Teko CLASSES ..... the support

REAL NUMBERS

1.1 DIVISIBILITY :

A non-zero integer **'a'** is said to divide an integer **'b'** if there exists an integer **'c'** such that **b= ac.** The integer **'b'** is called dividend, integer **'a'** is known as the divisor and integer **'c'** is known as the quotient.

For example, 5 divides 35 because there is an integer 7 such that 35 = 5 × 7.

If a non-zero integer **'a'** divides an integer **b,** then it is written as **a | b** and read as **'a a divides b', a/b** is written to indicate that **b** is not divisible by **a.**

1.2 EUCLID'S DIVISION LEMMA :

Let **'a'** and **'b'** be any two positive integers. Then, there exists unique integers **'q'** and **'r'** such that **a = b + r,** where **0 ≤ r b.** If **b|a,** than **r = 0.**

Ex.1 Show that any positive odd integer is of the form 6q + 1 or, 6q + 3 or, 6q + 5, where q is some integer.

Sol. Let 'a' be any positive integer and b = 6. Then, by Euclid's division lemma there exists integers 'a' and 'r' such that

$a = 6q + r$, where $0 \le r < 6$.

$\Rightarrow$ $a = 6q$ or, $a = 6q + 1$ or, $a = 6q + 2$ or, $a = 6a + 3$ or, $a = 6q + 4$ or, $a = 6q + 5$.

$[\therefore 0 \le r < 6 \Rightarrow r = 0, 1,2,3,4,5]$

$\Rightarrow$ $a = 6q + 1$ or, $a = 6q + 3$ or, $a = 6q + 5$.

$[\therefore$ a is an odd integer, $\therefore \therefore 6q, a \ne 6q + 2, a \ne 6q + 4]$

Hence, any odd integer is of the form 6q + 1 or, 6q + 3 or, 6q + 5.

Ex.2 Use Euclid's Division Lemma to show that the cube of any positive integer is of the form 9m, 9m + 1 or 9 m + 8, for some integer q.

Sol, Let x be any positive integer. Then, it is of the form 3q or, 3q + 1 or, 3 + 2.

Case - I When $x = 3q$ $\Rightarrow$ $x^3 = (3q)^3 = 27q^3 = 9(3q^3) = 9m$, where $m = 9q^3$

Case - II when $x = 3q + 1$ $\Rightarrow$ $x^3 = (3q + 1)^3$ $\Rightarrow$ $x^3 = 2q^3 + 27q^2 + 9q + 1$

$\Rightarrow$ $x^3 = 9q (3q^2 + 3q + 1) + 1$ $\Rightarrow$ $x^3 = 9m + 1$, where $m = q (3q^2 + 3q + 1)$.

Case -III when $x = 3q + 2$

$\Rightarrow$ $x^3 = (3q + 2)^3$ $\Rightarrow$ $x^3 = 27q^3 + 54q^2 + 36q + 8$ $\Rightarrow$ $x^3 = 9q(3q^2 + 6q + 4) + 8$

$\Rightarrow$ $x^3 = 9m + 8$, where $m = 3q^2 + 6q + 4)$ Hence, x^3 is either of the form 9m of 9m + 1 or 9m + 8.

Ex.3 Prove that the square of any positive integer of the form 5q + 1 is of the same form.

Sol. Let x be any positive's integer of the form 5q + 1.

When $x = 5q + 1$ $x^2 = 25q^2 + 10q + 1$

$x^2 = 5(5q + 2) + 1$

Let $m = q (5q + 2)$.

$x^2 = 5m + 1$. Hence, x^2 is of the same form i.e. $5m + 1$.

1.3 EUCLID'S DIVISION ALGORITHM :

If **'a'** and **'b'** are positive integers such that **a = bq + r,** then every common divisor of **'a'** and **'b'** is a common divisor of **'b'** and **'r'** and vice-versa.

Ex.4 Use Euclid's division algorithm to find the H.C.F. of 196 and 38318.

Sol. Applying Euclid's division lemma to 196 and 38318.

38318 = 195 × 196 + 98

196 = 98 × 2 + 0 The remainder at the second stage is zero. So, the H.C.F. of 38318 and 196 is 98.

Ex.5 If the H.C.F. of 657 and 963 is expressible in the form 657x + 963 × (-15), find x.

Sol. Applying Euclid's division lemma on 657 and 963.

963 = 657 × 1 + 306   657 = 306 × 2 + 45
306 = 45 × 6 + 36   45 = 36 × 1 + 9
36 = 9 × 4 + 0   So, the H.C.F. of 657 and 963 is 9.
Given : 657x + 963 × (-15) = H.C.F. of 657 and 963.
657 x + 963 × (-15) = 9
657 x = 9 + 963 × 15
657 x = 14454 $x = \frac{14454}{657} = 22.$

Ex.6 What is the largest number that divides 626, 3127 and 15628 and leaves remainders of 1, 2 and 3 respectively.

Sol. Clearly, the required number is the H.C.F. of the number 626 - 1 = 625, 3127 - 2 3125 and 15628 - 3 = 15625.
15628 - 3 = 15625.
Using Euclid's division lemma to find the H.C.F. of 625 and 3125.
3125 = 625 × 5 + 0   Clearly, H.C.F. of 625 and 3125 is 625.
Now, H.C.F. of 625 and 15625   15625 = 625 × 25 + 0
So, the H.C.F. of 625 and 15625 is 625.
Hence, H.C.F. of 625, 3125 and 15625 is 625.
Hence, the required number is 625.

Ex.7 144 cartons of coke cans and 90 cartons of Pepsi cans are to be stacked is a canteen. If each stack is of same height and is to contains cartons of the same drink, what would be the greatest number of cartons each stack would have ?

Sol. In order to arrange the cartons of the same drink is the same stack, we have to find the greatest number that divides 144 and 90 exactly. Using Euclid's algorithm, to find the H.C.F. of 144 and 90.
144 = 90 × 1 + 54
90 = 54 × 1 + 36
54 = 36 × 1 + 18
36 = 18 × 2 + 0 So, the H.C.F. of 144 and 90 is 18.   Number of cartons in each stack = 18.

1.4 FUNDAMENTAL THEOREM OF ARITHMETIC :

Every composite number can be expressed as a product of primes, and this factorisation is unique, except for the order in which the prime factors occurs.

SOME IMPORTANT RESULTS :

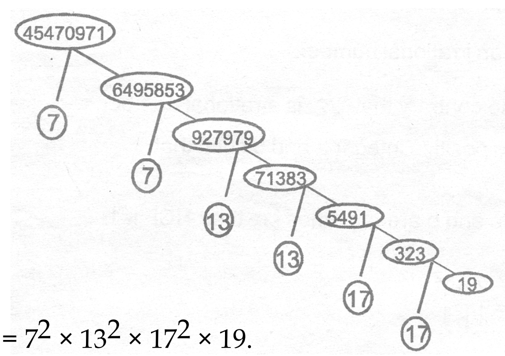
(i) Let **'p'** be a prime number and **'a'** be a positive integer. If **'p'** divides **a^2**, then **'p'** divides **'a'.**

(ii) Let **x** be a rational number whose decimal expansion terminates. Then, **x** can be expressed in the form $\frac{p}{q}$, where **p** and **q** are co-primes, and prime factorisation of **q** is of the form **$2^m \times 5^n$**, where **m, n** are non-negative integers.

(iii) Let $x = \frac{p}{q}$ be a rational number, such that the prime factorisation of q is not of the form $2^m \times 5^n$ where **m, n** are non - negative integers. Then, x has a decimal expansion which is non - terminating repeating.

Ex.8 Determine the prime factors of 45470971.

Sol.



∴ $45470971 = 7^2 \times 13^2 \times 17^2 \times 19.$

Ex.9 Check whether 6^n can end with the digit 0 for any natural number.

Sol. Any positive integer ending with the digit zero is divisible by 5 and so its prime factorisations must contain the prime 5.

$6^n = (2 \times 3)^n = 2^n \times 3^n$

$\Rightarrow$ The prime in the factorisation of 6^n is 2 and 3.

$\Rightarrow$ 5 does not occur in the prime factorisation of 6^n for any n.

$\Rightarrow$ 6^n does not end with the digit zero for any natural number n.

Ex.10 Find the LCM and HCF of 84, 90 and 120 by applying the prime factorisation method.

Sol. $84 = 2^2 \times 3 \times 7$, $90 = 2 \times 3^2 \times$ and $120 = 2^3 \times 3 \times 5$.

Prime factors	Least exponent
2	1
3	1
5	0
7	0

$\therefore$ HCF $= 2^1 \times 3^1 = 6$.

Common prime factors	Greatest exponent
2	3
3	2
5	1
7	1

$\therefore$ LCM $= 2^3 \times 3^3 \times 5^1 \times 7^1$

$= 8 \times 9 \times 5 \times 7 = 2520.$

Ex.11 In a morning walk three persons step off together, their steps measure 80 cm, 85 cm and 90 cm respectively. What is the minimum distance each should walk so that they can cover the distance in complete steps ?

Sol. Required minimum distance each should walk so, that they can cover the distance in complete step is the L.C.M. of 80 cm, 85 cm and 90 cm

$80 = 2^4 \times 5$ $\quad$ $85 = 5 + 17$ $\quad$ $90 = 2 \times 3^2 \times 5$

$\therefore$ LCM $= 2^4 \times 3^2 \times 5^1 \times 17^1$

LCM $= 16 \times 9 \times 5 \times 17$

LCM = 12240 cm, = 122 m 40 cm.

Ex.12 Prove that $\sqrt{2}$ is an irrational number.

Sol. Let assume on the contrary that $\sqrt{2}$ is a rational number.

Then, there exists positive integer a and b such that

$\sqrt{2} = \frac{a}{b}$ where, a and b are co primes i.e. their HCF is 1.

$\Rightarrow$ $(\sqrt{2})^2 = \left(\frac{a}{b}\right)^2$ $\quad\Rightarrow\quad$ $2 = \frac{a^2}{b^2}$

$\Rightarrow$ $a^2 = 2b^2$ $\quad\Rightarrow\quad$ a^2 is multiple of 2

a is a multiple of 2 ....(i)

$\Rightarrow$ a = 2c for some integer c.

$\Rightarrow$ $a^2 = 4c^2$ $\quad\Rightarrow\quad$ $2b^2 = 4c^2$

$\Rightarrow$ $b^2 = 2c^2$ $\quad\Rightarrow\quad$ b^2 is a multiple of 2

b is a multiple of 2 ....(ii)

From (i) and (ii), a and b have at least 2 as a common factor. But this contradicts the fact that a and b are co-prime. This means that $\sqrt{2}$ is an irrational number.

Ex.13 Prove that $3 - \sqrt{5}$ is an irrational number.

Sol. Let assume that on the contrary that $3 - \sqrt{5}$ is rational.

Then, there exist co-prime positive integers a and b such that,

$$3 - \sqrt{5} = \frac{a}{b}$$

$\Rightarrow$ $3 - \frac{a}{b} = \sqrt{5}$ $\quad\Rightarrow\quad$ $\frac{3b - a}{b} = \sqrt{5}$

$\Rightarrow$ $\sqrt{5}$ is rational [$\because$ a,b, are integer $\therefore \frac{3b-a}{b}$ is a rational number]

This contradicts the fact that $\sqrt{5}$ is irrational

Hence, $3-\sqrt{5}$ is an irrational number.

Ex.14 Without actually performing the long division, state whether $\frac{13}{3125}$ has terminating decimal expansion or not.

Sol. $\frac{13}{3125}=\frac{13}{2^0\times 5^5}$ This, shows that the prime factorisation of the denominator is of the form $2^m \times 5^n$.
Hence, it has terminating decimal expansion.

Ex.15 What can you say about the prime factorisations of the denominators of the following rationals :
(i) 43.123456789 (ii) $43.\overline{123456789}$

Sol. (i) Since, 43.123456789 has terminating decimal, so prime factorisations of the denominator is of the form $2^m \times 5^n$, where m, n are non - negative integers.
(ii) Since, $43.\overline{123456789}$ has non-terminating repeating decimal expansion. So, its denominator has factors other than 2 or 5.

DAILY PRACTICE ROBLEMS # 1

SUBJECTIVE DPP 1.1

1. Use Euclid's division algorithm to find the HCF of :
(i) 56 and 814 (ii) 6265 and 76254

2. Find the HCF and LCM of following using Fundamental Theorem of Arithmetic method.
(i) 426 and 576 (ii) 625, 1125 and 2125

3. Prove that $\sqrt{3}$ is an irrational number. **[CBSE - 2008]**

4. Prove that $\sqrt{5}$ is irrational number. **[CBSE - 2008]**

5. Prove that $5+\sqrt{2}$ is irrational.

6. Prove that $\sqrt{2}+\sqrt{3}$ is irrational.

7. Can we have any $n \in N$, where 7^n ends with the digit zero.

8. Without actually performing the long division, state whether the following rational number will have a terminating decimal expansion or non - terminating decimal expansion :
(i) $\frac{77}{210}$ (ii) $\frac{15}{1600}$

9. An army contingent of 616 members is to march behind and army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

10. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point ?

11. Write a rational number between $\sqrt{2}$ and $\sqrt{3}$. **[CBSE - 2008]**

12. Use Euclid's' Division Lemma to show that the square of any positive integer is either of the form 3m of 3m + 1 for some integer m. **[CBSE - 2008]**

ANSWERS

(Sujective DPP 1.1)

1. **(i)** 2 **(ii)** 179 **2.** (i) 6,40896 (ii) 125, 95625
7. No **8.** (i) Non-terminating (ii) Terminating **9.** 8 columns
10. 36 minutes **11.** $\frac{3}{2}$

Teko CLASSES ..... the support

LINEAR EQUATIONS IN TWO VARIABLES

2.1 LINEAR EQUATIONS IN TWO VARIABLES :

An equation of the form Ax + By + C = 0 is called a linear equation.

Where A is called coefficient of x, B is called coefficient of y and C is the constant term (free form x & y)

A, B, C, $\in$ R [$\in \rightarrow$ belongs, to R $\rightarrow$ Real No.]

But A and B ca not be simultaneously zero.

If A $\neq$ 0, B = 0 equation will be of the form Ax + C = 0. [Line | | to Y-axis]

If A = 0, B $\neq$ 0, equation will be of the form By + C = 0. [Line | | to X-axis]

If A $\neq$ 0, B $\neq$ 0, C = 0 equation will be of the form Ax + By = 0. [Line passing through origin]

If A $\neq$ 0, B $\neq$ C, C $\neq$ 0 equation will be of the form A x + By + C = 0.

It is called a linear equation in two variable because the two unknown (x & y) occurs only in the first power, and the product of two unknown equalities does not occur.

Since it involves two variable therefore a single equation will have infinite set of solution i.e. indeterminate solution. So we require a pair of equation i.e. simultaneous equations.

Standard form of linear equation : (Standard form refers to all positive coefficient)

$a_1x + b_1y + c_1 = 0$ ....(i)

$a_2x + b_2y + c_2 = 0$ ....(ii)

For solving such equations we have three methods.

(i) Elimination by substitution (ii) Elimination by equating the coefficients

(iii) Elimination by cross multiplication.

2.1 Elimination By Substitution :

Ex.1 Solve x + 4y = 14 .....(i)

7x - 3y = 5 ....(ii)

Sol. From equation (i) x = 14 - 4y ....(iii)

Substitute the value of x in equation (ii)

$\Rightarrow$ 7 (14 - 4y) - 3y = 5 $\Rightarrow$ 98 - 28y - 3y = 5

$\Rightarrow$ 98 - 31y = 5 $\Rightarrow$ 93 = 31y $\Rightarrow$ $y = \dfrac{93}{31} \Rightarrow y = 3$

Now substitute value of y in equation (iii)

$\Rightarrow$ 7x - 3 (3) = 5 $\Rightarrow$ 7x - 3 (3) = 5

$\Rightarrow$ 7x = 14 $\Rightarrow$ $x = \dfrac{14}{7} = 2$ So, solution is x = 2 and y = 3

2.1 (b) Elimination by Equating the Coefficients :

Ex.2 Solve 9x - 4y = 8 .....(i)

13x + 7y = 101 ....(ii)

Sol. Multiply equation (i) by 7 and equation (ii) by 4, we get

Add 63x – 28y = 56

52x + 28y = 404

115x = 460 $\Rightarrow$ $x = \dfrac{460}{115} \Rightarrow x = 4$.

Substitute x = 4 in equation (i)

9 (4) - 4y = 8 $\Rightarrow$ 36 - 8 = 4y $\Rightarrow$ 28 = 4y $\Rightarrow$ $y = \dfrac{28}{4} = 7$

So, solution is x = 4 and y = 7.

2.1 (c) Elimination by Cross Multiplication :

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$ $\left[\because \dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}\right]$

$$\begin{array}{cccc} b_1 & c_1 & a_1 & b_1 \\ b_2 & c_2 & a_2 & b_2 \end{array}$$ [Write the coefficient in this manner]

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{a_2c_1 - a_1c_2} = \frac{1}{a_1b_2 - a_2b_1} \Rightarrow \therefore \frac{x}{b_1c_2 - b_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\Rightarrow \quad x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$$

Also, $\frac{y}{a_2c_1 - a_1c_2} = \frac{1}{a_1b_2 - a_2b_1}$ $\therefore$ $y = \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}$

Ex.3 Solve $3x + 2y + 25 = 0$ .....(i)
$x + y + 15 = 0$ ....(ii)

Sol. Here, $a_1 = 3$ $b_1 = 2$, $c_1 = 25$
$a_2 = 1$ $b_2 = 1$, $c_2 = 15$

$$\therefore \begin{array}{cccc} 2 & 25 & 3 & 2 \\ 1 & 15 & 1 & 1 \end{array}$$

$$\frac{x}{2 \times 15 - 25 \times 1} = \frac{y}{25 \times 1 - 15 \times 3} = \frac{1}{3 \times 1 - 2 \times 1}; \frac{x}{30 - 25} = \frac{y}{25 - 45} = \frac{1}{3 - 2}$$

$$\frac{x}{5} = \frac{y}{-20} = \frac{1}{1} \qquad .....(i)$$

$$\frac{x}{5} = 1, \frac{y}{-20} = \frac{1}{1}$$

X = 5, y = - 20 So, solution is x = 5 and y = - 20.

2.2 CONDITIONS FOR SOLVABILITY (OR CONSISTENCY) OF SYSTEM OF EQUATIONS:

2.2 (a) Unique Solution :

Two lines $a_1 + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, if the denominator $a_1b_2 - a_2b_1 \neq 0$ then the given system of equations have unique solution (i.e. only one solution) and solutions are said to be consistent.

$\therefore$ $a_1b_2 - a_2b_1 \neq 0$ $\Rightarrow$ $\frac{a_1}{b_2} \neq \frac{b_1}{b_2}$

2.2 (b) No Solution :

Two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, if the denominator $a_1b_2 - a_2b_1 = 0$ then the given system of equations have no solution and solutions are said to be consistent.

$\therefore$ $a_1b_2 - a_2b_1 \neq 0 \Rightarrow$ $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

2.2 (c) Many Solution (Infinite Solutions)

Two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = -$ then system of equations has many solution and solutions are said to be consistent.

Ex.4 Find the value of 'P' for which the given system of equations has only one solution (i.e. unique solution).
$Px - y = 2$ ....(i)
$6x - 2y = 3$ ....(ii)

Sol. $a_1 = P$, $b_1 = -1$, $c_1 = -2$
$a_2 = 6$ $b_2 = -2$, $c_2 = -3$

Conditions for unique solution is $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow$ $\frac{P}{6} \neq \frac{+1}{+2}$ $\Rightarrow$ $P \neq \frac{6}{2}$ $\Rightarrow$ $P \neq 3$ $\therefore$ P can have all real values except 3.

Ex.5 Find the value of k for which the system of linear equation
$kx + 4y = k - 4$
$16x + ky = k$ has infinite solution.

Sol. $a_1 = k$, $b_1 = 4$, $c_1 = -(k - 4)$
$a_2 = 16$, $b_2 = k$, $c_2 = -k$

Here condition is $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{k}{16} = \frac{4}{k} = \frac{(k-4)}{(k)}$ $\Rightarrow$ $\frac{k}{16} = \frac{4}{k}$ also $\frac{4}{k} = \frac{k-4}{k}$

$\Rightarrow$ $k^2 = 64$ $\Rightarrow$ $4k = k^2 - 4k$

$\Rightarrow$ $k = \pm 8$ $\Rightarrow$ $k(k-8) = 0$

k = 0 or k = 8 but k = 0 is not possible other wise equation will be one variable.

$\therefore$ k = 8 is correct value for infinite solution.

Ex.6 Determine the value of k so that the following linear equations has no solution.

$(3x + 1)\, x + 3y - 2 = 0$

$(k^2 + 1)\, x + (k - 2)\, y - 5 = 0$

Sol. Here $a_1 = 3k + 1$, $b_1 = 3$ and $c_1 = -2$

$a_2 = k^2 + 1$, $b_2 = k - 2$ and $c_2 = -5$ For no solution, condition is $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\frac{3k+1}{k^2+1} = \frac{3}{k-2} \neq \frac{-2}{-5}$ $\Rightarrow$ $\frac{3k+1}{k^2+1} = \frac{3}{k-2}$ and $\frac{3}{k-2} \neq \frac{2}{5}$

Now, $\frac{3k+1}{k^2+1} = \frac{3}{k-2}$

$\Rightarrow$ $(3k + 1)(k - 2) = 3(k^2 + 1)$ $\Rightarrow$ $3k^2 - 5k - 2 = 3k^2 + 3$

$\Rightarrow$ $-5k - 2 = 3$ $\Rightarrow$ $-5k = 5$

$\Rightarrow$ $k = -1$ Clearly, $\frac{3}{k-2} \neq \frac{2}{5}$ for $k = -1$.

Hence, the given system of equations will have no solution for k = - 1.

DAILY PRACTIVE PROVBLEMS # 2

OBJECTIVE DPP - 2.1

1. The equations $3x - 5y + 2 = 0$, and $6x + 4 = 10y$ have :

(A) No solution (B) A single solution
(C) Two solutions (D) An infinite number of solution

2. If p + q = 1 and the ordered pair (p, q) satisfy $3x + 2y = 1$ then is also satisfies :

(A) $3x + 4y = 5$ (B) $5x + 4y = 4$ (C) $5x + 5y = 4$ (D) None of these.

