THIS FILE CONTAINS

MATCH THE COLUMNS

(COLLECTION # 2)

Very Important Guessing Questions

For IIT JEE 2011 With Detail Solution

Junior Students Can Keep It Safe For Future IIT-JEEs

➔ ***Mix Topics***

For Collection # 1 (Next File)

TOPIC = MATCH THE COLUMNS

MATCH THE COLUMNS

Q. 1 **Column-I** **Column-II** **(codeV3T2PBQ2)**

(A) Number of integral values of m satisfying the relation, $17^2 + n^4 = m^2$ is (where $n \in I$) (P) 1

(B) If the distance from the orthocenter to the vertex of the triangle, is k times the distance from the circumcentre to the middle point of the opposite side, ten k equals (Q) 2

(C) The remainder when 3^{37} is divided by 80 is equal to (R) 3

(D) Number of values of k for which the equations $x^3 + x^2 - kx - k = 0$ $x^3 - x^2 - 3x + k = 0$ have a common root, is (S) 4

Q. 2 **Column-I** **Column-II** **(codeV3T4PBQ2)**

(A) All the five digit numbers in which each successive digit exceeds its predecessor are arranged in the increasing order. The $(105)^{th}$ number does not contain the digit (P) 1

(B) Possible integers in the range of the function (Q) 2

$f(x) = \frac{4}{x} = \tan\left(\frac{\pi x}{8}\right)$ in the interval $[1, 2]$ can be

(C) Let z_1, z_2, z_3 be three complex number such that (R) 3

$|z_1| = |z_2| = |z_3| = 1$ and $\frac{z_1^2}{z_2 z_3} + \frac{z_2^2}{z_1 z_3} + \frac{z_3^2}{z_1 z_2} + 1 = 0$ then $|z_1 + z_2 + z_3|$ can take the value equal to

(D) If $y = \log_a\left(\frac{x+2}{3x}\right)$, $a \in (0, 1)$ is positive, then x can be equal to (S) 4

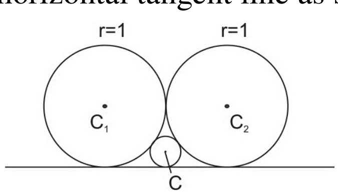
Q. 3 **Column-I** **Column-II** **(codeV3T4PBQ3)**

(A) Box contains 2 one rupee, 2 five rupee, 2 ten rupee and 2 twenty rupee coin. Two coins are drawn at random simultaneously. The probability that their sum is Rs. 20 or more, is (P) 1/3

(B) An unfair coin has the property that when flipped 4 times, it has the same probability of showing up 2 heads and 2 tails (in any order) or 3 heads and 1 tail (in any order). The probability of getting a head in any one flip, is (Q) 11/28

(C) If $A + B + C = \pi$ and $\sin\left(A + \frac{C}{2}\right) = 4\sin\frac{C}{2}$, then $\tan\frac{A}{2}\tan\frac{B}{2} =$ (R) 1/2

(D) Two congruent circles each of radii one are externally tangent as shown. A circle with centre C is inscribed within circle C_1, circle C_2 and the horizontal tangent line as shown. The diameter of the circle C is (S) 3/5



Q. 4 Column I Column II **(codeV3T8PBQ1)**

(A) Let $A = a_{ij}$ be a 3×3 matrix where $a_{ij} = \begin{cases} 2\cos t, & \text{if } i = j \\ 1 & \text{if } |i-j| = 1 \\ 0 & \text{otherwise} \end{cases}$ (P) ½

If D denotes the determinant of the coefficient matrix then maximum value of 'D' is

(B) $\lim_{n\to\infty} \frac{1}{\sqrt{n}}\left(1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}+..........+\frac{1}{\sqrt{n}}\right)$ has the equal to (Q) 1

(C) The product of all value of 'x' which satisfy $\sin\left(4\tan^{-1}x\right) = \frac{24}{25}$, is (R) 2

