV3T10PAQ4)

(A) 0 (B) $\dfrac{1}{2009}$ (C) $\dfrac{1}{2008}$ (D) 2009

COMPREHENSION TYPE

Paragraph for question nos. 1 to 3

Consider a function $y=f(x)$ satisfying the equation $\tan^{-1}y=\tan^{-1}x+C$ where $y=1$ when $x=0$. Now answer the following :

(codeV3T1PAQ9to11)

Q. 1 The domain of the explicit form of the function is

(A) $(-\infty, 1)$ (B) $R-\{-1, 1\}$ (C) $(-1, 1)$ (D) $[0, \infty)$

Q. 2 Range of the function is

(A) $R-\{-1\}$ (B) $(-1,\infty)$ (C) $[1,\infty)$ (D) $(-\infty,1]$

Q. 3 For the function $y=f(x)$ which one of the following does not hold good?

(A) $f(x)$ is injective (B) $f(x)$ is neither odd nor even

(C) $f(x)$ is a period (D) explicit form of $f(x)$ is $\frac{x+1}{x-1}$.

Paragraph for question nos. 4 to 6

Consider a quadratic function $f(x)=ax^2+bx+c,\ (a,b,c\in R,\ a\neq 0)$ and satisfying the following conditions.

(codeV3T3PAQ11to13)

(i) $f(x-4)=f(2-x)\ \forall x\in R$ and $f(x)\geq x\ \forall x\in R$ (ii) $f(x)\leq\left(\frac{x+1}{2}\right)^2\ \forall x\in(0,2)$

(iii) The minimum value of $f(x)$ is zero.

Q. 4 The value of the leading coefficient of the quadratic polynomial is

(A) 1/4 (B) 1/3 (C) 1/2 (D) 1

Q. 5 $f'(1)$ has the value equal to

(A) 1/4 (B) 1/3 (C) 1/2 (D) 1

Q. 6 The area enclosed by $y=f(x)$ and the coordinate axes is

(A) 1/12 (B) 1/4 (C) 1/3 (D) ½

Paragraph for Question Nos. 7 to 9

Let $y=f(x)$ satisfies the equation $f(x)=(e^{-x}+e^{x})\cos x-2x-\int_0^x(x-t)f'(t)\,dt.$ **(codeV3T4PAQ17to19)**

Q. 7 y satisfies the differential equation

(A) $\frac{dy}{dx}+y=e^x(\cos x-\sin x)-e^{-x}(\cos x+\sin x)$ (B) $\frac{dy}{dx}-y=e^x(\cos x-\sin x)+e^{-x}(\cos x+\sin x)$

(C) $\frac{dy}{dx}+y=e^x(\cos x+\sin x)-e^{-x}(\cos x-\sin x)$ (D) $\frac{dy}{dx}-y=e^x(\cos x-\sin x)+e^{-x}(\cos x-\sin x)$

Q. 8 The value of $f'(0)+f''(0)$ equals

(A) -1 (B) 2 (C) 1 (D) 0

Q. 9 $f(x)$ as a function of x equals

(A) $e^{-x}(\cos x-\sin x)+\frac{e^x}{5}(3\cos x+\sin x)+\frac{5}{2}e^{-x}$ (B) $e^{-x}(\cos x+\sin x)+\frac{e^x}{5}(3\cos x-\sin x)-\frac{5}{2}e^{-x}$

(C) $e^{-x}(\cos x-\sin x)+\frac{e^x}{5}(3\cos x-\sin x)+\frac{5}{2}e^{-x}$ (D) $e^{-x}(\cos x+\sin x)+\frac{e^x}{5}(3\cos x-\sin x)-\frac{5}{2}e^{-x}$

ASSERTION REASON TYPE

Q. 1 Let f :defined from $(1/4,\infty)\to R^+$ as, $f(x)=\log_{1/4}\left(x-\frac{1}{4}\right)+\frac{1}{2}\log_4(16x^2-8x+1)$

(codeV3T1PAQ15)

Statement-1: $f(x)$ is neither injective not surjective.

because Statement-2 : $f(x)$ is a constant function.

Q. 2 Consider the function $f(x)=\sin^{-1}\left(\frac{1+x^2}{2x}\right)$

(codeV3T2PAQ16)

Statement-1: Range of the function $f(x)$ has exactly two elements.

because Statement-2 : Domain of the function $f(x)$ has exactly two elements.

Q. 3 Consider the function $f(x)=\frac{1}{2\{-x\}}-\{x\}$ where $\{x\}$ denotes the fractional part of *x* and *x* is not an integer :

(codeV3T2PAQ17)

Statement-1: The minimum value of $f(x)$ is $\sqrt{2}-1$

because Statement-2 : If the product of two positive numbers is a constant the minimum value of their sum is 2 times the square root of their product.

Q. 4 Consider the function $f(x)=\ln\ln\left(\frac{x}{4e}+\frac{e^3}{x}\right)$

(codeV3T2PAQ18)

Statement-1: The range of the function $f(x)$ is R^+.

because Statement-2 : For two positive reals a and b, $\frac{a+b}{2}\geq\sqrt{ab}$

Q. 5 Let $f(x)=\begin{cases} x+\{x\}+x\sin\{x\} & \text{for } x\neq 0 \\ 0 & \text{for } x=0 \end{cases}$ where $\{x\}$ denotes the fractional part function.

Statement-1: $f(x)$ is neither derivable nor continuous at $x=0$. **(codeV3T3PAQ7)**

because Statement-2 : Non derivability $\Rightarrow$ discontinuity

Q. 6 **Statement-1:** The line $\frac{x}{a}+\frac{y}{b}=1$ touches the curve $y=be^{-\frac{x}{a}}$ at some point $x=x_0$

because Statement-2 : $\frac{dy}{dx}$ exists at $x=x_0$. **(codeV3T4PAQ1)**

Q. 7 **Statement-1** : $\lim_{x\to\pi/2}\frac{\sin\left(\cot^2 x\right)}{(\pi-2x)^2}=\frac{1}{2}$ **(codeV3T6PAQ4)**

because Statement - 2 : $\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1$ and $\lim_{\theta\to 0}\frac{\tan\theta}{\theta}=1$, where θ is measured in radians.

