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विध्न विचारत भीरु जन, नहीं आरम्भे काम, विपति देख छोड़े तुरंत मध्यम मन कर श्याम।
पुरुष सिंह संकल्प कर, सहते विपति अनेक, 'बना' न छोड़े ध्येय को, रघुबर राखे टेक।।

रचितः मानव धर्म प्रणेता
सद्‌गुरु श्री रणछोड़दासजी महाराज

# SOLUTION OF IIT JEE-09

# BY SUHAAG SIR

## & HIS STUDENTS OF CLASS MOVING 11th TO 12th

| | |
|---|---|
| Abhishek Bhardwaj | School MVM |
| Abhishek Singh | School MVM |
| Avantika Garde | School KV-1 |
| Sangya Thakur | School KV-1 |
| Rahul Mishra | School Model T. T. Nagar |

# Solution also Available of IIT JEE 2009

## Paper-1 & 2 on website : www.iitjeeiitjee.com, www.tekoclasses.com or come at our institute

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## MATHEMATICS PAPER-2

**Question sequence as per Paper CODE-7**

## SECTION-1

### Single Correct Choice Type

This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

20. If the sum of first n terms of an A. P. is $cn^2$, then the sum of squares of these ***n*** terms is

(A) $\frac{n(4n^2-1)c^2}{6}$ (B) $\frac{n(4n^2+1)c^2}{3}$ (C) $\frac{n(4n^2-1)c^2}{3}$ (D) $\frac{n(4n^2+1)c^2}{6}$

21. The locus of the orthocentre of the triangle formed by the lines $(1+p)x - py + p(1+p) = 0$, $(1+q)x - qy + q(1+q) = 0$, and $y = 0$, where $p \neq q$, is

(A) a hyperbola (B) a parabola (C) an ellipse (D) a straight line

22. The normal at a point P on the ellipse $x^2 + 4y^2 = 16$ meets the x-axis at Q. If M is the mid point of the line segment PQ, then the locus of M intersects the latus rectums of the given ellipse at the points

(A) $\left(\pm\frac{3\sqrt{5}}{2}, \pm\frac{1}{7}\right)$ (B) $\left(\pm\frac{3\sqrt{5}}{2}, \pm\frac{\sqrt{19}}{4}\right)$ (C) $\left(\pm 2\sqrt{3}, \pm\frac{1}{7}\right)$ (D) $\left(\pm 2\sqrt{3}, \pm\frac{4\sqrt{3}}{7}\right)$

23. A line with positive direction cosines passes through the point P(2, –1, 2) and makes equal angles with the coordinate axes. The line meets the plane, $2x + y + z = 9$, at point Q. The length of the line segment PQ equals

(A) 1 (B) $\sqrt{2}$ (C) $\sqrt{3}$ (D) 2

## SECTION-II

### Multiple Correct Choice Type

This section contains 5 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** is/are correct.

24. For $0<\theta<\pi/2$, the solution (s) of $\sum_{m=1}^{6} \operatorname{cosec}\left(\theta + \frac{(m-1)\pi}{4}\right) \operatorname{cosec}\left(\theta + \frac{m\pi}{4}\right) = 4\sqrt{2}$ is (are)

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{12}$ (D) $\frac{5\pi}{12}$

25. An ellipse intersects the hyperbola $2x^2 - 2y^2 = 1$ orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. If the axes of the ellipse are along the coordinate axes, then

(A) Equation of ellipse is $x^2 + 2y^2 = 2$ (B) The foci of ellipse are $(\pm 1, 0)$

(C) Equation of ellipse is $x^2 + 2y^2 = 4$ (D) The foci of ellipse are $(\pm\sqrt{2}, 0)$

26. The tangent PT and the normal PN to the parabola $y^2 = 4ax$ at a point P on it meet its axis at points T and N, respectively. The locus of the centroid of the triangle PTN is a parabola whose

(A) vertex is $\left(\frac{2a}{3}, 0\right)$ (B) directrix is $x = 0$ (C) latus rectum is $\frac{2a}{3}$ (D) focus is (a, 0)

27. If $I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)} dx$, n = 0, 1, 2, ....., then

(A) $I_n = I_{n+2}$ (B) $\sum_{m=1}^{10} I_{2m+1} = 10\pi$ (C) $\sum_{m=1}^{10} I_{2m} = 0$ (D) $I_n = I_{n+1}$

