Paper - I

41. The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line $4x-5y=20$ to the circle $x^2+y^2=9$ is

(a) $20\left(x^2+y^2\right)-36x+45y=0$

(b) $20\left(x^2+y^2\right)+36x-45y=0$

(c) $36\left(x^2+y^2\right)-20x+45y=0$

(d) $36\left(x^2+y^2\right)+20x-45y=0$

Sol. (A) $20\left(x^2+y^2\right)-36x+45y=0$

SUHAG SHORT TRICK TAKE A POINT (0,-4)ON LINE

It's very simple no need for detalied solution.

42. The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is

(a) 75 (b) 150
(c) 210 (d) 243

Sol. (A) 75

Concept make bundals 1,1,3
1,2,2

$$\frac{\lfloor 5}{\lfloor 1 \lfloor 1 \lfloor 3 \lfloor 2}\times\lfloor 3+\frac{\lfloor 5}{\lfloor 1 \lfloor 2 \lfloor 2 \lfloor 2}\times\lfloor 3 = 150$$

43. Let $f(x)=\begin{cases} x^2\left|\cos\dfrac{\pi}{x}\right|, & x\neq 0, \\ 0, & x=0 \end{cases}$ $x\in IR$,

then f is

(a) differentiable both at x = 0 and at x = 2
(b) differentiable at x = 0 but not differentiable at x = 2
(c) not differentiable at x = 0 but differentiable at x = 2
(d) differentiable neither at x = 0 nor at x = 2

Sol. (B)

differentiable at x = 0 but not differentiable at x = 2

44. The function $f:[0,3]\to[1,29]$, defined by $f(x)=2x^3-15x^2+36x+1$, is

(a) One-one and onto
(b) onto but not one-one
(c) one-one but not onto
(d) neither one-one nor onto

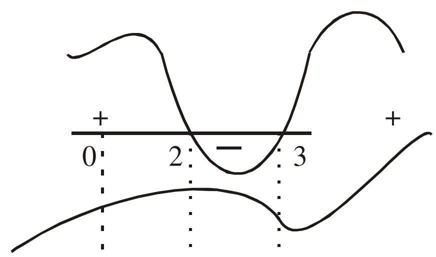
Sol. (B)

$$f(x)=2x^3-15x^2+36+1$$

$$f(x)=6x^2-30x+36$$

$$f'(x)=6\left[x^2-5x+6\right]=6(x-2)(x-3)$$

wavy curve $f'(x)$ then f(x)



45. If $\lim\limits_{x\to\infty}\left(\dfrac{x^2+x+1}{x+1}-ax-b\right)=4,$ then

(a) $a=,b=4$ (b) $a=1,b=-4$
(c) $a=2,b=-3$ (d) $a=2,b=3$

Sol. (B)

$$\lim_{x\to\infty}\left(\frac{x^2+x+1}{x+1}-ax-b\right)=4,$$

$$\lim_{x\to\infty}\left(\frac{x^2+x+1-ax^2-ax-bx-b}{x+1}\right)=4$$

$$\lim_{x\to\infty}\left(\frac{x^2(1-a)+x(1-a-b)+1-b}{x+1}\right)=4$$

$$\left\{\lim_{x\to\infty}\frac{x(-b)+1-b}{x+1}=4\right.$$

$$-b=4\to b=-4$$

46. Let z be a complex number such that the imaginary part of z is nonzero and $a=z^2+z+1$ is real. Then a cannot take the value

(a) – 1 (b) $\dfrac{1}{3}$

(c) $\frac{1}{2}$ (d) $\frac{3}{4}$

Sol. (D)

$a = z^2 + z + 1$

Let $z = x + 1y$ and given $y \neq 0$

$a = (x+y)^2 + (x+1y) + 1$

$= x^2 - y^2 + 2ixy + x + iy + 1$

$a = (x^2 - y^2 + x + 1) + i(2xy + y)$

$a \in R$ So, $2xy + y = 0$ or $x = \frac{-1}{2}$

since $y \neq 0$ $z \neq x$

Thus $a \neq (x)^2 + x + 1 = \left(-\frac{1}{2}\right)^2 - \frac{1}{2} + 1 = 3/4$

47. The ellipse $E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$ is inscribed in a rectangle R whose sides are parallesl to the coordinate axes. Another ellipse E_2 passing through the point (0,4) circumscribes the rectangle R. The eccentricity of the ellipse E_2 is

