THIS FILE CONTAINS

TRIGONOMETRY

(COLLECTION # 2)

Very Important Guessing Questions For IIT JEE 2011 With Detail Solution

<u>Junior Students Can Keep It Safe For Future IIT-JEEs</u>

- ***Compound Angle***
- ***Trigonometric Equation & Inequations***
- ***Solution of Triangle***

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For Collection # 1 Question (Next File)

- ***Single Correct Answer Type Question***
- ***Comprehension Type Questions***
- ***Assertion Reason Type Question***
- ***More Correct Answers Type Questions***
- ***Subjective (Up to 4 Digits)***
- ***Detiail Solution By Genuine Method (But In) Classroom I Will Give <u>Short Tricks</u>)***

For Collection # 2

- ***Same As Above***

TOPIC = TRIGONOMETRY

SINGLE CORRECT TYPE

Q. 1 Number of solution to the equation $\cos^3 x - 3\cos x \sin^2 x = \cos 3x$ which is in the interval $(0, 1)$, is

(A) 1 (B) 2 (C) 3 (D) more than **(codeV3T1PAQ4)**

Q. 2 The value of $x \in (0, 1)$ satisfying the equation $\tan\left(\sin^{-1} x\right) = 3,$ is **(codeV3T2PAQ2)**

(A) 0.3 (B) 0.03 (C) $\frac{3\sqrt{10}}{10}$ (D) 0.9

Q. 3 General values of *x* satisfying $2\sin x + 1 > 0$ is **(codeV3T2PAQ7)**

(A) $2n\pi - \frac{\pi}{6} < x < 2n\pi + \frac{7\pi}{6}$ (B) $2n\pi + \frac{\pi}{6} < x < 2n\pi + \frac{5\pi}{6}$

(C) $2n\pi - \frac{\pi}{6} < x < 2n\pi + \frac{\pi}{6}$ (D) $2n\pi + \frac{2\pi}{3} < x < 2n\pi + \frac{4\pi}{6}$

Q. 4 Triangle ABC is isosceles with $AB = AC$ and $\angle CAB = 106°.$ Point M is the interior of the triangle so that $\angle MBA = 7°$ and $\angle MAB = 23°.$ The number of degrees in $\angle AMC$ is equal to

(A) 87° (B) 67° (C) 74° (D) 83° **(codeV3T3PAQ6)**

Q. 5 The equation $\sin x + x\cos x = 0$s has at least one root in **(codeV3T4PAQ1)**

(A) $\left(-\frac{\pi}{2}, 0\right)$ (B) $(0, \pi)$ (C) $\left(\pi, \frac{3\pi}{2}\right)$ (D) $\left(0, \frac{\pi}{2}\right)$

Q. 6 Let $S = \sin\sqrt{2} - \sin\sqrt{3}$ and $C = \cos\sqrt{2} - \cos\sqrt{3}$ then which one of the following is correct ?

(A) $S > 0$ and $C > 0$ (B) $2^{f(4)+g(4)} = 4 = 2^2$ **(codeV3T9PAQ5)**

(C) $S < 0$ and $C > 0$ (D) $S < 0$ and $C < 0$

Q. 7 Let X be the set of all solution to the equation $\cos x.\sin\left(x + \frac{1}{x}\right) = 0.$ Number of real numbers contained by X in the interval $(0 < x < \pi)$, is **(codeV3T10PAQ1)**

(A) 0 (B) 1 (C) 2 (D) more than 2

Q. 8 If f is positive and continuous such that the area under the curve $y = f(x)$ from $x = 0$ to $x = a$ has the value $\frac{a^2}{2} + \frac{a}{2}\sin a + \frac{\pi}{2}\cos a$ then $f\left(\frac{\pi}{2}\right)$ has the value equal to **(codeV3T10PAQ6)**

(A) $\frac{1}{2}$ (B) $\pi + \frac{1}{2}$ (C) $\pi - \frac{1}{2}$ (D) $\frac{\pi}{2} + 1$

ASSERTION REASON TYPE

Q. 1 A triangle is inscribed in a circle of radius R. The length of the sides of the triangle are 7, 8 and 9 units.

Statement-1 : The radius R has an irrational value.

