Total Que 60, Each Subj 20; PAPER I CODE 2; Total Marks 264; Each Subject 88

Phy: Q.1 to Q.20; Chemistry: Q.21 to Q.40; Maths: Q.41 to Q.60

Solution by Suhag Kariya also with help of their students Anurag Gupta, Lucky Dubey, Manas Deep Viswakarma

Q.41. Let n be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that all the girls stand consecutively in the queue. Let m be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that exactly four girls stand consecutively in the queue. Then the value of $\frac{m}{n}$ is

Sol. (n) Let a packet of 5 girls & differen iteams as 5 boys

(Class room concept Que.) So 5 boy + 1 packet = 6 items.

$n = \lfloor 6 \lfloor 5$

(m) Let a packet of 4 girl & 1 girls saperat.
Settle 5 boys in row. $\lfloor 5$.

↗ B_1 ↗ B_2 ↗ B_3 ↗ B_4 ↗ B_5 ↗

Single girl = 6 choice.

Packet of 4 gr = 5.($\lfloor 4$) ↑ आपस में

$m = \lfloor 5 .6.5.\lfloor 4 \times {}^5C_1$

So $\frac{m}{n} = \frac{\lfloor 5 \cdot 6 \cdot 5 \cdot \lfloor 4 \times {}^5C_1}{\lfloor 6 \cdot \lfloor 5} = 1 \times 5$

Que 41 (5)

Q.42. If the normals of the parabola $y^2 = 4x$ drawn at the end points of its latus rectum are tangents to the circle $(x-3)^2 + (y+2)^2 = r^2$, then the value of r^2

Sol. $L \to (1,2) \to M_{T(1,2)} \to \frac{dy}{dx} = \frac{4}{2y} = \frac{4}{2 \cdot 2} = 1$, $M_{N(1,2)} = -1$,

(2) Eq of normal $= (y-2) = -1(x-1) \Rightarrow x + y - 3 = 0$, $p = r$

$\left| \frac{3-2-3}{\sqrt{1^2+1^2}} \right| = r$ ⟹ $r^2 = \frac{4}{2}$, $r^2 = 2$

Ans (2)

P.T.O.

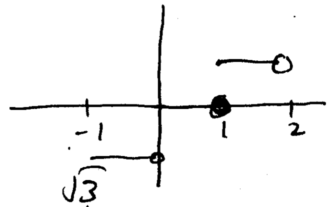
code 2

Q.43 Let $f:R\to R$ be a function defined by $f(x)=\begin{cases}[x] & ; x\le 2\\ 0 & x>2\end{cases}$

If $I=\int_{-1}^{2}\frac{x f(x^2)\,dx}{2+f(x+1)}$

Soln:

Graph of $f(x)$



$$I=\int_{-1}^{2}\frac{x f(x^2)\,dx}{2+f(x+1)}$$

$$I=\int_{-1}^{1}\frac{x\cdot 0\,dx}{1+f(x+1)}+\int_{1}^{\sqrt2}\frac{x\cdot 1\,dx}{2}+\int_{\sqrt2}^{\sqrt3}\frac{x\cdot 0\,dx}{2+0}+\int_{\sqrt3}^{2}\frac{x\cdot 0}{2+0}dx$$

$$I=0+\int_{1}^{\sqrt2}\frac{x}{2}dx+0$$

$$I=\left(\frac{x^2}{4}\right)_{1}^{\sqrt2}$$

$$I=\frac{1}{4}$$

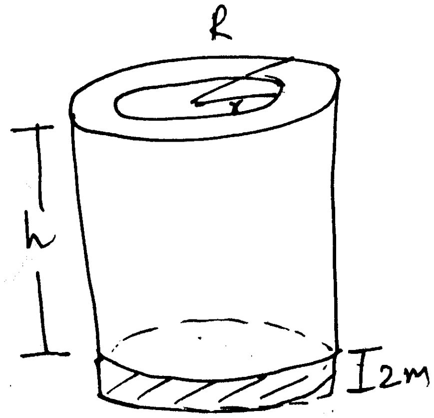
so $4I-1=4\times\frac{1}{4}-1=0$

Q.44 A cylindrical container is to be made from certain solid material with the following constraints. Its inner volume is fixed $V\,mm^3$, has a 2 mm thick solid wall and is open at the top. The bottom of the container is a solid disc of thickness 2 mm and is of radius equal to the outer radius of container. If the volume of material used is min. when __

Que44 Start from page 2 (code2) (Page 3)

.....when the inner radius of the container is 10 mm, then the value of $\frac{V}{250\pi}$ is....

Sol.