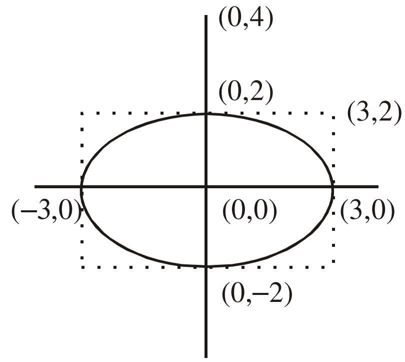
(a) $\frac{\sqrt{2}}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) $\frac{1}{2}$ (d) $\frac{3}{4}$

Sol. (C)

Form figure one vertex of rectangle is (3,2)

Let $E_2 : \frac{x^2}{a^2} + \frac{y^2}{16} = 1$



It passes thorugh (3,2)

$\frac{9}{a^2} + \frac{4}{16} = 1$, $\frac{9}{a^2} = 1 - \frac{1}{4} = \frac{3}{4}$

$\frac{9}{a^2} = \frac{3}{4}$ or $\frac{3}{a^2} = \frac{1}{4} \Rightarrow a^2 = 12$

Thus $a^2 = 16(1 - e^2)$, $\frac{12}{16} = 1 - e^2$

$1 - e^2 = \frac{3}{4}$, $e^2 = \frac{1}{4}$, or $e = \frac{1}{2}$

48. Let $P[a_{ij}]$ be a 3×3 matrix and let $Q = [b_{ij}]$, where $b_{ij} = 2^{i+j} a_{ij}$ for $1 \leq i, j \leq 3$. If the determinant of P is 2, then the determinant of the matrix Q is

(a) 2^{10} (b) 2^{11}

(c) 2^{12} (d) 2^{13}

Sol. (D) SUHAG SHORT TRICKS

$$P = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad Q = \begin{bmatrix} 2^2.2 & 0 & 0 \\ 0 & 2^4.1 & 0 \\ 0 & 0 & 2^6.1 \end{bmatrix}$$

So $|Q| = 2^{13}$

49. The intergral $\int \frac{\sec^2 x}{(\sec + \tan x)^{\frac{9}{2}}} dx$ equals (for some arbitray constant K)

(a) $-\frac{1}{(\sec x + \tan x)^{\frac{11}{2}}} \left\{ \frac{1}{11} - \frac{1}{7} (\sec x + \tan x)^2 \right\} + K$

(b) $\dfrac{1}{(\sec x+\tan x)^{\frac{11}{2}}}\left\{\dfrac{1}{11}-\dfrac{1}{7}(\sec x+\tan x)^2\right\}+K$

(c) $-\dfrac{1}{(\sec x+\tan x)^{\frac{11}{2}}}\left\{\dfrac{1}{11}+\dfrac{1}{7}(\sec x+\tan x)^2\right\}+K$

(d) $\dfrac{1}{(\sec x+\tan x)^{\frac{11}{2}}}\left\{\dfrac{1}{11}+\dfrac{1}{7}(\sec x+\tan x)^2\right\}+K$

Sol. C

50. The point P is the intersection of the straight line joining the points $Q(2,3,5)$ and $R(1,-1,4)$ with the plane $5x-4y-z=1$. If S is the foot of the perpendicular drawn from the point $T(2,1,4)$ to QR, then the length of the segment PS is

(a) $\dfrac{1}{\sqrt{2}}$ (b) $\sqrt{2}$

(c) 2 (d) $2\sqrt{2}$

Sol. A

51. Let $\theta,\varphi\in[0,2\pi]$ be such that

$$2\cos\theta(1-\sin\varphi)=\sin^2\theta\left(\tan\frac{\theta}{2}+\cos\frac{\theta}{2}\right)\cos\varphi-1,$$

$$\tan(2\pi-\theta)>0 \text{ and } -1<\sin\theta<-\frac{\sqrt{3}}{2}.$$

Then φ cannot satisfy

(a) $0<\varphi<\dfrac{\pi}{2}$ (b) $\dfrac{\pi}{2}<\varphi<\dfrac{4\pi}{3}$

(c) $\dfrac{4\pi}{3}<\varphi<\dfrac{3\pi}{2}$ (d) $\dfrac{3\pi}{2}<\varphi<2\pi$

Sol.ANS. A,C,D

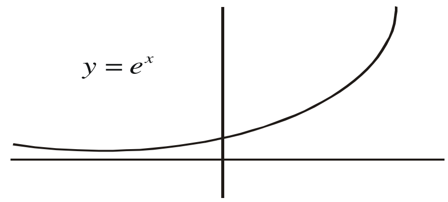
52. Let S be the area of the region enclosed by $y=e^{-x^2}$, $y=0$, and $x=1$. Then