3. If x = y, 3x - y = 4 and x + y + x = 6 then the value of z is :

(A) 1 (B) 2 (C) 3 (D) 4

4. The system of linear equation ax + by = 0, cx + dy = 0 has no solution if :

(A) ad - bc > 0 (B) ad - bc < 0 (C) ad + bc = 0 (D) ad - bc = 0

5. The value of k for which the system kx + 3y = 7 and 2x - 5y = 3 has no solution is :

(A) $7 \;\&\; k = -\frac{3}{14}$ (B) $4 \;\&\; k = \frac{3}{14}$ (C) $\frac{6}{5} \;\&\; k \neq \frac{14}{3}$ (D) $-\frac{6}{5} \;\&\; k \neq \frac{14}{3}$

6. If 29x + 37y = 103, 37x + 29y = 95 then :

(A) x = 1, y = 2 (B) x = 2, y = 1 (C) x = 2, y = 3 (D) x = 3, y = 2

7. On solving $\frac{25}{x+y} - \frac{3}{x-y} = 1, \frac{40}{x+y} + \frac{2}{x-y} = 5$ we get :

(A) x = 8, y = 6 (B) x = 4, y = 6 (C) x = 6, y = 4 (D) None of these

8. If the system 2x + 3y - 5 = 0, 4x + ky - 10 = 0 has an infinite number of solutions then :

(A) $k = \frac{3}{2}$ (B) $k \neq \frac{3}{2}$ (C) $k \neq 6$ (D) $k = 6$

9. The equation x + 2y = 4 and 2x + y = 5

(A) Are consistent and have a unique solution (B) Are consistent and have infinitely many solution
(C) are inconsistent (B) Are homogeneous linear equations

10. If $\frac{1}{x} - \frac{1}{y} = \frac{1}{z}$ then z will be :

(A) y - x (B) x - y (C) $\frac{y-x}{xy}$ (D) $\frac{xy}{y-x}$

SUBJECTIVE DPP 2.2

Solve each of the following pair of simultaneous equations.

1. $\frac{x}{3} + \frac{y}{12} = \frac{7}{2}$ and $\frac{x}{6} - \frac{y}{8} = \frac{6}{8}$ **2.** 0.2 x + 0.3y = 0.11 = 0, 0.7x - 0.5y + 0.08 = 0

3. $3\sqrt{2}x - 5\sqrt{3}y + \sqrt{5} = 0$; $2\sqrt{3}x + 7\sqrt{2}y - 2\sqrt{5} = 0$

4. $\frac{x}{3}+y=1.7$ and $\frac{11}{x+\frac{y}{3}}=10\forall\left[x+\frac{y}{3}\neq 0\right]$

5. Prove that the positive square root of the reciprocal of the solutions of the equations $\frac{3}{x}+\frac{5}{y}=29$ and $\frac{7}{x}-\frac{4}{y}=5(x\neq 0,\ y\neq 0)$ satisfy both the equation $2(\sqrt{3}x+4)-3(4y-5)=5\ \&\ 7\left(\frac{9x}{\sqrt{3}}+8\right)+5(7y-25)=64$.

6. For what value of a and b, the following system of equations have an infinite no. of solutions. 2x + 3y = 7; (a-b) x + (a +b) + b - 2

7. Solve : (i) $\frac{7}{x^3}-\frac{6}{2^y}=15;\frac{8}{3^x}=\frac{9}{2^y}$ (ii) 119x - 381y = 643; 381x - 119y = -143

8. Solve: $\frac{bx}{a}-\frac{ay}{b}$ + a + b = 0; bx - ay + 2ab = 0

9. Solve : $\frac{1}{3x}+\frac{1}{5y}=1;\ \frac{1}{5x}+\frac{1}{3y}=1\frac{2}{15}$

10. Solve x - y + z = 6
x - 22y - 2z = 5
2x + y - 3z = 1

11. Solve, px + qy = r and qx = 1 + r

12. Find the value of k for which the given system of equations
(A) has a Unique solution. (B) becomes consistent.
(i) 3x + 5y = 12 (ii)3x - 7y = 6
4x - 7y = k 21x - 49y = l - 1

13. Find the value of k for which the following system of linear equation becomes infinitely many solution. or represent the coincident lines.
(i) 6x + 3y = k - 3 (ii) x + 2y + 7 = 0
2k x + 6y = 6 2x + ky + 14 = 0

14. Find the value of k or C for which the following systems of equations be in consistent or no solution.
(i) 2 x ky + k + 2 = 0 (ii) Cx + 3y = 3
kx + 8y + 3k = 0 12x + Cy = 6

15. Solve for x and y : (a - b) x + (a + b) y = a^2 - 2ab - b^2
(a + b) (x + y) = a^2 + b^2 **[CBSE - 2008]**

16. Solve for x and y :
37x + 43y = 123
43x + 37y = 117 **[CBSE - 2008]**

Teko CLASSES ..... the support

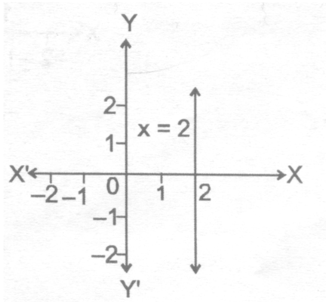
LINEAR EQUATIONS IN TWO VARIABLES

3.1 GRAPHICAL SOLUTION OF LINEAR EQUATIONS IN TWO VARIABLES :
Graphs of the type (i) ax = b

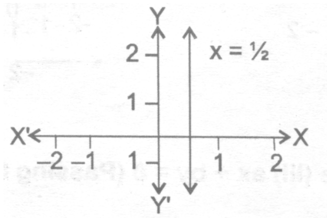
Ex.1 Draw the graph of following : (i) x = 2, (ii) 2x = 1 (iii) x + 4 = 0 (iv) x = 0

Sol.

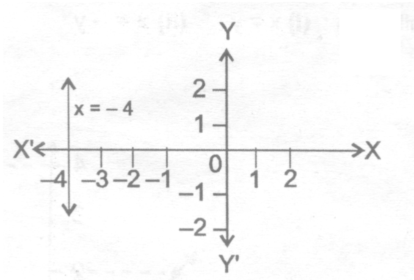
(i) x = 2



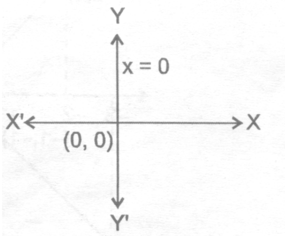
(ii) $2x = 1 \Rightarrow x = \frac{1}{2}$



(iii) $x + 4 = 0 \Rightarrow x = -4$



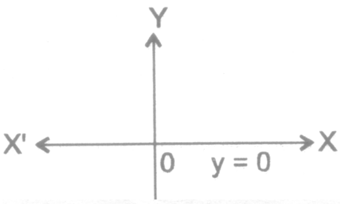
(iv) $x = 0$



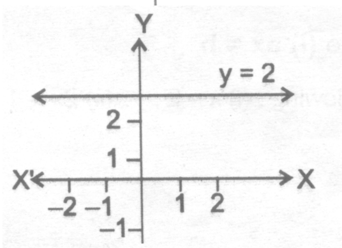
Graphs of the type (ii) ay = b.

Ex.2 Draw the graph of following : (i) $y = 0$, (ii) $y - 2 = 0$, (iii) $2y + 4 = 0$

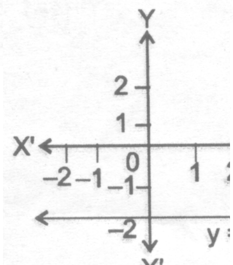
(i) $y = 0$



(ii) $y - 2 = 0$



(iii) $2y + 4 = 0 \Rightarrow y = -2$



Graphs of the type (iii) ax + by = 0 (Passing through origin)

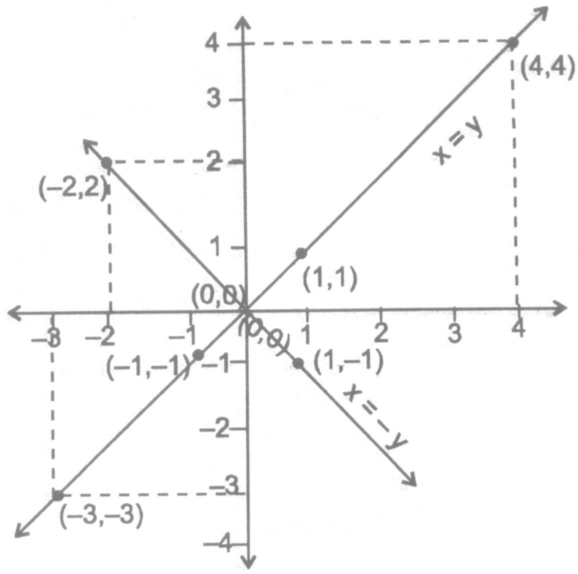
Ex.3 Draw the graph of following : (i) $x = y$ (ii) $x = -y$

Sol. (i) $x - y$

x	1	4	-3	0
y	1	4	-3	0

(ii) $x = -y$

x	1	-2	0
y	-1	2	0



Graphs of the Type (iv) ax + by + c = 0. (Making Interception x - axis, y-axis)

Ex.4 Solve the following system of linear equations graphically : x - y = 1, 2x + y = 8. Shade the area bounded by these two lines and y-axis. Also, determine this area.

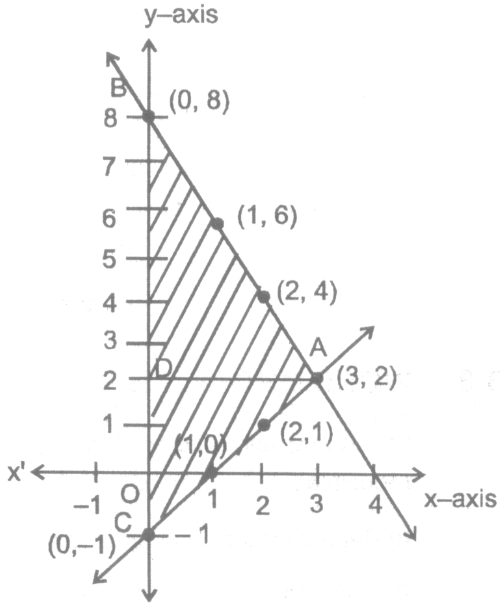
Sol. (i) $x - y = 1$
$x - y + 1$

x	0	1	2
y	-1	0	1

(ii) $2x + y = 8$
$y = 8 - 2x$

X	0	1	2
Y	8	6	4

(ii) $2x + y = 8$

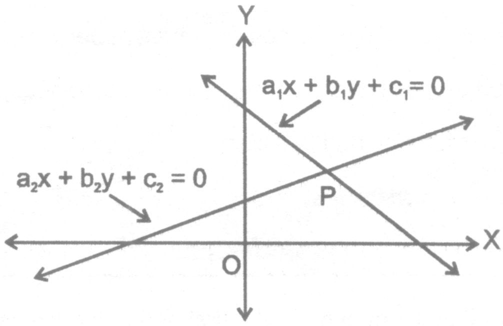


Solution is x = 3 and y = 2
Area of is x = 3 and y = 2
Area of $\Delta ABC = \frac{1}{2} \times BC \times AD$
$= \frac{1}{2} \times 9 \times 3 = 13.5$ Sq. unit.

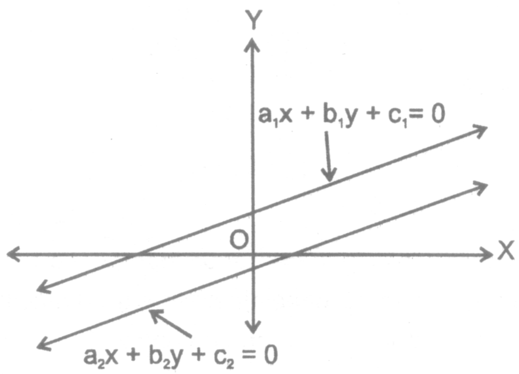
3.2 NATURE OF GRAPHICAL SOLUTION :

Let equations of two lines are $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$.

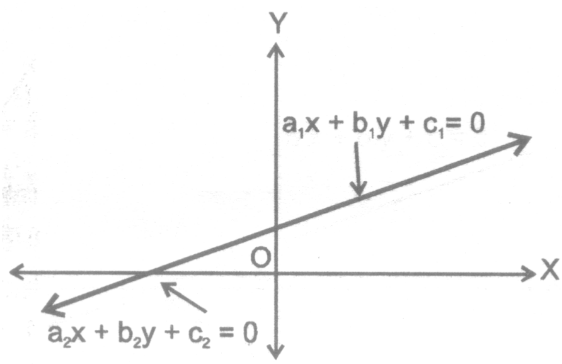
(i) Lines are consistent (**unique solution)** i.e. they meet at one point condition is $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$



(ii) Lines are inconsistent **(no solution)** i.e. they do not meet at one point condition is $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$



(iii) Lines are coincident **(infinite solution)** i.e. overlapping lines (or they are on one another) condition is
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$



3.3 WORD PROBLEMS:

For solving daily - life problems with the help of simultaneous linear equation in two variables or equations reducible to them proceed as :-

(i) Represent the unknown quantities by same variable x and y, which are to be determined.

(ii) Find the conditions given in the problem and translate the verbal conditions into a pair of simultaneous linear equation.

(iii) Solve these equations & obtain the required quantities with appropriate units.

Type of Problems :

(i) Determining two numbers when the relation between them is given,

(ii) Problems regarding fractions, digits of a number ages of persons.

(iii) Problems regarding current of a river, regarding time & distance.

(iv) Problems regarding menstruation and geometry.

(v) Problems regarding time & work

(vi) Problems regarding mixtures, cots of articles, porting & loss, discount et.

Ex.5 Find two numbers such that the sum of twice the first and thrice the second is 89 and four times the first exceeds five times the second by 13.

Sol. Let the two numbers be x and y.

Then, equation formed are $2x + 3y = 89$ ....(i)

$4x - 5y = 13$ ...(ii)

On solving eq. (i) & (ii) we get

$x = 22$

$y = 15$

Hence required numbers are 22 & 15.

Ex.6 The numerator of a fraction is 4 less than the denominator If the numerator is decreased and the denominator is increased by 1, then the denominator is eight time the numerator, find the reaction.

Sol. Let the numerator and denominator of a fraction be x and y

Then, equation formed are $y - x = 4$ ....(i)

$y + 1 = 8 (x - 2)$ ....(ii)

On solving eq. (i) & (ii) we get

$x = 3$

and $y = 7$ Hence, fractions is $\frac{3}{7}$.

Ex.7 A number consists of two digits, the sum of the digits being 12. If 18 is subtracted from the number, the digits are reversed. Find the number

Sol. Let the two digits number be $1y + x$

Then, equations formed are

$10y + x - 18 = 10x + y \Rightarrow y - x = 2$ ......(i)

and $x + y = 12$ ......(ii)

On solving eq. (i) & (ii) we get

$x = 5$

and $y = 7$ Hence number is 75.

Ex.8 The sum of a two - digit number and the number obtained by reversing the order of its digits is 165. If the digits differ by 3, find the number

Sol. Let unit digit be x ten's digit be y no. will be $10y + x$.

Acc. to problem $(10y + x) + (10x + y) = 165$

$\Rightarrow x + y = 15$ ...(i)

and $x - y = 3$ ...(ii)

or $-(x - y) = 3$ ....(iii)

On solving eq. (i) and (ii)

we gets $= 9$ and $y = 6$ $\therefore$ The number will be 69. **Ans.**

On solving eq. (i) and (iii)

we gets $x = 6$ and $y = 9$ $\therefore$ The number will be 96. **Ans.**

Ex.9 Six years hence a men's age will be three times the age of his son and three years ago he was nine times as old as his son. Find their present ages

Sol. Let man's present age be x yrs & son's present age be 'y' yrs.

According to problem $x + 6 = 3 (y + 6)$ [After 6 yrs]

and $x - 3 = 9 (y - 3)$ [Before 3 yrs.]

On solving equation (i) & (ii) we gets $x = 30$, $y = 6$.

So, the present age of man = 30 years, present age of son = 6 years.

Ex.10 A boat goes 12 km upstream and 40 km downstream in 8 hrs. It can go 16 km. upstream and 32 km downstream in the same time. Find the speed of the boat it still water and the speed of the stream.

Sol. Let the speed of the boat in still water be x km/hr and the speed of the stream be y km/hr then speed of boat in downstream is (x + y) km/hr and the speed of boat upstream is (x - y) km/hr.

Time taken to cover 12 km upstream $= \frac{12}{x-y}$ hrs.

Time taken to cover 40 km downstream $= \frac{40}{x+y}$ hrs.

But, total time taken 8 hr

$\therefore \quad \frac{12}{x-y} + \frac{40}{x+y} = 8$ .....(i)

Time taken to cover 16 km upstream $= \frac{16}{x-y}$ hrs.

Time taken to cover 32 km downstream $= \frac{32}{x+y}$ hrs.

Total time taken = 8 hr

$\therefore \quad \frac{16}{x-y} + \frac{32}{x+y} = 8$ .....(ii)

Solving equation (i) & (ii) we gets x = 6 and y = 2.

Hence, speed of boat in still water = 6 km/hr and speed of stream = 2 km/hr.

Ex.11 Ramesh travels 760 km to his home partly by train and partly by car. He taken 8 hr, if he travels 160 km by train and the rest by car. He takes 12 minutes more, if he travels 240 km by train and the rest by car. Find the speed of train and the car.

Sol. Let the speed of train be x km/hr & car be y km/hr respectively.

Acc. to problem $\frac{160}{x} + \frac{600}{y} = 8$ ....(i)

$\frac{240}{x} + \frac{520}{y} = \frac{41}{5}$ ....(ii)

Solving equation (i) & (ii) we gets x = 80 and y = 100.

Hence , speed ot train = 80 km/hr and speed of car = 100km/hr.

Ex.12 Points A and B are 90 km apart from each other on a highway. A car starts from A and another from B at the same time. If they go in the same direction, they meet in 9 hrs and if they go in opposite direction, they meet in $\frac{9}{7}$ hrs. Find their speeds.

Sol. Let the speeds of the cars starting from A and B be x km/hr and y km/hr respectively.

Acc to problem $9x - 90 = 9y$ ....(i)

& $\frac{9}{7}x + \frac{9}{7}y = 90$ .....(ii) Solving (i) & (ii) we gets x = 40 & y = 30.

Hence, speed of car starting from point A = 40 km/hr & speed of car starting from point B = 30 km/hr.

Ex.13 In a cyclic quadrilateral ABCD, $\angle A = (2x + 11)^0$, $\angle B = (y + 12)^0$, $\angle C = (3y + 6)^0$ and $\angle D = (5x - 25)^0$, find the angles of the quadrilateral.

Sol. Acc. to problem $(2x + 11)^0 + (3y + 6)^0 = 180^0$

$(y + 12)^0 + (5x - 25)^0 = 180^0$

Solving we get $x = \frac{416}{13}$ & $y = \frac{429}{13}$ $\Rightarrow$ x = 32 and y = 33

$\therefore \quad \angle A = 75^0, \angle B = 45^0, \angle C = 105^0, \angle D = 135^0$

Ex.15 A vessel contains mixture of 24ℓ milk and 6ℓ water and a second vessel contains a mixture of 15ℓ milk & 10ℓ water. How much mixture of milk and water should be taken from the first and the second vessel separately and kept in a third vessel so that the third vessel may contain a mixture of 25ℓ milk and 10ℓ water ?

Sol. Let x ℓ of mixture be taken from Ist vessel & y ℓ of the mixture be taken from 2nd vessel and kept in 3rd vessel so that (x + y) ℓ of the mixture in third vessel may contain 25 ℓ of milk & 10ℓ of water.

A mixture of x ℓ from 1st vessel contains $\frac{24}{30}x = \frac{4}{5}x\ell$ of milk & $\frac{x}{5}\ell$ of water and a mixture of $y\ell$ from 2nd vessel contains $\frac{3y}{5}\ell$ of milk & $\frac{2y}{5}\ell$ of water.

$\therefore \quad \frac{4}{5}x+\frac{3}{5}y=25$ ....(i) $\quad \frac{x}{5}+\frac{2}{5}y=10$ ....(ii)

Solving (i) & (ii) x = 20 litres and y = 15 litres.

Ex.15 A lady has 25 p and 50 p coins in her purse. If in all she has 40 coins totaling Rs. 12.50, find the number of coins of each type she has.

Sol. Let the lady has x coins of 25 p and y coins of 50 p.

Then acc. to problem $x + y = 40$ .....(i)

and $25x + 50y = 1250$ ....(ii

Solving for x & y we get x = 30 (25 p coins) & y = 10 (50 P coins).

Ex.16 Students of a class are made to stand in rows. If one student is extra in a row, there would be 2 rows less. If one students is less in row, there would be 3 rows more. Find the total number of students in the class.

Sol. Let x be the original no. of rows & y be the original no. of student s in each row.

$\therefore$ Total no. of students = xy.

Acc. to problem

$(y + 1)(x - 2) = xy$ ....(i)

and $(y - 1)(x + 3) = xy$ ....(ii) Solving (i) & (ii) to get

$x = 12$ & $y = 5$ $\therefore$ Total no. of students = 60

Ex.17 A man started his job with a certain monthly salary and earned a fixed increment every year. If his salary was Rs. 4500 after 5 years. of service and Rs. 5550 after 12 years of service, what was his starting salary and what his annual increment.

Sol. Let his initial monthly salary be Rs x and annual increment be Rs y.

Then, Acc. to problem $x + 5y = 4500$ ......(i)

$x + 12y = 5550$ ....(ii)

Solving these two equations, we get x= Rs. 3750 y = Rs 150.

Ex.18 A dealer sold A VCR and a TV for Rs. 38560 making a profit of 12% on CVR and 15% on TV. By selling them for Rs. 38620, he would have realised a profit of 15% on CVR and 12% on TV. Find the cost price of each.

Sol. Let C.P. of CVR be Rs x & C.P. of T.V. be Rs y.

Acc. to problem $\frac{112}{100}x+\frac{115}{100}y=38560$ .....(i)

and $\frac{115}{100}x+\frac{112}{100}y=38620$ ....(ii)

Solving for x & y we get x = Rs. 18000 & y = Rs. 16000.

DAILY PRACTIVE PROBLEMS # 3

OBJECTIVE DPP 3.1

1. The graphs of $2x + 3y - 6 = 0$, $4x - 3y - 6 = 0$, $x = 2$ and $y = \frac{2}{3}$ intersects in :

(A) Four points (B) one point (C) two point (D) infinite number of points

2. The sum of two numbers is 20, their product is 40. The sum of their reciprocal is :

(A) $\frac{1}{2}$ (B) 2 (C) 4 (D) $\frac{1}{10}$

3. If Rs. 50 is distributed among 150 children giving 50 p to each boy and 25 p to each girl. Then the number of boys is :

(A) 25 (B) 40 (C) 36 (D) 50

4. In covering a distance of 30 km. Amit takes 2 hrs. more than suresh. If Amit doubles his speed, he would take one hour less than suresh. Amits' speed is :

(A) 5 km/hr. (B) 7.5 km/hr. (C) 6 km/hr. (D) 6.2 km/hr.