Q. 8 Consider $f(x)=\sin^{-1}\left(\sec\left(\tan^{-1}x\right)\right)+\cos^{-1}\left(\operatorname{cosec}\left(\cot^{-1}x\right)\right)$ **(codeV3T6PAQ4)**

Statement-1 : Domain of $f(x)$ is a singleton

because Statement - 2 : Range of the function $f(x)$ is a singleton

Q. 9 Let $h(x)=f_1(x)+f_2(x)+f_3(x)+......+f_n(x)$ where $f_1(x), f_2(x), f_3(x), ..., f_n(x)$ are real value functions of x: **(codeV3T6PAQ7)**

Statement-1 : $f(x)=|\cos|x||+\cos^{-1}\operatorname{sgn}(x)+|\ln x|$ is not differentiable at 3 points in $(0, 2\pi)$

because Statement - 2 : Exactly one function $f_i(x)$, $i=1,2, ..., n$ not differentiable and the rest of the function differentiable at $x=a$ makes $h(x)$ not differentiable at $x=a$.

Q. 10 Let $f(x)=\frac{\ln x}{x}$ $x>0$, then **(codeV3T7PAQ7)**

Statement-1 : $f\left(\frac{5}{2}\right)>f\left(\frac{9}{2}\right)$

because Statement - 2 : $f(x_1)>f(x_2)\ \forall\ x_1\in(2, 4)$ and $x_1\in(2, 4)$ and $x_2\in(0, 2)\cup(4, \infty)$

Q. 11 Let the line $ax+by+c=0$ is a tangent to $f(x)=x-x^2+x^3$, where $a, b \neq 0$, then

Statement-1 : $ab<0$ **(codeV3T7PAQ9)**

because

Statement - 2 : Exactly one distinct tangent can be drawn to curve $y=f(x)$ passing through any point $(1/3, k)$

Q. 12 Consider the functions $f(x)=x-\sin x$ and $g(x)=x^4$.

(codeV3T8PAQ11)

Statement-1 : $f(x)$ is increasing for $x>0$ as well as $x<0$ and $g(x)$ is increasing for $x>0$ and decreasing for $x<0$.

because Statement - 2 : If an odd function is known to be increasing on the interval $x>0$ then it is increasing for $x<0$ also and if an even function is increasing for $x>0$ then it is decreasing for $x<0$.

Q. 13 Consider the function $f(x)=\begin{cases} x\cos\frac{1}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}$ in $[-1, 1]$

(codeV3T8PAQ13)

Statement-1 : LMVT is not applicable to $f(x)$ in the indicated interval.

because Statement - 2 : As $f(x)$ is neither continuous nor differentiable in $[-1, 1]$

Q. 14 Consider the polynomial function $f(x)=\frac{x^7}{7}-\frac{x^6}{6}+\frac{x^5}{5}-\frac{x^4}{4}+\frac{x^3}{3}-\frac{x^2}{2}+x$

(codeV3T9PAQ16)

Statement-1 : The equation $f(x)=0$ can not have two or more roots.

because Statement - 2 : Rolles theorem is not applicable for $y=f(x)$ on any interval $[a, b]$ where $a, b\in R$

Q. 15 Consider the function $f(x)=x^2-|x^2-1|+2||x|-1|+2|x|-7$.

(codeV3T9PAQ17)

Statement-1 : f is not differentiable at $x=1, -1$ and 0.

because Statement - 2 : $|x|$ not differentiable at $x=0$ and $|x^2-1|$ is not differentiable at $x=1$ and -1.

Q. 16 **Statement-1** : The function $f(x)=a\sqrt{x-1}+b\sqrt{2x-1}-\sqrt{2x^2-3x+1}$ where $a+2b=2$ and $a, b\in R$ always has a root in $(1, 5)$ only if $b\geq 0$. **(codeV3T10PAQ16)**

because Statement - 2 : If $f(x)$ is continuous in $[a, b]$ and $f(a)$ and $f(b)$ have opposite signs then $f(x)$ has a root in (a, b)

Q. 17 Consider the function $f(x)=x^2-|x^2-1|+2||x|-1|+2|x|-7$. **(codeV3T10PAQ18)**

Statement-1 : The area bounded by the graph of $y=f(x)$ and the x-axis is $\frac{56}{3}$

because Statement - 2 : If $f(x)=f(-x)$ then area enclosed by $y=f(x)$ and positive x-axis is equal to be area enclosed by $y=f(x)$ and negative x-axis.

MORE THAN ONE MAY CORRECT TYPE

Q. 1 Which of the following function(s) is/are periodic?

(codeV3T2PAQ21)

(A) $f(x)=\frac{2^x}{2^{[x]}}$ where [ ] denotes greatest function

(B) $g(x)=\text{sgn}\{x\}$ where $\{x\}$ denotes the fractional part function

(C) $h(x)=\sin^{-1}\left(\cos\left(x^2\right)\right)$ (D) $k(x)=\cos^{-1}\left(\sqrt{\sin x}\right)$

Q. 2 Which of the following statement(s) is/are True for the function $f(x)=(x-1)^2(x-2)+1$ defined on $[0, 2]$? **(codeV3T6PAQ9)**

(A) Range of f is $\left[\frac{23}{27}, 1\right]$

(B) The coordinates of the turning point of the graph of $y=f(x)$ occur at $(1, 1)$ and $\left(\frac{5}{3}, \frac{23}{27}\right)$

(C) The value of p for which the equation $f(x)=p$ has 3 distinct solutions lies in interval $\left(\frac{23}{27}, 1\right)$

(D) The area enclosed by $y=f(x)$, the lines $x=0$ and $y=1$ as x varies from 0 to 1 is $\frac{7}{12}$.