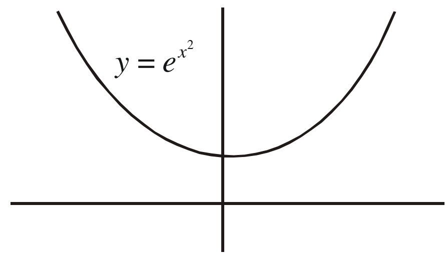
(a) $S\ge\dfrac{1}{e}$ (b) $S\ge 1-\dfrac{1}{e}$

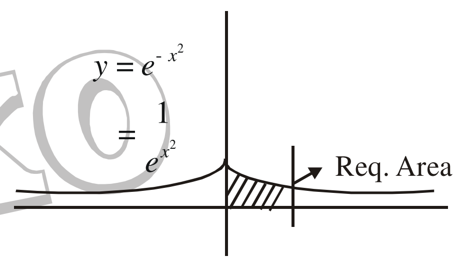
(c) $S\ge\dfrac{1}{4}\left(1+\dfrac{1}{\sqrt{e}}\right)$

(d) $S\ge\dfrac{1}{\sqrt{2}}+\dfrac{1}{\sqrt{e}}\left(1-\dfrac{1}{\sqrt{2}}\right)$

Sol. (A,B,D)







$\int e^{-x}dx$, $\quad -\left[e^{-x}\right]_0^1$

$-\left[e^{-1}-e^{0}\right]$ $\quad \left(1-\dfrac{1}{e}\right)$

$S\ge\dfrac{1}{e}$ & $S\ge 1-\dfrac{1}{e}$

53. A ship is fitted with three engines E_1, E_2 and E_3. The engines function independently of each other with respective probabilities $\dfrac{1}{2},\dfrac{1}{4}$ and $\dfrac{1}{4}$. For the shop to be operational at least two of its engines must function. Let X denote the event that the ship to be operational at least two of its engines must function. Let X denote the event that the ship is operational and let X_1, X_2 and X_3 denote respectively the events that the engines E_1, E_2 and E_3 are functioning. Which of the following is (are) true ?

(a) $P\left[X_1^c \mid X\right]=\frac{3}{16}$

(b) P {Exac tly two engines of the ship are functioning $\mid x]=\frac{7}{8}$

(c) $P\left[X \mid X_2\right]=\frac{5}{16}$

(d) $P\left[X \mid X_1\right]=\frac{7}{16}$

Sol. B,D

54. Tangents are drawn to the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$, parallel to the straight line $2x-y=1$. The points of contact of the tangents on the hyperbol are

(a) $\left(\frac{9}{2\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ (b) $\left(-\frac{9}{2\sqrt{2}}-\frac{1}{\sqrt{2}}\right)$

(c) $\left(3\sqrt{3},-2\sqrt{3}\right)$ (d) $\left(-3\sqrt{3},2\sqrt{2}\right)$

Sol. (A,B)

Let P be $\left(3\sec\theta, 2\tan\theta\right)$

tangent at P

$$\frac{x\sec\theta}{3}-\frac{y\tan\theta}{2}=1$$

55. If y(x) satisfies the differential equation $y'-y\tan x=2x\sec x$ and $y=(0)=0$, then

(a) $y\left(\frac{\pi}{4}\right)=\frac{\pi^2}{8\sqrt{2}}$ (b) $y'\left(\frac{\pi}{4}\right)=\frac{\pi^2}{18}$

(c) $y\left(\frac{\pi}{3}\right)=\frac{\pi^2}{9}$ (d) $y'\left(\frac{\pi}{3}\right)=\frac{4\pi}{3}+\frac{2\pi^2}{3\sqrt{3}}$

Sol. (A,D)

If y(x) satisfies :

$$\frac{dy}{dx}-y.\tan x=2x.\sec x \text{ and } y(0)=0$$

$$\frac{dy}{dx}+\left(-\tan x\right).y=2x.\sec x$$

If $=e^{\int -\tan x dx}=e^{-\ln\sec x}=\cos x.$

$$y.\cos x=\int 2x.\sec x\cos x dx+c$$

$$y.\cos x=\int 2x dx+c$$

$$y.\cos x=x^2+c$$

$y(0)=0$ so $c=0$

$$y\cos x=x^2$$

$$y=x^2.\sec x$$

$$y'=2x.\sec x+x^2\sec x\tan x$$

(a) $y\left(\frac{\pi}{4}\right)=\frac{\pi^{\frac{2}{16}}}{\frac{1}{\sqrt{2}}}$ $(correct)$