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28. For the function $f(x) = x\cos\frac{1}{x}, x \ge 1,$

(A) for at least one $\boldsymbol{x}$ in the interval $[1,\infty), f(x+2) - f(x) < 2$

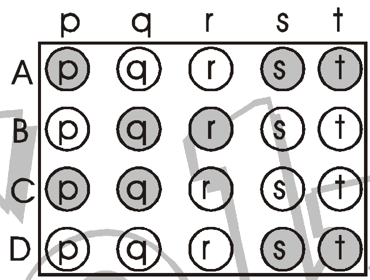
(B) $\lim\limits_{x\to\infty} f'(x) = 1$ (C) for all x in the interval $[1,\infty), f(x+2) - f(x) < 2$

(D) f'(x) is strictly decreasing in the interval $[1, \infty)$

## SECTION-III

### Matrix -Match Type

This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statements is Column I are labelled A, B, C and D, while the statements in Column II are labelled p, q, r, s and t. Any given statement in Column I can have correct matching with ONE OR MORE statement(s) in Column II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the folloiwng example:
If the correct matches are A - p, s and t; B - q and r; C - p and q; and D - s and t; then the correct darkening of bubbles will look like the following.



29. Match the statements/expressions given in Column I with the values given in Column II

| | Column I | Column II |
|---|---|---|
| (A) | The number of solutions of the equation $xe^{\sin x} - \cos x = 0$ in the interval $\left(0, \frac{\pi}{2}\right)$ | (p) 1 |
| | | (q) 2 |
| (B) | Value(s) of $\boldsymbol{k}$ for which the planes kx + 4y + z = 0, 4x + ky +2z = 0 and 2x + 2y + z = 0 intersect in a straight line | (r) 3 |
| | | (s) 4 |
| (C) | Value(s) of $\boldsymbol{k}$ for which $\lvert x-1\rvert + \lvert x-2\rvert + \lvert x+1\rvert + \lvert x+2\rvert = 4k$ has integer solution(s) | (t) 5 |
| (D) | If y' = y + 1 and y(0) = 1, then value(s) of y (ln 2) | |

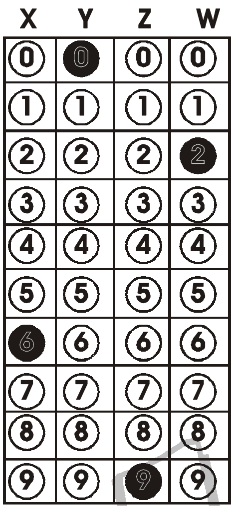
30. Match the statements/expressions given in Column I with the values given in Column II

| | Column I | Column II |
|---|---|---|
| (A) | Root(s) of the equation $2\sin^2\theta + \sin^2 2\theta = 2$ | (p) $\pi/6$ |
| (B) | Points of discontinuity of the function $f(x) = \left[\frac{6x}{\pi}\right]\cos\left[\frac{3x}{\pi}\right]$, | (q) $\pi/4$ |
| | Where [y] denotes the largest integer less than or equal to y | (r) $\pi/3$ |
| (C) | Volume of the parallelopiped with its edges represented by the vectors | (s) $\pi/2$ |
| | $\hat{i}+\hat{j}, \hat{i}+2\hat{j}$ and $\hat{i}+\hat{j}+\pi\hat{k}$ | (t) $\pi$ |
| (D) | Angle between vecotrs $\vec{a}$ and $\vec{b}$ where $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors satisfying $\vec{a}+\vec{b}+\sqrt{3}\vec{c} = \vec{0}$ | |

## SECTION-IV

### Integer Answer Type

This section contains 8 questions. The answer to each of the questions is a single digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened. For example, if the correct answers to question numbers X, Y, Z and W(say) are 6, 0, 9 and 2, respectively, then the correct darkening of bubbles will look like the following:




31. Let $f : R \to R$ be a continuous function which satisfies $f(x)\int_0^x f(t)\,dt$. Then the value of $f(\ln 5)$ is

32. If the function $f(x) = x^3 + e^{\frac{x}{2}}$ and $g(x) = f^{-1}(x)$, then the value of $g'(1)$ is

33. The maximum value of the function $f(x) = 2x^3 - 15x^2 + 36x - 48$ on the set $A = \{x | x^2 + 20 < 9x\}$ is

34. Let ABC and ABC' be two non-congruent triangles with sides AB = 4, $AC = AC' = 2\sqrt{2}$ and angle B = 30º. The absolute value of the difference between the areas of these triangles is