Q. 3 Consider the function $f(x)$ and $g(x)$, both defined from $R\to R$ and are defined as $f(x)=2x-x^2$ and $g(x)=x^n$ where $n\in N$. If the area between $f(x)$ and $g(x)$ is ½ then n is a divisor of
(codeV3T10PAQ19)

(A) 12 (B) 15 (C) 20 (D) 30

Q. 4 Let $\frac{dy}{dx}+y=f(x)$ where y is a continuous function of x with $y(0)=1$ and $f(x)=\begin{bmatrix} e^{-x} & \text{if } 0\le x\le 2 \\ e^{-2} & \text{if } x>2 \end{bmatrix}$ Which of the following hold(s) good ?
(codeV3T10PAQ21)

(A) $y(1)=2e^{-1}$ (B) $y'(1)=-e^{-1}$ (C) $y(3)=-2e^{-3}$ (D) $y'(3)=-2e^{-3}$

MATCH THE COLUMNS

Q. 1 Let $g(x)=\begin{cases} \frac{\pi}{2}-2\tan^{-1}f(x) & f(x)\in(-1, 1) \\ -\frac{\pi}{2}-2\tan^{-1}f(x) & f(x)\in(-\infty,-1) \\ \frac{3\pi}{2}-2\tan^{-1}f(x) & f(x)\in(1, \infty) \end{cases}$ where $f(x)=\begin{cases} x+1 & x\le 0 \\ 1-x^2 & 0<x<2 \\ x-5 & x\ge 2 \end{cases}$

Column-I	Column-II
(A) Values of x for which derivation of $g(x)$ w.r.t. $f(x)$ is $-\frac{1}{13}$ is/are	(P) – 6
(B) Values of x for which $f(x)$ has local maximum or local minimum is/are	(Q) 0
(C) Values of k for which $f(x)+k=0$ has 2 positive and one negative root is/are	(R) 2

(codeV3T6PBQ2)

(D) Values of x for which $g'(x) < 0$ is/are (S) 10

Q. 2 Column I Column II
(codeV3T8PBQ1)

(A) Let $A = a_{ij}$ be a 3×3 matrix where $a_{ij} = \begin{cases} 2\cos t, & \text{if } i = j \\ 1 & \text{if } |i-j| = 1 \\ 0 & \text{otherwise} \end{cases}$ (P) ½

If D denotes the determinant of the coefficient matrix then maximum value of 'D' is

(B) $\lim_{n\to\infty} \frac{1}{\sqrt{n}}\left(1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}+..........+\frac{1}{\sqrt{n}}\right)$ has the equal to (Q) 1

(C) The product of all value of 'x' which satisfy $\sin\left(4\tan^{-1}x\right) = \frac{24}{25}$, is (R) 2

(D) Let (S) 4

$I_1 = \int_0^{\frac{\pi}{2}} \cos\theta.f\left(\sin\theta + \cos^2\theta\right)d\theta$ and $I_2 = \int_0^{\frac{\pi}{2}} \sin 2\theta.\ f\left(\sin\theta + \cos^2\theta\right)d\theta$

then $\frac{I_1}{I_2}$ is equal to

Q. 3 Column I Column II
(codeV3T8PBQ3)

(A) Let $A_1, A_2, A_3, ..., A_n$ are the vertices of a polygon of sides n. If the number of pentagons that can be constructed by joining these vertices such that none of the side of the polygon is also the side of the pentagon is 36, then the value of n is equal to (P) 8

(B) Number of polynomials of the form $x^3 + ax^2 + bx + c$ which are divisible by $\left(x^2+1\right)$ where $a, b, c \in \{1, 2, 3, ..., 9, 10\}$, is equal to (Q) 10

(C) For $n \geq 2, n \in N$, number of solutions of the equation $\bar{z} + z^6 = i\left(\bar{z} - z^6\right)$ is (R) 11

(D) Let f and g be function defined for all non negative integers such that for each x (S) 12

(a) $f(x)$ is a non negative integer, (b) $0 \leq g(x) < 1$ and (c) $2^{f(x)+g(x)} = x$ the value of $f(4) + f(1000)$ equals

SOLUTION

SINGLE CORRECT TYPE

Q. 1 (A) least value is 14 which occurs when $x \in [2, 8]$]

Q. 2 (B) $f'(3^+) = \lim\limits_{h\to 0} \dfrac{f(3+h)-f(3)}{h} = \lim\limits_{h\to 0} \dfrac{(2-e^h)-1}{h} = -\lim\limits_{h\to 0}\left(\dfrac{e^h-1}{h}\right) = -1$

$f'(3^-) = \lim\limits_{h\to 0} \dfrac{f(3-h)-f(3)}{-h} = \lim\limits_{h\to 0} \dfrac{\sqrt{10-(3-h)^2}-1}{-h} = -\lim\limits_{h\to 0} \dfrac{\sqrt{1+(6h-h^2)}-1}{-h}$

$= \lim\limits_{h\to 0} \dfrac{(6h-h^2)}{-h\left(\sqrt{1+6h-h^2}+1\right)} = \lim\limits_{h\to 0} \dfrac{h(h-6)}{h\left(\sqrt{1+6h-h^2}+1\right)} = \dfrac{-6}{2} = -3$

Hence $f'(3^+) \ne f'(3^-)$ $\Rightarrow$ (B)]

Q. 3 (C) Sol $\sum\limits_{k=0}^{2009} g(k) = g(0)+g(1)+g(2)+.........+g(2009) = ?$

Now $\left.\begin{array}{l} f(k)=\frac{k}{2009} \\ f(2009-k)=\frac{2009-k}{2009}\end{array}\right] \Rightarrow f(k)+f(2009-k)=1$ ……(1)

again $g(k) = \dfrac{f^4(k)}{(1-f(k))^4+f^4(k)}$ …….(2)

Again $g(2009-k) = \dfrac{f^4(2009-k)}{\left(1-f[2009-k]^4+f^4(2009-k)\right)} = \dfrac{[1-f(k)]^4}{(f(k))^4+(1-f(k))^4}$ ……..(3)

(2) + (3) gives

$\therefore$ $g(k)+g(2009-k) = \dfrac{f^4(k)+(1-f(k))^4}{(f(k))^4+(1-f(k))^4} = 1$

$\therefore$ $g(0)+g(2009)=1$ $\Rightarrow$ $g(1)+g(2008)=1$ $\Rightarrow$ $g(2)+g(2007)=1$

: : :

. . .