(D)Let $I_1 = \int_0^{\frac{\pi}{2}} \cos\theta . f\left(\sin\theta + \cos^2\theta\right) d\theta$ and $I_2 = \int_0^{\frac{\pi}{2}} \sin 2\theta . \ f\left(\sin\theta + \cos^2\theta\right) d\theta$ (S) 4

then $\frac{I_1}{I_2}$ is equal to

Q. 5 Column-I Column-II **(codeV3T9PBQ1)**

(A) Number of solution of the equation $\cos^{-1}(\cos x) = [x]$ where $[\ ]$ denotes the greatest integer function, is (P) 1

(B) The number of real common tangents to the circle $5x^2 + 5y^2 = 16$ and the hyperbola $3x^2 - y^2 = 48$, is (Q) 2

(C) The number of real common normals to parabola $y^2 = 4x$ and the circle $(x-1)^2 + (y-1)^2 = 1$, is (R) 4

(S) 5

Q. 6 **Column-I** **Column-II** **(codeV3T9PBQ2)**

(A) Let $f(x) = x^3 + ax^2 + ax + 1$ has local extrema at $x = \alpha, \beta$ where $\alpha < \beta$. If $f(\alpha) + f(\beta) = 2$ then the value of 'a' equals (P) 2

(B) If $x = \tan\left(\frac{5\pi}{12}\right)$ then the value of the expression $y = x^6 - 2\sqrt{3}x^5 - x^4 + x^3 - 4x^2 + 2x - \sqrt{3}$ equals (Q) 9/2

(C) Number of natural numbers *n*, less than or equal to 24 such that $n!$ is divisible by $(1+2+3+......+n)$ is (R) 9

(D) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ then $A^2 = \lambda A + \mu I$, where $(\lambda + \mu)$ equals (S) 16

Q. 7 Column-I | Column-II **(codeV3T10PBQ1)**

(A) ABC is a triangle with $B = 90°$ and $A = 30°$, P, Q and R are on AB, BC and CA respectively such that PQR is an equilateral triangle. If Q is the midpoint of BC and $BC = 4$, then the side of the equilateral triangle PQR is $\sqrt{K}$. The value of K is equal to — (P) 5

(B) A drawer contains several pairs of socks. Not wanting to count Mr. M asked miss K "how many pairs of socks are there in the drawer?". Miss K, not wanting to give answer replied. "well, each sock has exactly one matching sock and the probability that two socks drawn from the drawer form a matching pair is 1/15." Then the answer to Mr. M's question is — (Q) 6

(C) If w is one of the imaginary cube root of unity then the sum
$1(2-w)(2-w^2)+2(3-w)(3-w^2)+..........(n-1)(n-w)(n-w^2)=220$
The value of n equals — (R) 7

(S) 8

SOLUTION
MATCH THE COLUMNS

Q. 1 () Sol. (A) R; (B) Q; (C) R; (D) R

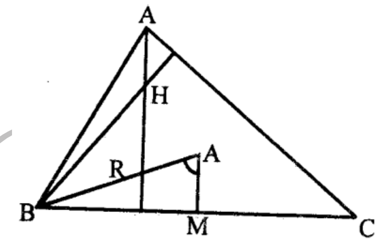
[Sol. (A) $17^2 = (m-n)^2(m+n^2) = 1.17^2 \Rightarrow$ Hence $m - n^2 = 1$

$m + n^2 = 17^2$

$2m = 290 \quad \Rightarrow \quad m = 145$

$n^2 = 144 \quad \Rightarrow \quad n = 12$

Also $m = \pm 17 \quad \& \quad n = 0$



(B) $\left.\begin{matrix} AH = 2R\cos A \\ AM = R\cos A \end{matrix}\right] \Rightarrow k = 2$

(C) $3^{37} = 3.3^{36} = 3(3^4)^9$

$\therefore \quad 3^{37} = 3(1+80)^9 \Rightarrow = 3\left[1 + {}^9C_1.80 + ... + {}^9C_9.80^9\right]$

$= 3 + 80I$ when I is an integer $\Rightarrow$ remainder is 3 Ans.