(codeV3T6PAQ6)

because

Statement - 2 : Area of the triangle has an irrational value.

Q. 2 A circle is circumscribed about an equilateral triangle ABC and a point P on the minor arc joining A and B, is chosen. Let $x = PA$, $y = PB$ and $z = PC$. (z is larger than both x and y.)

Statement-1 : Each of the possibilities $(x + y)$ greater than z, equal to z or less than z is possible for some P. **(codeV3T9PAQ15)**

because

Statement - 2 : In a triangle ABC, sum of the two sides of a triangle is greater than the third and the third side is greater than the difference of the two.

MORE THAN ONE MAY CORRECT TYPE

Q. 1 The sum $\sum_{n=0}^{\infty}\operatorname{cosec}^{-1}\sqrt{4n^4+1}$ is **(codeV3T1PAQ20)**

(A) $\pi+\tan^{-1}(-1)$ (B) $2\cos^{-1}(0)$ (C) $4\tan^{-1}(1)$ (D) $\cot^{-1}\left(\frac{1}{2}\right)+\cot^{-1}\left(\frac{1}{3}\right)$

Q. 2 If $\tan\alpha$ satisfies the relation $\tan^2\alpha-a\tan\alpha+1=0$ where $a>0$ and $0<\alpha<\frac{\pi}{4}$, then identify the correct statements **(codeV3T6PAQ10)**

(A) $\sin 2\alpha=\frac{2}{a}$ (B) $\sin\alpha+\cos\alpha=\sqrt{\frac{a+2}{a}}$ (C) $a\geq 2$ (D) $\sin\alpha-\cos\alpha=\sqrt{\frac{a-2}{a}}$

Q. 3 In a ΔABC let **(codeV3T7PAQ20)**

A, s, a, b, c denote the area of triangle ABC, semi perimeter, length of BC, AC and AB respectively. h_a, h_b, h_c - length of the heights of the triangle from the vertex A, B and C respectively. r_a, r_b, r_c – lengths of radius of inscribed circles that are tangent to BC, AC and AB respectively. and r – radius of an inscribed circle, then which of the following relations holds good ?

(A) $r=\frac{3}{\frac{1}{r_a}+\frac{1}{r_b}+\frac{1}{r_c}}$ (B) $A^2=r.r_a.r_b.r_c$ (C) $r=\frac{1}{\frac{1}{h_a}+\frac{1}{h_b}+\frac{1}{h_c}}$ (D) $A=\frac{c.r_a.r_b}{r_a+r_b}$

Q. 4 Let x_1, x_2, x_3, x_4 be four non zero numbers satisfying the equation **(codeV3T7PAQ23)**

$\tan^{-1}\frac{a}{x}+\tan^{-1}\frac{b}{x}+\tan^{-1}\frac{c}{x}+\tan^{-1}\frac{d}{x}=\frac{\pi}{2}$ Then which of the following relation(s) hold good?

(A) $\sum_{i=1}^{4}x_i=a+b+c+d$ (B) $\sum_{i=1}^{4}\frac{1}{x_i}=0$ (C) $\prod_{i=1}^{4}x_i=abcd$

(D) $(x_1+x_2+x_3)(x_2+x_3+x_4)(x_3+x_4+x_1)(x_4+x_1+x_2)=abcd$

SOLUTION

SINGLE CORRECT TYPE

Q. 1 (D) Sol $\cos^3 - 3\cos x\sin^2 x = 4\cos^3 x - 3\cos x$

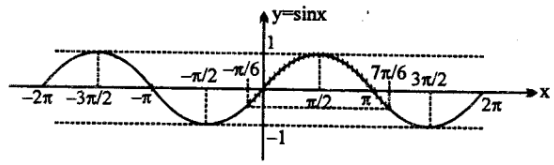
$\cos^3 = 4\cos^3 x - 3\cos x\left(1-\sin^2 x\right) \Rightarrow 4\cos^3 - 3\cos^3 x \Rightarrow \cos^3 x$