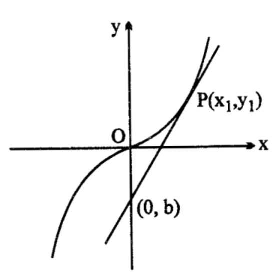
$\underline{g(1004)+g(1005)=1}$ $\Rightarrow$ $\sum\limits_{k=0}^{2009} g(k) = 1005$ $\Rightarrow$ [C]]

Q. 4 (C) Sol $g(x) = f(-x+f(f(x)))$; $f(0)=0$; $f'(0)=2$

$g'(x) = f(-x+f(f(x)))[-1+f'(f(x)).f'(x)]$

$g'(0) = f'(f(0)).[-1+f'(0).f'(0)] \Rightarrow f'(0)[-1+(2)(2)] \Rightarrow (2)(3) = 6$ Ans.]

Q. 5 (C) Sol $f(x) = \dfrac{41x^3}{3}$



$f'(x) = 41x^2 \Rightarrow f'(x)|_{x_1,y_1} = 41x_1^2 \Rightarrow \therefore$ $41x_1^2 = 2009 = 7^2.41$

$x_1^2 = 49$ $\Rightarrow$ $x_1 = 7;\ y_1 = \dfrac{41.7^3}{3}(x_1 \ne -7, \text{ think!})$

Now $\dfrac{y_1-b}{x_1-0} = 2009$ $\Rightarrow$ $y_1 - b = 7.2009 = 7^3.41$

$b = \dfrac{41.7^3}{3} - 7^3.41 = \dfrac{41.7^3}{3}(-2) = -\dfrac{82.7^3}{3}$ **Ans.]**

Q. 6 (D) Sol $f(x)=\int \frac{x^{2009}}{\left(1+x^2\right)^{1006}}dx \Rightarrow$ Put $1+x^2=t \quad \Rightarrow \quad 2xdx=dt$

$I=\frac{1}{2}\int \frac{(t-1)^{1004}dt}{t^{1006}}=\frac{1}{2}\int \left(1-\frac{1}{t}\right)^{1004}.\frac{1}{t^2}dt \quad \Rightarrow$ put $\quad 1-\frac{1}{t}=y \quad \Rightarrow \quad \frac{1}{t^2}dt=dy$

$\therefore \quad I=\frac{1}{2}\int y^{1004}dy=\frac{1}{2}\frac{y^{1005}}{1005}+C=\frac{1}{2010}\left(\frac{t-1}{t}\right)^{1005}+C$

Q. 7 (B) Sol $S=\frac{1+2^{2008}+3^{2008}+.........+n^{2008}}{n^{2009}}$

$Tr=\frac{1}{n}\frac{r^{2008}}{n^{2008}}=\frac{1}{n}.\left(\frac{r}{n}\right)^{2008} \quad \Rightarrow \quad S=\int x^{2008}dx=\frac{1}{2009}$]

COMPREHENSION OR PARAGRAPH

Q. 1 [A] **Q. 2** [B] **Q. 3** [D]

[Sol. (1) $\tan^{-1}y=\tan^{-1}x+C \quad \Rightarrow \quad x=0;\ y=1 \quad \Rightarrow \quad C=\frac{\pi}{4}$

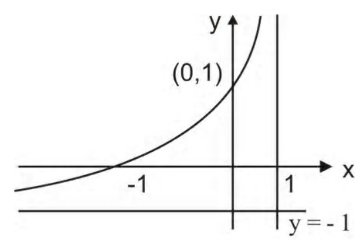
$\Rightarrow \quad \tan^{-1}y=\tan^{-1}x+\frac{\pi}{4}$ note : even $\frac{-\pi}{4}<\tan^{-1}x+\frac{\pi}{4}<\frac{\pi}{2}$;

$\frac{-\pi}{2}<\tan^{-1}x<\frac{\pi}{4}$; $-\infty<x<1\Rightarrow(A) \quad \Rightarrow \quad x<1 \quad \Rightarrow \quad (A)$

(3) $\because \quad y=\tan\left(\tan^{-1}+\frac{\pi}{4}\right)=\frac{x+1}{1-x} \quad \Rightarrow$ (D) is correct

(2) The graph of $f(x)$ is as shown.