5. If in a fraction 1 less from two times of numerator & 1 add in denominator then new fraction will be :

(A) $2\left(\frac{x-1}{y+1}\right)$ (B) $\frac{2(x+1)}{y+1}$ (C) $\left(\frac{x}{y}\right)$ (D) $\frac{2x-1}{y+1}$

SUBJECTIVE DPP 3.2

1. The denominator of a fraction is greater than its numerator by 7. If 4 is added to both its numerator and denominator, then it becomes $\frac{1}{2}$. Find the fraction.

2. In a certain number is divided by the sum of its two digits, the quotient is 6 and remainder is 3. If the digits are interchanged and the resulting number is divided by the sum of the digits, then the quotient is 4 and the remainder is 9. Find the number.

3. 2 men and 3 boys together can do a piece of work is 8 days. The same work si done in 6 days by 3 men and 2 boys together. How long would 1 boy alone or 1 man alone take to complete the work

4. The um of two no s is 18. the sum of their reciprocal is $\frac{1}{4}$. Find the numbers.

5. In a cyclic quadrilateral ABCD, $\angle A = (2x + 4)^0$, $\angle B = (y + 3)^0$, $\angle C = (2y + 10)^0$ and $\angle D = (4x - 5)^0$ then find out the angles of quadrilateral.

6. Solve graphically and find the pints where the given liens meets the y - axis : $2x + y - 11= 0$, $x - y - 1 = 0$.

7. Use single graph paper & draw the graph of the following equations. Obtain the vertices of the triangles so obtained : $2y - x = 8$, $5y - x = 14$ & $y - 2x = 1$.

8. Draw the graph of $x - y + 1 = 10$; $3x + 2y - 12 = 0$. Calculate, the area bounded by these lines and x - axis.

9. A man sold a chair and a table together for Rs. 1520 thereby making a profit of 25% on chair and 10% on table. By selling them together for Rs. 1535 he would have made a profit of 10% on the chair and 25% on the table. Find cost price of each.

10. A man went to the Reserve Bank of India with a note or Rs. 500. He asked the cashier to give him Rs. 5 and Rs. 10 notes in return. The cashier gave him 70 notes in all. Find how many notes of Rs. 5 and Rs. 10 did the man receive.

11. Solve graphically: $5x - 6y + 30 = 0$; $5x + 4y - 20 = 0$ Also find the vertices of the triangle formed by the above two lines and x -axis.

12. The sum of the digits of a two-digit number is 12. "The number obtained by interchanging the two digits exceeds the given number by 18. Find the number.

13. Draw the graphs of the following equations and solve graphically:
$3x + 2y + 6 = 0$; $3x + 8y - 12 = 0$
Also determine the co-ordinates of the vertices of the triangle formed by these lines and the x - axis.

14. A farmer wishes to purchase a number of sheep found the if they cost him Rs 42 a head, he would not have money enough by Rs 25; But if they cost him Rs 40 a head, he would them have Rs 40 more than he required; find the number of sheeps, and the money which he had.

ANSWERS

(Objective DPP 2.1)

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	D	A	B	D	D	A	C	D	A	D

(Subjective DPP 2.2)

1. $x = 9, y = 6$ **2.** $x = 0.1, y = 0.3$

3. $x = \frac{10\sqrt{5} - 7\sqrt{10}}{72}$ $y = \frac{2\sqrt{15} + 6\sqrt{10}}{72}$ **4.** $x = 0.6, y = 1.5$

6. $a = 5, b = 1$ **7.** (i) $x = -2, y = -3$ (ii) $x = -1, y = -2$

8. $x = -a, y = b$ **9.** $x = \frac{2}{3}, y = \frac{2}{5}$ **10.** $x = 3, y = -2, x = 1$

11. $x = \frac{q + r(p + q)}{p^2 + q^2}, y = \frac{r(q - p) - p}{p^2 + q^2}$ **12.** (a) k is any real number (b) $k = 41$

13. (a) $k = 6$ (b) $k = 4$ **14.** (a) $k = -4$ (b) $C = -6$

15. $x = a + b, y = -\frac{2ab}{a + b}$ **16.** $x = 1, y = 2$

(Objective DPP 3.1)

Que.	1	2	3	4	5
Ans.	B	A	D	A	D

(Subjective DPP 3.2)

1. 3/10 **2.** 75

3. One boy can do in 120 days and one man can do in 20 days.

4. No. 's are 12 and 6 **5.** $A = 70^0$, $B = 53^0$, $C = 110^0$, $D = 127^0$

6. $x = 4, y = 3$, Point of contact with x - axis (0, 11), (0, -1)

7. (-4, 2), (1, 3), (2,5) **8.** 37.5 Square units.

9. Chair = Rs. 600, Tables = Rs. 700 **10.** 5 rupees notes = 40 & 10 rupees notes = 30

11. (0,5) vertices (0,5) (-6,0), (4, 0) **12.** 57

13. $x = -4, y = 3$, Lines meets x-axis at (-2, 0) & (4, 0) **14.** 34 sheep, Rs 1400

POLYNOMIALS

4.1 POLYNOMIALS :

An algebraic expression f(x) of the form (fx) = $a_0 + a_1x + a_2x^2 + ...+ a_nx^n$, where $a_0, a_1, a_2.......a_n$ are real numbers and all the index of **x** are non-negative integers is called polynomials in **x** and the highest Index **n** in called the degree of the polynomial, if $a_n \neq 0$.

4.1 (a) Zero Degree Polynomial :

Any non-zero number is regarded as a polynomial of degree zero or zero degree polynomial. For example, **f(x) = a,** where $a \neq 0$ is a zero degree polynomial, since we can write **f(x) = a** as **f(x) = ax^0.**

4.1 (b) Constant Polynomial :

A polynomial of degree zero is called a constant polynomial. **For example,** f(x) = 7.

4.1 (c) Linear Polynomial :

A polynomial of **degree 1** is called a linear polynomial.

For example : p(x) = 4x - 3 and f(t) = $\sqrt{3}t + 5$ are linear polynomials.

4.1 (d) Quadratic Polynomial :

A polynomial of **degree 2** is called quadratic polynomial.

For example : f(x) = $2x^2 + 5x - \frac{3}{5}$ and g(y) = $3y^2 - 5$ are quadratic polynomials with real coefficients.

IMPORTANT FORMULAE :

$(x + a)^2 = x^2 + 2ax + a^2$

$(x - a)^2 = x^2 - 2ax + a^2$

$x^2 - a^2 = (x + a)(x - a)$

$x^3 + a^3 = (x + a)(x^2 - ax + a^2) = (x + a)^3 - 3xa(x + a)$

$x^3 - a^3 = (x - a)(x^2 + ax + a^2) = (x - a)^3 + 3xa(x - a)$

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

Special Case : If a + b + c = 0 then $a^3 + b^3 + c^3 = 3abc$.

4.2 GRAPH OF POLYNOMIALS :

In algebraic or in set theoretic language the graph of a polynomial f(x) is the collection (or set) of all points **(x, y),** where **y = f(x).** In geometrical or in graphical language the graph of a polynomial f(x) is a smooth free hand curve passing through points **x_1, y_1), (x_2, y_2), (x_3, y_3), ... etc.** where **y_1, y_2, y_3,....** are the values of the polynomial **f(x)** at **x_1, x_2, x_3,....** respectively.

In order to draw the graph of a polynomial **f(x),** follow the following algorithm.

ALGORITHM :

Step (i) Find the values **y_1, y_2, ... y_n** of polynomial f(x) on different points **x_1, x_2, ... x_n……….** and prepare a table that gives values of **y** or **f(x)** for various values of **x.**

x:	x_1	x_2	......	x_n	x_{n+1}	....
y = f(x)	$y_1=f(x_1)$	$y_2=f(x_2)$	....	$Y_n=f(x_n)$	$y_{n+1} = f(x_{n+1})$	.....

Step (ii) Plot that points **(x_1, y_1), (x_2, y_2), (x_3, y_3),.....(x_n, y_n)....** on rectangular co-ordinate system. In plotting these points use different scales on the X and Y axes.

Step (iii) Draw a free hand smooth curve passing through points plotted in **step 2** to get the graph of the polynomial **f(x).**

4.2 (a) Graph of a Linear Polynomial :

Consider a linear polynomial **f(x) = ax + b,** $a \neq 0$ Graph of **y = ax + b** is a straight line. That in why f(x) = ax + b) is called a linear polynomial. Since two points determine a straight line, so only two points need to plotted to draw the line **y = ax + b.** The line represented by **y = ax + b** crosses the X-axis at exactly one point, namely $\left(-\frac{b}{a}, 0\right)$.

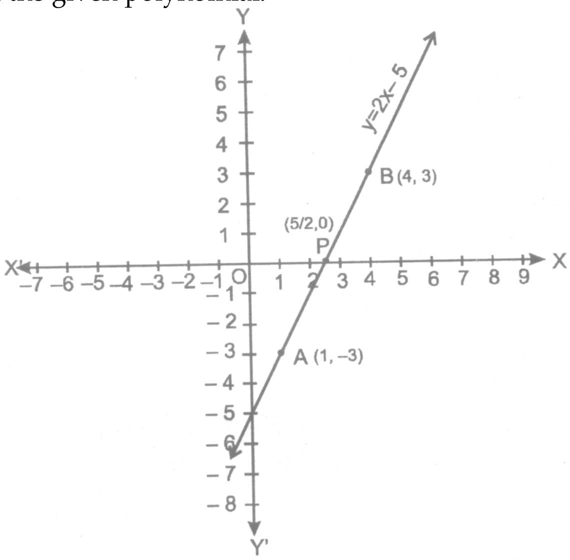
Ex.1 Draw the graph of the polynomial f(x) = 2x - 5. Also, find the coordinates of the point where it crosses X-axis.

Sol. Let y = 2x - 5.

The following table list the values of **y** corresponding to different values of **x.**

x	1	4
y	-3	3

The points A (1, - 3) and B (4, 3) are plotted on the graph paper on a suitable scale. A line is drawn passing through these points to obtain the graphs of the given polynomial.



4.2 (b) Graph of a Quadratic Polynomial :

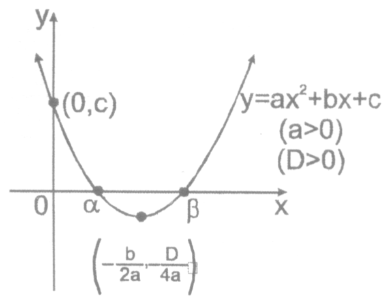
Let a,b,c be real numbers and $a \neq 0$. Then f(x) = ax^2 + bx + c is known as a quadratic polynomial in x. Graph of the quadratic polynomial i.e. he curve whose equation is y = ax^2 + bx + c, $a \neq 0$ Graph of a quadratic polynomial is always a parabola.

Let y = ax^2 + bx + c, where $a \neq 0$

$\Rightarrow$ $4ay = 4a^2x^2 + 4abx + 4ac$

$\Rightarrow$ $4ay = 4a^2x^2 + 4abx + b^2 - b^2 + 4ac$

$\Rightarrow$ $4ay = (2ax + b)^2 - (b^2 - 4ac)$



$\Rightarrow$ $4ay + (b^2 - 4ac) = (2ax + b)^2 \Rightarrow 4ay + (b^2 - 4ac) = 4a^2(x + b/2a)^2$

$\Rightarrow$ $4a\left\{y + \frac{b^2 - 4ac}{4a}\right\} = 4a^2\left(x + \frac{b}{2a}\right)^2$

$\Rightarrow$ $\left(y + \frac{D}{4a}\right) = a\left(a + \frac{b}{2a}\right)^2$ ...(i)

where **D = b^2 - 4ac** is the discriminate of the quadratic equation.

REMARKS :

Shifting the origin at $\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$, we have X = x - $\left(-\frac{b}{2a}\right)$ and Y = y - $\frac{(-D)}{4a}$

Substituting these values in (i), we obtain

Y = aX^2 **....(ii)**

which is the standard equation of parabola

Clearly, this is the equation of a parabola having its vertex at $\left(-\frac{b}{2a}, \frac{D}{4a}\right)$.

The parabola opens upwards or downwards according as a > 0 or a < 0.

4.3 SIGN OF QUADRTIV EXPRESSIONS :

Let α be a real root of **ax^2 + bx + c = 0.** Then $a\alpha^2 + b\alpha + c = 0$ Point $(\alpha, 0)$ lies on y = ax^2 + bx + c. Thus, every real root of ax^2 + bx + c= 0 represents a point of intersecting of the parabola with the X-axis.

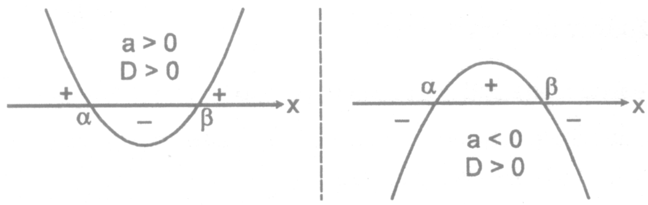
Conversely, if the parabola y = $ax^2 + bx + c$ intersects the X-axis at a point $(\alpha,0)$ then $(\alpha,0)$ satisfies the equation y = $ax^2 + bx + c$

$\Rightarrow$ $a\alpha^2 + b\alpha + c = 0$ [α is a real root of $ax^2 + bx + c = 0$]

Thus, the intersection of the parabola y = $ax^2 + bx + c$ with X-axis gives all the real roots of $ax^2 + bx + c = 0$. Following conclusions may be drawn :-

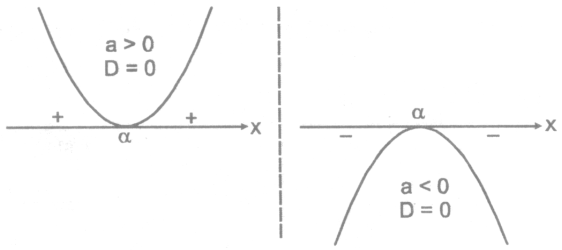
(i) If D>0, the parabola will intersect the x-axis in two distinct points and vice-versa.

The parabola meets x-axis at $\alpha = \dfrac{-b-\sqrt{D}}{2a}$ and $\beta = \dfrac{-b+\sqrt{D}}{2a}$



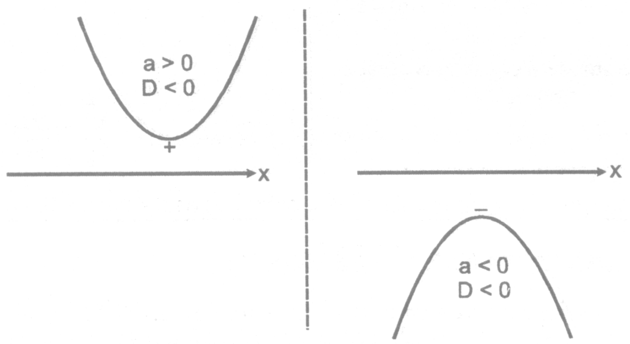
Roots are real & distinct

(ii) If D = 0, the parabola will just touch the x-axis at one point and vice-versa.



Roots are equal

(iii) If D<0, the parabola will not intersect x-axis at all and vice-versa.



Roots are imaginary

REMARKS

★ $\forall x \in R, y > 0$ only if $a > 0$ & $D \equiv b^2 - 4ac < 0$

★ $\forall x \in R, y < 0$ only if $a < 0$ & $D \equiv b^2 - 4ac < 0$

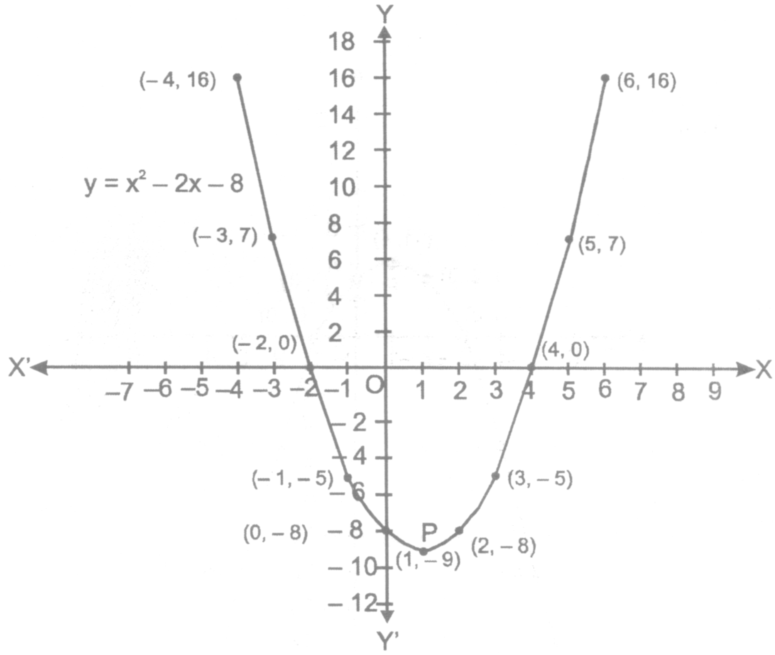
Ex.2 Draw the graph of the polynomial f(x) = $x^2 - 2x - 8$

Sol. Let y = $x^2 - 2x - 8$.

The following table gives the values of y or f(x) for various values of x.

x	-4	-3	-2	-1	0	1	2	3	4	5	6
$y = x^2 - 2x - 8$	16	7	0	-5	-8	-9	-8	-5	0	7	16

Let us plot the points (-4, 16), (-3, 7), (-2, 0), (-1, -5), (0, - 8), (1, - 9), (2, - 8), (3, - 5), (4, 0), (5, 7) and (6, 16) on a graphs paper and draw a smooth free hand curve passing through these points. The curve thus obtained represents the graphs of the polynomial f(x) = $x^2 - 2x - 8$. This is called a parabola. The lowest point P, called a minimum points, is the vertex of the parabola. Vertical line passing through P is called the axis of the parabola. Parabola is symmetric about the axis. So, it is also called the line of symmetry.



Observations :

Fro the graphs of the polynomial $f(x) = x^2 - 2x - 8$, following observations can be drawn :

(i) The coefficient of x^2 in $f(x) = x^2 - 2x - 8$ is 1 (a positive real number) and so the parabola opens upwards.(ii) $D = b^2 - 4ac = 4 + 32 = 36 > 0$. So, the parabola cuts X-axis at two distinct points.

(iii) On comparing the polynomial $x^2 - 2x - 8$ with $ax^2 + bx + c$, we get a = 1, b = - 2 and c = - 8.

The vertex of the parabola has coordinates (1, -9) i.e. $\left(\frac{-b}{2a}, \frac{-D}{4a}\right)$, where $D \equiv b^2 - 4ac$.

(iv) The polynomial $f(x) = x^2 - 2x - 8 = (x - 4)(x + 2)$ is factorizable into two distinct linear factors (x - 4) and (x + 2). So, the parabola cuts X-axis at two distinct points (4, 0) and (-2, 0). the x-coordinates of these points are zeros of f(x).

Ex.3 Draw the graphs of the quadratic polynomial $f(x) = 3 - 2x - x^2$.

Sol. Let y = f(x) or, $y = 3 - 2x - x^2$.

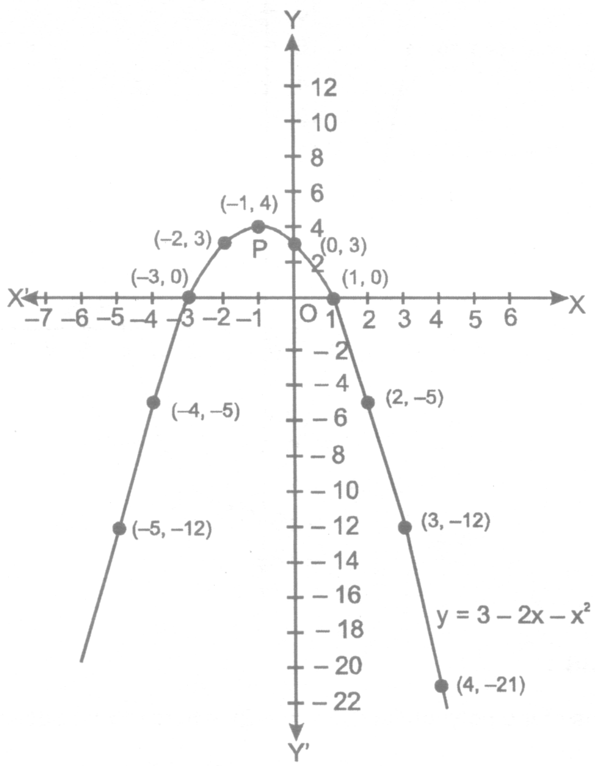
Let us list a few values of $y = 3 - 2x - x^2$ corresponding to a few values of x as follows :

x	-5	-4	-3	-2	-1	0	1	2	3	4
$y = 3 - 2x - x^2$	-12	-5	0	3	4	3	0	-5	-12	-21

Thus, the following points lie on the graph of the polynomial $y = 2 - 2x - x^2$:

(-5, -12), (-4, -5), (-3, 0), (-2, 4), (-1, 4), (0, 3), (1, 0), (2, - 5), (3, -12) and (4, - 21).

Let plot these points on a graph paper and draw a smooth free hand curve passing through these points to obtain the graphs of $y = 3 - 2x - x^2$. The curve thus obtained represents a parabola, as shown in figure. The highest point P(-1, 4), is called a maximum points, is the vertex of the parabola. Vertical line through P is the axis of the parabola. Clearly, parabola is symmetric about the axis.



Observations : Following observations from the graph of the polynomial $f(x) = 3 - 2x - x^2$ is as follows :

(i) The coefficient of x^2 in $f(x) = 3 - 2x - x^2$ is - 1 i.e. a negative real number and so the parabola opens downwards.

(ii) $D \equiv b^2 - 4ax = 4 + 12 = 16 > 0$. So, the parabola cuts x-axis two distinct points.

(iii) On comparing the polynomial $3 - 2x - x^2$ with $ax^2 + bc + c$, we have a = - 1, b = - 2 and c = 3. The vertex of the parabola is at the point (-1, 4) i.e. at $\left(\frac{-b}{2a}, \frac{-D}{4a}\right)$, where $D = b^2 - 4ac$.

(iv) The polynomial $f(x) = 3 - 2x - x^2 = (1 - x)(x + 3)$ is factorizable into two distinct linear factors (1 - x) and (x + 3). So, the parabola cuts X-axis at two distinct points (1, 0) and (-3, 0). The co-ordinates of these points are zeros of f(x).