(D) $x^2(x+1) = k(x+1)$ $\quad\quad [Ans.\ 3;\ k \in \{-1, 0, 3\}]$

$(x+1)(x^2-k) = 0 \quad \Rightarrow \quad x = -1 \ or \ x^2 = k$

If $x = -1$ is a common root then

$x^3 - x^7 = 3x - k \Rightarrow -1 - 1 = -3 - k$

$k = -1$ for $k = -1$ the equation is $x^3 - x^2 - 3x - 1 = 0$

If $x \neq -1$, then $x = \sqrt{k}$ or $-\sqrt{k}$

If $x = \sqrt{k}$ then $k\sqrt{k} - k - 3\sqrt{k} + k = 0$

$k = 0 \quad or \quad k = 3]$

Q. 2 (A) P, R, S; (B) R, S; (C) P, Q; (D) Q, R, S]

(A) Starting with 1

1				

2 3 4 5 6 7 8 9

$= {}^8C_4 = 70$

Starting with 2

2				

3 4 5 6 7 8 9

$= {}^7C_4 = 35 \Rightarrow$ Total = 105

$(105)^{th}$ number 26789 $\Rightarrow$ $(105)^{th}$ does not contain 1, 3, 4, 5 $\Rightarrow$ **P,R,S**

(B) $f(x) = \frac{4}{x} + \tan\left(\frac{\pi x}{8}\right) \Rightarrow f'(x) = -\frac{4}{x^2} + \frac{\pi}{8}\sec^2\left(\frac{\pi x}{8}\right)$ decreasing in $[1, 2]$

$\Rightarrow \quad f(x)]_{min} = f(2) = 2 + 1 = 3 \Rightarrow f(x)]_{max} = f(1) = 4 + \sqrt{2} - 1 = 3 + \sqrt{2}$

range $\left[3, 3+\sqrt{2}\right] \quad \Rightarrow \quad$ possible integer $\{3, 4\} \Rightarrow$ **R,S**

(C) Given $|z_1| = |z_2| = |z_3| = 1 \Rightarrow \quad z_1 = \frac{1}{z_1}$ etc.

also $\frac{z_1^2}{z_2 z_3} + \frac{z_2^2}{z_1 z_3} + \frac{z_3^2}{z_1 z_2} + 1 = 0 \quad \Rightarrow \quad (z_1)^3 + (z_2)^3 + z_1 z_2 z_3 = 0$

$\Rightarrow \quad (z_1)^3 + (z_2)^3 + (z_3)^3 - 3z_1 z_2 z_3 = -4z_1 z_2 z_3$

$$(z_1+z_2+z_3)\left[(z_1)^2+(z_2)^2+(z_3)^2-\sum z_1z_2\right]=-4z_1z_2z_3$$

$$\sum z_1\left[\left(\sum z_1\right)^2-3\sum z_1z_2\right]=-4z_1z_2z_3$$

Let $z_1+z_2+z_3=z \quad \Rightarrow \quad \overline{z_1}+\overline{z_2}+\overline{z_3}=\overline{z} \Rightarrow z\left[z^2-3\sum z_1z_2\right]=-4z_1z_2z_3$

$$z^3=3z\sum z_1z_2-4z_1z_2z_3 \Rightarrow z^3=z_1z_2z_3\left[3z\left(\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right)-4\right]+z_1z_2z_3\left[3z\left(\overline{z_1}+\overline{z_2}+\overline{z_3}\right)-4\right]$$

$$z^3=z_1z_2z_3\left[3|z|^2-4\right] \Rightarrow \therefore \quad |z|^3=\left|3|z|^2-4\right| \quad ....(1)$$

now if $|z|\ge\frac{2}{\sqrt{3}}$ then $|z|^3=3|z|^2-4 \quad \Rightarrow \quad |z|^3-3|z|^2+4=0$

$\Rightarrow \quad |z|^2(|z|-2)-|z|(|z|-2)-2(|z|-2)=0 \quad \Rightarrow \quad (|z|-2)(|z|^2-|z|-2)=0$

$\Rightarrow \quad (|z|-2)(|z|-2)(|z|+1)=0 \quad \Rightarrow \quad |z|=2$ or $|z|=-1$ (rejected)

now if $0<|z|<\frac{2}{\sqrt{3}}$ then equation (1) becomes

$|z|^3=4-3|z|^2 \quad \Rightarrow \quad |z|^3+3|z|^2-4=0$

$\Rightarrow \quad |z|^2(|z|-1)+4|z|(|z|-1)+4(|z|-1)=0 \quad \Rightarrow \quad (|z|-1)(|z|^2+4|z|+4)=0$