Hence it is an identity $\Rightarrow$ infinite solution $\Rightarrow$ (D)]

Q. 2 (C) Sol $\tan\left(\sin^{-1} x\right) = 3$ Let $\sin^{-1} x = \theta \Rightarrow \sin\theta = x \Rightarrow \tan\theta = 3$

$\frac{x}{\sqrt{1-x^2}} = 3 \Rightarrow x^2 = 9 - 9x^2 \Rightarrow 10x^2 = 9 \Rightarrow x = \frac{3\sqrt{10}}{10}$ **Ans.]**

Q. 3 (A) Sol Plot the graph of
$y = \sin x;\ y = -1/2$



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Q. 4 (D) Sol $\frac{x}{\sin 7°} = \frac{a}{\sin 150°}$

$x = 2a\sin 7°$ … (1)

Using cosine rule in ΔAMC, $\Rightarrow y^2 = x^2 + a^2 - 2ax\cos 83° = 4a^2\sin^2 7° + a^2 - 4a^2\sin 7°$

$y^2 = a^2 \Rightarrow y = a \Rightarrow$ Hence $\angle AMC = 83°$ **Ans.]**

Q. 5 (B) Sol Let $f(x) = \sin x + x\cos x$

consider $g(x) = \int_0^x (\sin t + t\cos t)\,dt = t\sin t]_0^x = x\sin x$

$g(x) = x\sin x$ which is differentiable

now $g(0) = 0$ and $g(\pi) = 0$, using Rolles Theorem

hence $\exists$ at least one $x \in (0, \pi)$ where $g'(x) = 0$

i.e. $x\cos x + \sin x = 0$ for atleast one $x \in (0, \pi)$ Ans. $\Rightarrow$ (B)]

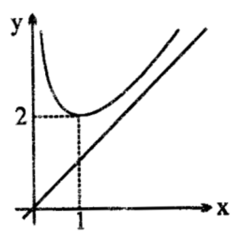
Q. 6 (A) Sol $\sqrt{2} + \sqrt{3} > \pi \quad (1.414 + 1.732 = 3.146 > \pi)$

$\therefore \quad \frac{\sqrt{2}+\sqrt{3}}{2} > \frac{\pi}{2}$; Also $0 < \sqrt{3} - \sqrt{2} < \frac{\pi}{4} \quad \left(\sqrt{3} - \sqrt{2} = 0.318 < \frac{\pi}{4}\right)$

now $\sin\sqrt{2} - \sin\sqrt{3} = 2\underbrace{\cos\frac{\sqrt{2}+\sqrt{3}}{2}}_{-ve}\underbrace{\sin\frac{\sqrt{2}-\sqrt{3}}{2}}_{-ve} > 0$

and $\cos\sqrt{2} - \cos\sqrt{3} = 2\underbrace{\sin\frac{\sqrt{2}+\sqrt{3}}{2}}_{+ve}\underbrace{\sin\frac{\sqrt{3}-\sqrt{2}}{2}}_{+ve} > 0$] $\Rightarrow$ (A)]

Q. 7 (D) Sol $\cos x.\sin\left(x + \frac{1}{x}\right) = 0$



$\cos x = 0 \Rightarrow x = \pi/2$

$\sin\left(x + \frac{1}{x}\right) = 0 \Rightarrow x + \frac{1}{x} = n\pi,\ n \in I$

If $x \in (0, 1)$ then $x + \frac{1}{x} \in (2, \infty)$ for $x > 0$

Hence there are infinite solution]..