Hence range is $(-1,\infty) \quad \Rightarrow$ **(B)**



Q. 4 [A] **Q. 5** [D] **Q. 6** [A]

[Sol. Since minimum value is zero hence touches the x-axis and mouth opening upwards i.e., $a>0$ given $f(x-4)=f(2-x) \Rightarrow x\to x+3 \Rightarrow$

$f(x-1)=f(-1-x) \Rightarrow f(-1+x)=f(-1-x)$

Hence *f* is symmetric about the line $x=-1$

$\therefore \quad f(x)=a(x+1)^2$

Now given $f(x)\geq x\ \forall\ x$

$f(1)\geq 1 \quad \ldots(1)$ and $f(x)\leq\left(\frac{x+1}{2}\right)^2$ in (0, 2) $\quad \Rightarrow \quad f(1)\leq 1$

From (1) and (2)

$f(1)=1 \quad \Rightarrow$ now $f(x)=a(x+1)^2$

$f(1)=4a=1 \quad \Rightarrow \quad a=\frac{1}{4} \quad \Rightarrow \therefore \quad f(x)=\frac{(x+1)^2}{4}$ now proceed]

Q. 7 [A] **Q. 8** [D] **Q. 9** [C]

[Sol. $f(0)=2 \Rightarrow f(x)=\left(e^x+e^{-x}\right)\cos x-2x-\left[x\int_0^x f'(t)dt-\int_0^x tf'(t)dt\right]$

$f(x)=\left(e^x+e^{-x}\right)\cos x-2x-\left[x\left(f(x)-f(0)\right)-\left\{t.f(t)\Big|_0^x-\int_0^x f(t)dt\right\}\right]$

$f(x)=(e^x+e^{-x})\cos x-2x-xf(x)+2x+\left[xf(x)-\int_0^x f(t)\,dt\right] \Rightarrow f(x)=(e^x+e^{-x})\cos x-\int_0^x f(t)\,dt$ ..(1)

differentiating equation (1)

$f'(x)+f(x)+\cos x(e^x-e^{-x})-(e^x+e^{-x})\sin x$

Hence $\frac{dy}{dx}+y=e^x(\cos x-\sin x)-e^{-x}(\cos x+\sin x)$ **Ans. (i)**

(ii) $f'(0)+f(0)=0-2.0=0$ **Ans (ii)**

(iii) I.F. of DE (1) is e^x

$y.e^x=\int e^{2x}(\cos x-\sin x)dx-\int(\cos x+\sin x)dx$ $y.e^x=\int e^{2x}(\cos x-\sin x)dx-(\sin x-\cos x)+C$

Let $I=\int e^{2x}(\cos x-\sin x)dx=e^{2x}(A\cos x+B\sin x)$

Solving $A=3/5$ and $B=-1/5$ and $C=2/5$

$\therefore$ $y=e^x\left(\frac{3}{5}\cos x-\frac{1}{5}\sin x\right)-(\sin x-\cos x)e^{-x}+\frac{2}{5}e^{-x}$ **Ans. (iii)]**

ASSERTION REASON TYPE

Q. 1 (B) Sol $f(x)=\log_{1/4}\left(x-\frac{1}{4}\right)+\frac{1}{2}\log_4\left(x^2-\frac{x}{2}+\frac{1}{16}\right)$ $\left(x>\frac{1}{4}\right)$

$=\log_{1/4}\left(x-\frac{1}{4}\right)+1+\log_4\left(x-\frac{1}{4}\right) \Rightarrow =-\log_4\left(x-\frac{1}{4}\right)+\log_4\left(x-\frac{1}{4}\right)+1=1$ $\Rightarrow$ f is constant

Hence f is many one as well into. Also range is a singleton $\Rightarrow$ *f* is constant but a constant function can be anything $\Rightarrow$ not the correct explanation]

Q. 2 (B) Sol Domain is $\{-1, 1\}$ and range is $\left\{-\frac{\pi}{2},\frac{\pi}{2}\right\}$ and domain having two elements $\not\Rightarrow$ range must have two elements]

Q. 3 (A) Sol $f(x)=\frac{1}{2\{-x\}}-\{x\},\ x\notin 1$

Using $\{x\}+\{-x\}=1$ if $x\notin I\Rightarrow\{x\}=1-\{-x\}$

$\therefore$ $f(x)=\frac{1}{2\{-x\}}-(1-\{-x\})=\{x\}+\frac{1}{2\{-x\}}-1 \Rightarrow f(x)/_{min.}=2.\frac{1}{\sqrt{2}}-1=\sqrt{2}-1$]

Q. 4 (D) Sol $\frac{x}{4e}+\frac{e^3}{x}\ge 2\sqrt{\frac{x}{4e}.\frac{e^3x}{x}}=e$ Hence range is $[0,\infty)$ $\Rightarrow$ $S-1$ is false]

Q. 5 (C)

Q. 6 (B) Line touches the curve at $(0, b)$ and $\left.\frac{dy}{dx}\right]_{x=0}$ also exists but even if $\frac{dy}{dx}$ fails to exist. tangent line can be drawn.]

Q. 7 (D) $\lim_{x\to\pi/2}\frac{\sin(\cot^2 x)}{\cot^2 x}.\frac{\cot^2 x}{(\pi-2x)^2}$; put $x=\frac{\pi}{2}-h \Rightarrow \lim_{x\to 0}\frac{\tan^2 h}{4h^2}=\frac{1}{4}$]

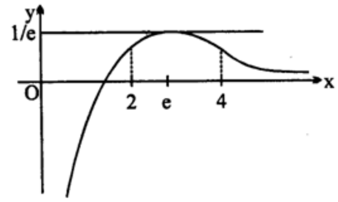
Q. 8 (B) Range of f is $\left\{\frac{\pi}{2}\right\}$ and domain of f is $\{0\}$. Hence if domain of f is singleton then angle has to be a singleton.

If $S-2$ and $S-1$ are reverse then the answer will be B.]