$V = \pi r^2 h$ (constant) $\quad R - r = 2$

$h = \frac{V}{\pi r^2}$

$V_{metal} = \underbrace{\pi h(R^2 - r^2)}_{cylinder} + \underbrace{\pi R^2 2}_{disc}$

$V_{metal} = \pi \times \frac{V}{\pi r^2}\left((2+r)^2 - r^2\right) + 2\pi(2+r)^2$

$= \frac{V}{r^2}\left(4 + r^2 + 4r - r^2\right) + 2\pi(r^2 + 4 + 4r)$

$= 4V\left(\frac{1}{r^2} + \frac{1}{r}\right) + 2\pi(r^2 + 4r + 4)$

$\frac{dV_{met}}{dr} = 4V\left(\frac{-2}{r^3} - \frac{1}{r^2}\right) + 2\pi(2r + 4)$

Given, min. at $r = 10 \Rightarrow \left(\frac{dV}{dr}\right)_{r=10} = 0$

$0 = 4V\left(\frac{-2}{1000} - \frac{1}{100}\right) + 2\pi(20 + 4)$

$\frac{48V}{1000} = 48\pi$

$\frac{V}{250\pi} = 4$ $\quad$ (4) Ans

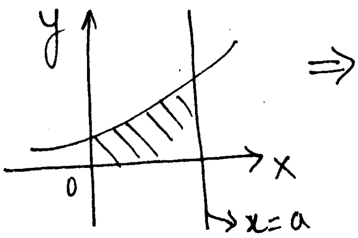
P.T.O.

Q.45. let $F(x) = \int_x^{x^2+\frac{\pi}{6}} 2\cos^2 t\,dt$ for all $x \in R$ (Code 2)

and $f : \left[0, \frac{1}{2}\right] \to [0, \infty)$ be a continous function. for $a \in \left[0, \frac{1}{2}\right]$, if $F'(a) + 2$ is the area of reagion by $x = 0$, $y = 0$, $y = f(x)$ and $x = a$ then $f(0)$ is

Sol. 45. Ans 3

Given.



$\Rightarrow \int_0^a f(x)dx = F'(a) + 2$

$\Rightarrow \boxed{f(x) = F''(x)}$

$F(x) = \int_x^{x^2+6} 2\cos^2 t$

$\Rightarrow F'(x) = 2\cdot 2x\cos^2(x^2+6) + 1\cdot 2\cos^2(x)$ [By Leibnitz Theorem]

$\Rightarrow F''(x) = 4x[\sim\sim] + 4\cos^2(x^2+\frac{\pi}{6}) - 4\cos x\sin x$

$x = 0$ (whole zero)

Putting $x = 0$

$F''(0) = 4\left(\cos^2\left(\frac{\pi}{6}\right)\right) = 4 \times \frac{3}{4} = 3$ Ans (3)

Q.46. The num of distinct solutions of eq.

$\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$ in $x \in [0, 2\pi]$ is.

Sol. Ans 8

$\frac{5}{4}\cos^2 2x + 1 - 2\cos^2 x\sin^2 x + 1 - 3\cos^2 x\sin^2 x = 2$

$\frac{5}{4}\cos^2 2x - 5\cos^2 x\sin^2 x = 0$

$\cos^2 2x - 4\cos^2 x\sin^2 x = 0 \to \cos^2 2x - \sin^2 2x = 0$

$\cos 4x = 0$

$4x = (2n+1)\frac{\pi}{2} \to x = (2n+1)\frac{\pi}{8}$

$n \to 0, 1, 2, 3, 4, 5, 6, 7$

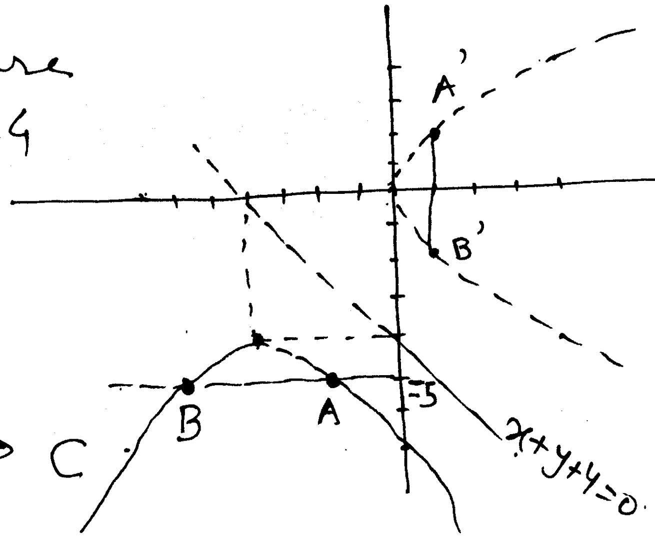
Ans (8) P.T.O

Q.47. Let the curve C be the mirror image of the parabola $y^2 = 4x$, with respect to the line $x+y+4=0$. If A and B are the points of intersection of C with the line $y=-5$, then the distance between A and B is.

Sol Ans 4 According to figure 4th quadrant $AB = 4$ because I & II quadrant A' & B' has dist. $= 4$

To improve knowledge of graph, visit special class on graph on www.MathsBySuhag.com videos

Req. → C



Q.48. The minimum number of times a fair coin needs to be tossed, so that the probability of getting at least two heads is at least 0.96 is ____

Ans ⑧

Sol. required value $^nC_2\left(\frac{1}{2}\right)^n + {^nC_3}\left(\frac{1}{2}\right)^n + \dots + {^nC_n}\left(\frac{1}{2}\right)^n \geq 0.96$

$$\frac{1}{2^n}\left[{^nC_2} + {^nC_3} + {^nC_4} + \dots + {^nC_n}\right] \geq 0.96$$

$$\frac{1}{2^n}\left[2^n - 1 - n\right] \geq 0.96$$

$$2^n - 1 - n \geq 0.96\, 2^n$$

$$2^n(1-0.96) \geq 1+n$$

$$2^n(0.04) \geq 1+n$$

$$\frac{2^n}{1+n} \geq 25$$

n	$\frac{2^n}{1+n}$
0	1
1	1
2	$\frac{4}{3}$
3	2
4	$\frac{16}{3}$
5	$32/6$
6	$64/7$
7	$128/8$
8	$256/9$

Ans. min. value of $n = 8$

P.T.O.

