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ENJOY **MATHEMATICS** WITH **SUHAAG SIR**

IIT-JEE, AIEEE (WITH 12TH, 11TH & DROPPER)

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विघ्न विचारत भीरु जन, नहीं आरम्भे काम, विपति देख छोड़े तुरंत मध्यम मन कर श्याम।
पुरुष सिंह संकल्प कर, सहते विपति अनेक, 'बना' न छोड़े ध्येय को, रघुबर राखे टेक।।

रचित: मानव धर्म प्रणेता
सद्गुरु श्री रणछोड़दासजी महाराज

SOLUTION OF IIT JEE-09

BY SUHAAG SIR

& HIS STUDENTS OF CLASS MOVING 11th TO 12th

Abhishek Bhardwaj	School MVM
Abhishek Singh	School MVM
Avantika Garde	School KV-1
Sangya Thakur	School KV-1
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Solution also Available of IIT JEE 2009

Paper-1 & 2 on
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MATHEMATICS PAPER-1

Question sequence as per Paper CODE-5

SECTION-1

Single Correct Choice Type

This section contains 8 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

21. The number of seven digit integers, with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only, is

(A) 55 (B) 66 (C) 77 (D) 88

22. Let $z = x + iy$ be acomplex number where x and y are integers. Then the area of the rectangle whose vertices are the roots of the equation $z\bar{z}^3 + \bar{z}z^3 = 350$ is

(A) 48 (B) 32 (C) 40 (D) 80

23. Tangents drawn from the point P(1, 8) to the cirlce $x^2+y^2-6x-4y-11=0$ touch the cirlce at the points A and B. The equation of the circumcircle of the triangle PAB is

(A) $x^2+y^2+4x-6y+19=0$ (B) $x^2+y^2-4x-10y+19=0$

(C) $x^2+y^2-2x+6y-29=0$ (D) $x^2+y^2-6x-4y+19=0$

24. The line passing through the extremity A of the major axis and extremity B of the minor axis of the ellipse $x^2+9y^2=9$, meets its auxiliary circle at the point M. Then the area of the triangle with vertices at A, M and the origin O is

(A) $\frac{31}{10}$ (B) $\frac{29}{10}$ (C) $\frac{21}{10}$ (D) $\frac{27}{10}$

25. Let $z = \cos\theta + i\sin\theta$. Then the value of $\sum_{m=1}^{15} \mathrm{Im}\left(z^{2m-1}\right)$ at $\theta = 2^\circ$ is

(A) $\frac{1}{\sin 2^\circ}$ (B) $\frac{1}{3\sin 2^\circ}$ (D) $\frac{1}{2\sin 2^\circ}$ (D) $\frac{1}{4\sin 2^\circ}$

26. Let f be a non-negative function defined on the interval [0,1]. If $\int_0^x \sqrt{1-\left(f'(t)\right)^2}\,dt = \int_0^x f(t)dt,\ 0 \le x \le 1,$ and f(0) = 0, then

(A) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$ (B) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

(C) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$ (D) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

27. Let P(3, 2, 6) be a pint in space and Q be a point on the line $\vec{r} = \left(\hat{i}-\hat{j}+2\hat{k}\right)+\mu\left(-3\hat{i}+\hat{j}+5\hat{k}\right)$. Then the value of μ for which the vector $\overrightarrow{PQ}$ is parallel to the plane x –4y + 3z = 1 is

(A) $\frac{1}{4}$ (B) $-\frac{1}{4}$ (C) $\frac{1}{8}$ (D) $-\frac{1}{8}$

28. If $\vec{a},\vec{b},\vec{c}$ and $\vec{d}$ are unit vectors such that $\left(\vec{a}\times\vec{b}\right).\left(\vec{c}\times\vec{d}\right)=1$ and $\vec{a}.\vec{c}=\frac{1}{2}$, then

(A) $\vec{a},\vec{b},\vec{c}$ are non-coplanar (B) $\vec{b},\vec{c},\vec{d}$ are non-coplanar

(C) $\vec{b},\vec{d}$ are non-parallel (D) $\vec{a},\vec{d}$ are parallel and $\vec{b},\vec{c}$ are parallel

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SECTION-II

Multiple Correct Choice Type

This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** is/are correct.