35. Let p(x) be a polynomial of dgree 4 having extremum at x = 1, 2 and $\lim_{x\to\infty}\left(1+\frac{p(x)}{x^2}\right) = 2$. Then the value of p(2) is

36. Let (x, y , z) be points with integer coordinates satisfying the system of homogeneous equations:

$$3x - y - z = 0$$
$$-3x + z = 0$$
$$-3x + 2y + z = 0$$

Then the number of such points for which $x^2 + y^2 + z^2 < 100$ is

37. The centres of two circles $C_1$ and $C_2$ each of unit radius are at a distance of 6 units from each other. Let P be the mid point of the line segment joining the centres of $C_1$ and $C_2$ and C be a circle touching circles $C_1$ and $C_2$ externally. If a common tangent to $C_1$ and C passing through P is also a common tangent to $C_2$ and C, then the radius of the circle C is

38. The smallest value of $k$, for which both the roots of the equation $x^2 - 8kx + 16(k^2 - k + 1) = 0$ are real, distinct and have values at least 4, is

# SOLUTION-IIT JEE-2009 (PAPER-2)

20. (C) If $S_{nAP} = Cn^2 \{S_1 = C\}$ = T1,

$S_2 = 4C = T_1 + T_2$

$T_2 = 4C - T1 = 4C - C = 3C.$

AP is $\Rightarrow$ C, 3C, 5C, 7C ..........

Sum of $C^2$, $9C^2$, $25C^2$, $49C^2$.............

$S_1' = T_1' = C^2$ Put n = 1

$S_2' = T_1' + T_2' = C^2 + 9C^2 = 10C^2$

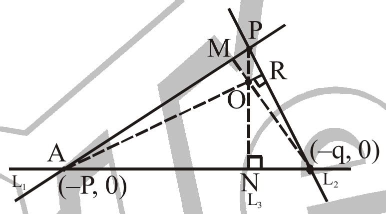
Put n = 2

Check the option Only C is correct.

21. (D) [This is our class room notes que.]

Findcutting point 'p' of line $L_1$ & $L_2$. Now find eq. of PN & AR cutting point of PN & AR is ortho center of $\Delta$PAB Let (h, k)

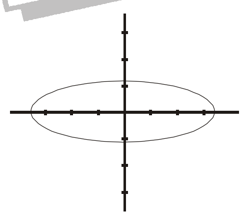
check it will be




h+k = 0

x+y = 0 Straight Line.

22. $x^2+4y^2 = 16 \Rightarrow \frac{x^2}{4^2}+\frac{y^2}{2^2}$



Co-ordinates of 'P' (4cosθ, 2sinθ)

Slope at P of curve = $-\frac{\cot\theta}{2}$

Slope of $\perp$ at P = 2 tanθ.

eq. of PQ $\Rightarrow$ y–2sinθ = w tanθ (x–4 tanθ)

Co-ordinates of Q(3cosθ, 0)

M = Mid point of PQ = $\left(\frac{7\cos\theta}{2}, \sin\theta\right)$

Locus of M is $\frac{4x^2}{49}+y^2=1$ cutting point of curve & latus ract. is $\left(\pm 2\sqrt{3}, \pm\frac{1}{7}\right)$

23. (C) DR's of line (+1, +1, +1), it passes through (2, –1, 2)

Eq. $\frac{x-2}{1}=\frac{y+1}{1}=\frac{z-2}{1}=k$, Point on line which cuts the plane 2x + y + z = 0 (k+2, k–1, k+2) satisfy plane eq. k = 1 So point on plane is (3, 0, 3)

Distance between points

$=\sqrt{(3-2)^2+(0+1)^2+(3-2)^2}$

$=\sqrt{1+1+1}=\sqrt{3}$

24. (C, D) It is formula based question.

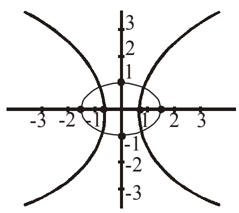
$f(\theta)=\frac{1}{\sin\theta_1-\sin\theta_2}$ Have difference is $(\theta_2-\theta_1)=\frac{\pi}{4}$. So multiply & divide by sinπ/4.