(code2) Que 49 to Q58 are More than One May correct Type

Q49. In R^3, consider the planes $P_1: y=0$ & $P_2: x+z=1$; let P_3 be a plane, different from P_1 & P_2, which passes through the intersection of P_1 & P_2. If the distance of the points $(0,1,0)$ from P_3 is 1 and the distance of a point (α,β,γ) from P_3 is 2, then which of the following relation is(are) true? A) $2\alpha+\beta+2\gamma+2=0$ B) $2\alpha-\beta+2\gamma+4=0$ C) $2\alpha+\beta-2\gamma-10=0$ D) $2\alpha-\beta+2\gamma-8=0$

Sol Let the plane be $x+z-1+\lambda y=0$

(B), (D)

From $(0,1,0)$ distance is 1

$$\frac{0+0-1+\lambda}{\sqrt{1^2+1^2+\lambda^2}}=1$$

$$(\lambda-1)^2=\lambda^2+2$$

Solve $\boxed{\lambda=\frac{-1}{2}}$

Thus plane is $x-\frac{y}{2}+z-1=0$ or $2x-y+2z-2=0$

Given from (α,β,γ) distance is 2

Putting (α,β,γ)

$$\left|\frac{2\alpha-\beta+2\gamma-2}{\sqrt{2^2+2^2+1^2}}\right|=2$$

$$2\alpha-\beta+2\gamma-2=\pm 6$$

$\Rightarrow 2\alpha-\beta+2\gamma-8=0 \longrightarrow$ (B)

& $2\alpha-\beta+2\gamma+4=0 \longrightarrow$ (D)

Q.50. If R^3, let L be a straight line passing through the origin. Suppose that all the points on L are at a constant distance from two planes $P_1: x+2y-z+1=0$; $P_2: 2x-y+z-1=0$. Let M be the locus of the feet of the perpendiculars drawn from the points on L to the plane P_1. Which of the following points lie(s) on M?

A) $\left(0, -\frac{5}{6}, -\frac{2}{3}\right)$ B) $\left(-\frac{1}{6}, -\frac{1}{3}, \frac{1}{6}\right)$ C) $\left(-\frac{5}{6}, 0, \frac{1}{6}\right)$ D) $\left(-\frac{1}{3}, 0, \frac{2}{3}\right)$

Sol Since line is parallel to P_1 & P_2 thus the product of its dr's is zero

P_1 & L: $a+2b-c=0$

P_2 & L: $2a-b+c=0$

$$\frac{a}{1} = \frac{-b}{3} = \frac{c}{-5} = K$$

Now pts. on line be $K, -3K, -5K$

Foot on plane P_1 is

$$\frac{x-K}{1} = \frac{y+3K}{2} = \frac{z+5K}{-1} = -\frac{(K-6K+5K+1)}{1^2+2^2+1^2}$$

Suhag Sir Short Trick

$x = \frac{-1}{6}+K$, $y = -\frac{1}{3}-3K$, $z = +\frac{1}{6}-5K$

By cross checking Options

(A) & (B) satisfied fully $K=\frac{1}{6}$ & $K=0$

P.T.O.

Q.51. Let P and Q be distinct points on the parabola $y^2 = 2x$ such that a circle with PQ as diameter passes through the vertex O of the parabola. If P lies in the first quadrant and the area of the triangle ΔOPQ is $3\sqrt{2}$, then which of the following is (are) the co-ordinates of P?
A) $(4, 2\sqrt{2})$ B) $(9, 3\sqrt{2})$ C) $\left(\frac{1}{4}, \frac{1}{\sqrt{2}}\right)$ D) $(1, \sqrt{2})$.

Sol. Since PQ is diameter, & OPQ is triangle, also OPQ passing a circle so $\angle POQ = 90°$, PQ will pass through $(4a, 0)$, $P(t_1)$ & $Q(t_2)$; $t_1 t_2 = -\frac{4a}{a} = -4$

Ans (A, D)

$P(at^2, 2at)$

$$y^2 = 4ax;\ y^2 = 2x;\ 4a = 2;\ a = \frac{1}{2}$$

$t_2 = -\frac{4}{t_1}$

$$P\left(\frac{t_1^2}{2}, t_1\right), Q\left(\frac{8}{t_1^2}, \frac{-4}{t_1}\right), O(0,0)$$

$$\text{Area of } \Delta = \left|\frac{1}{8a}(y_2 - y_1)(y_1 - y_3)(y_3 - y_2)\right|$$

$$\Delta = \left|\frac{1}{8 \cdot \frac{1}{2}}\left(t_1 + \frac{4}{t_1}\right)\left(\frac{4}{t_1}\right)(t_1)\right|$$

$$3\sqrt{2} = \Delta = \left|\frac{1}{4}\left(t_1 + \frac{4}{t_1}\right) \cdot 4\right| = \left|t_1 + \frac{4}{t_1}\right|$$