Q. 9 (A) Sol $y = |\ln x|$ not differentiable at $x = 1$

$y = |\cos|x||$ is not differentiable at $x = \frac{\pi}{2}, \frac{3\pi}{2}$

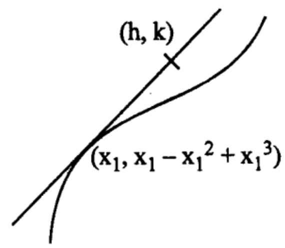
$y = \cos^{-1}(\text{sgn}\, x) = \cos^{-1}(1) = 0$ differentiable $\forall\ x \in (0, 2\pi)$]



Q. 10 (A) $f'(x) = \frac{1-\ln x}{x^2}$; note that $f(2) = f(4) \Rightarrow f$ is increasing $x \in (0, e)$ and f is decreasing (e, ∞)]

Q. 11 (B) Sol $f'(x) = 1 - 2x + 3x^2 > 0$

$\Rightarrow \quad -\frac{a}{b} > 0 \quad \Rightarrow \quad ab < 0$



$\frac{x_1^3 - x_1^2 + x_1 - k}{x_1 - 1/3} = 3x_1^2 - 2x_1 + 1$

$g(x_1) = 2x_1^3 - 2x_1^2 + \frac{2}{3}x_1 + k - \frac{1}{3}$

$g'(x_1) = 6x_1^2 - 4x_1 + \frac{2}{3} = \frac{2}{3}(3x_1 - 1)^2$]

Q. 12 (A) **Q. 23** (C) **Q. 24** (A)

Sol Let $f(x) = 0$ has two roots say $x = r_1$ and $x = r_2$ where $r_1, r_2 \in [a, b]$

$\Rightarrow \quad f(r_1) = f(r_2)$

Hence $\exists$ there must exist some $c \in (r_1, r_2)$ where $f'(c) = 0$

but $f'(x) = x^6 - x^5 + x^4 - x^3 + x^2 - x + 1$

for $x \geq 1$, $f'(x) = (x^6 - x^5) + (x^4 - x^3) + (x^2 - x) + 1 > 0$

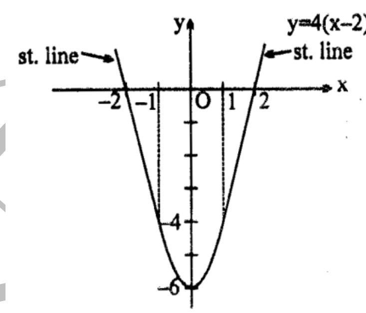
for $x \leq 1$, $f'(x) = (1 - x) + (x^2 - x^3) + (x^4 - x^5) + x^6 > 0$

hence $f'(x) > 0$ for all x

$\therefore$ Rolles theorem fails $\Rightarrow$ $f(x) = 0$ can not have two or more roots.]

Q. 13 (D) $f(x) = x^2 - |x^2 - 1| + 2||x| - 1| + 2|x| - 7$

$f(-x) = f(x) \Rightarrow$ Area $x < 0$ = area $x > 0$



Case - I : for $0 < x < 1$

$y = x^2 - (1 - x^2) + 2(1 - x) + 2x - 7 = 2(x^2 - 3)$

If $- < x < 0$

$f(x) = 2(x^2 - 3)$ now $f'(0^+) = f'(0^-) = 0$

for $x > 1 \Rightarrow f(x) = x^2 - (x^2 - 1) + 2(x - 1) + 2x - 7$

$\Rightarrow f(x) = 4(x - 2)$

note $\lim_{x \to 1} f(x) = -4 = f(1) \Rightarrow$ f is continuous. Also $f'(1^-) = f'(1^+) = 4$

$\Rightarrow$ f is derivable at $x = 1$]

Q. 14 (D) Sol Let $b > 0$, then $f(1) = b > 0$ and

$f(5) = 2a + 3b - 6 = 2(a + 2b) - b - 6 = 4 - b - 6 = -(2 + b) < 0$

Hence by IVT, $\exists$ some $c \in (1, 5)$ s.t. $\Rightarrow$ $f(c) = 0$

If $b = 0$ then $a = 2$

$f(x) = 2\sqrt{x-1} - \sqrt{2x^2 - 3x + 1} = 0 \Rightarrow \quad 4(x - 1) = 2x^2 - 3x + 1 = (2x - 1)(x - 1)$

$(x-1)(2x-5)=0 \Rightarrow x=\frac{5}{2}$

Hence $f(x)=0$ if $x=\frac{5}{2}$ which lies in $(1, 5)$

If $b<0$, $f(1)=b<0$ and

$f(2)=a+b\sqrt{3}-\sqrt{3}(a+2b)+(\sqrt{3}-2)b-\sqrt{3} \Rightarrow (2-\sqrt{3})-(2-\sqrt{3})b$

$=(2-\sqrt{3})(1-b)>0$ (as $b<0$)

Hence $f(1)$ as $f(2)$ have opposite signs

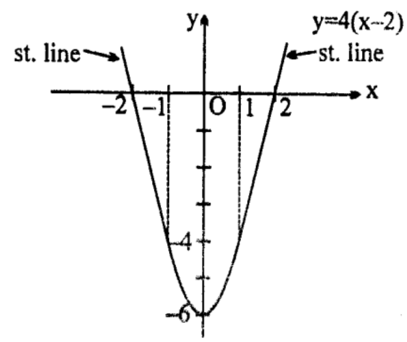
$\exists$ some $c\in(1, 2)\subset(1, 5)$ for which $f(c)=0$

$\Rightarrow$ Statement -1 is valid for all $b\in R$ $\Rightarrow$ statement -1 is false.

Statement -2 is obviously true $\Rightarrow$ (D)]

Q. 15 (D) $f(x)=x^2-|x^2-1|+2||x|-1|+2|x|-7$

$f(-x)=f(x) \Rightarrow$ Area for $x<0=$ area of $x>0$



Case-I : for $0<x<1$

$y=x^2-(1-x^2)+2(1-x)+2x-7=2(x^2-3)$

For $x>1$

$f(x)=x^2-(x^2-1)+2(x-1)+2x-7$

$f(x)=4(x-2)$

note $\lim_{x\to 1} f(x)=-4=f(1)$

$\Rightarrow$ f is continuous $\forall$ $x\in R$. Also $f'(1^{-1})=f'(1^{+})=4$

$\Rightarrow$ f is derivable at $x=1$

Area bounded by the $y=f(x)$ and +ve x-axis is

$$\text{Area }=\left|2\int_0^1(x^2-3)dx\right|+2=\left|2\left(\frac{1}{2}-3\right)\right|+2=\frac{16}{3}+2=\frac{22}{3}$$

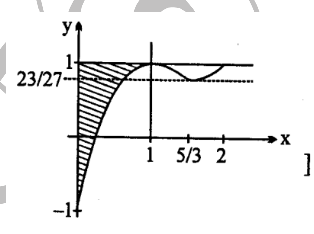
$\therefore$ Area bounded by the $f(x)$ and x-axis $=2\left(\frac{22}{3}\right)=\frac{44}{3}$ Ans.]