$=\frac{1}{\sin\frac{\pi}{4}}\sum_{m=1}^{6}\frac{\sin\frac{\pi}{4}}{\sin\theta_1\sin\theta_2}$

$=\sqrt{2}\sum_{m=1}^{6}(\cot\theta_2-\cot\theta_1)$

Simplyfy it Ans will be $\frac{\pi}{12}$ & $\frac{5\pi}{12}$.

25. (A, B) Hyperbola is rectangular




$e_H=\sqrt{2}$ So $e_E=\frac{1}{\sqrt{2}}$ according to fig. option A & B correct.

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26. (A, D)

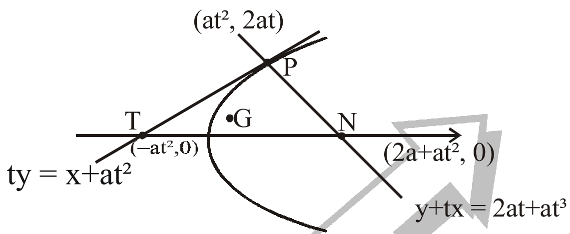
G: $h = \dfrac{at^2 - at^2 + 2a + at^2}{3}$

$k = \dfrac{2at + 0 + 0}{3}$

For locus h $\to$ x, k $\to$ y Put value at K (elivinate)

$(3x - 2a) = a\left(\dfrac{3y}{2a}\right)^2$

$y^2 = 4\left(\dfrac{a}{3}\right)\left(x - \dfrac{2a}{3}\right)$




Directrix x = a/3
L. R. 4a/3
F(a, 0)

$V\left(\dfrac{2a}{3}, 0\right)$

27. (A, B, C)

$$I_n = \int_{-\pi}^{0} \frac{\sin nx\, dx}{(1+\pi^x)\sin x} + \int_0^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x}$$

x $\to$ (-x)

$$I_n = \int_0^{\pi} \frac{\pi^x \sin nx}{(1+\pi^x)\sin x} + \int_0^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x}$$

$I_n = \displaystyle\int_0^{\pi} \frac{\sin nx}{\sin x}$ Now simplyfy and check option ans are A, B, C

28. (B, C, D)

$f(x) = x.\cos\dfrac{1}{x} \Rightarrow f'(x) = \cos\dfrac{1}{x} + \dfrac{1}{x}\sin\left(\dfrac{1}{x}\right)$

Option B is correct.

$$f''(x) = +\frac{1}{x^2}\sin\frac{1}{x} + \left(-\frac{1}{x^2}\right)\sin\frac{1}{x} - \frac{1}{x^2}\cos\frac{1}{x}$$

becous $x \ge 1; d''(x)$ is -ve so

f'(x) is strictly decreasing. Check for option C also correct discorrec here. Ans. (B, C, D)

29.

| | p | q | r | s | t |
|---|---|---|---|---|---|
| A | **p** | q | r | s | t |
| B | p | **q** | r | **s** | t |
| C | p | **q** | **r** | **s** | **t** |
| D | p | q | **r** | s | t |

(A) f(x) = x.e$^{\sin x}$, f'(x) = x.e$^{\sin x}$.cosx–sinx.
f'(x) $\to$+ve in 0 to $\pi$/2, so f(x) is strictly incresing.
f(0) = –ve, f($\pi$/2) is +ve so only one possible root.

(B) $\begin{vmatrix} k & 4 & 1 \\ 4 & k & 2 \\ 2 & 2 & 1 \end{vmatrix} = 0 \quad \Rightarrow K = 2 \,\&\, K = 4.$

(C) Just put the values of x = 1, 2, 3, .... you will get $k \ge \dfrac{3}{2}$ then it will convest x = k. So ans are 2, 3, 4, 5 [q, r, s, t]

(D) Function should be
y = 2e$^x$–1

y' = 2e$^x$ = y+1 so $y\left(\log_e^2\right)$

$= 2e^{\log_e^2} - 1 = 2.2 - 1 = 3$

30.

| | p | q | r | s | t |
|---|---|---|---|---|---|
| A | p | **q** | r | **s** | t |
| B | **p** | q | **r** | **s** | **t** |
| C | p | q | r | s | **t** |
| D | p | q | **r** | s | t |

(A) Put only $\pi/4$ & $\pi/2$ applicable.

(B) [k] $\rightarrow$ non continous when k is integer

$\left(\frac{6x}{\pi}\right) or \left(\frac{3x}{\pi}\right)$ become interer on p, r, s, t.