$3\sqrt{2} t_1 = t_1^2 + 4$

$0 = t_1^2 - 3\sqrt{2} t_1 + 4$

$0 = t_1^2 - 2\sqrt{2} t_1 - \sqrt{2} t_1 + 4$

$0 = t_1(t_1 - 2\sqrt{2}) - \sqrt{2}(t_1 - 2\sqrt{2})$

$0 = (t_1 - 2\sqrt{2})(t_1 - \sqrt{2})$

$t_1 = 2\sqrt{2}$ Ans (A) | $t_1 = \sqrt{2}$ Ans (D)

$-3\sqrt{2} = t_1 + \frac{4}{t_1}$

$0 = t_1^2 + 3\sqrt{2} t_1 + 4$

$0 = (t_1 + 2\sqrt{2})(t_1 + \sqrt{2})$

$t_1 = -2\sqrt{2}$ ✗ | $t_1 = -\sqrt{2}$ ✗

P.T.O.

(Code 2)

Q.52. Let $y(x)$ be a solution of differential Eqn. $(1+e^x)\cdot y' + y\cdot e^x = 1$. If $y(0)=2$, then which of following statements is (are) true?

A) $y(-4)=0$ B) $y(-2)=0$ C) $y(x)$ has critical point in the interval $(-1,0)$ D) $y(x)$ has no critical point in $(-1,0)$.

Ans (A&C)

Sol.

$$(1+e^x)\frac{dy}{dx} + y\cdot e^x = 1$$

$$\frac{dy}{dx} + y\frac{e^x}{1+e^x} = \frac{1}{1+e^x}$$

$$\frac{dy}{dx} + y\cdot P = Q$$

$$\text{I.F.} = e^{\int \frac{e^x}{1+e^x}dx} = e^{\ln(1+e^x)} = 1+e^x$$

$$y\cdot(IF) = \int Q\cdot(IF)\,dx$$

$$y(1+e^x) = \int \frac{1}{1+e^x}(1+e^x)dx + c$$

$$y(1+e^x) = \int 1\,dx + c$$

$$y(1+e^x) = x + c$$

$x \to 0$, $y \to 2$

$$2(1+e^0) = 0 + c$$

$$4 = c$$

$$y(1+e^x) = x+4$$

$$y = \left(\frac{x+4}{1+e^x}\right)$$

x	y	
−4	0	A ✓
−2	not 0	B ✗

$$\frac{dy}{dx} = \frac{1+e^x-(x+4)(e^x)}{(1+e^x)^2}$$

$$0 = \frac{1+e^x-xe^x-4e^x}{\sim}$$

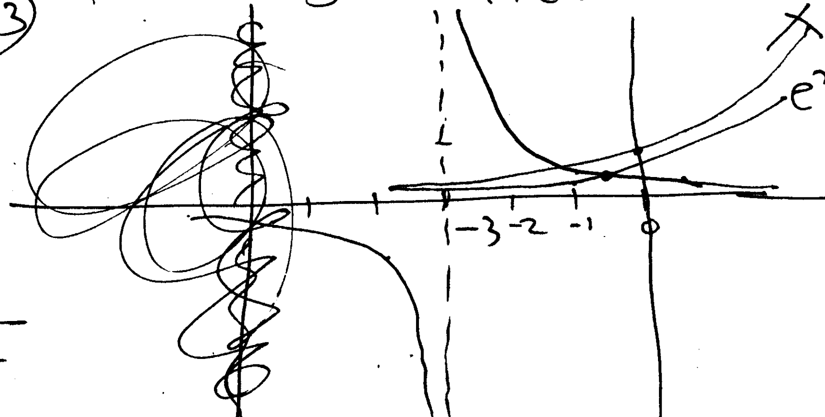
$$0 = 1 - xe^x - 3e^x$$

$$0 = 1 - e^x(x+3)$$

$$e^x(x+3) = 1$$

$$e^x = \frac{1}{x+3}$$

$$\frac{1}{e} = \frac{1}{2.7} \quad \left\{ \frac{1}{2} \right.$$



Also Ans (C)

P.T.O.

Q=53. Consider the family of all circles whose centers lie on the straight lines $y=x$, if this family of circles is represented by diff. eq. $Py''+Qy'+1=0$ where P & Q are function of x, y, y', then which of the statements is (are) true?