4.4 GRAPH OF A CUBIC POLYNOMIAL :

Graphs of a cubic polynomial does not have a fixed standard shape. Cubic polynomial graphs will always cross X-axis at least once and at most thrice.

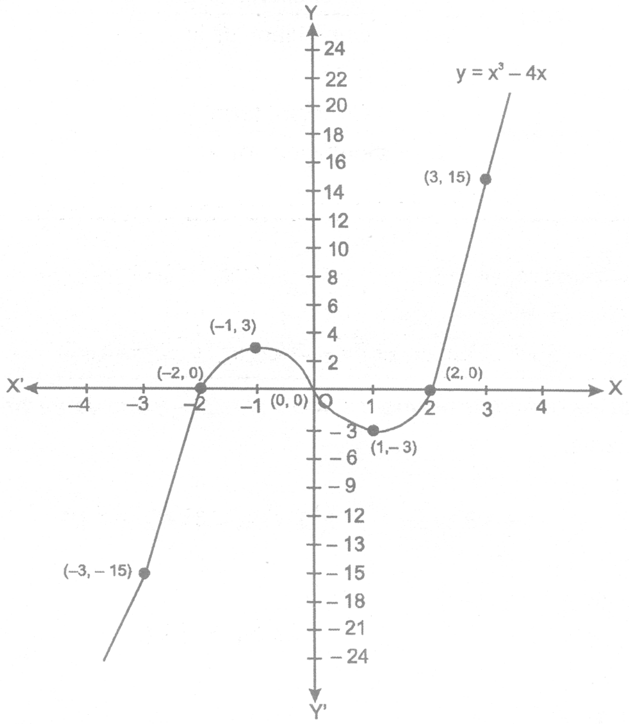
Ex.4 Draw the graphs of the polynomial $f(x) = x^3 - 4x$.

Sol. Let y = f(x) or, $y = x^2 - 4x$.

The values of y for variable value of x are listed in the following table :

x	-3	-2	-1	0	1	2	3
$y = x^3 - 4x$	-15	0	3	0	-3	0	15

Thus, the curve $y = x^3 - 4x$ passes through the points (-3, -15), (-2, 0), (-1, 3), (0 ,0), (1, -3), (2, 0), (3, 15), (4, 48) etc.Plotting these points on a graph paper and drawing a free hand smooth curve through these points, we obtain the graph of the given polynomial as shown figure.



Observations :

For the graphs of the polynomial $f(x) = x^3 - 4x$, following observations are as follows :-

(i) The polynomial $f(x) = x^3 - 4x = x(x^2 - 4) = x(x - 2)(x + 2)$ is factorizable into three distinct linear factors. The curve y = f(x) also cuts X-axis at three distinct points.

(ii) We have, f(x) = x (x - 2) (x + 2) Therefore 0, 2 and -2 are three zeros of f(x). The curve y = f(x) cuts X-axis at three points O (0, 0), P(2, 0) and Q (-2, 0).

4.5 RELATIONSHIP BETWEEN ZEROS AND COEFFICIENTS OF A QUADRATIC POLYNOMIAL :

Let α and β be the zeros of a quadratic polynomial $f(x) = ax^2 + bx + c$. By facto r theorem $(x - \alpha)$ and $(x - \beta)$ are the factors of f(x).

$\therefore$ $f(x) = k(x - \alpha)(x - \beta)$ are the factors of f(x)

$\Rightarrow$ $ax^2 + bx + c = k\{x^2 - (\alpha + \beta)x + \alpha\beta\}$

$\Rightarrow$ $ax^2 + bx + c = kx^2 - k(\alpha + \beta)x + k\alpha\beta$

Comparing the coefficients of x^2, x and constant terms on both sides, we get

$a = k, b = -k\ (\alpha+\beta)$ and $k\alpha\beta$

$\Rightarrow \quad \alpha+\beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a} \quad \Rightarrow \quad \alpha+\beta = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$ and $\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Hence,

$$\text{Sum of the zeros} = -\frac{b}{a} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\text{Product of the zeros} = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

REMAKRS :

If α and β are the zeros of a quadratic polynomial f(x). The , the polynomial f(x) is given by

$f(x) = k\{x^2 - (\alpha+\beta)x + \alpha\beta\}$

or $f(x) = k\{x^2$ - (Sum of the zeros) x + Product of the zeros}

Ex.5 Find the zeros of the quadratic polynomial $f(x) = x^2 - 2x - 8$ and verify and the relationship between the zeros and their coefficients.

Sol. $f(x) = x^2 - 2x - 8$

$\Rightarrow f(x) = x^2 - 4x + 2x - 8 \quad \Rightarrow \quad f(x) = x(x-4) + 2(x-4)]$

$\Rightarrow f(x) = (x-4)(x+2)$

Zeros of f(x) are given by f(x) = 0

$\Rightarrow x^2 - 2x - 8 = 0 \quad \Rightarrow \quad (x-4)(x+2) = 0$

$\Rightarrow x = 4$ or $x = -2$

So, $\alpha = 4$ and $\beta = -2$

$\therefore$ sum of zeros $\alpha+\beta$

$= 4 - 2 = 2$

Also, sum of zeros $= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = \frac{-(-2)}{1} = 2$

So, sum of zeros $= \alpha+\beta = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Now, product of zeros $= \alpha\beta \quad = (4)(-2) = -8$

Also, product of zeros $= \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{-8}{1} = -8$

$\therefore \quad$ Product of zeros $= \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \alpha\beta$.

Ex.6 Find a quadratic polynomial whose zeros are $5+\sqrt{2}$ and $5-\sqrt{2}$

Sol. Given $\alpha = 5+\sqrt{2}, \beta = 5-\sqrt{2}$

$\therefore \quad f(x) = k\{x^2 - x(\alpha+\beta) + \alpha\beta\}$ Here, $\alpha+\beta = 5+\sqrt{2}+5-\sqrt{2} = 10$

and $\alpha\beta = (5+\sqrt{2})(5-\sqrt{2})$ $= 25 - 2 = 23$

$\therefore \quad f(x) = k\{x^2 - 10x + 23\}$, where, k is any non-zero real number.

Ex.7 Sum of product of zeros of quadratic polynomial are 5 and 17 respectively. Find the polynomial.

Sol. Given : Sum of zeros = 5 and product of zeros = 17

So, quadratic polynomial is given by

$\Rightarrow f(x) = k\{x^2 - x$(sum of zeros) + product of zeros}

$\Rightarrow f(x) = k\{x^2 - 5x + 17\}$, where, k is any non-zero real number,

4.6 RELATIONSHIP BETWEEN ZEROS AND COEFFICIENTS OF A CUBIC POLYNOMIAL :

Let α, β, γ be the zeros of a cubic polynomial $f(x) = ax^3 + bx^2 + cx + d$, $a \neq 0$ Then, by factor theorem, $a-\alpha, x-\beta$ and $x-\gamma$ are factors of f(x). Also, f(x) being a cubic polynomial cannot have more than three linear factors.

$\therefore \quad f(x) = k(x-\alpha)(x-\beta)(x-\gamma) \quad \Rightarrow \quad ax^3 + bx^2 + cx + d = k\ (x-\alpha)(x-\beta)(x-\gamma)$

$\Rightarrow ax^3 + bx^2 + cx + d = k\{x^3 - (\alpha+\beta+\gamma)x^2 + (\alpha\beta+\beta\gamma+\gamma\alpha)x - \alpha\beta\gamma\}$

$\Rightarrow$ $ax^3 + bx^2 + cx + d = k\,x^3 - k\,(\alpha+\beta+\gamma)x^2 + k(\alpha\beta+\beta\gamma+\gamma\alpha)x - k\alpha\beta\gamma$

Comparing the coefficients of x^3, x^2, x and constant terms on both sides, we get

$a = k,\ b = -k\,(\alpha+\beta+\gamma), c = (\alpha\beta+\beta\gamma+\gamma\alpha) \text{ and } d = -k(\alpha\beta\gamma)$

$\Rightarrow \quad \alpha+\beta+\gamma = -\dfrac{b}{a} \qquad \Rightarrow \quad \alpha\beta+\beta\gamma+\gamma\alpha = \dfrac{c}{a}$

And, $\alpha\beta\gamma = -\dfrac{d}{a} \qquad \Rightarrow$ Sum of the zeros $= -\dfrac{b}{a} = -\dfrac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$

$\Rightarrow$ Sum of the products of the zeros taken two at a time $= \dfrac{c}{a} = \dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$

$\Rightarrow$ Product of the zeros $= -\dfrac{d}{a} = -\dfrac{\text{Constant term}}{\text{Coefficient of } x^3}$

REMARKS :

Cubic polynomial having α, β and γ as its zeros is given by

$f(x) = k\,(x-\alpha)(x-\beta)(x-\gamma)$

$f(x) = k\,\{x^3 - (\alpha+\beta+\gamma)x^2 + (\alpha\beta+\beta\gamma+\gamma\alpha)x - \alpha\beta\gamma\}$ where k is any non-zero real number.

Ex.8 Verify that $\dfrac{1}{2}, 1 - 2$ are zeros of cubic polynomial $2x^3 + x^2 - 5x + 2$. Also verify the relationship between, the zeros and their coefficients.

Sol. $f(x) = 2x^3 + x^2 - 5x + 2$

$f\left(\dfrac{1}{2}\right) = 2\left(\dfrac{1}{2}\right)^3 + \left(\dfrac{1}{2}\right)^2 - 5\left(\dfrac{1}{2}\right) + 2 \qquad = \dfrac{1}{4} + \dfrac{1}{4} - \dfrac{5}{2} + 2$

$f(1) = 2()^3 + (1)^2\ 5(1) + 2 = 2 + 1 - 5 + 2 = 0.$

$f(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2 = -16 + 4 + 10 + 2 = 0.$

Let $\alpha = \dfrac{1}{2}, \beta = 1$ and $\gamma = -2$

Now, Sum of zeros $= \alpha+\beta+\gamma$

$= \dfrac{1}{2} + 1 - 2 = -\dfrac{1}{2}$

Also, sum of zeros $= -\dfrac{(\text{Coefficient of } x^2)}{\text{Coefficient of } x^3} = -\dfrac{1}{2}$

So, sum of zeros $= \alpha+\beta+\gamma = -\dfrac{(\text{Coefficient of } x^2)}{\text{Coefficient of } x^3}$

Sum of product of zeros taken two at a time $= \alpha\beta+\beta\gamma+\gamma\alpha$

$= \dfrac{1}{2}\times 1 + 1\times(-2) + (-2)\times\dfrac{1}{2} = -\dfrac{5}{2}$

Also, $\beta\beta+\beta\gamma+\gamma\alpha = \dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^3} = \dfrac{-5}{2}$

So, sum of product of zeros taken two at a time = $\alpha\beta+\beta\gamma+\gamma\alpha = \dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$

Now, Product of zeros = $\alpha\beta\gamma \quad = \left(\dfrac{1}{2}\right)(1)(-2) = -1$

Also, product of zeros = $\dfrac{\text{Constant term}}{\text{Coefficient of } x^3} = \dfrac{-2}{2} = -1$

$\therefore$ Product zeros = $\alpha\beta\gamma = -\dfrac{\text{Constant term}}{\text{Coefficient of } x^3}$

Ex.9 Find a polynomial with the sum, sum of the product of its zeros taken two at a time, and product its zeros as 3, -1 and -3 respectively.

Sol. Given $\alpha+\beta+\gamma = 3, \alpha\beta+\beta\gamma+\gamma\alpha = -1$ and $\alpha\beta\gamma = -3$

So, polynomial $f(x) = k\,\{x^3 - x^2\,(\alpha+\beta+\gamma) + x(\alpha\beta+\beta\gamma+\gamma\alpha) - \alpha\beta\gamma\}$

$f(x) = k\,\{x^3 - 3x^2 - x + 3\}$, where k is any non-zero real number.

4.7 VALUE OF A POLYNOMIAL :

The value of a polynomial **f(x)** at **x =** α is obtained by substituting $x = \alpha$ in the given polynomial and is denoted by $f(\alpha)$.

For example : If $f(x) = 2x^3 - 13x^2 + 17x + 12$ then its value at x = 1 is

$f(1) = 2(1)^3 - 13(1)^2 + 17(1) + 12 = 2 - 13 + 17 + 12 = 18$.

4.8 ZEROS OF ROOTS OF A POLYNOMIAL :

A real number **'a'** is a zero of a polynomial **f(x),** if **f(a) = 0,** Here **'a'** is called a root of the equation **f(x) = 0.**

Ex.10 Show that x = 2 is a root of $2x^3 + x^2 - 7x - 6$

Sol. $p(x) = 2x^3 + x^2 - 7x - 6$.

Then, $p(2) = 2(2)^3 + (2)^2 - 7(2) - 6 = 16 + 4 - 14 = 0$

Hence x = 2 is a root of p(x).

Ex.11 If $x = \frac{4}{3}$ is a root of the polynomial $f(x) = 6x^3 - 11x^2 + kx - 20$ then find the value of k.

Sol. $f(x) = 6x^3 - 11x^2 + kx - 20$

$$f\left(\frac{4}{3}\right) = 6\left(\frac{4}{3}\right)^3 - 11\left(\frac{4}{3}\right)^2 + k\left(\frac{4}{3}\right) - 20 = 0$$

$$\Rightarrow \quad 6\left(\frac{64}{27}\right) - 11\left(\frac{16}{9}\right) + \frac{4k}{3} - 20 = 0 \Rightarrow 6\left(\frac{64}{27}\right) - 11\left(\frac{16}{9}\right) + \frac{4k}{3} - 20 = 0$$

$\Rightarrow$ 128 - 176 + 12k - 180 = 0 $\qquad \Rightarrow$ 12k + 128 - 356 = 0

$\Rightarrow$ 12k = 228 $\quad \Rightarrow$ k = 19.

Ex.12 If x = 2 & x = 0 are roots of the polynomials $(f)x = 2x^3 - 5x^2 + ax + b$, then find the values of a and b/

Sol. $f(2) = 2(2)^3 - 5(2)^2 + a(2) + b = 0$

$\Rightarrow$ 16 - 20 + 2a + b = 0 $\Rightarrow$ 2a + b = 4 .....(i)

$\Rightarrow$ $f(0) = 2(0)^3 - 5(0)^2 + a(0) + b = 0 \Rightarrow$ b = 0

$\Rightarrow$ 2a = 4 $\Rightarrow$ a = 2, b = 0.

4.9 FACTOR THEOREM :

Let **p(x)** be a polynomial of degree greater than or equal to 1 and **'a'** be a real number such that **p(a) = 0.** then **(x - a)** is a factor of **p(x).** Conversely, if **(x - a)** is a factor of **p(x)**, then **p(a) = 0.**

Ex.13 Show that x + 1 and 2x - 3 are factors of $2x^3 - 9x^2 + x + 12$.

Sol. To prove that (x + 1) and (2x - 3) are factors of $p(x) = 2x^3 - 9x^2 + x + 12$ it is sufficient to show that p(-1) and $p\left(\frac{3}{2}\right)$ both are equal to zero.

$p(-1) = 2(-1)^3 - 9(-1)^2 + (-1) + 12 = - 2 - 9 - 1 + 12 = - 12 + 12 = 0$

And $p\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^3 - 9\left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right) + 12$

$$= \frac{27}{4} - \frac{81}{4} + \frac{3}{2} + 12 = \frac{27 - 81 + 6 + 48}{4} = \frac{-81 + 81}{4} = 0$$

Ex.14 Find α and β if x + 1 and x + 2 are factors of $p(x) = x^3 + 3x^2 - 2\alpha x + \beta$.

Sol. x + 1 and x + 2 are the factor of p(x).

Then, p(-1) = 0 & p(-2) = 0

Therefore, $p(-1) = (-1)^3 + 3(-1)^2 - 2\alpha(-1) + \beta = 0$

$\Rightarrow \quad -1 + 3 + 2\alpha + \beta = 0 \Rightarrow \beta = -2\alpha - 2$ ....(i)

$p(-2) = (-2)^3 + 3(-2)^2 - 2\alpha(-2) + \beta = 0$

$\Rightarrow$ $-8 + 12 + 4\alpha + \beta = 0 \Rightarrow \beta = -4\alpha - 4$ ....(ii)

From equation (1) and (2)

$-2\alpha - 2 = -4\alpha - 4 \qquad \Rightarrow \qquad 2\alpha = -2 \Rightarrow \alpha = -1$

Put α = -1 equation (1) $\Rightarrow$ β = - 2 (-1) - 2 = 2 - 2 = 0. Hence $\alpha = -1, \beta = 0$

Ex.15 What must be added to $3x^3 + x^2 - 22x + 9$ so that the result is exactly divisible by $3x^2 + 7x - 6$.

Sol. Let $p(x) = 3x^3 + x^2 - 22x + 9$ and $q(x) = 3x^2 + 7x - 6$

We know if p(x) is divided by q(x) which is quadratic polynomial then the remainder be r(x) and degree of r(x) is less than q(x) or Divisor.

$\therefore$ By long division method

Let we added ax + b (linear polynomial) in p(x), so that p(x) + ax + b is exactly divisible by $3x^2 + 7x - 6$.
Hence, $p(x) + ax + b = s(x) = 3x^3 - x^2 - 22x + 9 + ax + b = 3x^3 + x^2 x(22 - a) + (9 + b)$.

$$\begin{array}{r|l} & x-2 \\ \hline 3x^2+7x-6 & 3x^3+x^2-x(22-a)+9+b \\ & -3x^3 \pm 7x^2{}_{+} -6x \\ \hline & -6x^2+6x-(22-a)x+9+b \\ & -6x^2x(-16+a)+9+b \\ & {}_{+}-6x^2{}_{+}-14x \pm 12 \\ \hline & x(-2+a)+(b-3)=0 \end{array}$$

Hence, x(a - 2) + b - 3 = 0. x + 0
$\Rightarrow$ a - 2 = 0 & b - 3 = 0 $\Rightarrow$ a = 2 and b = 3

Hence if in p(x) we added 2x + 3 then it is exactly divisible by $3x^2 + 7x - 6$.

Ex.16 What must be subtracted from $x^3 - 6x^2 - 15x + 80$ so that the result is exactly divisible by $^2 + x - 12$.

Sol. Let ax + b be subtracted from $p(x) = x^3 - 6x^2 - 15x + 80$ so that it is exactly divisible by $x^2 + x - 12$.

$\therefore$ $s(x) = x^3 - 6x^2 - 15x + 80 - (ax + b)$
$= x^3 - 6x^2 - (15 + a)x + (80 - b)$
Dividend = Divisor × quotient + remainder
But remainder will be zero.
$\therefore$ Dividend = Divisor × quotient
$\Rightarrow$ $s(x) = (x^2 + x - 12) \times$ quotient $\Rightarrow$ $s(x) = x^3 - 6x^2 - (15 + a)x + (80 - b)$

$$\begin{array}{r|l} & x-7 \\ \hline x^2+x-12 & x^3-6x^2-x(15+a)+80-b \\ & {}_{-}x^3 \pm x^2{}_{\mp}12x \\ \hline & -7x^2+12x-(15+a)x+80-b \\ & -7x^2+(-3-a)+80-b \\ & {}_{\mp}7x^2{}_{\mp}7x \qquad \pm 84 \\ \hline & x(4-a)+(-4-b)=0 \end{array}$$

Hence, x(4 - a) + (-4 -b) = 0.x + 0
$\Rightarrow$ 4 - a = 0 & (-4 - b) = 0 $\Rightarrow$ a = 4 and b = - 4
Hence, if in p(x) we subtract 4x - 4 then it is exactly divisible by $x^2 + x - 12$.

Ex.17 Using factor theorem, factorize : $p(x) = 2x^4 - 7x^3 - 13x^2 + 63x - 45$.

Sol. 45 $\Rightarrow$ ±1,±3,±5,±9,±15,±45
If we put x = 1 in p(x)
$p(1) = 2(1)^4 - 7(1)^3 - 13(1)^2 + 63(1) - 45$
p(1) = 2 - 7 - 13 + 63 - 45 = 65 - 65 = 0
$\therefore$ x = 1 or x - 1 is a factor of p(x).
Similarly if we put x = 3 in p(x)
$p(3) = 2(3)^4 - 7(3)^3 - 13(3)^2 + 63(3) - 45$
p(3) = 162 - 189 - 117 + 189 - 45 = 162 - 162 = 0
Hence, x = 3 or (x - 3) = 0 is the factor of p(x).
$p(x) = 2x^4 - 7x^3 - 13x^2 + 63x - 45$
$\therefore$ $p(x) = 2x^3 (x - 1) - 5x^2 (x - 1) - 18x(x - 1) + 45(x - 1)$
$\Rightarrow$ $p(x) = (x - 1) (2x^3 - 5x^2 - 18x + 45)$ $\Rightarrow$ $p(x) = (x - 1) (2x^3 - 5x^2 - 18x + 45)$
$\Rightarrow$ $p(x) = (x - 1) [2x^2 -(x - 3) + x(x - 3) - 15(x - 3)]$ $\Rightarrow$ $p(x) = (x - 1) (x - 3) (2x^2 + x - 15)$
$\Rightarrow$ $p(x) = (x - 1) (x - 3) (2x^2 + 6x - 5x -15)$ $\Rightarrow$ $p(x) = (x - 1) (x - 3)[2x(x + 3) - 5(x + 3)]$
$\Rightarrow$ p(x) = (x - 1) (x - 3) (x + 3) (2x - 5).

4.10 REMAINDER THEOREM :

Let **p(x)** be any polynomial of degree greater than or equal to one and **'a'** be any real number. If **p(x)** is divided by **x - a),** then the remainder is equal to **p(a).**

Let q(x) be the quotient and r(X) be the remainder when p(x) is divided by (x - a), then

Dividend = Divisor × Quotient + Remainder

Ex.18 Find the remainder when $f(x) = x^3 - 6x^2 + 2x - 4$ is divided by $g(x) = 1 - 2x$.

Sol. $1 - 2x = 0 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$

$$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 - 6\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right) - 4 = \frac{1}{8} - \frac{3}{2} + 1 - 4 = \frac{1-12+8-32}{8} = -\frac{35}{8}$$

Ex.19 Apply division algorithm to find the quotient q(x) and remainder r(x) on dividing $f(x) = 10x^4 + 17x^3 - 62x^2 + 30x - 3$ by $b(x) = 2x^2 - x + 1$.

Sol.

$$\begin{array}{r|l}
 & 5x^2 + 11x - 28 \\
2x^2 - x + 1 & 10x^4 + 17x^3 - 62x^2 + 30x - 3 \\
 & \underline{\ _-10x^4 \ _+ - 5x^3 \ _\pm 5x^2} \\
 & 22x^3 - 67x^2 + 30x - 3 \\
 & \underline{\ _-22x^3 \ _+ - 11x^2 \ _\pm 11x} \\
 & -56x^2 + 19x - 3 \\
 & \underline{\ _+-56x^2 \ _\pm 28x \ _+ - 28} \\
 & -9x + 25
\end{array}$$

So, quotient $q(x) = 5x^2 + 11x - 28$ and remainder $r(x) = -9x + 25$.