Q. 8 (A) Sol $\int_0^a f(x)\,dx = \frac{a^2}{2} + \frac{a}{2}\sin a + \frac{\pi}{2}\cos a$

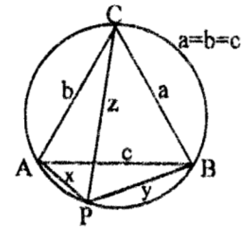
Differentiating with respect to 'a'

$f(a) = a + \frac{a}{2}\cos a = \frac{\sin a}{2} - \frac{\pi}{2}\sin a \Rightarrow f\left(\frac{\pi}{2}\right) = \frac{\pi}{2} + 0 + \frac{1}{2} - \frac{\pi}{2} = \frac{1}{2}$ **Ans.]**

ASSERTION REASON TYPE

Q. 1 (A) Sol $\Delta = 12\sqrt{5}$ using Heroes formula $R = \dfrac{21\sqrt{5}}{10}$]

Q. 2 (D) Sol **Using Tolemy's theorem for a cyclic quadrilateral**



$(z).(AB) = ax + by \quad \Rightarrow \quad z.c = ax + by$

but $a = b = c \quad \Rightarrow$ hence $x + y = z$ is true always

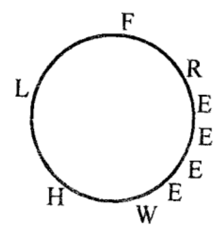
$\Rightarrow$ $S-1$ is false and S-2 is true]

MORE THAN ONE MAY CORRECT TYPE

Q. 1 A, D Sol. $S = \dfrac{\pi}{2} + \underbrace{\sum_{n=1}^{\infty} \text{cosec}^{-1}\sqrt{4n^4+1}}_{S_1};$ $\quad S_1 = \sum_{n=1}^{\infty} \tan^{-1}\dfrac{1}{2n^2} = \dfrac{\pi}{4}$ (verify)

Hence $S = \dfrac{3\pi}{4} \quad \Rightarrow \quad A, D]$

Q. 2 A, B **Sol.** $\dfrac{1}{a} = \dfrac{\tan\alpha}{1+\tan^2\alpha}$



$\Rightarrow \quad \sin 2\alpha = \dfrac{2}{\alpha}, \ (\sin\alpha + \cos\alpha)^2 = 1 + \sin 2\alpha = 1 + \dfrac{1}{a}$

$\Rightarrow \quad \sin\alpha + \cos\alpha = \sqrt{\dfrac{a+2}{a}} \quad \Rightarrow \ D = a^2 - 4 \ge 0 \quad \Rightarrow a \ge 2$

$(\sin\alpha - \cos\alpha)^2 = 1 - \dfrac{2}{a} = \dfrac{a-2}{a} \quad \Rightarrow \quad \sin\alpha < \cos\alpha \Rightarrow \sin\alpha - \cos\alpha = -\sqrt{\dfrac{a-2}{a}}$]

Q. 3 [B, C, D] Verify each alternative.

Q. 4 [B, C, D] **[Sol.** Let $\tan^{-1}\dfrac{a}{x} = \alpha \quad \Rightarrow \quad \tan\alpha = \dfrac{a}{x}$ etc.

$\alpha + \beta + \gamma + \delta = \dfrac{\pi}{2} \quad \Rightarrow \quad \tan(\alpha+\beta+\gamma+\delta) = \tan\dfrac{\pi}{2}$

$\dfrac{S_1 - S_3}{1 - S_2 + S_4} = \infty \quad \Rightarrow \quad 1 - S_2 + S_4 = 0 \quad \Rightarrow \quad S_4 - S_2 + 1 = 0$

How, $S_4 = \tan\alpha.\tan\beta.\tan\gamma.\tan\delta = \dfrac{abcd}{x^4} \quad \Rightarrow \quad S_2 = \sum \tan\alpha.\tan\beta = \dfrac{\sum ab}{x^2}$

$x^4 - \sum abx^2 + abcd = 0$ (roots x_1, x_2, x_3, x_4)

$\therefore \quad \dfrac{abcd}{x^4} - \dfrac{\sum ab}{x^2} + 1 = 0 \quad \Rightarrow$

$\therefore \quad x_1 + x_2 + x_3 + x_4 = 0 \quad \ldots(1)$

$\sum x_1x_2x_3 \quad \ldots(2)$

$\underbrace{x_1x_2x_3}_{\text{non zero}}\left[\dfrac{1}{x_1} + \dfrac{1}{x_2} + \dfrac{1}{x_3} + \dfrac{1}{x_4}\right] = 0$

$x_1x_2x_3x_4 = abcd \quad \Rightarrow \quad (C)]$