A) $P = y+x$ B) $P = y-x$
C) $P+Q = 1-x+y+y'+(y')^2$ D) $P-Q = x+y-y'-(y')^2$

Sol. Centers lie of $x=y$, So Center is (k,k) and radius is r, so eq. of circle is $(x-k)^2+(y-k)^2=r^2$

Ans (B, C)

$$2(x-k)+2(y-k)\cdot y'=0$$

$$\frac{x-k}{y-k}=\frac{-y'}{1}$$

$$x-k=y'k-yy'$$

$$x+yy'=k(1+y')$$

$$\frac{x+yy'}{1+y'}=k$$

$$2(1-0)+2(y'-0)\cdot y'+2(y-k)y''=0$$

$$2+2(y')^2+2yy''-2ky''=0$$

$$1+(y')^2+y\cdot y''-k\cdot y''=0$$

$$1+(y')^2+y(y'')-\left(\frac{x+yy'}{1+y'}\right)y''=0$$

$$1(1+y')+(y')^2(1+y')+y(y'')(1+y')-xy''-yy'y''=0$$

$$1+y'+(y')^2+(y')^3+yy''+y\cdot y''y'-xy''-y\cdot y'y''=0$$

$$1+y'+(y')^2+(y')^3+y(y'')+0-xy''=0$$

$$(1+y'+(y')^2)y'+1+(y-x)y''=0$$

$\rightarrow Q$, $\rightarrow P$

Ans (B)(C) Only

P.T.O

Q.54. let $g: R \to R$ be a differential function with $g(0)=0$, $g'(0)=0$ (← single diff.), and $g'(1) \neq 0$, let

$$f(x) = \begin{cases} \frac{x}{|x|} g(x) & ; x \neq 0 \\ 0 & ; x = 0 \end{cases}$$

and $h(x) = e^{|x|}$ for all $x \in R$

let $(f \circ h)(x)$ denote $f(h(x))$ and $(h \circ f)(x) = h(f(x))$, Then which of the following is (are) True?

A) f is differentiable at $x=0$ B) h is diff'ble at $x=0$

C) foh ——"—— D) hof ——"—— $x=0$

Sol.

Ans (AD)

$$f(x) = \begin{cases} g(x) & : x > 0 \\ 0 & ; x = 0 \\ -g(x) & ; x < 0 \end{cases}$$

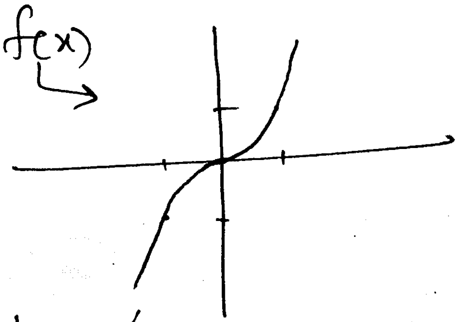
SUHAG SHORT Trick

according to condition let $g(x) = x^2$; $g(0) = 0$

$g'(0) = 0$; $g'(1) \neq 0$

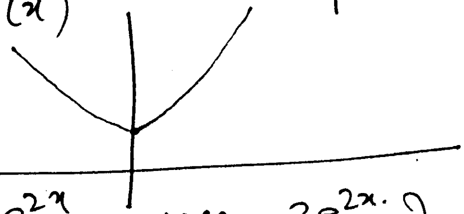
$$f(x) = \begin{cases} x^2 & ; x > 0 \\ 0 & ; x = 0 \\ -x^2 & ; x < 0 \end{cases}$$

f(x) →



$$h(x) = \begin{cases} e^{x} & ; x \geq 0 \\ e^{-x} & ; x \leq 0 \end{cases}$$

h(x)



$foh = f(h(x)) = \begin{cases} (h(x))^2 = (e^{x})^2 = e^{2x} \xrightarrow{diff} 2e^{2x} \\ \& \ (e^{-x})^2 = e^{-2x} \longrightarrow -2 \cdot e^{-2x} \end{cases}$ not diff.

$hof = h(f(x)) = e^{|f(x)|} = e^{x^2} \longrightarrow e^{x^2} \cdot 2x \longrightarrow$ differentiable

Q.55. Let $f(x) = \sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$ for all $x \in R$ and $g(x) = \frac{\pi}{2}\sin x$ for all $x \in R$. Let $fog(x)$, $(gof)(x)$ are there simple meaning. Then which of the following is (are) true.

A) Range of f is $\left[-\frac{1}{2}, \frac{1}{2}\right]$ B) Range of fog is $\left[-\frac{1}{2}, \frac{1}{2}\right]$

C) $\lim_{x\to 0}\frac{f(x)}{g(x)} = \frac{\pi}{6}$ D) There is an $x \in R$, for $(gof)(x) = 1$

Sol. Ans (ABC)

$f(x) = \sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$

$-1 \le \sin x \le 1$ → $-\frac{\pi}{2} \le g(x) \le \frac{\pi}{2}$ → fog simply full range of f will begin (B) Ans.

$-\frac{\pi}{2} \le \frac{\pi}{2}\sin x \le +\frac{\pi}{2}$

$-1 \le \sin\left(\frac{\pi}{2}\sin x\right) \le 1$

$-\frac{\pi}{6} \le \frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right) \le \frac{\pi}{6}$

$-\frac{1}{2} \le f(x) \le \frac{1}{2}$ → Ans (A)

(C) Check

Suhag Short Trick

$$\lim_{x\to 0}\frac{\sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)}{\frac{\pi}{2}\sin x} = \lim_{x\to 0}\frac{\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)}{\frac{\pi}{2}\cdot x} = \frac{\frac{\pi}{6}\cdot\frac{\pi}{2}\sin x}{\frac{\pi}{2}\cdot x}$$

Ans (C)

(D) Check. $gof = g(f(x))$

$-\frac{1}{2} \le f(x) \le \frac{1}{2}$

$g\left(-\frac{1}{2}\right) \le g(f(x)) \le g\left(\frac{1}{2}\right)$

$-\frac{\pi}{2}\sin\frac{1}{2}$ ≯ $\frac{\pi}{2}\sin\frac{1}{2}$

$-\frac{\pi}{4}$ ≯ $\frac{\pi}{4}$ → So not D

P.T.O.