$\Rightarrow \quad (|z|-1)(|z|+2)^2=0 \quad \Rightarrow \quad |z|=+1$ or $|z|=-2$ (rejected)

hence $|z|=\{1, 2\}$ where $|z|=|z_1+z_2+z_3| \quad \Rightarrow$ **A, B**

NOTE : $z_1=1;\ z_2=i$ and $z_3=-i$

and $z_1=1;\ z_2=-w$ and $z_3=w^2$ also gives that result

(D) $\frac{x+2}{3x}>0 \quad \Rightarrow \quad x>0$ or $x<-2$ (domain)

Given $0<a<1$ then $\log_a\left(\frac{x+2}{3x}\right)>0 \quad \Rightarrow \quad 0<\frac{x+2}{3x}<1$

$\frac{2-2x}{3x}<0 \quad \Rightarrow \quad \frac{x-1}{x}>0 \quad \Rightarrow \quad x>1$ or $x<0$

$\Rightarrow \quad x\in\{2, 3, 4\} \quad \Rightarrow$ **Q, R, S**

Q. 3 (A) R; (B) S; (C) S; (D) R

$P(A)=1-P(\text{Total value is}<20)=1-\frac{{}^6C_2-{}^2C_2}{{}^8C_2}=1-\frac{14}{28}=1-\frac{1}{2}=\frac{1}{2}$ Ans. $\Rightarrow$ (R)

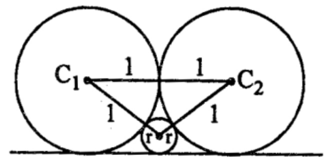
(selecting any two of 1,1,5,5, 10, 10 & when both 10 & 10 are taken)

(B) Let $P(H)=p; \quad P(T)=1-p \Rightarrow \therefore\ {}^4C_2p^2(1-p)^2={}^4C_3p^3(1-p)$

$6(1-p)=4p \quad (p\ne 0, 1)\Rightarrow 3-3p=2p\Rightarrow 5p=3 \Rightarrow \quad p=3/5$ Ans. $\Rightarrow$ (S)

(c) $\dfrac{\sin\left(A+\dfrac{C}{2}\right)}{\sin\left(\dfrac{C}{2}\right)} = 4.$ Now apply C/D and then proceed

and get $\dfrac{4-1}{4+1} = \dfrac{3}{5}$ Ans. $\Rightarrow$ (S)



(D) $(1+r)^2 = (1-r)^2 + 1$

$2r = 1 - 2r$

$4r = 1 \Rightarrow r = 1/4$ $\Rightarrow$ diameter = 1/2 Ans. $\Rightarrow$ (R)]

Q. 4 (A) S; (B) R; (C) Q; (D) Q

(A) $D = \begin{vmatrix} 2\cos t & 1 & 0 \\ 1 & 2\cos t & 1 \\ 0 & 1 & 2\cos t \end{vmatrix} = 2\cos t\left[4\cos^2 t - 1\right] - \left[2\cos t\right] = 2\cos t\left[4\cos^2 t - 2\right]$

$D = \cos t.\cos 2t$

$\Rightarrow$ $D_{max} = 4$ which occurs when $t = 0, 2\pi, 4\pi,$............ $\Rightarrow$ (S)]

(B) $T_{r+1} = \dfrac{1}{\sqrt{rn}} = \dfrac{1}{n}.\dfrac{1}{\sqrt{r/n}}$; $\therefore$ $L = \int_0^1 \dfrac{1}{\sqrt{x}}dx = 2$ Ans. $\Rightarrow$ (R)

(C) put $\tan^{-1} x = \theta$ $\Rightarrow$ $x \tan\theta$

$\sin 4\theta = \dfrac{24}{25}$ $\Rightarrow$ $2\sin 2\theta.\cos 2\theta = \dfrac{24}{25}$