Now, dividend = Quotient × Divisor + Remainder

$= (5x^2 + 11x - 28)(2x^2 - x + 1) + (-9x + 25)$

$= 10x^4 - 5x^3 + 5x^2 + 22x^3 - 11x^2 + 11x - 56x^2 + 28x - 28 - 9x + 25$

$= 10x^4 + 17x^3 - 62x^2 + 30x - 3$

Hence, the division algorithm is verified.

Ex.20 Find all the zeros of the polynomial $f(x) = 2x^4 - 2x^3 - 7x^2 + 3x + 6$, if two of its zeros are $-\sqrt{\frac{3}{2}}$ and $\sqrt{\frac{3}{2}}$.

Sol. Since $-\sqrt{\frac{3}{2}}$ and $\sqrt{\frac{3}{2}}$ are zeros of f(x).

Therefore, $\left(x + \sqrt{\frac{3}{2}}\right)\left(x - \sqrt{\frac{3}{2}}\right) = \left(x^2 - \frac{3}{2}\right) = \frac{2x^2 - 3}{2}$ or $2x^2 - 3$ is a factor of f(x).

$$\begin{array}{r|l}
 & x^2 - x - 2 \\
2x^2 - 3 & 2x^4 - 2x^3 - 7x^2 + 3x + 6 \\
 & \underline{\ _-2x^4 \qquad \ _\mp 3x^2} \\
 & -2x^3 - 4x^2 + 3x + 6 \\
 & \underline{\ _\mp 2x^3 \qquad \ _\mp 3x} \\
 & -4x^2 + 6 \\
 & -4x^2 + 6 \\
 & \underline{+ \qquad -} \\
 & 0
\end{array}$$

$\therefore$ $2x^4 - 2x^3 - 7x^2 + 3x + 6 = (2x^2 - 3)(x^2 - x - 2)$

$= (2x^2 - 3)(x - 2)(x + 1)$

$= 2\left(x + \sqrt{\frac{3}{2}}\right)\left(x - \sqrt{\frac{3}{2}}\right)(x-2)(x+1)$ So, the zeros are $-\sqrt{\frac{3}{2}}, \sqrt{\frac{3}{2}}, 2, -1$

DAILY PRACTICE PROBLESM # 4

OBJECTIVE DPP - 4.1

1. If $4x^4 - 3x^3 - 3x^2 + x - 7$ is divided by $1 - 2x$ then remainder will be

(A) $\frac{57}{8}$ (B) $-\frac{59}{8}$ (C) $\frac{55}{8}$ (D) $-\frac{55}{8}$

2. The polynomials $ax^3 + 3x^2 - 3$ and $2x^3 - 5x + a$ when divided by $(x - 4)$ leaves remainders R_1 & R_2 respectively then value of '**a**' if $2R_1 - R_2 = 0$.

(A) $-\frac{18}{127}$ (B) $\frac{18}{127}$ (C) $\frac{17}{127}$ (D) $-\frac{17}{127}$

3. A quadratic polynomial is exactly divisible by $(x + 1)$ & $(x + 2)$ and leaves the remainder 4 after division by $(x + 3)$ then that polynomial is

(A) $x^2 + 6x + 4$ (B) $2x^2 + 6x + 4$ (C) $2x^2 + 6x - 4$ (D) $x^2 + 6x - 4$

4. The values of a & b so that the polynomial $x^3 - ax^2 - 13x + b$ is divisible by $(x - 1)$ & $(x + 3)$ are

(A) a = 15, b = 3 (B) a = 3, b = 15 (C) c = - 3, b = 15 (D) a = 3, b = - 15

5. Graph of quadratic equation is always a -

(A) straight line (B) circle (C) parabola (D) Hyperbola

6. If the sign of '**a**' is positive in a quadratic equation then its graph should be =

(A) parabola open upwards (B) parabola open downwards
(C) parabola open leftwards (D) can't be determined

7. The graph of polynomial $y = x^3 - x^2 + x$ is always passing through the point -

(A) (0, 0) (B) (3, 2) (C) (1, -2) (D) all of these

8. How many time, graph of the polynomial $f(x) = x^3 - 1$ will intersect X-axis -

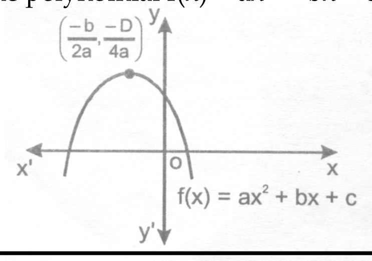
(A) 0 (B) 1 (C) 2 (D) 4

9. Which of the following curve touches X-axis -

(A) $x^2 - 2x + 4$ (B) $3x^2 - 6x + 1$ (C) $4x^2 - 16x + 9$ (D) $25x^2 - 20x + 4$

10. In the diagram given below shows the graphs of the polynomial $f(x) = ax^2 + bx + c$, then

(A) a < 0, b < 0 and c > 0
(B) a < 0, b < 0 and c < 0
(C) a < 0, b > 0 and c > 0
(D) a < 0, b > 0 and c < 0



SUBJECTIVE DPP 4.2

1. Draw the graph of following polynomials.

a. $f(x) = -3$ **b.** $f(x) = x - 4$ **c.** $f(x) = |x + 2|$
d. $f(x) = x^2 - 9$ **e.** $f(x) = 2x^2 - 4x + 5$ **f.** $f(x) = x(2 - 3x) + 1$
g. $f(x) = x^3 - x^2$ **h.** $f(x) = x^3 + 2x$

2. Find the zeros of quadratic polynomial $p(x) = 4x^2 + 24x + 36$ and verify the relationship between the zeros and their coefficients.

3. Find a quadratic polynomial whose zeros are 5 and - 5.

4. Sum and product of zeros of a quadratic polynomial are 2 and $\sqrt{5}$ respectively. Find the quadratic polynomial.

5. Find a quadratic polynomial whose zeros are $3+\sqrt{5}$ and $3-\sqrt{5}$.

6. Verify that $-5, \frac{1}{2}, \frac{3}{4}$ are zeros of cubic polynomial $4x^3 + 20x + 2x - 3$. Also verify the relationship between the zeros and the coefficients.

7. Divide $64y^3 - 1000$ by $8y - 20$.

8. If α, β are zeros of $x^2 + 5x + 5$, find the value of $\alpha^{-1} + \beta^{-1}$.

9. Apply the division algorithm to find the quotient and remainder on dividing $p(x) = x^4 - 3x^2 + 4x + 5$ by $g(x) = x^2 + 1 - x$.

10. On dividing $x^3 - 3x^2 + x + 2$ by polynomial g(x), the quotient remainder were x 2 and $-2x + 4$, respectively. Find g(x).

11. α, β, γ are zeros of cubic polynomial $x^3 - 12x^2 + 44x + c$. If α, β, γ are in A.P., find the value of c.

12. Obtain all the zeros of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeros are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

13. What must be added to $x^3 - 3x^2 - 12x + 19$ so that the result is exactly divisible by $x^2 + x - 6$?

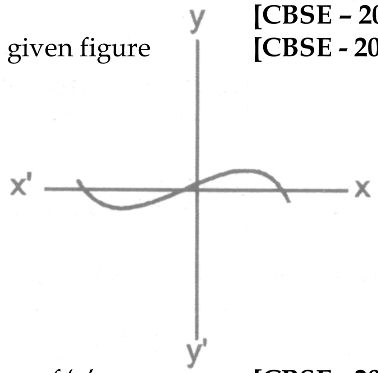
14. What must be subtracted from $x^4 + 2x^3 - 13x^2 - 12x + 21$ so that the result is exactly divisible by $x^2 - 4x + 3$?

15. If α, β are zeros of quadratic polynomial $kx^2 + 4x + 4$, find the value of k such that $(\alpha+\beta)^2 - 2\alpha\beta = 24$.

16. Find the quadratic polynomial sum of whose zeros is 8 and their product is 12. Hence find f the zeros of the polynomial. **[CBSE - 2008]**

17. Is $x = -4$ a solution of the equations $2x^2 + 5x - 12 = 0$ > **[CBSE – 2008]**

18. Write the number of zeros of the polynomial $y = f(x)$ whose graph is given figure **[CBSE - 2008]**



19. If the product of zeros of the polynomial $ax^2 - 6x - 6$ is 4, find the value of 'a'. **[CBSE - 2008]**

ANSWERS

(Objective DPP 4.1)

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	B	B	B	C	A	A	B	D	A

(Subjective DPP 4.2)

2. -3, -3

3. $k\{x^2 - 25\}$

4. $k\{x^2 - 2x + \sqrt{5}\}$

5. $k\{x^2 - 6x + 4\}$

7. $8y^2 + 20y + 50$

8. -1

9. Quotient = $x^2 + x - 3$, Remainder = 8

10. $x^2 - x + 1$

11. $c = -48$

12. $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}$, -1 and -1

13. $2x + 5$

14. $2x - 3$

15. $k = \frac{2}{3}, -1$

16. $k\{x^2 - 8x + 12\}$ and zeros are 6 & 2.

17. Yes

18. No. of zeros = 3

19. $a = -\frac{3}{2}$

TRIANGLES

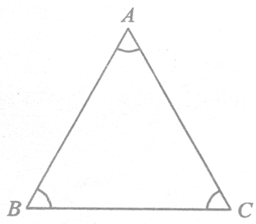
8.1 CONGRUENT AND SIMILAR FIGURES:

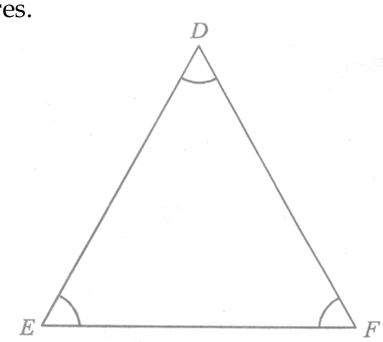
Two geometric figures having the same shape and size are known as congruent figures. Geometric figures having the same shape but different sizes are known as similar figures.

8.2 SIMILAR TRIANGLES:

Two triangles **ABC** and **DEF** are said to be similar if their

(i) Corresponding angles are equal.
i.e. $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$
And,





(ii) Corresponding sides are proportional i.e. $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

8.2 (a) Characteristic Properties of Similar Triangles :

(i) (AAA Similarity) If two triangles are equiangular, then they are similar.
(ii) (SSS Similarity) If the corresponding sides of two triangles are proportional, then they are similar.
(iii) (SAS Similarity) If in two triangle's one pair of corresponding sides are proportional and the included angles are equal then the two triangles are similar.

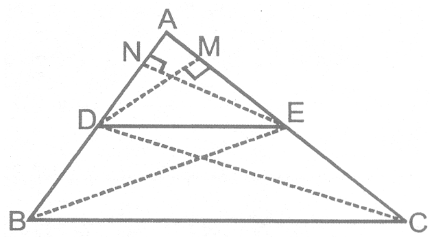
8.2 (b) Results Based Upon Characteristic Properties of Similar Triangles :

(i) If two triangles are equiangular, then the ratio of the corresponding sides is the same as the ratio of the corresponding medians.
(ii) If two triangles are equiangular, then the ratio of the corresponding sides is same at the ratio of the corresponding angle bisector segments.
(iii) if two triangles are equiangular then the ratio of the corresponding sides is same at the ratio of the corresponding altitudes.
(vi) If one angle of a triangle is equal to one angle of another triangle and the bisectors of these equal angles divide the opposite side in the same ratio, then the triangles are similar.
(v) If two sides and a median bisecting one of these sides of a triangle are respectively proportional to the two sides and the corresponding median of another triangle, then the triangles are similar.
(vi) If two sides and a median bisecting the third side of a triangle are respectively proportional to the corresponding sides and the median another triangle, then two triangles are similar.

8.3 THALES THEOREM (BASIC PROPROTIONALITY THEOREM) :

Statement : If a line is drawn parallel to one side of a triangle to intersect the other sides in distinct points, then the other two sides are divided in the same ratio.

Given: A triangle ABC in which a line parallel to side BC intersects other two sides AB and AC at D and E respectively.



To Prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Construction : Join BE and CD and draw $DM \perp AC$ and $EN \perp AB$.

Proof : Area of ΔADE $(= \frac{1}{2}$ base × height$)$ = $\frac{1}{2} AD \times EN$.

Area of ΔADE is denoted as are (ADE)

So, $ar(ADE) = \frac{1}{2} DB \times EN$ And $ar(BDE) = \frac{1}{2} DB \times EN$,

Therefore, $\frac{ar(ADE)}{ar(BDE)} = \frac{\frac{1}{2} AD \times EN}{\frac{1}{2} DB \times EN} = \frac{AD}{DB}$ ….(i)

Similarly, $ar(ADE = \frac{1}{2} AE \times DM$ and $ar(DEC = \frac{1}{2} EC \times DM$.

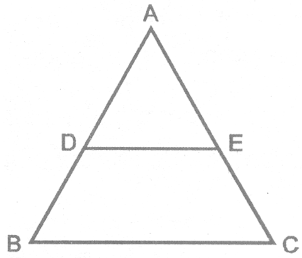
And $$\frac{ar(ADE)}{ar(DEC)} = \frac{\frac{1}{2}AE \times DM}{\frac{1}{2}EC \times DM} = \frac{AE}{EC} \quad ......(ii)$$

Note that Δ BDE and Δ DEC are on the same base DE and between the two parallel lines BC and DE.

So, ar(BDE) = ar(DEC) .....(iii)

Therefore, from (i), (ii) and (iii), we have :

$$\frac{AD}{DB} = \frac{AE}{EC}$$ **Hence Proved.**



Corollary : If in a ΔABC, a line DE | | BC, intersects AB in D and AC in E, then

(i) $\frac{DB}{AD} = \frac{EC}{AE}$ (ii) $\frac{AB}{AD} = \frac{AC}{AE}$ (ii) $\frac{AD}{AB} = \frac{AE}{AC}$

(iv) $\frac{AB}{DB} = \frac{AC}{EC}$ (v) $\frac{DB}{AB} = \frac{EC}{AC}$

8.3 (a) Converse of Basic Proportionality Theorem :

If a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

8.3 (b) Some Important Results and Theorems :

(i) The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

(ii) In a triangle **ABC**, if **D** is a point on **BC** such that **D** divides **BC** in the ratio **AB : AC,** then **AD** is the bisector of **∠A.**(iii) The external bisector of an angle of a triangle divides the opposite sides externally in the ratio of the sides containing the angle.

(iv) The line drawn from the mid-point of one side of a triangle parallel to another side bisects the third side. (v) The line joining the mid-points of two sides of a triangle is parallel to the third side.

(vi) The diagonals of a trapezium divide each other proportionally.

(vii) If the diagonals of a quadrilateral divide each other proportionally, then it is a trapezium.

(viii) Any line parallel to the parallel sides of a trapezium divides the non-parallel sides proportionally.

(ix) If three or more parallel lines are intersected by two transversal, then the intercepts made by them on the transversal are proportional.

Ex.1 In a ΔABC, D and E are points on the sides AB and AC respectively such that DE | | BC. If AD = 4x - 3, AE = 8x - 7, BD = 3x - 1 and CE = 5x - 3, find the value of x. **[CBSE - 2006]**

Sol. In ΔABC, we have

DE| |BC

$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC}$ [By Basic Proportionality Theorem]

$\Rightarrow \quad \frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$

$\Rightarrow \quad 20x^2 - 15x - 12x + 9 = 24x^2 - 21x - 8x + 7$

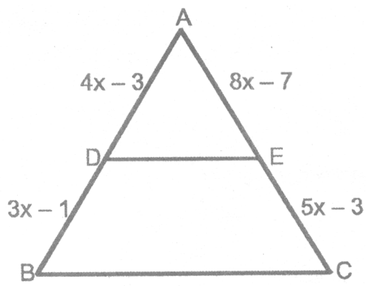
$\Rightarrow \quad 20x^2 - 27x + 9 = 24x^2 - 29x + 7$

$\Rightarrow \quad 4x^2 - 2x - 2 = 0$

$\Rightarrow \quad 2x^2 - x - 1 = 0$

$\Rightarrow \quad (2x + 1)(x - 1) = 0$

$\Rightarrow \quad x = 1 \text{ or } x = -\frac{1}{2}$



So, the required value of x is 1. [$x = -\frac{1}{2}$ is neglected as length can not be negative].

Ex.2 D and E are respectively the points on the sides AB and AC of a ΔABC such that AB = 12 cm, AD = 8 cm, AE = 12 cm and AC = 18 cm, show that DE | | BC.

Sol. We have,

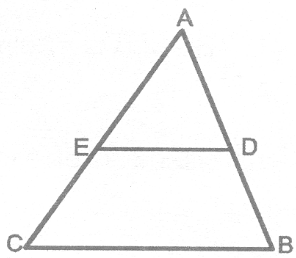
AB = 12 cm, AC = 18 m, AD = 8 cm and AE = 12 cm.

$\therefore$ BD = AB - AD = (12 - 8) cm = 4 cm

CE = AC - AE = (18 12) cm = 6 cm

Now, $\frac{AD}{BC} = \frac{8}{4} = \frac{2}{1}$

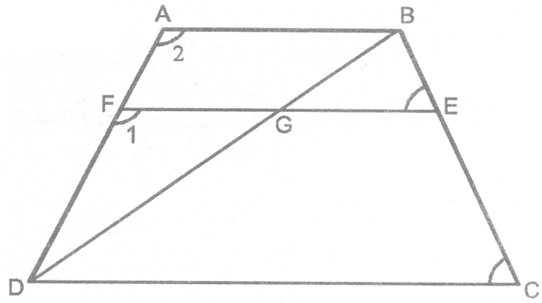
And, $\frac{AE}{CE} = \frac{12}{6} = \frac{2}{1} \quad \Rightarrow \quad \frac{AD}{BD} = \frac{AE}{CE}$



Thus, DE divides sides AB and AC of ΔABC in the same ratio. Therefore, by the conserve of basic proportionality theorem we have DE | | BC.

Ex.3 In a trapezium ABCD AB | | DC and DC = 2AB. EF drawn parallel to AB cuts AD in F and BC in E such that $\frac{BE}{EC} = \frac{3}{4}$. Diagonal DB intersects EF at G. Prove that 7FE = 10AB.

Sol. In ΔDFG and ΔDAB,



$\angle 1 = \angle 2$ [Corresponding ∠s ∴ AB | | FG]

$\angle FDG = \angle ADB$ [Common]

∴ ΔDFG ~ ΔDAB [By AA rule of similarity]

∴ $\frac{DF}{DA} = \frac{FG}{AB}$ .....(i)

Again in trapezium ABCD

EF | | AB | | DC

∴ $\frac{AF}{DF} = \frac{BE}{EC}$

⇒ $\frac{AF}{DF} = \frac{3}{4}$ $\left[\because \frac{BE}{EC} = \frac{3}{4}(\text{given})\right]$

⇒ $\frac{AF}{DF} = 1 = \frac{3}{4} + 1$ ⇒ $\frac{AF+DF}{DF} = \frac{7}{4}$

⇒ $\frac{AD}{DF} = \frac{7}{4}$ ⇒ $\frac{DF}{AD} = \frac{4}{7}$ ....(ii)

From (i) and (ii), we get

$\frac{FG}{AB} = \frac{4}{7}$ i.e. FG = $\frac{4}{7}AB$ ......(iii)

In ΔBEG and ΔBCD, we have

$\angle BEG = \angle BCD$ [Corresponding angle ∴ EG | | CD]

$\angle GBE = \angle DBC$ [Common]

∴ ΔBEG ~ ΔBCD [By AA rule of similarity]

∴ $\frac{BE}{BC} = \frac{EG}{CD}$

∴ $\frac{3}{7} = \frac{EG}{CD}$ $\left[\because \frac{BE}{EG} = \frac{3}{7} \text{ i.e.. } \frac{EC}{BE} = \frac{4}{3} \Rightarrow \frac{EC+BE}{BE} = \frac{4+3}{3}\right] \Rightarrow \frac{BC}{BE} = \frac{7}{3}$

∴ $EG = \frac{3}{7}CD = \frac{3}{7}(2\,AB)$ $[\because CD = 2AB\ (\text{given})]$

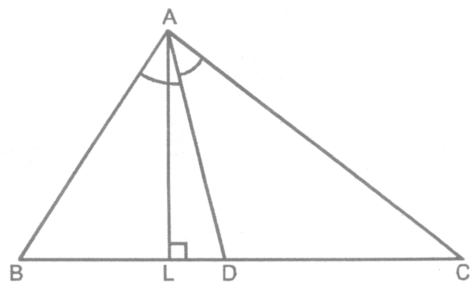
∴ $EG = \frac{6}{7}AB$ .....(iv)

Adding (iii) and (iv), we get

$FG + EG = \frac{4}{7}AB + \frac{6}{7}AB = \frac{10}{7}AB$

⇒ $EF = \frac{10}{7}AB$ i.e., $7EF = 10AB$. **Hence proved.**

Ex.4 In ΔABC, if AD is the bisector of ∠A, prove that $\frac{\text{Area }(\Delta ABD)}{\text{Area }(\Delta ACD)} = \frac{AB}{AC}$



Sol. In ΔABC, AD is the bisector of ∠A.

∴ $\frac{AB}{AC} = \frac{BD}{DC}$ ....(i) [By internal bisector theorem]

From A draw AL ⊥ BC

∴ $\frac{\text{Area }(\Delta ABD)}{\text{Area }(\Delta ACD)} = \frac{\frac{1}{2}BD.AL}{\frac{1}{2}DC.AL} = \frac{BD}{DC} = \frac{AB}{AC}$ [From (i)] **Hence Proved.**

Ex.5 $\angle BAC = 90^0$, AD is its bisector. IF DE ⊥ AC, prove that DE × (AB + AB) = AB × AC.

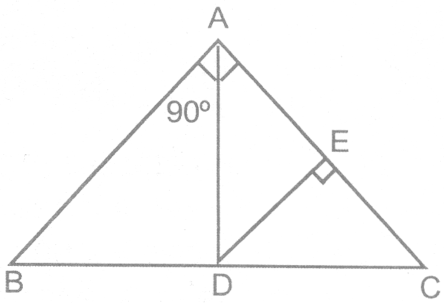
Sol. It is given that AD is the bisector of ∠A of ΔABC.