Q.56. Let ΔPQR be a triangle. Let $\vec{a} = \vec{QR}$, $\vec{b} = \vec{RP}$, $\vec{c} = \vec{PQ}$. If $|\vec{a}| = 12$, $|\vec{b}| = 4\sqrt{3}$, $\vec{b}\cdot\vec{c} = 24$, then which of the following is (are) true?

A) $\frac{|c|^2}{2} - |\vec{a}| = 12$

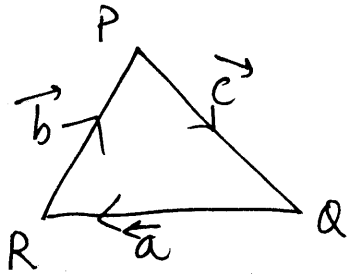
B) $\frac{|c|^2}{2} + |a| = 30$

C) $|\vec{a}\times\vec{b} + \vec{c}\times\vec{a}| = 48\sqrt{3}$

D) $\vec{a}\cdot\vec{b} = -72$.

Sol. By Δ law

(A, C, D)



$\vec{a}+\vec{b}+\vec{c} = 0$

$\vec{b}+\vec{c} = -\vec{a}$

$|\vec{b}+\vec{c}|^2 = |\vec{a}|^2$

$|b|^2 + |c|^2 + 2\vec{b}\cdot\vec{c} = |\vec{a}|^2$

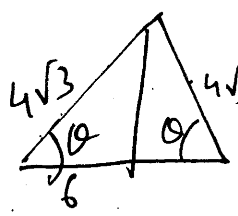
$\Rightarrow 48 + |c|^2 + 48 = 144$

$|\vec{c}| = 4\sqrt{3}$

(A) $\frac{48}{2} - 12 = 12$ ✓

(B) $\frac{48}{12} + 12 \neq 30$ ✗

(C)



$\sin\theta = \frac{1}{2}$

$\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}$

$= 12 \times 4\sqrt{3} \times \frac{1}{2} = 24\sqrt{3}$

$\vec{c}\times\vec{a} = |\vec{c}| * |\vec{a}| \sin\theta = 24\sqrt{3}$

$|\vec{a}\times\vec{b} + \vec{c}\times\vec{a}| = 48\sqrt{3}$ ✓

(D) $\vec{a}+\vec{b} = -\vec{c}$

$|\vec{a}+\vec{b}|^2 = |\vec{c}|^2 \Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b} = |c|^2$

$\Rightarrow 144 + 48 + 2\vec{a}\cdot\vec{b} = 48 \rightarrow \vec{a}\cdot\vec{b} = -72$

(D) ✓

Q.57. Let X, Y be two arbitrary, 3x3 non zero, skew symmetric matrices and Z be an arbitrary 3x3, non zero, symmetric matrix. Then which of the following matrices is (are) skew symmetric?

A) $Y^3Z^4 - Z^4Y^3$ B) $X^{44} + Y^{44}$ C) $X^4Z^3 - Z^3X^4$ D) $X^{23} + Y^{23}$

Sol (C,D)

A) $Y^3Z^4 - Z^4Y^3$

$Y^3 \to -Y^3$ [Skew]

(C,D) $2Y^3Z^4$ [Non-skew symmetric]

B) $X^{44} + Y^{44} = \text{symm.}$ (symm. + symm)

C) $X^4Z^3 - Z^3X^4$

$X^4Z^3 - Z^3X^4 = 0$ so skew symm (X^4 symm, X^4 sym)

D) $X^{23} + Y^{23} = \text{Skew}$ (skew + skew)

P.T.O.

Q.58 Which of the following values of α satisfy the equation

(B, C)

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2 \\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2 \end{vmatrix} = -648\alpha \ ?$$

A) −4 B) 9
C) −9 D) 4

Sol. $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_2$

$$\begin{vmatrix} (1+\alpha)^2 & \alpha(3\alpha+2) & \alpha(5\alpha+2) \\ (\alpha+\alpha)^2 & \alpha(3\alpha+4) & \alpha(5\alpha+4) \\ (3+\alpha)^2 & \alpha(3\alpha+6) & \alpha(5\alpha+6) \end{vmatrix} = -648\alpha$$

Taking α common $R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_2$

$$\alpha^2 \begin{vmatrix} (1+\alpha)^2 & 3\alpha+2 & 5\alpha+2 \\ (2\alpha+3) & 2 & 2 \\ (2\alpha+5) & 2 & 2 \end{vmatrix} = -648\alpha$$

~~$R_3 \to R_3 - R_2$,~~ $C_3 \to C_3 - C_2$

$$\alpha^2 \begin{vmatrix} (1+\alpha)^2 & 3\alpha+2 & 2\alpha \\ 2\alpha+3 & 2 & 0 \\ 2\alpha+5 & 2 & 0 \end{vmatrix} = -648\alpha$$

$$\alpha^2 \times 2\alpha[4\alpha+6-4\alpha-10] = -648\alpha$$

$$2\alpha^3[+4] = +648\alpha$$

$$\alpha^2 = \frac{648}{8}$$

$$\alpha^2 = 81$$

$$\alpha = \pm 3$$

Ans (B, C)

P.T.O.