29. If $\frac{\sin^4 x}{2}+\frac{cos^4 x}{3}=\frac{1}{5}$, then

(A) $\tan^2 x=\frac{2}{3}$ (B) $\frac{\sin^8 x}{8}+\frac{\cos^8 x}{27}=\frac{1}{125}$

(C) $\tan^2 x=\frac{1}{3}$ (D) $\frac{\sin^8 x}{8}+\frac{\cos^8 x}{27}=\frac{2}{125}$

30. Area of the region bounded by the curve $y=e^x$ and lines $x=0$ and $y=e$ is

(A) $e-1$ (B) $\int_1^e \ln(e+1-y)\,dy$ (C) $e-\int_1^1 e^x dx$ (D) $\int_1^e \ln y\,dy$

31. Let $L=\lim_{x\to 0}\frac{a-\sqrt{a^2-x^2}-\frac{x^2}{4}}{x^4}, a>0$. If L is finite, then

(A) a = 2 (B) a = 1 (C) L = 1/64 (D) L = 1/32

32. In a triangle ABC with fixed base BC, The vertex A moves such that $\cos B+\cos C=4\sin^2\frac{A}{2}$. If a, b and c denote the lengths of the sides of the triangle opposite to the angles A, B and C, respectively, Then

(A) b + c = 4a (B) b + c = 2a

(C) locus of point A is an ellipse (D) locus of point A is a pair of straight lines

SECTION-III

Comprehension Type

This section contains 2 groups of questions. Each group has 3 multiple choice questions based on a paragraph. Each question has 4 choices (A), (B), (C) and (D) for its anwer, out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 33 to 35

A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required.

33. The probability that X = 3 equals

(A) 25/216 (B) 25/36 (C) 5/36 (D) 125/216

34. The probability that $X\geq 3$ equals

(A) 125/216 (B) 25/36 (C) 5/36 (D) 25/216

35. The conditional probability that $X\geq 6$ given $X>3$ equals

(A) 125/216 (B) 25/216 (C) 5/36 (D) 25/36

Paragraph for Question Nos. 36 to 38

Let $\mathcal{A}$ be the set of all 3×3 symmetric matrices all of whose entries are either 0 or 1. Five of these entries are 1 and four of them are 0.

36. The number of matrices in $\mathcal{A}$ is

(A) 12 (B) 6 (C) 9 (D) 3

37. The number of matrices A in $\mathcal{A}$ for which the system of linear equations $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has a unique solution, is

(A) less than 4 (B) at least 4 but less than 7
(C) at least 7 but less than 10 (D) at least 10

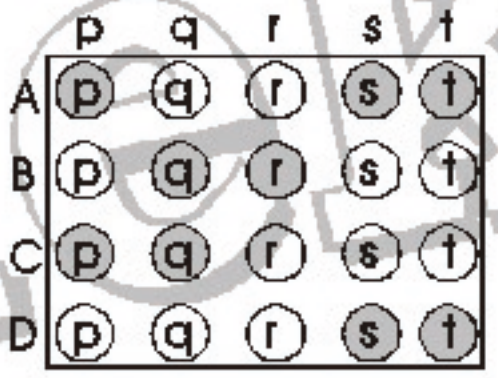
38. The number of matrices A in $\mathcal{A}$ for which the system of linear equations $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is inconsistent, is

(A) 0 (B) more than 2 (C) 2 (D) 1

SECTION-IV

Matrix -Match Type

This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statements is Column I are labelled A, B, C and D, while the statements in Column II are labelled p, q, r, s and t. Any given statement in Column I can have correct matching with ONE OR MORE statement(s) in Column II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the folloiwng example:
If the correct matches are A - p, s and t; B - q and r; C - p and q; and D - s and t; then the correct darkening of bubbles will look like the following.



39. Match the conics in Column I with the statements/expressions in Column II.

	Column I		Column II
(A)	Circle	(p)	The locus of the point (h, k) for which the line $hx + ky = 1$ touches the circle $x^2+y^2 = 4$
(B)	Parabola	(q)	Points z in the complex plane satisfying $\|z + 2\| - \|z - 2\| = \pm 3$
(C)	Ellipse	(r)	Point of the conic have parametric representation $x = \sqrt{3}\left(\frac{1-t^2}{1+t^2}\right), y = \frac{2t}{1+t^2}$
(D)	Hyperbola	(s)	The eccentricity of the conic lies in the interval $1 \le x < \infty$
		(t)	Points z in the complex plane satisfying $\mathrm{Re}(z + 1)^2 = \|z\|^2 + 1$

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40. Match the statements/expressions in Column I with the open intervals in Column II.