$2\dfrac{2x}{1+x^2}.\dfrac{1-x^2}{1+x^2} = \dfrac{24}{25}$ $\Rightarrow$ $6x^4 + 25x^3 + 12x + 6 = 0$

$\Rightarrow$ $x_1x_2x_3x_4 = 1$ Ans. $\Rightarrow$ (**Q**)

(D) $I_1 = \int_0^{\pi/2} \cos\theta.f\left(\sin\theta + 1 - \sin^2\theta\right)d\theta$

put $\sin\theta = t$ $\Rightarrow$ $\cos\theta d\theta = dt$ $\Rightarrow I_1 = \int_0^1 f\left(t+1-t^2\right)dt$ ....(1)

Now, $I_2 = \int_0^{\pi/2} 2\sin\theta\cos\theta.f\left(\sin\theta + 1 - \sin^2\theta\right)d\theta$

put $\sin\theta = t$ $\Rightarrow$ $\cos\theta d\theta = dt$

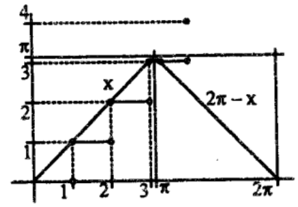
$I_2 = \int_0^1 2t.f\left(t+1-t^2\right)dt$ ....(2)

$= \int_0^1 (1-t).f\left(1-t+1-(1-t)^2\right)dt$ (Using King)

$I_2 = 2\int_0^1 (1-t).f\left(1+t-t^2\right)dt \Rightarrow I_2 = 2\left[\int f\left(1+t-t^2\right)dt - \int_0^1 t.f\left(1+t-t^2\right)dt\right]$ .....(3)

$(2)+(3)$, $\Rightarrow 2I_2 = 2\int_0^1 f\left(1+t-t^2\right)dt \Rightarrow I_2 = \int_0^1 f\left(1+t-t^2\right)dt \Rightarrow$ $I_1 = I_2 \Rightarrow \dfrac{I_1}{I_2} = 1$ Ans.]]

Q. 5 (A) Q; (B) R; (C) P



(A) $2\pi - x = 3$

$x = 2\pi + 3$

Hence the solutions are

$\{0, 1, 2, 3, 2\pi - 3\}$ $\Rightarrow$ 5 **Ans.**

(B) $S : x^2 + y^2 = \frac{16}{5}$

$H : \frac{x^2}{16} - \frac{y^2}{48} = 1$ (dividing by 48)

tangent to H is , $y = mx \pm \sqrt{16m^2 - 48}$

use $p = r$

$$\left|\frac{\sqrt{16m^2 - 48}}{\sqrt{1+m^2}}\right| = \frac{4}{\sqrt{5}} \Rightarrow 5\left(16m^2 - 48\right) = 16\left(1 + m^2\right) \quad \Rightarrow \quad 64m^2 = 16 \times 16$$

$m^2 = 4 \Rightarrow$ $m = \pm 2$

$\Rightarrow$ 4 common tangent $\Rightarrow$ (S) [for each value3 of m there are two parallel tangents]

(C) $P : y^2 = 4x$

$S : (x-1)^2 + (y-1)^2 = 1$

Normal to P: $y = mx - 2m - m^3$ passes through the centre of the circle $(1, 1)$

$1 = m - 2m - m^3$

$m^3 + m + 1 = 0 \Rightarrow$ $f'(m) > 0$ $\Rightarrow$ one real root

$\therefore$ one common normal $\Rightarrow$ (P)]

Q. 6 (A) S; (B) P; (C) S; (D) R

(A) $f'(x) = 3x^2 + 2ax + a = 0 \,{}^{\alpha}_{\beta}$; $\alpha + \beta = -\frac{2a}{3}$ and $\alpha\beta = \frac{a}{3}$

given $f(\alpha) + f(\beta) = 2$

$$\left(\alpha^3 + a\alpha^2 + a\alpha + 1\right) + \left(\beta^3 + a\beta^2 + a\beta + 1\right) = 2$$

$$\left(\alpha^3 + \beta^3\right) + a\left(\alpha^2 + \beta^2\right) + a(\alpha + \beta) = 0$$

$$(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) + a\left[(\alpha + \beta)^2 - 2\alpha\beta\right] + a(\alpha + \beta) = 0$$

$$-\frac{8a^3}{27} - a\left(\frac{2a}{3}\right) + a\left[\frac{4a^2}{9} - \frac{2a}{3}\right] - \frac{2a^2}{9} = 0$$