Q. 59 Match Matrix type (code 2)

(A) $\vec{a} = \alpha\hat{i} + \beta\hat{j}$ $\quad \vec{b} = \sqrt{3}\hat{i} + \hat{j}$

(Q, P)

projection of $\vec{a}$ on $\vec{b}$

$= \frac{\vec{a}\cdot\vec{b}}{|b|} = \frac{\sqrt{3}\alpha + \beta}{2} = \pm\sqrt{3}$

$\sqrt{3}\alpha + \beta = \pm 2\sqrt{3}$

$\sqrt{3}(2 + \sqrt{3}\beta) + \beta = \pm 2\sqrt{3}$

$\beta = 0$ or $-1/\sqrt{3}$

$\alpha = 2 + \sqrt{3}\beta$

$= 2 + 0 = 2$, option (Q) or $\alpha = 2 + \sqrt{3}\times\left(\frac{-1}{\sqrt{3}}\right) = -2$

$|\alpha| = 1$ option (P)

(B) $f(x) = \begin{cases} -3ax^2 - 2 & x < 1 \\ bx + a^2 & x \geq 1 \end{cases}$

(P, Q)

(1) For continuity

$-3a - 2 = b + a^2$ at $x = 1$

For differentiablity

L-H-D = R-H-D

$-6ax = b$ at $x = 1$

$b = -6a$

$-3a - 2 = -6a + a^2$

$a^2 - 3a + 2 = 0$

$a = 1, 2$ option (P, Q)

(C) (PQST)

$(3 - 3\omega + 2\omega^2)^{4n+3} + (2 + 3\omega - 3\omega^2)^{4n+3} + (-3 + 2\omega + 3\omega^2)^{4n+3} = 0$

$\Rightarrow (1 - 5\omega)^{4n+3} + (-5\omega^2 + \omega)^{4n+3} + (\omega^2 - 5)^{4n+3} = 0$ $\quad [\omega + \omega^2 + 1 = 0]$

$\Rightarrow (1 - 5\omega)^{4n+3} + \omega^{4n}(-5\omega + 1) + (\omega^2)^{4n}(1 - 5\omega) = 0$

$(1 - 5\omega)^{4n+3}[1 + \omega^{4n} + \omega^{8n}] = 0$

so n must not be a multiple of 3

$= 1, 2, 4, 5$ (P, Q, S, T)

P.T.O.

Q. 59 Match Matrix

(D) a & b in H.P $\frac{2ab}{a+b}=4$

(Q, T) a, 5, q, b in A.P

$A-3d,\ A-d,\ A+d,\ A+3d$

$$\frac{2(A-3d)(A+3d)}{(A-3d)(A+3d)}=4$$

$$A^2-9d^2=4A \Rightarrow A^2-4A-9d^2=0 \quad \text{(I)}$$

a, 5, q in A.P

~~A−d+5~~ $A-3d+A+d=10$

$$\Rightarrow 2A-2d=5$$

$$A=5+d \quad \text{(II)}$$

from (I) & (II)

$$25+d^2+10d-20-4d-9d^2=0$$

$$8d^2-6d-5=0$$

$$d=\frac{5}{4},\ \frac{-1}{2}$$

$$|q-a|=|A+d-A+3d|=|4d|$$

$$=\left|4\times\frac{5}{4}\right|,\ \left|4\times\frac{-1}{2}\right|$$

$$=5,\ 2$$ ~~Neglect~~

P.T.O.

Q.60. Match Matrix Type

(A) $\lambda = \dfrac{\sin X\cos Y - \cos X \sin Y}{\sin Z} = \dfrac{ak\cos Y - \cos X(bk)}{ck}$ [sine rule]

$= \dfrac{a\cos Y - b\cos X}{c} \Rightarrow \dfrac{a\left(\frac{a^2+c^2-b^2}{2ab}\right) - b\left(\frac{b^2+c^2-a^2}{2bc}\right)}{c}$

Given that

$2(a^2-b^2)=c^2$

$\therefore \dfrac{a^2-b^2}{c^2} = \dfrac{1}{2}$

$\Rightarrow \dfrac{a^2+c^2-b^2 - b^2-c^2+a^2}{2c^2}$

$\Rightarrow \dfrac{2a^2-2b^2}{2c^2} = \dfrac{a^2-b^2}{c^2} = \dfrac{1}{2}$

(P, R, S)

Now

$\cos(n\pi\lambda)=0 \Rightarrow n\lambda\pi = (2m+1)\dfrac{\pi}{2} \Rightarrow \dfrac{n}{2} = \dfrac{(2m+1)}{2}$

$n =$ odd. $\therefore$ from options (P)(R)(S) are correct.