Column I	Column II
(A) Interval contained in the domain of definition of non-zero solutions of the differential equation $(x-3)^2 y' + y = 0$	(p) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
(B) Interval containing the value of the integral $\int_1^5 (x-1)(x-2)(x-3)(x-4)(x-5)\,dx$	(q) $\left(0, \frac{\pi}{2}\right)$
(C) Interval in which at least one of the points of local maximum of $\cos^2 x + \sin x$ lies	(r) $\left(\frac{\pi}{8}, \frac{5\pi}{4}\right)$
(D) Interval in which $\tan^{-1}(\sin x + \cos x)$ increasing	(s) $\left(0, \frac{\pi}{8}\right)$
	(t) $(-\pi, \pi)$

SOLUTION-IIT JEE-2009 (PAPER-1)

21. (C) 1, 2, 3

3	2	1	1	1	1	1

$\frac{\lfloor 7}{\lfloor 5} = 7.6 = 42$

2	2	2	1	1	1	1

$\frac{\lfloor 7}{\lfloor 3 \lfloor 4} = 7 \times 5 = 35$

$42+35 = 77$

22. (A) $(x+iy)(x-iy)^3 + (x-iy)(x+iy)^2 = 350$

$(x-iy)(x-iy)[(x-iy)^2+(x-iy)^2] = 350$

$[x^2+y^2][x^2-y^2+x^2-y^2] = 350$

$(x^4 - y^4) = 175$

$x^4 = 175 + y^4$

Satisfies (4, 3) (–4, 3) (–4, –3) (4, –3)

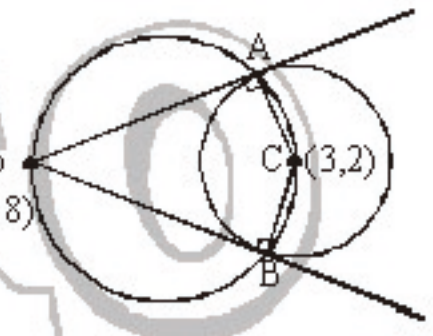
so area is $8 \times 6 = 48$

23. (B) $x^2+y^2-6x-4y-11=0$

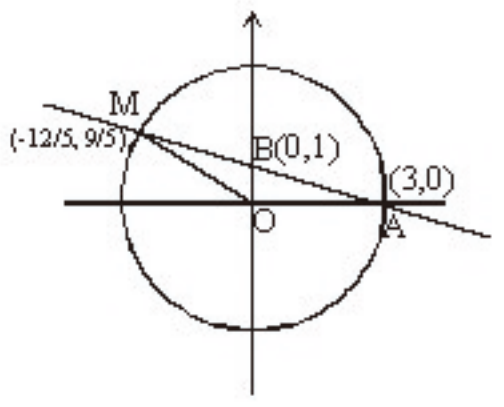
By diametrical form

$(x-3)(x-1) + (y-8)(y-2)$

$x^2+y^2-4x-10y+19 = 0$



24. (D)



Equation of line AB is $x = -3y + 3$

It cuts the auxiliry circle at M.

area of Triangle AOM is 27/10

25. (D) $z = \cos\theta + i\sin\theta$

$z = e^{i\theta}$

$\sum_{m=1}^{15} Im\left(\left(e^{i2^\circ}\right)^{2m-1}\right)$

$\sum_{m=1}^{15} Im\left(\frac{e^{i4^\circ M}}{e^{i2^\circ}}\right)$

$$\frac{1}{e^{i2^0}}\left[e^{i4^0}+e^{i8^0}+e^{i12^0}+e^{i16^0}+.....e^{i60^0}\right]$$

$$\frac{e^{i4^0}}{e^{i2^0}}\left[1+e^{i4^0}+e^{i8^0}+e^{i12^0}+.....e^{i56^0}\right]$$

$$e^{i2^0}\left[\frac{\left(e^{i4^0}\right)^{15}-1}{e^{i4^0}-1}\right]=\frac{e^{i2^0}\left[\frac{1}{2}+i\frac{\sqrt{3}}{2}-1\right]}{\left[\cos 4^0+i\sin 4^0-1\right]}$$

$$\frac{e^{i2^0}\left[-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right]}{-\left[2\sin^2 2^0-2i\sin 2^0\cos 2^0\right]}$$

$$\frac{\left[-\frac{\sqrt{3}}{2}-i\frac{1}{2}\right]}{-2\sin 2^0}=1/(4\sin 2^0)$$

26. due to less time period now we will show on our web site

27. (A) Coordinate of Q $x=-3\mu+1$, $y=+\mu-1$, $z=+5\mu+2$

d"s dr of PQ $-3\mu+1-3$, $+\mu-1-2$, $+5\mu+2-6$

$(-3\mu-2)(1)+(\mu-3)(-4)+(5\mu-4)(3)=0$

$\mu=1/4$

28. due to less time period now we will show on our web site

29. (A, B)

$$\frac{\sin^4 x}{2}+\frac{\cos^4 x}{3}=\frac{1}{5}$$

$15\sin^4 x+10\cos^4 x=6$

$15[\sin^4 x]+10[1-\sin^2 x]^2=6$

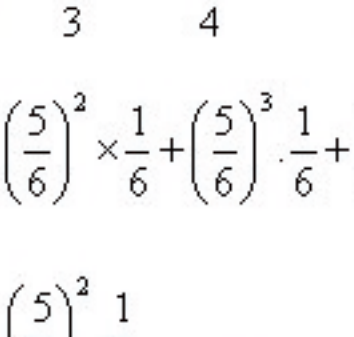
$15[\sin^4 x]+10[1+\sin^4 x-\sin^2 x]=6$

$25\sin^4 x-20\sin^2 x+4=0$

$[5\sin^2 x-2][5\sin^2 x-2]=0$

$[5\sin^2 x-2]^2=0$

$$\sin x=\pm\frac{\sqrt{2}}{\sqrt{5}}$$



You can put values in (A, B) and check.