(b) $y'\left(\frac{\pi}{2}\right)=\frac{2\pi}{4}.\sqrt{2}+\frac{\pi^2}{16}.\sqrt{2}$ $(wrong)$

(c) $y\left(\frac{\pi}{3}\right)=\frac{\frac{\pi^2}{9}}{\frac{1}{2}}$ $(wrong)$

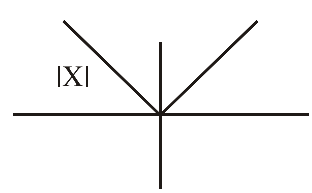
(d) $y'\left(\frac{\pi}{3}\right)=2.\frac{\pi}{3}.2+\frac{\pi}{9}.2.\sqrt{3}$

$=\frac{4\pi}{3}+\frac{\pi^2.2}{3\sqrt{3}}$ $(correct)$

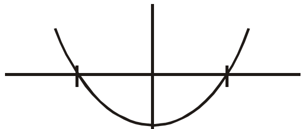
56. Let $f: IR\to IR$ be difined as $f(x)=\left|x\right|+\left|x^2-1\right|$. The total number of points at which f attains either a local maximum or a local minimum is

Sol. (5)

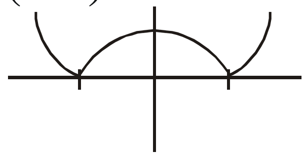
$f(x)=\left|x\right|+\left|x+1\right|$



$x^2 - 1 = (x-1)(x+1)$



$(x^2 - 1)$



57. The value of

$$6+\log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\ldots\ldots}}}\right) \text{ is}$$

Sol. Let $y=\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{\sqrt{3}}\ldots\ldots}}}$

$$y=\sqrt{4-\frac{1}{3\sqrt{2}}y} \qquad y^2=4-\frac{1}{3\sqrt{2}}y$$

$$y^2+\frac{y}{3\sqrt{2}}-4=0 \qquad 3\sqrt{2}y^2+y-12\sqrt{2}=0$$

$$y=\frac{-1\pm17}{6\sqrt{2}}=-\frac{3}{\sqrt{2}},\ \frac{16}{6\sqrt{2}}$$

$$y=\frac{4\sqrt{2}}{3}=\frac{8}{3\sqrt{2}}$$

$$6+\log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}\left(\frac{4\sqrt{2}}{3}\right)\right)$$

$$6+\log_{\frac{3}{2}}\left(\frac{4}{9}\right) \qquad 6+(-2)=4$$

58. Let $p(x)$ be a real polynomial of least degree which has a local maximum at x = 1 and a lacal minimum at x = 3. If $p(1)=6$ and $p(3)=2$, then $p'(0)$ is s

Sol. ANS. 9

59. If $\vec{a},\vec{b}$ and $\vec{c}$ are unit vectors satisfying $|\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2=9$, then $|2\vec{a}+5\vec{b}+5\vec{c}|$

Sol. $|a|^2+|b|-2|a||b|\cos\alpha$

So, $9=6-2(\cos\alpha+\cos\beta+\cos\gamma)$

$$-\frac{3}{2}=\cos\alpha+\cos\beta+\cos\gamma$$

SUHAG SHORT TRICKS

ASSUME ANGLE ALL SAME 120 DEGREE

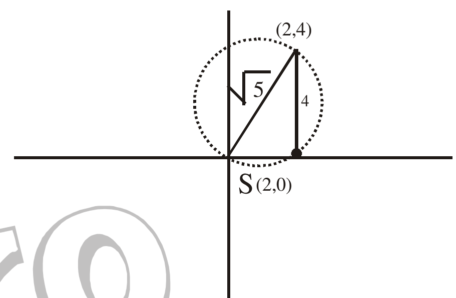
In think Ans is 3

60. Let S be the focus the parabola $y^2=8x$ and let PQ be the common chord of the circle $x^2+y^2-2x-4y=0$ and the given parabola. The area of the triangle PQS is

Sol. ANS 4

Let S be the focus ....

Put in circle



$$x=\frac{8}{y^2}, \qquad y^2=8x, \qquad y^2=4.2.x$$

$$s=(2,0), \qquad \frac{1}{2}.2.4=4$$