ANSWERSHEET (TOPIC = TRIGONOMETRY) COLLECTION #2

Question Type = A.Single Correct Type

Q. 1 (D) Sol $\cos^3 - 3\cos x\sin^2 x = 4\cos^3 x - 3\cos x$

$\cos^3 = 4\cos^3 x - 3\cos x\left(1-\sin^2 x\right)$

$= 4\cos^3 - 3\cos^3 x$

$= \cos^3 x$

Hence it is an identity $\Rightarrow$ infinite solution $\Rightarrow$ (D)]

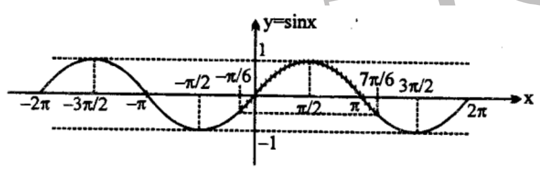
Q. 2 (C) Sol $\tan\left(\sin^{-1} x\right) = 3$

Let $\sin^{-1} x = \theta \Rightarrow \sin\theta = x$

$\tan\theta = 3$

$\dfrac{x}{\sqrt{1-x^2}} = 3 \Rightarrow x^2 = 9 - 9x^2 \Rightarrow 10x^2 = 9 \Rightarrow x = \dfrac{3\sqrt{10}}{10}$ Ans.]

Q. 3 (A) Sol Plot the graph of $y = \sin x;\ y = -1/2$



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Q. 4 (D) Sol $\dfrac{x}{\sin 7°} = \dfrac{a}{\sin 150°}$

$x = 2a\sin 7°$ … (1)

Using cosine rule in ΔAMC,

$y^2 = x^2 + a^2 - 2ax\cos 83° = 4a^2\sin^2 7° + a^2 - 4a^2\sin 7°$

$y^2 = a^2 \Rightarrow y = a$

Hence $\angle AMC = 83°$ **Ans.]**

Q. 5 (B) Sol Let $f(x) = \sin x + x\cos x$

consider $g(x) = \int_0^x (\sin t + t\cos t)\,dt = t\sin t]_0^x = x\sin x$

$g(x) = x\sin x$ which is differentiable

now $g(0) = 0$ and $g(\pi) = 0$, using Rolles Theorem

hence $\exists$ at least one $x \in (0, \pi)$ where $g'(x) = 0$

i.e. $x\cos x + \sin x = 0$ for atleast one $x \in (0, \pi)$ Ans. $\Rightarrow$ (B)]

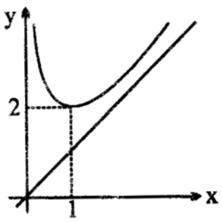
Q. 6 (A) Sol $\sqrt{2}+\sqrt{3}>\pi \quad (1.414+1.732=3.146>\pi)$

$\therefore \quad \frac{\sqrt{2}+\sqrt{3}}{2}>\frac{\pi}{2}$; Also $0<\sqrt{3}-\sqrt{2}<\frac{\pi}{4} \quad \left(\sqrt{3}-\sqrt{2}=0.318<\frac{\pi}{4}\right)$

now $\sin\sqrt{2}-\sin\sqrt{3}=2\underbrace{\cos\frac{\sqrt{2}+\sqrt{3}}{2}}_{-ve}\underbrace{\sin\frac{\sqrt{2}-\sqrt{3}}{2}}_{-ve}>0$

and $\cos\sqrt{2}-\cos\sqrt{3}=2\underbrace{\sin\frac{\sqrt{2}+\sqrt{3}}{2}}_{+ve}\underbrace{\sin\frac{\sqrt{3}-\sqrt{2}}{2}}_{+ve}>0$] $\Rightarrow$ (A)]

Q. 7 (D) Sol



$\cos x.\sin\left(x+\frac{1}{x}\right)=0$

$\cos x=0 \quad \Rightarrow \quad x=\pi/2$

$\sin\left(x+\frac{1}{x}\right)=0 \quad \Rightarrow \quad x+\frac{1}{x}=n\pi,\ n\in I$

If $x\in(0,1)$ then $x+\frac{1}{x}\in(2,\ \infty)$ for $x>0$

Hence there are infinite solution]..