∴ $\frac{AB}{AC} = \frac{BD}{DC}$

⇒ $\frac{AB}{AC} + 1 = \frac{BD}{DC} + 1$ [Adding 1 on both sides]

$\Rightarrow \quad \dfrac{AB+AC}{AC} = \dfrac{BD+DC}{DC}$

$\Rightarrow \quad \dfrac{AB+AC}{AC} = \dfrac{BC}{DC}$ ....(i)



In Δ's CDE and CBA, we have

$\angle DCE = \angle BCA$ [Common]

$\angle DEC = \angle BAC$ [Each equal to 90^0]

So, by AA-criterion of similarity

$\Delta\, CDE \sim \Delta\, CBA$

$\Rightarrow \quad \dfrac{CD}{CB} = \dfrac{DE}{BA} \quad \Rightarrow \quad \dfrac{AB}{DE} = \dfrac{BC}{DC}$ ....(ii)

From (i) and (ii), we have

$\Rightarrow \quad \dfrac{AB+AC}{AC} = \dfrac{AB}{DE} \quad \Rightarrow \quad DE \times (AB + AC) = AB \times AC.$

Ex.6 In the given figure, PA, QB and RC are each perpendicular to AC. Prove that $\dfrac{1}{x} + \dfrac{1}{z} = \dfrac{1}{y}$

Sol. In ΔPAC, we have BQ||AP

$\Rightarrow \quad \dfrac{BQ}{AP} = \dfrac{CB}{CA}$ [$\therefore$ ΔCBQ ~ ΔCAP]

$\Rightarrow \quad \dfrac{y}{x} = \dfrac{CB}{CA}$ ...(i)

In ΔACR, we have BQ||CR

$\Rightarrow \quad \dfrac{BQ}{CR} = \dfrac{AB}{AC}$ [$\therefore$ Δ ABQ ~ ΔACR]

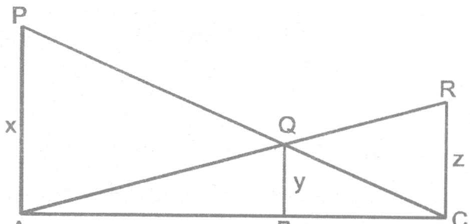
$\Rightarrow \quad \dfrac{y}{z} = \dfrac{AB}{AC}$ ...(ii)

Adding (i) and (ii), we get

$\dfrac{y}{x} + \dfrac{y}{z} = \dfrac{CB}{AC} + \dfrac{AB}{AC}$

$\Rightarrow \quad \dfrac{y}{x} + \dfrac{y}{z} = \dfrac{AB+BC}{AC} \quad \Rightarrow \quad \dfrac{y}{x} + \dfrac{y}{z} = \dfrac{AC}{AC}$

$\Rightarrow \quad \dfrac{y}{x} + \dfrac{y}{z} = 1 \quad \Rightarrow \quad \dfrac{1}{x} + \dfrac{1}{z} = \dfrac{1}{y}$



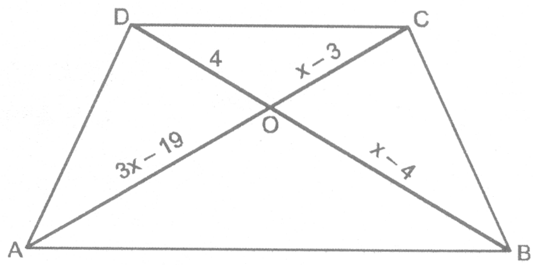
Hence Proved.

Ex.7 In the given figure, AB||CD. Find the value of x.

Sol. Since the diagonals of a trapezium divide each other proportionally.

$\therefore \quad \dfrac{AO}{OC} = \dfrac{BO}{OD}$

$\Rightarrow \quad \dfrac{3x-19}{x-3} = \dfrac{x-4}{4}$



$\Rightarrow \quad 12x - 76 = x^2 - 4x - 3x + 12$

$\Rightarrow \quad x^2 - 19x + 88 = 0$

$\Rightarrow \quad x^2 - 11x - 8x + 88 = 0$

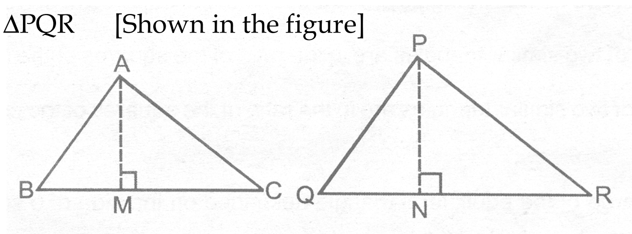
$\Rightarrow \quad (x - 8)(x - 11) = 0$

$\Rightarrow \quad x = 8$ or $x = 11$.

8.4 AREAS OF SIMILAR TRIANGLS :

Statement : The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Given : Two triangles ABC and PQR such that ΔABC ~ ΔPQR [Shown in the figure]



To Prove : $\dfrac{ar(ABC)}{ar(PQR)} = \left(\dfrac{AB}{PQ}\right)^2 = \left(\dfrac{BC}{QR}\right)^2 = \left(\dfrac{CA}{RP}\right)^2$

Construction : Draw altitudes AM and PN of the triangle ABC an PQR.

Proof : $ar(ABC) = \frac{1}{2}BC \times AM$

And $ar(PQT) = \frac{1}{2}QR \times PN$

So, $\frac{ar(ABC)}{ar(PQR)} = \frac{\frac{1}{2}BC\times AM}{\frac{1}{2}QR\times PN} = \frac{BC\times AM}{QR\times PN}$ ....(i)

Now, in Δ ABM and Δ PQN,

And $\angle B = \angle Q$ [As Δ ABC ~ ΔPQR]

$\angle M = \angle N$ [90^0 each]

So, ΔABM ~ ΔPQN [AA similarity criterion]

Therefore, $\frac{AM}{PN} = \frac{AB}{PQ}$ ....(ii)

Also, ΔABC ~ ΔPQR [Given]

So, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$ .....(iii)

Therefore, $\frac{ar(ABC)}{ar(PQR)} = \frac{BC}{QR}\times\frac{AB}{PQ}$ [From (i) and (ii)]

$= \frac{AB}{PQ}\times\frac{AB}{PQ}$ [From (iii)]

$= \left(\frac{AB}{PQ}\right)^2$

Now using (iii), we get $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$

8.4 (a) Properties of Areas of Similar Triangles :

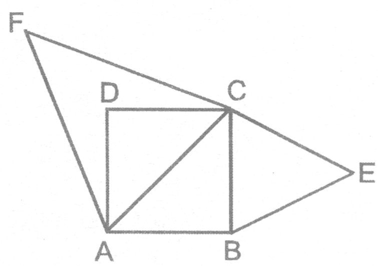
(i) The areas of two similar triangles are in the ratio of the squares of corresponding altitudes.
(ii) The areas of two similar triangles are in the ratio of the squares of the corresponding medians.
(iii) The area of two similar triangles are in the ratio of the squares of the corresponding angle bisector segments.

Ex.8 Prove that the area of the equilateral triangle described on the side of a square is half the area of the equilateral triangle described on this diagonals. **[CBSE - 2001]**

Sol. **Given :** A square ABCD. Equilateral triangles ΔBCE and ΔACF have been described on side BC and diagonals AC respectively.

To prove : Area (ΔBCE) $= \frac{1}{2}$. Area (ΔACF)

Proof : Since ΔBCE and ΔACF are equilateral. Therefore, they are equiangular (each angle being equal to 60^0) and hence ΔBCE ~ ΔACF.

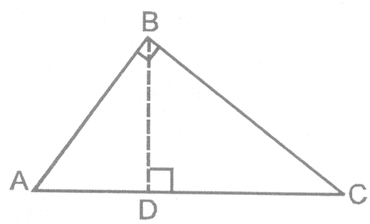


$\Rightarrow \frac{Area(\Delta BCE)}{Area(\Delta ACF)} = \frac{BC^2}{AC^2}$

$\Rightarrow \frac{Area(\Delta BCE)}{Area(\Delta ACF)} = \frac{BC^2}{(\sqrt{2}BC)^2} = \frac{1}{2}$ $\left[\begin{array}{l}\because \text{ABCD is a square} \\ \because \text{Diagonal} = \sqrt{2}(\text{side}) \\ \Rightarrow AC = \sqrt{2}BC\end{array}\right]$

$\Rightarrow \frac{Area(\Delta BCE)}{Area(\Delta ACF)} = \frac{1}{2}$ **Hence Proved.**

8.5 PYTHAGOREOUS THEOREM :



Statement : In a right triangle, the square of the hypotenuse is equal to the sum of the square of the other two sides.

Given : A right triangle ABC, right angled at B.

To prove : $AC^2 = AB^2 + BC^2$

Construction : $BD \perp AC$

Proof : Δ ADB & ΔABC

$\angle DAB = \angle CAB$ [Common]

$\angle BDA = \angle CBA$ [90^0 each]

So, $\Delta ADB \sim \Delta ABC$ [By AA similarity]

$\frac{AD}{AB} = \frac{AB}{AC}$ [Sides are proportional]

or, $AD \,.\, AC = AB^2$ .....(i)

Similarly $\Delta BDC \sim \Delta ABC$

So, $\frac{CD}{BC} = \frac{BC}{AC}$

or $CD \,.\, AC = BC^2$ .....(ii)

Adding (i) and (ii),

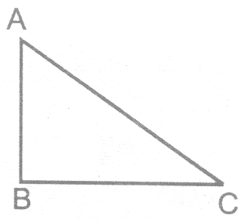
$AD \,.\, AC + CD \,.\, AC = AB^2 + BC^2$

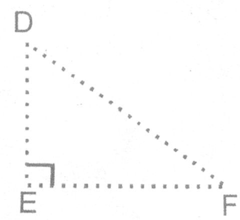
or, $AC\,(AD + CD) = AB^2 + BC^2$

or $AC.AC = AB^2 + BC^2$

or, $\mathbf{AC^2 = AB^2 + BC^2}$ **Hence Proved.**

8.5 (a) Converse of Pythagoreans Theorem :





Statement : In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle.

Given : A triangle ABC such that $AC^2 = AB^2 + BC^2$

Construction : Construct a triangle DEF such that DE = AB, EF = BC and $\angle E = 90^0$

Proof : In order to prove that $\angle B = 90.^0$, it is sufficient to show Δ ABC ~ Δ DEF. For this we proceed as follows Since Δ DEF is a right - angled triangle with right angle at E. Therefore, by Pythagoras theorem, we have

$DF^2 = DE^2 + EF^2$

$\Rightarrow$ $DF^2 = AB^2 + BC^2$ [$\therefore$ DE = AB and EF = BC (By construction)]

$\Rightarrow$ $DF^2 3 = AC^2$ [$\therefore$ $AB^2 + BC^2 = AC^2$ (Given)]

$\Rightarrow$ DF = AC .....(i)

Thus, in Δ ABC and Δ DEF, we have

AB = DE, BC = EF [By construction]

And AC = DF [From equation (i)]

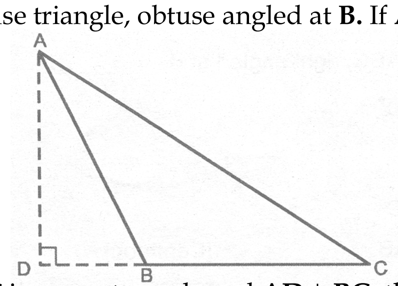
$\therefore$ $\Delta ABC \cong \Delta DEF$ [By SSS criteria of congruency]

$\Rightarrow$ $\angle B = \angle E = 90^0$

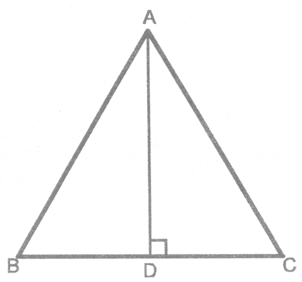
Hence, ΔABC is a right triangle, right angled at B.

8.5 (b) Some Results Deduced From Pythagoreans Theorem :

(i) In the given figure **ΔABC** is an obtuse triangle, obtuse angled at **B.** If **AD $\perp$ CD,** then $\mathbf{AC^2 = AB^2 + BC^2 + 2BC \,.\, BC}$



(ii) In the given figure, if **$\angle$B** of **ΔABC** is an acute angle and **AD $\perp$ BC,** then $\mathbf{AC^2 = AB^2 + BC^2 - 2BC \,.\, BD}$



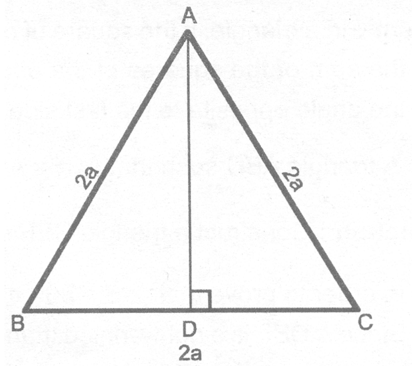
(iii) In any triangle, the sum of the squares of any two sides is equal to twice the square of half of the third side together with twice the square of the median which bisects the third side.

(iv) Three times the sum of the squares of the sides of a triangle is equal to four times the sum of the squares o the medians of the triangle.

Ex.9 In a ΔABC, AB = BC = CA = 2a and $AD \perp BC$. Prove that **[CBSE - 2002]**

(i) $AD = a\sqrt{3}$ **(ii)** area $(\Delta ABC) = \sqrt{3}\,a^2$

Sol. (i) Here, $AD \perp BC$.



Clearly, ΔABC is an equilateral triangle.

Thus, in ΔABD and ΔACD

AD = AD [Common]

$\angle ADB = \angle ADC$ [90^0 each]

And AB = AC

$\therefore$ by RHS congruency condition

$\Delta ABD \cong \Delta ACD$

$\Rightarrow$ BD = DC = a

Now, ΔABD is a right angled triangle

$\therefore$ $AD = \sqrt{AB^2 - BD^2}$ [Using Pythagoreans Theorem]

$AD = \sqrt{4a^2 - a^2} = \sqrt{3}\,a$ or $a\sqrt{3}$

(ii) Area $(\Delta ABC) = \frac{1}{2} \times BC \times AD$

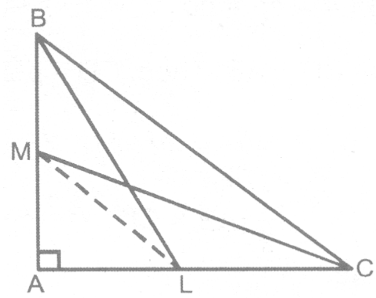
$= \frac{1}{2} \times 2a \times a\sqrt{3}$

$= a^2\sqrt{3}$

Ex.10 BL and Cm are medians of ΔABC right angled at A. Prove that $4(BL^2 + CM^2) = 5\,BC^2$

[CBSE-2006]

Sol. In ΔBAL



$BL^2 = AL^2 + AB^2$ ....(i) [Using Pythagoreans theorem]

and In ΔCAM

$CM^2 = AM^2 + AC^2$ .....(ii) [Using Pythagoreans theorem]

Adding (1) and (2) and then multiplying by 4, we get

$4(BL^2 + CM^2) = 4(AL^2 + AB^2 + AM^2 + AC^2)$

$=$ $4\{AL^2 + AM^2 + (AB^2 + AC^2)\}$ [$\therefore$ Δ ABC is a right triangle]

$=$ $4(AL^2 + AM^2 + BC^2)$

$=$ $4(ML^2 + BC^2)$ [$\therefore$ ΔLAM is a right triangle]

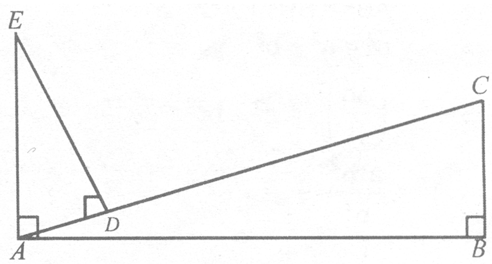
$=$ $4ML^2 + 4\,BC^2$

[A line joining mid-points of two sides is parallel to third side and is equal to half of it, ML = BC/2]

$=$ $BC^2 + 4BC^2 = 5BC^2$ **Hence proved.**

Ex.11 In the given figure, $BC \perp AB$, $AE \perp AB$ and $DE \perp AC$. Prove that DE.BC = AD.AB.

Sol. In ΔABC and ΔEDA,



We have

$\angle ABC = \Delta ADE$ [Each equal to 90^0]

$\angle ACB = \angle EAD$ [Alternate angles]

$\therefore$ By AA Similarity

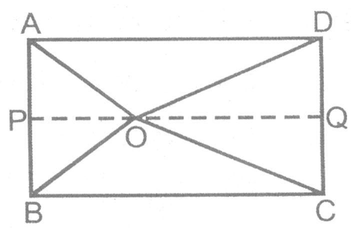
$\Delta ABC \sim \Delta EDA$

$\Rightarrow$ $\frac{BC}{AB} = \frac{AD}{DE}$

$\Rightarrow$ DE.BC = AD.AB. **Hence Proved.**

Ex.12 O is any point inside a rectangle ABCD (shown in the figure). Prove that $OB^2 + OD^2 = OA^2 + OC^2$

Sol. Through O, draw PQ | | BC so that P lies on A and Q lies on DC. **[CBSE - 2006]**



Now, PQ | | BC

Therefore, $PQ \perp AB$ and $PQ \perp DC$ [$\angle B = 90^0$ and $\angle C = 90^0$]

So, $\angle BPQ = 90^0$ and $\angle CQP = 90^0$

Therefore, BPQC and APQD are both rectangles.

Now, from Δ OPB,

$OB^2 = BP^2 + OP^2$ ....(i)

Similarly, from Δ ODQ,

$$OD^2 = OQ^2 + DQ^2 \qquad ....(ii)$$

From Δ OQC, we have

$$OC^2 = OQ^2 + CQ^2 \qquad ...(iii)$$

And form Δ OAP, we have

$$OA^2 = AP^2 + OP^2 \qquad ....(iv)$$

Adding (i) and (ii)

$$OB^2 + OD^2 = BP^2 + OP^2 + OQ^2 + DQ^2$$

$$= CQ^2 + OP^2 + OQ^2 + AP^2$$

[As BP = CQ and DQ = AP]

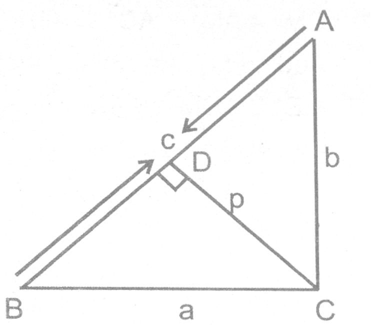
$$= CQ^2 + OQ^2 + OP^2 + AP^2$$

$$= OC^2 + OA^2 \quad \text{[From (iii) and (iv)]}$$

Hence Proved.

Ex.13 ABC is a right triangle, right-angled at C. Let BC = a, CA b, AB = c and let p be the length of perpendicular
form C on AB, prove that

(i) $cp = ab$ (ii) $\dfrac{1}{p^2} = \dfrac{1}{a^2} + \dfrac{1}{b^2}$



Sol. Let CD ⊥ AB. Then CD = p

∴ Area of ΔABC = $\dfrac{1}{2}$ (Base × height)

$= \dfrac{1}{2}$ (AB × CD) $= \dfrac{1}{2}cp$

Also,

Area of Δ ABC = $\dfrac{1}{2}$(BC × AC) = $\dfrac{1}{2}ab$

∴ $\dfrac{1}{2}cp = \dfrac{1}{2}ab$

⇒ CP = AB.

(ii) Since Δ ABC is a right triangle, right angled at C.

∴ $AB^2 = BC^2 + AC^2$

⇒ $c^2 = a.. + b^2$

⇒ $\left(\dfrac{ab}{p}\right)^2 = a^2 + b^2$ $\left[\because cp = ab \Rightarrow c = \dfrac{ab}{p}\right]$

⇒ $\dfrac{a^2b^2}{p^2} = a^2 + b^2$ ⇒ $\dfrac{1}{p^2} = \dfrac{1}{b^2} + \dfrac{1}{a^2}$ ⇒ $\dfrac{1}{p^2} = \dfrac{1}{a^2} + \dfrac{1}{b^2}$

Ex.14 In an equilateral triangle ABC, the side B is trisected at D. Prove that 9 $AD^2 = 7AB^2$.

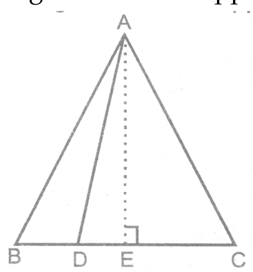
Sol. ABC be can equilateral triangle and D be point on BC such that **[CBSE - 2005]**

$BC = \dfrac{1}{3}BC$ (Given)

Draw AE ⊥ BC, Join AD.