MORE THAN ONE MAY CORRECT TYPE

Q. 1 A, B, D [Hint.
A=1; A = 1; B = 1; C = aperiodic; D = 2π]

Q. 2 B, C, D

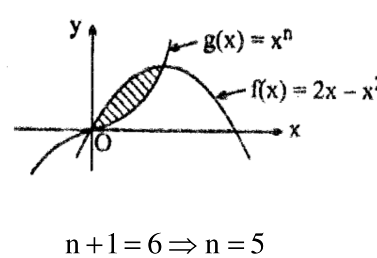
[Sol. The graph of $y=f(x)=(x-1)^2(x-2)+1$



$f(1)=f(2)=1$ and $f(0)=-1 \Rightarrow$ Verify alternatives]

Q. 3 B, C, D

[Sol. Solving $f(x)=2x-x^2$ and $g(x)=x^n$



We have $2x-x^2=x^n \Rightarrow x=0$ and $x=1$

$$A=\int_0^1(2x-x^2-x^n)dx-x^2-\frac{x^3}{3}-\frac{x^{n+1}}{n+1}\Bigg]_0^1=1-\frac{1}{3}-\frac{1}{n+1}=\frac{2}{3}-\frac{1}{n+1}$$

hence, $\frac{2}{3}-\frac{1}{n+1}=\frac{1}{2} \Rightarrow \frac{2}{3}-\frac{1}{2}=\frac{1}{n+1} \Rightarrow \frac{4-3}{6}=\frac{1}{n+1} \Rightarrow n+1=6 \Rightarrow n=5$

Hence n is a divisor of 15, 20, 30 $\Rightarrow$ B, C, D]

Q. 4 A, B, D **Sol.** $\frac{dy}{dx}+y=f(x)$

$\text{I.F.}=e^x \Rightarrow ye^x=\int e^x f(x)\,dx+C$

now if $0\le x\le 2$ then $ye^x=\int e^x e^{-x}dx+C \quad \Rightarrow \quad ye^x=x+C$

$x=0,\ \ y(0)=1, \qquad C=1 \quad \Rightarrow \quad \therefore \quad ye^x=x+1 \qquad \ldots.(1)$

$y=\frac{x+1}{e^x}; \qquad y(1)=\frac{2}{e}$ Ans. $\Rightarrow$ (A) is correct

$y'=\frac{e^x-(x+1)e^x}{e^{2x}} \Rightarrow y'(1)=\frac{e-2e}{e^2}=\frac{-e}{e^2}=-\frac{1}{e}$ Ans. $\Rightarrow$ (B) is correct

if $x>2 \quad \Rightarrow \quad ye^x=\int e^{x-2}dx \Rightarrow \quad ye^x=e^{x-2}+C \Rightarrow y=e^{-2}+Ce^{-x}$

as y is continuous

$\therefore \quad \lim\limits_{x\to 2}\frac{x+1}{e^x}=\lim\limits_{x\to 2}\left(e^{-2}+Ce^{-x}\right) \Rightarrow 3e^{-2}=e^{-2}+Ce^{-2} \quad \Rightarrow \quad C=2$

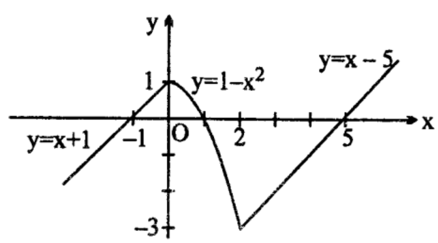
$\therefore$ for $x>2 \quad \Rightarrow \quad y=e^{-2}+2e^{-x}$ hence $y(3)=2e^{-3}+e^{-2}=e^{-2}\left(2e^{-1}+1\right)$

$y'=-2e^{-x} \quad \Rightarrow \quad y'(3)=-2e^{-3}$ Ans. $\Rightarrow$ (D) is correct]

MATCH THE COLUMNS

Q. 1 (A) P, S, (B) Q, R; (C) Q, R (D) P.S.

[Sol. Let $g(x)=\begin{cases}\frac{\pi}{2}-2\tan^{-1}f(x) & f(x)\in(-1,1)\\ -\frac{\pi}{2}-2\tan^{-1}f(x) & f(x)\in(-\infty,-1)\\ \frac{3\pi}{2}-2\tan^{-1}f(x) & f(x)\in(1,\infty)\end{cases}$



(A) $\frac{d(x)}{d(x)}=-\frac{2}{1+f^2(x)}=-\frac{1}{13} \quad \Rightarrow \quad f(x)\pm 5$

$\Rightarrow \quad x=-6,\,10 \quad \Rightarrow \quad x=-6,10 \quad \Rightarrow \quad$ P,S

(B) refer to graph of $y=f(x) \quad \Rightarrow$ **Q,R**

(C) $-k\in(-3,1) \quad \Rightarrow \quad k\in(-1,3) \quad \Rightarrow$ **Q, R**

(D) $g'(x)=\frac{-2f'(x)}{1+f^2(x)}<0 \Rightarrow \quad f'(x)>0 \quad \Rightarrow \quad x=-6,\,10 \quad \Rightarrow$ **P,S**]