Q. 8 (A) Sol $\int_0^a f(x)\,dx=\frac{a^2}{2}+\frac{a}{2}\sin a+\frac{\pi}{2}\cos a$

Differentiating with respect to 'a'

$f(a)=a+\frac{a}{2}\cos a=\frac{\sin a}{2}-\frac{\pi}{2}\sin a$

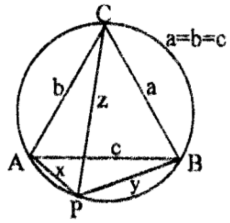
$f\left(\frac{\pi}{2}\right)=\frac{\pi}{2}+0+\frac{1}{2}-\frac{\pi}{2}=\frac{1}{2}$ **Ans.]**

Question Type = C.Assertion Reason Type

Q. 9 (A) Sol $\Delta=12\sqrt{5}$ using Heroes formula

$R=\frac{21\sqrt{5}}{10}$]

Q. 10 (D) Sol



Using Tolemy's theorem for a cyclic quadrilateral

$(z).(AB) = ax + by$

$z.c = ax + by$

but $a = b = c$

hence $x + y = z$ is true always

$\Rightarrow$ $S-1$ is false and S-2 is true]

Question Type = D.More than one may corect type

Q. 11 () Sol A, D

[Sol. $S = \frac{\pi}{2} + \underbrace{\sum_{n=1}^{\infty} \operatorname{cosec}^{-1}\sqrt{4n^4+1}}_{s_1};$ $S_1 = \sum_{n=1}^{\infty} \tan^{-1}\frac{1}{2n^2} = \frac{\pi}{4}$ (verify)

Hence $S = \frac{3\pi}{4}$ $\Rightarrow$ A, D]

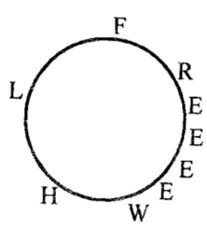
Q. 12 () Sol A, B

[Sol. $\frac{1}{a} = \frac{\tan\alpha}{1+\tan^2\alpha}$

$\Rightarrow$ $\sin 2\alpha = \frac{2}{\alpha}$, $(\sin\alpha + \cos\alpha)^2 = 1 + \sin 2\alpha = 1 + \frac{1}{a}$ $\Rightarrow$ $\sin\alpha + \cos\alpha = \sqrt{\frac{a+2}{a}}$

$D = a^2 - 4 \ge 0$ $\Rightarrow a \ge 2$

$(\sin\alpha - \cos\alpha)^2 = 1 - \frac{2}{a} = \frac{a-2}{a}$



$\sin\alpha < \cos\alpha \Rightarrow \sin\alpha - \cos\alpha = -\sqrt{\frac{a-2}{a}}$]

Q. 13 () Sol [B, C, D]

[Sol. Verify each alternative.

Q. 14 () Sol [B, C, D]

[Sol. Let $\tan^{-1}\frac{a}{x} = \alpha \quad \Rightarrow \quad \tan\alpha = \frac{a}{x}$ etc.

$$\alpha + \beta + \gamma + \delta = \frac{\pi}{2}$$

$$\tan(\alpha + \beta + \gamma + \delta) = \tan\frac{\pi}{2}$$

$$\frac{S_1 - S_3}{1 - S_2 + S_4} = \infty \quad \Rightarrow \quad 1 - S_2 + S_4 = 0 \quad \Rightarrow \quad S_4 - S_2 + 1 = 0$$

How, $S_4 = \tan\alpha.\tan\beta.\tan\gamma.\tan\delta = \frac{abcd}{x^4}$

$$S_2 = \sum\tan\alpha.\tan\beta = \frac{\sum ab}{x^2}$$

$$\therefore \quad \frac{abcd}{x^4} - \frac{\sum ab}{x^2} + 1 = 0$$

$$x^4 - \sum ab x^2 + abcd = 0 \quad \to \quad x_1, x_2, x_3, x_4$$

$$\therefore \quad x_1 + x_2 + x_3 + x_4 = 0 \qquad \ldots(1)$$

$$\sum x_1x_2x_3 \qquad \ldots(2)$$

$$\underbrace{x_1x_2x_3}_{\text{non zero}}\left[\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \frac{1}{x_4}\right] = 0$$

$x_1x_2x_3x_4 = abcd \quad \Rightarrow \quad$ (C)]