$\frac{4a^3}{27} = \frac{2}{3}$ $\Rightarrow$ $a = \frac{9}{2}$ **Ans.]**

(B) $x = 2 + \sqrt{3}$ $\Rightarrow$ $\frac{1}{x} = 2 - \sqrt{3}$

$y = x^5\left[x - 2\sqrt{3}\right] - x^4 + x^3 - 4x^2 + 2x - \sqrt{3} = x^5\underbrace{\left(2-\sqrt{3}\right)}_{=1/x} - x^4 + x^3 - 4x^2 + 2x - \sqrt{3}$

$= x^4 - x^4 + x^3 - 4x^2 + 2x - \sqrt{3} = x^2(x-4) + 2x - \sqrt{3} = x^2\left(\sqrt{3}-2\right) + 2x - \sqrt{3}$

$= x\left(\sqrt{3}+2\right)\left(\sqrt{3}-2\right) + 2x - \sqrt{3} = x(3-4) + 2x - \sqrt{3} = -x + 2x - \sqrt{3}$

$= x - \sqrt{3} = 2 + \sqrt{3} - \sqrt{3} = 2$ **Ans.**

(C) $\frac{2.n!}{n(n+1)}$ must be an integer

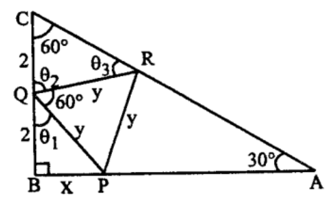
$\frac{2.(n-1)!}{(n+1)}$ must be an integer

If $(n+1)$ is an odd prime then the N^r will not be divisible by the denominator, otherwise it is divisible. Hence $n+1 \neq \underbrace{3, 5, 7, 11, 13, 17, 19, 23}_{=8}$

$\therefore$ our answer is $24 - 8 = 16$ **Ans.]**

Q. 7 (A) R; (B) S; (C) P

(A) $\theta_1 + \theta_2 = 120°$



ALSO $\theta_2 - \theta_3 = 0$

$\theta_1 = \theta_3 = \theta$ (say)

now in ΔQBP, $\sin\theta = \frac{x}{y} \Rightarrow y = \frac{x}{\sin\theta}$ ......(1)

also in ΔCQR, $\frac{2}{\sin\theta} = \frac{y}{\sin 60°} \Rightarrow \frac{2}{\sin\theta} = \frac{2y}{\sqrt{3}} \Rightarrow y = \frac{\sqrt{3}}{\sin\theta}$ ....(2)

from (1) and (2)

$\therefore \frac{x}{\sin\theta} = \frac{\sqrt{3}}{\sin\theta} \Rightarrow x = \sqrt{3}$

now, $x^2 + 4 = y^2 \Rightarrow y^2 = 7 \Rightarrow k = 7$ **Ans.]**

(B) Let the number of socks be 2n forming n-pairs of socks

$n(S) = {}^{2n}C_2$

$n(A) = {}^{n}C_2$ (number of ways of chosing exactly one pair out of n pairs

$\therefore P(A) = \frac{{}^{n}C_1}{{}^{2n}C_2} = \frac{1}{15} \Rightarrow \frac{n \times 2}{2n(2n-1)} = \frac{1}{15} \Rightarrow 2n - 1 = 15$

$\therefore 2n = 16$

$\therefore$ Number of pair of socks is 8 **Ans.**

(D) sum $= \sum_{n=1}^{n}(n-1)(n-w)(n-w^2) = \sum_{n=1}^{n}(n^3-1) = \left(\sum_{n=1}^{n} n^3\right) - n = 220$

or $\left[\frac{n(n+1)}{2}\right]^2 - n = 220$

if $n = 5$, **Ans.]**