BE = EC (Altitude drown from any vertex of an equilateral triangle bisects the opposite side)

So, $BE = EC = \dfrac{BC}{2}$



In Δ ABC

$$AB^2 = AE^2 + EB^2 \qquad .....(i)$$

$$AD^2 = AE^2 + ED^2 \qquad ....(ii)$$

From (i) and (ii)

$$AB^2 = AD^2 - ED^2 + EB^2$$

$$AB^2 = AD^2 - \frac{BC^2}{36} + \frac{BC^2}{4} \quad \left(\therefore BD + DE = \frac{BC}{2} \Rightarrow \frac{BC}{3} + DE = \frac{BC}{2} \Rightarrow DE = \frac{BC}{6}\right)$$

$$AB^2 + \frac{BC^2}{36} - \frac{BC^2}{4} = AD^2 \qquad \left(\therefore EB = \frac{BC}{2}\right)$$

$$AB^2 + \frac{AB^2}{36} - \frac{AB^2}{4} = AD^2 \qquad (\therefore AB = BC)$$

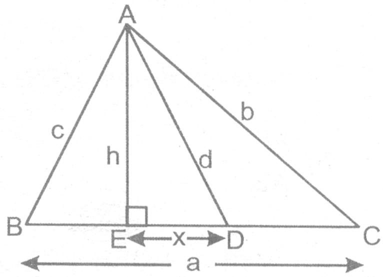
$$\frac{36AB^2 + AB^2 - 9AB^2}{36} = AD^2 \quad \Rightarrow \quad \frac{28AB^2}{36} = AD^2$$

$7AB^2 = 9AD^2$

DAILY PRACTIVE PROBLEMS # 8

OBJECTIVE DPP - 8.1

1. The perimeters of two similar triangles are 25 cm and 15 cm respectively. If one side of first triangle is 9 cm, then the corresponding side of the other triangle is
(A) 6.2 cm (B) 3.4 cm (C) 5.4 cm (D) 8.4 cm



2. In the following figure, $AE \perp BC$, D is the mid point of BC, hen x is equal to

(A) $\frac{1}{a}\left[b^2 - d^2 - \frac{a^2}{4}\right]$ (B) $\frac{h+d}{3}$

(C) $\frac{c+d-h}{2}$ (D) $\frac{a^2+b^2+d^2-c^2}{4}$

3. Two triangles ABC and PQR are similar, if $BC : CA : AB = 1 : 2 : 3$, then $\frac{QR}{PR}$ is

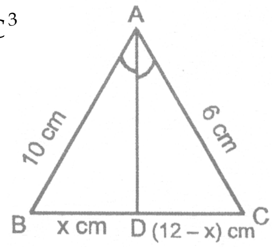
(A) $\frac{2}{3}$ (B) $\frac{1}{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{2}{3}$

4. In a triangle ABC, if angle B = 90^0 and D is the point in BC such that BD = 2 DC, then
(A) $AC^2 = AD^2 + 3\ CD^2$(B) $AC^2 = AD^2 + 5\ CD^2$ C) $AC^2 = AD^2 + 7\ CD^2$ (D) $AC^2 = AB^2 + 5\ BD^2$

5. P and Q are the mid points of the sides AB and BC respectively of the triangle ABC, right-angled at B, then

(A) $AQ^2 + CP^2 = AC^2$ (B) $AQ^2 + CP^2 = \frac{4}{5}AC^2$

(C) $AQ^2 + CP^2 = \frac{5}{4}AC^2$ (D) $AQ^2 + CP^2 = \frac{3}{5}AC^3$



6. In a ΔABC, AD is the bisector of $\angle A$, meeting side BC at D.
If AB = 10 cm, AC = 6 cm, BC = 12 cm, find BD.
(A) 3.3 (B) 18

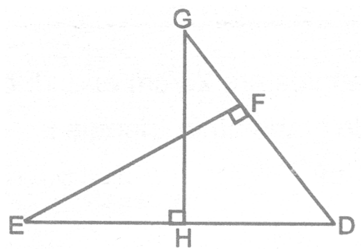
(C) 7.5 (D) 1.33

7. In a triangle ABC, a straight line parallel to BC intersects AB and AC at point D and E respectively. If the area of ADE is one-fifth of the area of ABC and BC = 10 cm, then DE equals
(A) 2 cm (B) $2\sqrt{5}$ cm (C) 4 cm (D) $4\sqrt{5}$ cm

8. ABC is a right-angle triangle, right angled at A . A circle is inscribed in it. The lengths of the two sides containing the right angle are 6 cm and 8 cm, then radius of the circle is
(A) 3 cm (B) 2 cm (C) 4 cm (D) 8 cm

SUBJECTIVE DPP - 8.2

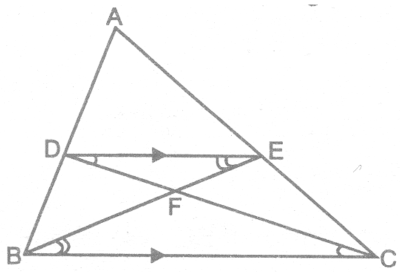
1. Given $\angle GHE = \angle DFE = 90^0$, DH = 8, DF = 12, DG = 3x - 1 and DE = 4x + 2.



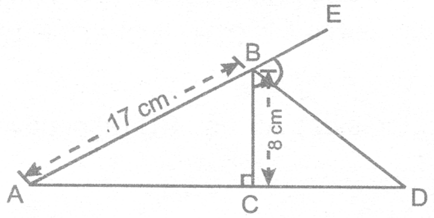
Find the lengths of segments DG and DE.

2. In the given **figure,** DE is parallel to the base BC of triangle ABC and AD : DB = 5 : 3. Find the ratio : -

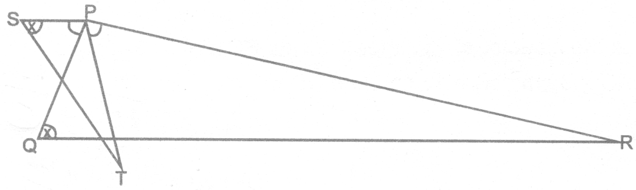
(i) $\frac{AD}{AB}$ (ii) $\frac{\text{Area of } \Delta DEF}{\text{Area of } \Delta CFB}$ **[CBSE - 2000]**



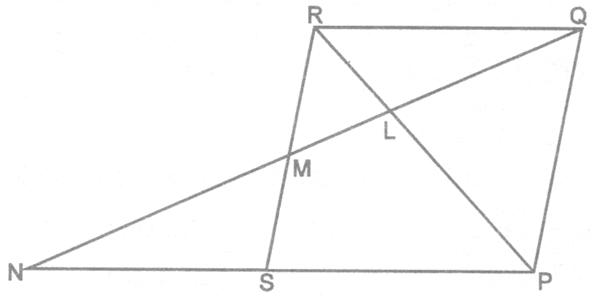
3. In **Figure,** ΔABC is a right-angled triangle, where $\angle ACB = 90^0$. The external bisector BD of $\angle ABC$ meets AC produced at D. If AB = 17 cm and BC = 8 cm, find the AC and BD.



4. In **figure,** $\angle QPS = \angle RPT$ and $\angle PST = \angle PQR$. Prove that $\Delta PST \sim \Delta PQR$ and hence find the ratio ST : PT, if PR : R = 4 : 5.



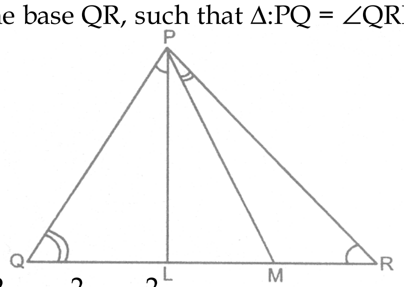
5. In the **figure,** PQRS is a parallelogram with PQ = 16 cm and QR = 10 cm. L is a point on PR such that RL : LP = 2 : 3. QL produced meets RS at M and PS produced at N.



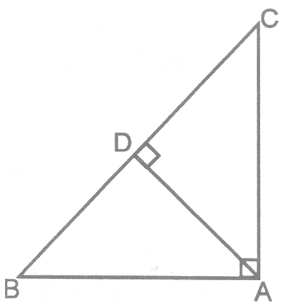
Find the lengths of PN and RM.

6. In Δ ABC, D and E are points on AB and AC respectively such that DE | | BC. If AD = 2.4 cm, AE = 3.2 cm, DE = 2 cm and BC = 5 cm, find BD and CE.

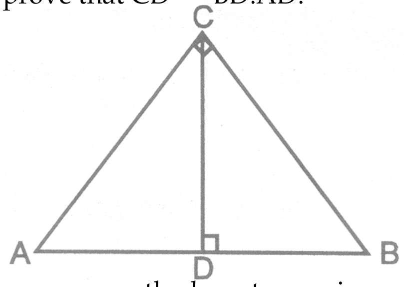
7. In a triangle PQR, L an DM are two points on the base QR, such that Δ:PQ = $\angle QRP$ and $\angle RPM = \angle RQP$. Prove that :
(i) $\Delta PQL \sim \Delta RPM$
(ii) QL × RM = PL × PM
(iii) $PQ^2 = QR \times QL$



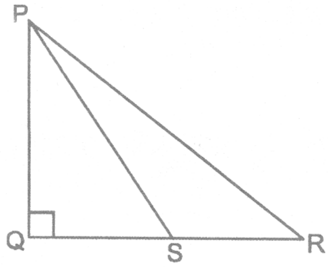
8. In **figure,** $\angle BAC = 90^0$, $AD \perp BC$. prove that $AB^2 = BD^2 - CD^2$.



9. In **figure,** $\angle ACB = 90^0$, $CD \perp AB$ prove that $CD^2 = BD.AD$.

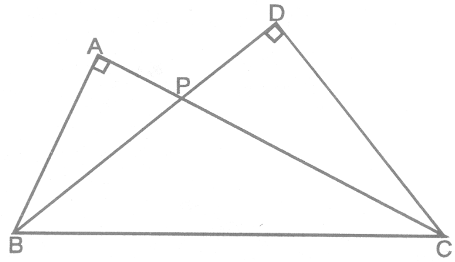


10. In a right triangle, prove that the square on the hypotenuse is equal to sum of the squares on the other two sides.
Using the above result, prove the following:
In **figure** PQR is a right triangle, right angled at Q. If QS = SR, show that $PR^2 = 4PS^2 - 3PQ^2$.

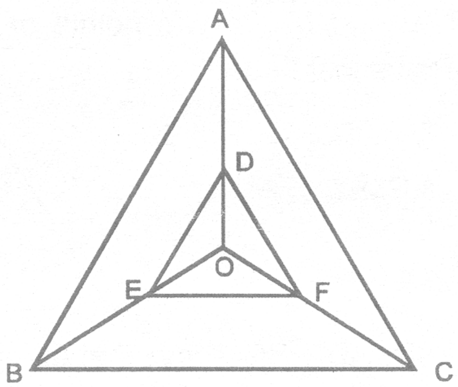


11. In Δ ABC, $\angle ABC = 135^0$. Prove that $AC^2 = AB^2 + BC^2 + 4ar\ (\Delta\ ABC)$.

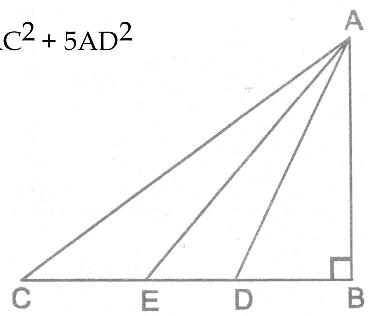
12. In **figure,** ABC and DBC are two right triangles with the common hypotenuse BC and with their sides AC and DB intersecting at P. Prove that AP × PC = DP × PB. **[CBSE - 2000]**



13. Any point O, inside ΔABC, in joined to its vertices. From a point D on AO, DE is drawn so that DE| |AB and EF| |BC as shown in figure. Prove that DF| |AC. **[CBSE-2002]**



14. In **figure,** D and E trisect BC. Prove that $8AE^2 = 3AC^2 + 5AD^2$ **[CBSE - 2006]**



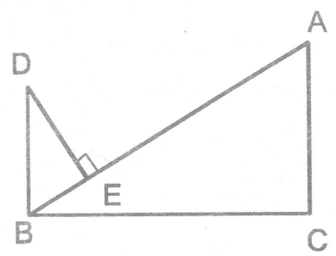
15. The perpendicular AD on the base BC of a ΔABC meets BC at D so that 2DB = 3CD. Prove that $5AB^2 = 5AC^2 + BC^2$. **[CBSE - 2007]**

16. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares on their corresponding sides.
Using the above, do the following :The diagonals of a trapezium ABCD, with AB || DC, intersect each other point O. If AB = 2 CD, find the ratio of the area of ΔAOB to the area of ΔCOD **[CBSE - 2008]**

17. D, E and F are the mid-points of the sides AB, BC and CA respectively of ΔABC. Find $\frac{ar(\Delta DEF)}{ar(\Delta ABC)}$. **[- 2008]**

18. D and E are points on the sides CA and CB respectively of ΔABC right-angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$.

19. In figure, $DB \perp BC$, $DE \perp AB$ and $AC \perp BC$. Prove that $\frac{BE}{DE} = \frac{AC}{BC}$ **[CBSE - 2008]**



ANSWERS

(Objective DPP # 8.1)

Que.	1	2	3	4	5	6	7	8
Ans.	C	A	B	B	C	C	B	B

(Subjective DPP # 8.2)

1. 20 unit & 30 unit **2.** (i) $\frac{5}{8}$ (ii) $\frac{25}{64}$

3. 15 cm., $\frac{8\sqrt{34}}{3}$ cm. **4.** 5 : 4 **5.** PN = 15 cm, RM = 10.67 cm.

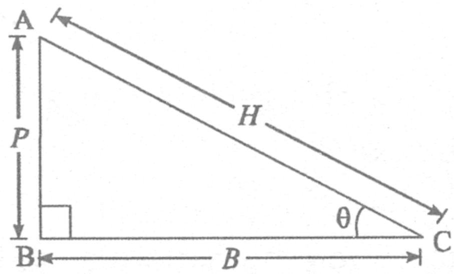
6. DB = 3.6 cm, CE = 4.8 cm **16.** 4 : 1 **17.** 1 : 4

➤➤➤TRIGONOMETRY⮜⮜⮜

11.1 TRIGONOMETRY :

Trigonometry means, the science which deals with the measurement of triangles.

11.1 (a) Trigonometric Ratios :



A right angled triangle is shown in **Figure.** $\angle B$ Is of **90^0** Side opposite to $\angle B$ be called **hypotenuse.** There are two other angles i.e. $\angle A$ and $\angle C$. It we consider $\angle C$ as θ, then opposite side to this angle is called **perpendicular** and side adjacent to θ is called base.

(i) **Six Trigonometry Ratio are :**

$$\sin\theta = \frac{\text{Perpenicular}}{\text{Hypotenuse}} = \frac{P}{H} = \frac{AB}{AC} \qquad \cos\text{es}\theta = \frac{\text{Hypoteuse}}{\text{Perpendicular}} = \frac{H}{P} = \frac{AC}{AB}$$

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{B}{H} = \frac{BC}{AC} \qquad \sec\theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{H}{B} = \frac{AC}{BC}$$

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{P}{B} = \frac{AB}{BC} \qquad \cot\theta = \frac{\text{Base}}{\text{Parpendicular}} = \frac{B}{P} = \frac{BC}{AB}$$

(ii) Interrelationship is Basic Trigonometric Ratio :

$$\tan\theta = \frac{1}{\cot\theta} \quad \Rightarrow \quad \cot\theta = \frac{1}{\tan\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \quad \Rightarrow \quad \sec\theta = \frac{1}{\cos\theta}$$

$$\sin\theta = \frac{1}{\cos\text{ec}\theta} \quad \Rightarrow \quad \cos\text{ec}\theta = \frac{1}{\sin\theta}$$

We also observe that

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \quad \Rightarrow \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

11.1 (b) Trigonometric Table :

$\theta \rightarrow$	0	30^0	45^0	60^0	90^0
Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
Cot	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
Sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
Cosec	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

11.1 (c) Trigonometric Identities :

(i) $\sin^2\theta + \cos^2\theta = 1$
(A) $\sin^2\theta = 1 - \cos^2\theta$
(B) $\cos^2\theta = 1 - \sin 2\theta$

(ii) $1 + \tan^2\theta = \sec^2\theta$
(A) $\sec^2\theta - 1 = \tan^2\theta$
(B) $\sec^2\theta - \tan^2\theta = 1$
(C) $\tan^2\theta - \sec^2\theta = -1$

(iii) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$
(A) $\operatorname{cosec}^2\theta - 1 = \cot^2\theta$
(B) $\operatorname{cosec}^2\theta - \cot^2\theta = 1$
(C) $\cot^2\theta - \operatorname{cosec}^2\theta = -1$

11.1 (d) Trigonometric Ratio of Complementary Angles :

$\sin(90-\theta) = \cos\theta$ $\qquad$ $\cos(90-\theta) = \sin\theta$

$\tan(90-\theta) = \cot\theta$ $\qquad$ $\cot(90-\theta) = \tan\theta$

$\sec(90-\theta) = \operatorname{cosec}\theta$ $\qquad$ $\operatorname{cosec}(90-\theta) = \sec\theta$

ILLUSTRATIONS :

EX.1 In the given triangle AB = 3 cm and AC = 5 cm. Find all trigonometric ratios.

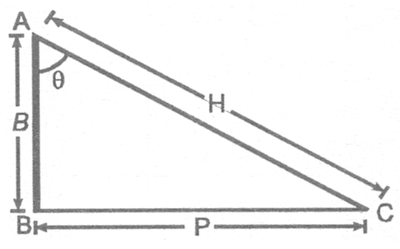
Sol. Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$\Rightarrow$ $5^2 = 3^2 + p^2$

$\Rightarrow$ $16 = p^2$ $\Rightarrow$ P = cm

Here P = 4 cm, B = 3 cm, H = 5 cm



$\therefore$ $\sin\theta = \frac{P}{H} = \frac{4}{5}$

$\cos\theta = \frac{B}{H} = \frac{3}{5}$

$\tan\theta = \frac{P}{B} = \frac{4}{3}$

$\cot\theta = \frac{B}{P} = \frac{3}{4}$

$\sec\theta = \frac{H}{B} = \frac{5}{3}$

$\operatorname{cosec}\theta = \frac{H}{P} = \frac{5}{4}$

Ex.2 If $\tan\theta = \frac{m}{n}$, then find $\sin\theta$.

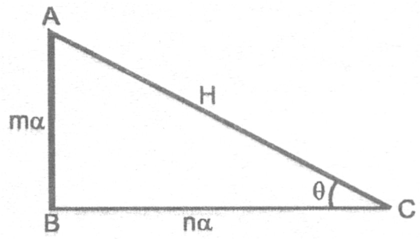
Sol. Let P = $m\alpha$ and B = $n\alpha$

$\therefore \quad \tan\theta = \frac{P}{B} = \frac{m}{n}$

$H^2 = P^2 + B^2$

$H^2 = m^2\alpha^2 + n^2\alpha^2$

$H = \alpha\sqrt{m^2 + n^2}$



$\therefore \quad \tan\theta = \frac{P}{H} = \frac{ma}{a\sqrt{m^2+n^2}}$

$\sin\theta = \frac{m}{\sqrt{m^2+n^2}}$

Ex.3 If cosec $A = \frac{13}{5}$ the prove than $\tan^2 A - \text{sing}^2 A = \sin^4 A \sec^2 A$.

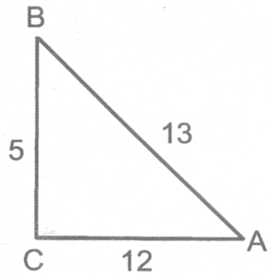
Sol. We hare coses $A = \frac{13}{5} = \frac{\text{Hypotenuse}}{\text{Perpendicular}}$

So, we draw a right triangle ABC, right angled at C such that hypotenuse AB = 13 units and perpendicular
BC = 5 units
B Pythagoras theorem,

$AB^2 = BC^2 + AC^2 \Rightarrow (13)^2 = (5)^2 + AC^2$

$AC^2 = 169 - 25 = 144$

$AC = \sqrt{144} = 12$ units



$\tan A = \frac{BC}{AC} = \frac{5}{12}$

$\sin A = \frac{BC}{AB} = \frac{5}{13}$

and $\sec A = \frac{AB}{AC} = \frac{13}{12}$

L.H.S. $\tan^2 A - \text{Sin}^2 A$

$= \left(\frac{5}{12}\right)^2 - \left(\frac{5}{13}\right)^2$

$= \frac{25}{144} - \frac{25}{169}$

$= \frac{25(169-144)}{144\times169}$

$= \frac{25\times25}{144\times169}$

R.H.S. $= \sin^4 A \times \sec^2 A$

$= \left(\frac{5}{13}\right)^4 \times \left(\frac{13}{12}\right)^2$

$= \frac{5^4\times13^2}{13^4\times12^2}$

$= \frac{5^4}{13^2\times12^2}$

$= \frac{25\times25}{144\times169}$

So, L.H.S. = R.H.S. **Hence Proved.**

Ex.4 In Δ ABC, right angled at B. AC + AB = 9 cm. Determine the value of cot C, cosec C, sec C.

Sol. In Δ ABC, we have

$(AC)^2 = (AB)^2 + BC^2$

$\Rightarrow \quad (9 - AB)^2 = AB^2 + (3)^2 \qquad [\because AC + AB = 9\text{cm} \Rightarrow AC = 9 - AB]$

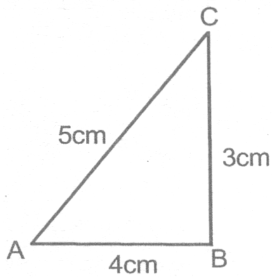
$\Rightarrow \quad (81 + AB^2 - 18AB = AB^2 + 9$

$\Rightarrow \quad 72 - 18\,AB = 0$

$\Rightarrow \quad AB = \frac{72}{18} = 4\text{cm}.$

Now, AC + AB = 9 cm

AC = 9 - 4 = 5 cm



So, $\cot C = \frac{BC}{AB} = \frac{3}{4}$, $\text{cosec}\, C = \frac{AC}{AB} = -\frac{5}{4}$, $\sec C = \frac{AC}{BC} = \frac{5}{3}$.

Ex.5 Given that cos (A - B) = cos A cos B + Sin B, find the value of cos 15^0.

Sol. Putting A = 45^0 and B = 30^0

We get $\cos(45^0 - 30^0) = \cos 45^0 \cos 30^0 + \sin 45^0 \sin 30^0$

$\Rightarrow \quad \cos 15^{0\cdot} = \frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\times\frac{1}{2}$

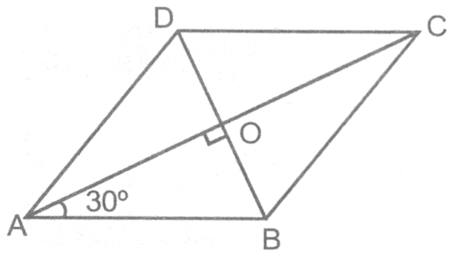
$\Rightarrow \quad \cos 15^0 = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$

Ex.6 A Rhombus of side of 10 cm has two angles of 60^0 each. Find the length of diagonals and also find its area.

Sol. Let ABCD be a rhombus of side 10 cm and $\angle BAD = \angle BCD = 60^0$. Diagonals of parallelogram bisect each other.