Q. 2 (A) Q; (B) S; (C) P; (D) R

[Sol. $f(x) = \frac{\ln x}{8} - ax + x^2;\ f'(x) = \frac{1}{8x} - a + 2x$ ....(1) $\Rightarrow$ $f'(x) = \frac{16x^2 - 8ax + 1}{8x}$

If $a = 1$, $f'(x) = \frac{(4x-1)^2}{8x} = 0 \Rightarrow$ $x = \frac{1}{4}$

Hence $x = 1/4$ is the point of inflection and $a = 1$ $\Rightarrow$ (C) $\Rightarrow$ (P)

now $f'(x) = 0$ gives $\frac{16x^2 - 8ax + 1}{8x} = 0$ or $16x^2 - 8ax + 1 = 0$

$x = \frac{8a \pm \sqrt{64a^2 - 64}}{32}$ $\Rightarrow$ $x = \frac{a + \sqrt{a^2 - 1}}{4}(a > 1)$ or $x = \frac{a - \sqrt{a^2 - 1}}{4}(a > 1)$

and $f''(x) = 2 - \frac{1}{8x^2}$ $\Rightarrow f''\left(\frac{a + \sqrt{a^2 - 1}}{4}\right) = 2 - \frac{16}{8\left(a + \sqrt{a^2 - 1}\right)^2} = 2 - \frac{2}{\left(a + \sqrt{a^2 - 1}\right)^2}$ $(a > 1)$

Hence for $a > 1$ and $x = \frac{a + \sqrt{a^2 - 1}}{4}$, f( ) has a local minima $\therefore$ **(B)** $\Rightarrow$ (S)

|||ly for $a > 1$ and $x = \frac{a - \sqrt{a^2 - 1}}{4}$

we have local mixima

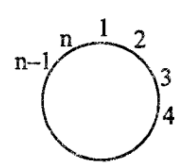
$\therefore$ **(A)** $\Rightarrow$ **(Q)** finally for $0 \le a < 1$

$f'(x) = \frac{16x^2 - 8ax + 1}{8x}$ $\Rightarrow \Delta = 64a^2 - 64 < 0$

Hence $f'(x) > 0$ $\Rightarrow$ f is monotonic $\Rightarrow$ **(D)** $\Rightarrow$ **(R)**

Q. 3 (A) S; (B) Q; (C) P; (D) R

[Sol. **(A)** **1st vertex** ${}^nC_1$ way



2 and n can not be taken. Remaining vertices are

$\underbrace{3, 4, 5 \ldots\ldots\ldots\ldots (n-1)}_{(n-3)\text{vertices}}$ $\Rightarrow$ $\underbrace{OOOO}_{\text{four to be taken}}$

$\underbrace{|x||x||x|\ldots\ldots|x|}_{(n-7)\text{not to be taken}}$ $\Rightarrow$ number of gaps $(n-6)$ out of which 4 can be selected in ${}^{n-6}C_4$ ways.

Hence required number of ways $\frac{{}^{n-6}C_4 . n}{5} = 36$

which is satisfied by $n = 12$ Ans. $\Rightarrow$ (S)

(B) $x^3 + ax^2 + bx + c \equiv (x^2+1)(x+k) = x^3 + kx^2 + x + k$

$\Rightarrow$ $b = 1$ and $a = c$

Now 'a' can be taken in 10 ways and as $a = c$ hence 'c' can be only in one way

Also $b = 1$. Hence total 10 Ans. $\Rightarrow$ (Q)

Alternatively: $-i - a + bi + c = 0 + 0i$

$\therefore$ $c - a + (b-1)i = 0 + 0i$ $\Rightarrow$ $a = c$ and $b = 1$]

(C) $z^6(1+i) = \bar{z}(i-1)$ ....(1)

$\therefore$ $|z|^6|1+i| = |\bar{z}||-1+i|$ $\Rightarrow$ $|z|^6 = |z|$ $\Rightarrow$ $|z| = 0$ or $|z| = 1$

if $|z| = 0$ $\Rightarrow$ $z = 0$

if $|z| = 1$ then $z\bar{z} = 1$ $\Rightarrow$ $\bar{z} = \frac{1}{z}$

hence equation (1) becomes

$$z^6(1+i) = \frac{1}{z}(-1+i) \Rightarrow z^7 = \frac{-1+i}{1+i} = \frac{(-1+i)(1-i)}{2} = i \Rightarrow z = \cos\frac{2m\pi + \frac{\pi}{2}}{7} + i\sin\frac{2m\pi + \frac{\pi}{2}}{7}$$

Where $m = 0, 1, 2, ..., 6$ are the other solutions

Total solutions = 8 Ans. $\Rightarrow$ (**P**)

(D) $2^{f(x)+g(x)} = x$ $\Rightarrow$ Put $x = 4$ $2^{f(4)+g(4)} = 4 = 2^2$

$f(4) + g(4) = 2 \Rightarrow$ $g(4) = 2 - f(4) \Rightarrow$ $\therefore$ $0 \le 2 - f(4) < -1$ $\Rightarrow$ $-2 \le f(4) < -1$

$1 < f(4) \le 2$ $\Rightarrow$ $f(4) = 2$ (as $f(x)$ is a non negative integer)

again put

$2^{f(1000)+g(1000)} = 1000$ $\Rightarrow$ $f(1000) + g(1000) = \log_2(1000)$ $\Rightarrow$ $g(1000) = \log_2(1000) - f(1000)$

$\therefore$ $0 \le \log_2 1000 - f(1000) < 1 \Rightarrow -\log_2 1000 \le -f(1000) < 1 - (\log_2 1000)$

$(\log_2 1000) - 1 \le f(1000) \le \log_2 1000 \Rightarrow$ $f(1000) = 9$ as f is an integer

Hence $f(4) + f(1000) = 11$ Ans. $\Rightarrow$ (**R**)]