(C) $\begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & \pi \end{vmatrix} = \pi$

(D) $\vec{a} + \vec{b} + \sqrt{3}\vec{c} = 0$

$\left(\vec{a} + \vec{b}\right)^2 = \left(-\sqrt{3}\vec{c}\right)^2$

$|a|^2 + |b|^2 + 2.|a||b|.\cos\theta = 3.|c|^2$

$1+1+2.1.1\cos\theta = 3$

$\cos\theta = 1/2, \quad \theta = \pi/3$

31. Ans. [0]. $f(x) = \int_0^x f(t)dt$;

$f(x) = c.e^x$.

But due to condition c = 0 so f(x) = 0 constant functions

32. Ans. (2) $f(x) = x^3 + e^{x/2}$

$g(x) = f^{-1}(x)$

$f(g(x)) = x$.

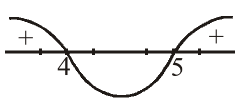
$f'(g(x)).g'(x) = 1$

$$g'(x) = \frac{1}{f'(g(x))}$$

g'(1) become '2'

33. Ans (7)

$x^2+20 \leq 9x$

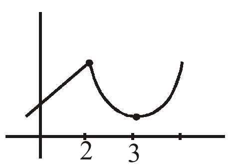



$\Rightarrow$ $x^2-9x+20 \leq 0$

$x^2-4x-5x+20 \leq 0$

$(x-5)(x-4) \leq 0$

$x \in [4,5]$




Now $f(x) = 2x^3-15x^2+36x-48$

$f'(x) = 6x^2-30x+36$

$0 = x^2-5x+6$

$0 = (x-2)(x-3)$


f(5) will be maxima = 7

34. (4) Using sine rule saperatly

$\sin C = \sin C' = \frac{1}{\sqrt{2}}$

$C = = \frac{\pi}{4}$ or $C' = \frac{3\pi}{4}$.

$Area_1 = 2\left(\sqrt{3}+1\right)$ & $Area_2 = 2\left(\sqrt{3}-1\right)$

difference = 4.

35. (0) $\lim_{x\to 0}\frac{P(x)}{x^2} = 1$

So P(0) = 0 L-Hospital Rule

P'(0) = 0

P''(0) = 2

If $P(x) = ax^4 + bx^3 + cx^2+dx+e$

Then Use above condition

e=0, d = 0, c = 1

$P(x) = ax^4+bx^3+x^2$

$P'(x)$ $4ax^3 + 3bx^2 + 2x$.

Use P'(1) = 0, P'(2) = 0

a = 1/4 & b = –1

so P(2) = 0

36. (7) Here $\Delta = \begin{vmatrix} 3 & -1 & -1 \\ -3 & 0 & 1 \\ -3 & 2 & 1 \end{vmatrix}$ is zero.

By use of eq. 2 assume x = k, y=0, z = 3k.

$x^2+y^2+z^2<100$

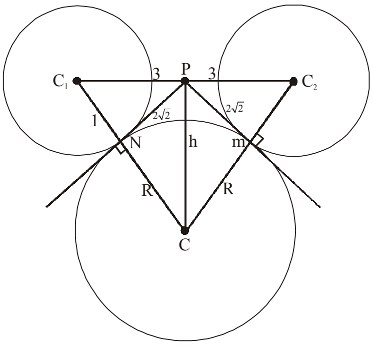
$\Rightarrow k^2 + 0^2 + 9k^2 < 100$

$-\sqrt{10} \leq k \leq \sqrt{10}$

Integer only. $-3 < k < 3$

–3, –2, –1, 0, 1, 2, 3

37. {8}




PM= $\sqrt{3^2-1^2}=2\sqrt{2}$

$\Delta$PMC $h^2=\left(2\sqrt{2}\right)^2+R^2$

$\Delta PC_1C$ $(R+1)^2=3^2+h^2$

Solve eq. get R = 8

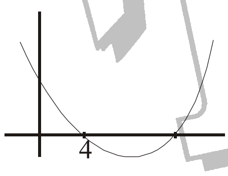
38. [2] Class room question.

$x^2-8kx+16(k^2-k+1)=0$

Roots are real

D > 0

$(-8k)^2-4.1.16\,(k^2-k+1)>0$




You will get K>1.

Now Condition f(4) > 0

When roots geterorequal to 4 $4^2-8.K.4+16(k^2-k+1)>0$

$K\in[1,2]$ Common value k = 2.