$\therefore n = 1, 3, 5 \ldots$

(B) $1+\cos 2X - 2\cos 2Y = 2\sin X\sin Y$

(P) $1+1-2\sin^2 X - 2 + 4\sin^2 Y = 2\sin X\sin Y$

$1+1-2a^2k^2-2+4b^2k^2 = 2ak\cdot bk$ [By Sine Rule]

$2-2a^2k^2-2+4b^2k^2 = 2abk^2$

$-a^2+2b^2 = ab$

$2b^2-ab-a^2=0$

$2b^2-2ab+ab-a^2=0$

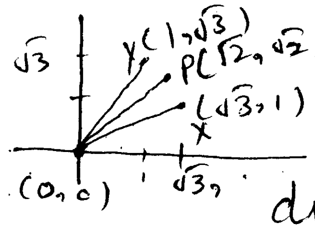
$2b(b-a)+a(b-a)=0 \rightarrow \dfrac{b}{a} = 1$ & $\dfrac{b}{a} = \dfrac{1}{2}$

P.T.O. →

Q.60 Match Matrix type

(C) $x = \sqrt{3}\hat{i} + \hat{j}$ $y = \hat{i} + \sqrt{3}\hat{j}$ $z = \beta\hat{i} + (1-\beta)\hat{j}$

(P, Q).



eq^n of OP = $y - x = 0$.

distance from $z(\beta, 1-\beta)$

$= \left|\frac{1-\beta-\beta}{\sqrt{2}}\right| = \frac{3}{\sqrt{2}}$.

$|1 - 2\beta| = 3$

$1 - 2\beta = \pm 3$

$\beta = 1, -2$

$|\beta| = 1, 2$ option P, Q

(B) & (T)

$F(1)$

$= F(0) - \frac{1}{2} 2 \cdot 1$

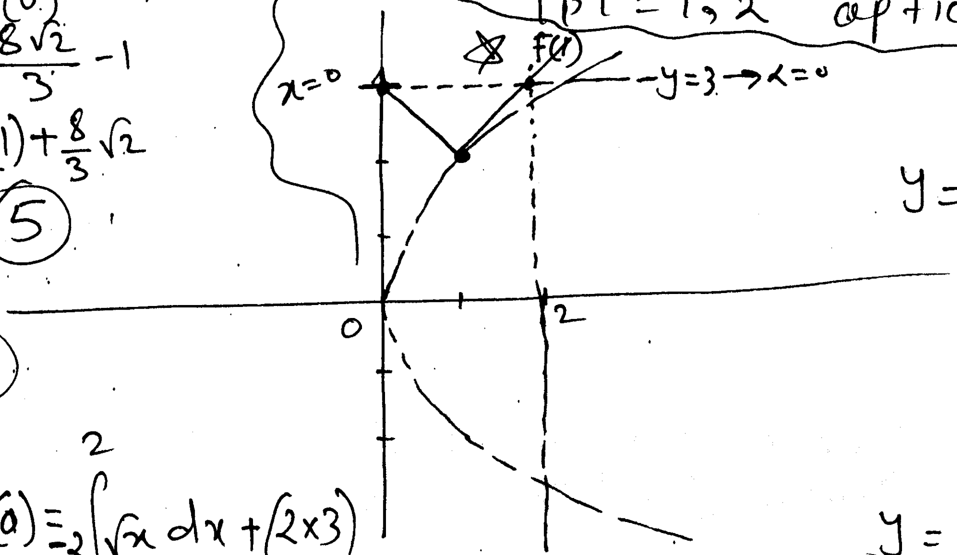
$= F(0) - 1$

$= \frac{-8\sqrt{2}}{3} - 1$

(D) $f(1) + \frac{8}{3}\sqrt{2}$

(5)

5 & 6



$\alpha = 0$

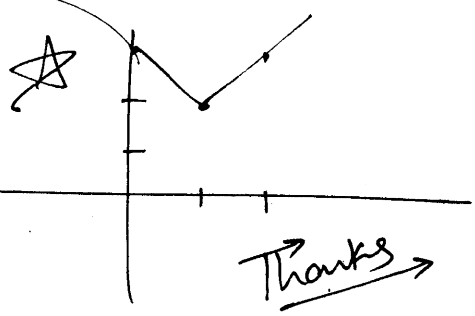
$y = |-1| + |-2| + 0$

$y = 3$

$2 \times \sqrt{2}$
2×1.4
2.8

$\alpha = 1$

$y = |x-1| + |x-2| + x$.



$F(0) = 2\int_0^2 \sqrt{x}\, dx + (2 \times 3)$

$= 2 \cdot \frac{x^{3/2}}{3/2} + 6$

$= -\frac{2 \cdot 2}{3}\left[x^{3/2}\right]_0^2 = -\frac{4}{3}\left[2^{3/2}\right] + 6$

$= -\frac{4}{3} \cdot 2\sqrt{2} = \frac{-8\sqrt{2}}{3} + 6$ so $F(0) + \frac{8}{3}\sqrt{2} = 6$

Thanks