30. due to less time period now we will show on our web site

31. (A,C) Just apply L-hospital two times we will get value a = 2 than further L-hospital final value is 1/64. (due to less time period now we will show de-tail on our web site)

32. due to less time period now we will show on our web site

33. (A) ${}^6P_3$

I	II	III
not	not	√

$$\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}=\frac{25}{216}$$

34.(B) 3 4 5 6 .....

$$\left(\frac{5}{6}\right)^2\times\frac{1}{6}+\left(\frac{5}{6}\right)^3.\frac{1}{6}+\left(\frac{5}{6}\right)^4\frac{1}{6}+\left(\frac{5}{6}\right)^5\frac{1}{6}+.....\infty$$

$$\frac{\left(\frac{5}{6}\right)^2.\frac{1}{6}}{1-\frac{5}{6}}=\frac{25}{36}$$

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35. (D) $$\frac{\left(\frac{5}{6}\right)^5 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^6 \cdot \frac{1}{6} + \ldots\ldots}{\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right) + \left(\frac{5}{3}\right)^4 \cdot \frac{1}{6} + \left(\frac{5}{3}\right)^5 \cdot \frac{1}{6} + \ldots}$$

$$\frac{\frac{\left(\frac{5}{6}\right)^5 \times \frac{1}{6}}{1-5/6}}{\frac{\left(\frac{5}{6}\right)^3 \times \left(\frac{1}{6}\right)}{1-5/6}}$$

$$\left(\frac{5}{6}\right)^2 = \frac{25}{36}$$

36. (A) $$\begin{bmatrix}1&0&0\\0&1&1\\0&1&1\end{bmatrix}\begin{bmatrix}1&1&0\\1&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&1\\0&1&0\\1&0&1\end{bmatrix}$$

$$\begin{bmatrix}1&0&1\\0&0&1\\1&1&0\end{bmatrix}\begin{bmatrix}1&1&0\\1&0&1\\0&1&0\end{bmatrix}\begin{bmatrix}1&1&1\\1&0&0\\1&0&0\end{bmatrix}$$

$$\begin{bmatrix}0&0&1\\0&0&1\\1&1&1\end{bmatrix}\begin{bmatrix}0&1&0\\1&0&1\\0&1&1\end{bmatrix}\begin{bmatrix}0&1&1\\1&0&0\\1&0&1\end{bmatrix}$$

$$\begin{bmatrix}0&0&1\\0&1&1\\1&1&0\end{bmatrix}\begin{bmatrix}0&1&0\\1&1&1\\0&1&0\end{bmatrix}\begin{bmatrix}0&1&1\\1&1&0\\1&0&0\end{bmatrix}$$

37. (B) $$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$

$$\begin{bmatrix}a_1&b_1&c_1\\a_2&b_2&c_2\\a_3&b_3&c_3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}d_1\\d_2\\d_3\end{bmatrix}$$

For uniq solution $\begin{bmatrix}a_1&b_1&c_1\\a_2&b_2&c_2\\a_3&b_3&c_3\end{bmatrix} \neq 0$

so answer is (B)

38. For uniq solution $\begin{bmatrix}a_1&b_1&c_1\\a_2&b_2&c_2\\a_3&b_3&c_3\end{bmatrix} = 0$

so answer is (B)

39.

	p	q	r	s	t
A	**p**	q	r	s	t
B	p	q	r	**s**	**t**
C	p	q	**r**	s	t
D	p	**q**	r	**s**	t

(p) length of perpendicular from centre = 2

$$\frac{0+0-1}{\sqrt{h^2+k^2}} = 2 \Rightarrow x^2+y^2 = \frac{1}{4} \text{ is circle}$$

(q) PA –PB = constant is defination of Hyperbola

(r) You assume t = tanθ then locus will be

$\frac{x^2}{3} + \frac{y^2}{1} = 1$ is ellipse.

(s) e = 1 parabola & e>1 hyperbola

(t) $z = (x+iy)$

$\text{Re}\,(x+iy+1)^2 = x^2+y^2+1$

$\text{Re}\,[(x+1)^2 - y^2 + 2ixy] = x^2+y^2+1$

$x = y^2$ is parabola

40. due to less time period now we will show on our web site