S, AO = OC and BO = OD

In right triangle AOB



$\sin 30^0 = \dfrac{OB}{AB}$ $\qquad\qquad$ $\cos 30^0 = \dfrac{OA}{AB}$

$\Rightarrow \quad \dfrac{1}{2} = \dfrac{OB}{10}$ $\qquad\qquad$ $\Rightarrow \quad \dfrac{\sqrt{3}}{2} = \dfrac{OA}{10}$

$\Rightarrow$ OB = 5 cm $\qquad\qquad$ $\Rightarrow$ $OA = 5\sqrt{3}$

$\therefore$ BD = 2 (OB) $\qquad\qquad$ $\Rightarrow$ AC = 2(OA)

$\Rightarrow$ BD = 2(5) $\qquad\qquad$ $\Rightarrow$ AC = 2 $(5\sqrt{3})$

$\Rightarrow$ BD = 10 cm $\qquad\qquad$ $\Rightarrow$ AC = $10\sqrt{3}$ cm

So, the length of diagonals AC = $10\sqrt{3}$ cm & BD = 10 cm

Area of Rhombus $= \dfrac{1}{2} \times AC \times BD$

$= \dfrac{1}{2} \times 10\sqrt{3} \times 10 \qquad = 50\sqrt{3}\text{cm}^2.$

Ex.7 Evaluate : $\dfrac{\sec^2 54^0 - \cot^2 36^0}{\operatorname{cosec}^2 57^0 - \tan^2 33^0} + 2\sin^2 38^0 \sec^2 52^0 - \sin^2 45^0 + \dfrac{2}{\sqrt{3}} \tan 17^0 \tan 60^0 \tan 73^0$

Sol. $\dfrac{\sec^2 54^0 - \cot^2 36^0}{\operatorname{cosec}^2 57^0 - \tan^2 33^0} + 2\sin^2 38^0 \sec^2 52^0 - \sin^2 45^0 + \dfrac{2}{\sqrt{3}} \tan 17^0 \tan 60^0 \tan 73^0$

$= \dfrac{\sec^2(90^0 - 36^0) - \cot^2 36^0}{\operatorname{cosec}^2(90^0 - 33^0) - \tan^2 33^0} + 2\sin^2 38^0 \sec^2(90^0 - 38^0) - \sin^2 45^0 + \dfrac{2}{\sqrt{3}} \tan(90^0 - 73^0)\tan 73^0 \tan 60^0$

$= \dfrac{\operatorname{cosec}^2 36^0 - \cot^2 36^0}{\sec^2 33^0 - \tan^2 33^0} + 2\sin^2 38^0 \operatorname{cosec}^2 38^0 - \left(\dfrac{1}{\sqrt{2}}\right)^2 + \dfrac{2}{\sqrt{3}} \cot 73^0 \tan 73^0 \times \sqrt{3}$

$= \dfrac{1}{1} + 2\sin^2 38^0 \times \dfrac{1}{\sin^2 38^0} - \dfrac{1}{2} + \dfrac{2}{\sqrt{3}} \times \dfrac{1}{\tan 73^0} \times 73^0 \times \sqrt{3} [\because \operatorname{cosec}^2\theta - \cot^2\theta = 1, \sec^2\theta - \tan^2\theta = 1]$

$= 1 + 2 - \dfrac{1}{2} + 2 = 5 - \dfrac{1}{2} \qquad = \dfrac{9}{2}.$

Ex.8 Prove that : $\operatorname{cosec}(65^0 + \theta) - \sec(25^0 - \theta) - \tan(55^0 - \theta) + \cot(35^0 + \theta) = 0$

Sol. $\operatorname{cosec}(65^0 + \theta) = \operatorname{cosec}\{90^0 - (25^0 - \theta)\} = \sec(25^0 - \theta)$ ....(i)

$\cot(35^0 + \theta) = \cot\{90^0 - (55^0 - \theta)\} = \tan(55^0 - \theta)$ .....(ii)

$\therefore$ **L.H.S.** $\operatorname{cosec}(65^0 + \theta) - \sec(25^0 - \theta) - \tan(55^0 - \theta) + \cot(35^0 + \theta)$

$= \sec(25^0 - \theta) - \sec(25^0 - \theta) - \tan(55^0 - \theta) + \tan(55^0 - \theta)$

$= 0$ [using (i) & (ii)] **R.H.S.**

Ex.9 Prove that : $\cot\theta - \tan\theta = \dfrac{2\cos^2\theta - 1}{\sin\theta\cos\theta}$

Sol. **L.H.S.** $\cot\theta - \tan\theta$

$= \dfrac{\cos\theta}{\sin\theta} - \dfrac{\sin\theta}{\cos\theta}$ $\qquad$ $\left[\because \cot\theta = \dfrac{\cos\theta}{\sin\theta}, \tan\theta = \dfrac{\sin\theta}{\cos\theta}\right]$

$= \dfrac{\cos^2\theta - \sin^2}{\sin\theta\cos\theta} \qquad = \dfrac{\cos^2\theta - (1 - \cos^2\theta)}{\sin\theta\cos\theta}$ $\qquad$ $[\because \sin^2\theta = 1 - \cos^2\theta]$

$= \dfrac{\cos^2\theta - 1 + \cos^2\theta}{\sin\theta\cos\theta} \qquad = \dfrac{2\cos^2\theta - 1}{\sin\theta\cos\theta}$ $\qquad$ **R.H.S.** $\qquad$ **Hence Proved.**

Ex.10 Prove that : (coses A – sin A) (sec A – cos A) (tan A + cot A) = 1.

Sol. L.H.S. (cosec A – sin A) (sec A – cos A) (tan A + cot A)

$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)\left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)$

$= \left(\frac{1-\sin^2 A}{\sin A}\right)\left(\frac{1-\cos^2 A}{\cos A}\right)\left(\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}\right)$

$= \left(\frac{\cos^2 A}{\sin A}\right)\left(\frac{\sin^2 A}{\cos A}\right)\left(\frac{1}{\sin A \cos A}\right)$ $[\because \sin^2 A + \cos^2 A = 1]$

$= 1$ **R.H.S.** **Hence Proved.**

Ex. 11 If $\sin\theta + \cos\theta = m$ and $\sec\theta + \operatorname{cosec}\theta = n$, then prove that $n(m^2 - 1) = 2m$.

Sol. **L.H.S.** $n(m^2 - 1)$

$= (\sec\theta + \operatorname{cosec}\theta)[(\sin\theta + \cos\theta)^2 - 1]$ $= \left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right)(\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1)$

$= \left(\frac{\cos\theta + \sin\theta}{\sin\theta\cos\theta}\right)(1 + 2\sin\theta\cos\theta - 1)$ $= \frac{(\cos\theta + \sin\theta)}{\sin\theta\cos\theta}(2\sin\theta\cos\theta)$

$= 2(\sin\theta + \cos\theta)$ $= 2m$ **R.H.S.** **Hence Proved.**

Ex.12 If $\sec\theta = x + \frac{1}{4x}$, then prove that $\sec\theta + \tan\theta = 2x$ or $\frac{1}{2x}$.

Sol. $\sec\theta = x + \frac{1}{4x}$ ....(i)

$\therefore$ $1 + \tan^2\theta = \sec^2\theta$

$\Rightarrow$ $\tan^2\theta = \sec^2\theta - 1$ $\Rightarrow$ $\tan^2\theta = \left(x + \frac{1}{4x}\right)^2 - 1$

$\Rightarrow$ $\tan^2\theta = x^2 + \frac{1}{16x^2} + 2\times x\times\frac{1}{4x} - 1$

$\Rightarrow$ $\tan^2\theta = x^2 + \frac{1}{16x^2} + \frac{1}{2} - 1$ $\Rightarrow$ $\tan\theta = \pm\left(x - \frac{1}{4x}\right)$

$\Rightarrow$ $\tan^2\theta = \left(x - \frac{1}{4x}\right)^2$ $\Rightarrow$ $\tan\theta = \pm\left(x - \frac{1}{4x}\right)$

So, $\tan\theta = x - \frac{1}{4x}$ ....(ii) or $\tan\theta = -\left(x - \frac{1}{4x}\right)$ ....(iii)

Adding equation (i) and (ii)

$\sec\theta + \tan\theta = x + \frac{1}{4x} + x - \frac{1}{4x}$

$\sec\theta + \tan\theta = 2x$

Adding equation (i) and (ii)

$\sec\theta + \tan\theta = x + \frac{1}{4x} - x + \frac{1}{4x}$

$= \frac{1}{2x}$ Hence, $\sec\theta + \tan\theta + 2x$ or $\frac{1}{2x}$.

Ex.13 If θ is an acute angle and $\tan\theta + \cot\theta = 2$ find the value of $\tan^9\theta + \cot^9\theta$

Sol. We have, $\tan\theta + \cot\theta = 2$

$\Rightarrow$ $\tan\theta + \frac{1}{\tan\theta} = 2$ $\Rightarrow$ $\frac{\tan^2\theta + 1}{\tan\theta} = 2$

$\Rightarrow$ $\tan^2\theta + 1 = 2\tan\theta$ $\Rightarrow$ $\tan^2\theta - 2\tan\theta + 1 = 0$

$\Rightarrow$ $(\tan\theta - 1)^2 = 0$ $\Rightarrow$ $\tan\theta - 1 = 0$

$\Rightarrow$ $\tan\theta = 1$ $\Rightarrow$ $\tan\theta = \tan 45^0$ $\Rightarrow$ $\theta = 45^0$

$\therefore$ $\tan^9\theta + \cot^9\theta$

$=$ $\tan^9 45^0 + \cot^9 45^0$ $=$ $(\tan 45)^9 + (\cot 45)^0$

$=$ $(1)^9 + (1)^9$ $=$ 2.

DAILY PRACTIVE PROBLEMS # 11

OBJECTIVE DPP - 11.1

1. If $\alpha+\beta=\frac{\pi}{2}$ and $\alpha=\frac{1}{3}$, then sin β is

(A) $\frac{\sqrt{2}}{3}$ (B) $\frac{2\sqrt{2}}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$

2. If 5 $\tan\theta=4$, then value of $\frac{5\sin\theta-3\cos\theta}{5\sin\theta+2\cos\theta}$ is

(A) $\frac{1}{3}$ (B) $\frac{1}{6}$ (C) $\frac{4}{5}$ (D) $\frac{2}{3}$

3. If 7 $\sin\alpha=24\ \cos\alpha; 0<\alpha<\frac{\pi}{2}$, then value of 14 $\tan\alpha-75\cos\alpha-7\sec\alpha$ is equal to

(A) 1 (B) 2 (C) 3 (D) 4

4. Given 3 $\beta+5\ \cos\ \alpha;\ \beta=5$, then the value of $(3\ \cos\ \beta-5\sin\beta)^2$ is equal to

(A) 9 (B) $\frac{9}{5}$ (C) $\frac{1}{3}$ (D) $\frac{1}{9}$

5. If $\tan\theta=4$, then $\left(\frac{\tan\theta}{\frac{\sin^3\theta}{\cos\theta}+\sin\theta\cos\theta}\right)$ is equal to

(A) 0 (B) $2\sqrt{2}$ (C) $\sqrt{2}$ (D) 1

6. The value of tan 5^0 tan 10^0 tan 15^0 20^0 .... tan 85^0, is

(A) 1 (B) 2 (C) 3 (D) None of these

7. As x increases from 0 to $\frac{\pi}{2}$ the value of cos x

(A) increases (B) decreases (C) remains constant (D) increases, then decreases

8. Find the value of x from the equation x sin $\frac{\pi}{6}\cos^2\frac{\pi}{4}=\frac{\cot^2\frac{\pi}{6}\sec\frac{\pi}{3}\tan\frac{\pi}{4}}{\text{cos ec}^2\frac{\pi}{4}\text{cos ec}\frac{\pi}{6}}$

(A) 4 (B) 6 (C) - 2 (D) 0

9. The area of a triangle is 12 sq. cm. Two sides are 6 cm and 12 cm. The included angle is

(A) $\cos^{-1}\left(\frac{1}{3}\right)$ (B) $\cos^{-1}\left(\frac{1}{6}\right)$ (C) $\sin^{-1}\left(\frac{1}{6}\right)$ (D) $\sin^{-1}\left(\frac{1}{3}\right)$

10. If $\alpha+\beta=90^0$ and $\alpha=2\beta$ then $\cos^2\alpha+\sin^2\beta$ equals to

(A) $\frac{1}{2}$ (B) 0 (C) 1 (D) 2

SBJECTIVE DPP - 11.2

1. Evaluate : (A) $\frac{\sin\theta\cos\theta\sin(90^0-\theta)}{\cos(90^0-\theta)}+\frac{\cos\theta\sin\theta\cos(90^0-\theta)}{\sin(90^0-\theta)}+\frac{\sin^2 27^0+\sin^2 63^0}{\cos^2 40^0+\cos^2 50^0}$

(B) $\cos 10^0\cos 2^0\cos 3^0--------\cos 180^0$

C) $\sin(50^0+\theta)-\cos(40^0-\theta)+\tan 1^0\tan 20^0\tan 70^0\tan 80^0\tan 89^0$

(D) $\frac{2}{3}(\cos^4 30^0-\sin^4 45^0)-3(\sin^2 60^0-\sec^2 45^0)+\frac{1}{4}\cot^2 30^0$

(E) $\frac{\cos^2 20^0+\cos^2 70^0}{\sec^2 50-\cot^2 40^0}+2\,\text{cos ec}^2 58^0-2\cot 58^0\tan 32^0-4\tan 13^0\tan 37^0\tan 45^0\tan 53^0\tan 77^0$

2. If cot $\theta=\frac{3}{4}$, prove that $\sqrt{\frac{\sec\theta-\text{cos ec}\theta}{\sec\theta+\text{cos ec}\theta}}=\frac{1}{\sqrt{7}}$.

3. If A + B = 90^0, prove that : $\sqrt{\frac{\tan A\tan B+\tan A\cot B}{\sin A\sec B}-\frac{\sin^2 B}{\cos^2 A}}=\tan A$

4. If A, B, C are the interior angles of a ΔABC, show that :

(i) $\sin\frac{B+C}{2}=\cos\frac{A}{2}$ (ii) $\cos\frac{B+C}{2}=\sin\frac{A}{2}$

Prove the following (Q, 5 to Q. 13)

5. $\tan^2\theta-\sin^2\theta=\tan^2\theta\sin^2\theta$
6. $(\sin\theta+\operatorname{cosec}\theta)+(\cos\theta+\sec\theta)^2=7+\tan^2\theta+\cot^2\theta$ **[CBSE - 2008]**
7. $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=\sec\theta\operatorname{cosec}\theta+1$
8. $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}=\sec\theta-\tan\theta$
9. $\frac{\sin A+\cos A}{\sin A+\cos A}+\frac{\sin A-\cos A}{\sin A+\cos A}=\frac{2}{\sin^2 A-\cos^2 A}=\frac{2}{1-2\cos^2 A}$
10. $(\sin\theta+\sec\theta)^2+(\cos\theta+\operatorname{cosec}\theta)^2=(1+\sec\theta\operatorname{cosec}\theta)^2$
11. $(1+\cot\theta-\operatorname{cosec}\theta)(1+\tan\theta+\sec\theta)=2$
12. $(\sin^8\theta-\cos^8\theta)=(\sin^2\theta-\cos^2\theta)(1-2\sin^2\theta\cos^2\theta)$
13. $\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}=\frac{1+\sin\theta}{\cos\theta}$
14. If $x=r\sin\theta\cos\phi,\ y=r\sin\theta\sin\phi,\ z=r\cos\theta$, then Prove that : $x^2+y^2+z^2=r^2$.
15. If $\cot\theta+\tan\theta=x$ and $\sec\theta-\cos\theta=y$, then prove that $(x^2y)^{2/3}-(xy^2)^{2/3}=1$
16. If $\sec\theta+\tan\theta=p$, then show that $\frac{p^2-1}{p^2+1}=\sin\theta$ **[CBSE - 2004]**
17. Prove that : $\tan^2 A-\tan^2 B=\frac{\cos^2 B-\cos^2 A}{\cos^2 B\cos^2 A}=\frac{\sin^2 A-\sin^2 B}{\sin^2 A\sin^2 B}$ **[CBSE - 2005]**
18. Prove that : $\frac{1}{\sec x-\tan x}-\frac{1}{\cos x}=\frac{1}{\cos x}=\frac{1}{\cos x}-\frac{1}{\sec x+\tan x}$ **[CBSE - 2005]**
19. Prove : $(1+\tan^2 A)+\left(1+\frac{1}{\tan^2 A}\right)=\frac{1}{\sin^2 A-\sin^4 A}$ **[CBSE - 2006]**
20. Evaluate :

$\tan 7^0\tan 23^0\tan 60^0\tan 67^0\tan 83^0+\frac{\cot 54^0}{\tan 36^0}+\sin 20^0\sec 70^0-2.$ **[CBSE - 2007]**

21. Without using trigonometric tables, evaluate the following :
$(\sin^2 65^0+\sin^2 25^0)+\sqrt{3}\,(\tan 5^0\tan 15^0\tan 30^0\tan 75^0\tan 85^0)$ **[CBSE - 2008]**
22. If $\sin 3\theta=\cos(\theta-60^0)$ and 3θ and $\theta-60^0$ are acute, find the value of θ **[CBSE - 2008]**
23. If $\sin\theta=\cos\theta$, find the value of θ. **[CBSE - 2008]**
24. If $7\sin^2\theta+3\cos^2\theta=4$, show that $\tan\theta=\frac{1}{\sqrt{3}}$ **[CBSE - 2008]**
25. Prove : $\sin\theta(1+\tan\theta)+\cos\theta(1+\cot\theta)=\sec\theta+\operatorname{cosec}\theta$. **[CBSE - 2008]**

ANSWERS

(Objective DPP - 11.1)

Qus.	1	2	3	4	5	6	7	8	9	10
Ans.	B	B	B	A	D	A	B	B	D	A

(Subjective DPP 11.2)

1. (A) 2 (B) 0 (C) 1 (D) $\frac{113}{24}$ (E) -1

20 $\sqrt{3}$ 21. 2 22. 24^0 23. 45^0

Teko CLASSES ..... the support

➤➤➤STATISTICS➤➤➤

14.1 INTRODUCTION :

The branch of science known as **statistics** has been used in India from ancient times. Statistics deals with
collection of numerical facts. i.e., data, their classification & tabulation and their interpretation.

14.2 MEASURES OF CENTRAL TENDANCY :

The commonly used measure of central tendency (or averages) are :

(i) **Arithmetic Mean (AM)** or **Simply Mean** (ii) **Median** (jjj) **Mode**

14.3 ARITHMETIC MEAN : Arithmetic mean of a set of observations is equal to their sum divided by the total number of observations.

Mean of raw data : $x_1, x_2, x_3, \ldots, x_n$ are the **n** values (or observations) the,

A.M. (Arithmetic mean) is

$$\bar{x} = \frac{x_1 + x_1 + \ldots + x_n}{n} = \frac{\sum_{i=1}^{n} x_i}{n} \qquad n\bar{x} \text{ - Sum of observations} = \frac{\sum_{i=1}^{n} x_i}{n}$$

i.e. product of mean & no. of items gives sum of observation.

Ex.1 The mean of marks scored by 100 students was found to be 40. Later on its was discovered that a score of 56 was misread as 83. Find the correct mean.

Sol. $n = 100,\ \bar{x} = 40$

$$\bar{x} = \frac{1}{n}\left(\sum x_i\right) \quad \Rightarrow \quad 40 = \frac{1}{100}\left(\sum x_i\right)$$

$\therefore$ Incorrect value of $\sum x_i = 4000.$ Now, Correct value of $\sum x_i = 4000 - 83 + 83 = 3970$

$\therefore$ Correct mean $= \dfrac{\text{correct value of } \sum x_i}{n} = \dfrac{3970}{100} = 39.7$ So, the correct mean is 39.7

Method for Mean of Ungrouped Data

x_i	f_i	F_1x_1
x_1	f_1	f_1x_1
x_2	f_2	f_2x_2
x_3	f_3	$f3x_3$
.	.	.
.	.	.
.	.	
	$\sum f_i =$	$\sum f_1x_1 =$

Grouped Frequency Distribution (Grouped)

(i) Direct method : for finding mean mean $\bar{x} = \dfrac{\sum f_i x_i}{\sum f_u}$

Ex2. Find the missing value of P for the following distribution whose mean is 12.58

x	5	8	10	12	P	20	25
y	2	5	8	22	7	4	2

Sol. Given $\bar{x}$ = 12.58 **Calculation of Mean :**

x_i	f_i	f_ix_i
5	2	10
8	5	40
10	8	80
12	22	264
P	7	7P
20	4	80
25	2	50
	$\sum f_i = 50$	$\sum f_ix_u = 524 + 7P$

$$\bar{x} = \frac{\sum f_ix_i}{\sum f_i}$$

$12.58 = \frac{524+7P}{}50$ 629 = 524 + 7P ; 7P = 105 ; P = 15.

Ex.3 Find the mean for the following distribution :

Marks	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	6	8	13	7	3	2	1

Sol.

Marks	Mid Values x_i	No. of students f_i	f_ix_i
10-20	15	6	90
20-30	25	8	200
30-40	35	13	455
40-50	45	7	345
50-60	55	3	165
60-70	65	2	130
70-80	75	1	75
		$\sum f_i = 40$	$\sum f_ix_i = 1430$

$$\bar{x} = \frac{\sum f_ix_i}{\sum f_i} = \frac{1430}{40} = \frac{143}{4} = 35.75$$

(ii) Deviation Method : (Assumed Mean Method)

$$\bar{x} = A + \frac{\sum f_id_i}{\sum f_i}$$

where, A = Assumed mean d_i = Deviation from mean (x_i - A)

Find the mean for the following distribution by using deviation method :

x_i	15	20	22	24	25	30	33	38
Frequency	5	8	11	20	23	18	13	2

Sol.

x_i	f_i	Let A = 25 $d_i = x_i - 25$	f_id_i
15	5	-10	-50
20	8	-5	-40
22	11	-3	-33

24	20	-1	-20
25	23	0	0
30	18	5	90
33	13	8	104
38	2	13	26
	$\sum f_i = 100$		$\sum f_i d_i = 77$

$$\bar{x} = A + \frac{\sum f_i d_i}{\sum f_i} = 25 + \frac{77}{100} = 25.77$$

(iii) Step - Deviation Method : $\bar{x} = A + \left(\frac{\sum f_i u_i}{\sum f_i}\right)h$

where, A = Assumed mean $u_i = \frac{x_i - A}{h}$, h = Width of class interval

Ex.5 Find the mean of following distribution with step - deviation method :

Class	10-15	15-20	20-25	25-30	30-35	35-40
Frequency	5	6	8	12	6	3

Sol. **Calculation of Mean :**

Class	x_i	f_i	Let A = 27.5 $u_i = \frac{x_i - 27.5}{5}$	$f_i u_i$
10-15	12.5	5	-3	-15
15-20	17.5	6	-2	-12
20-25	22.5	8	-1	-8
25-30	27.5	12	0	0
30-35	32.5	6	1	6
35-40	37.5	3	2	6
		$\sum f_i = 40$		$\sum f_i u_i = -23$

$\Rightarrow$ $\bar{x} = A + \left(\frac{\sum f_i u_i}{\sum f_i}\right)h$ $\Rightarrow$ $\bar{x} = 27.5 + \times\left(\frac{-23}{40}\right) = 24.625$

Ex. 6 The mean of the following frequency distribution is 62.8 and the sum of all frequencies is 50. Compute the missing frequency f_1 and f_2

Class	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	5	f_1	10	f_2	7	8

Sol. Let = 30, h = 20

Class	x_i	f_i	$u_i = \frac{x_i - A}{h}$	$f_i u_i$
0-20	10	5	-1	-5
20-40	30	f_1	0	0
40-60	50	10	+1	10
60-80	70	f_2	+2	$2f_2$
80-100	90	7	+3	21

100-120	**110**	**8**	**+4**	**32**
		$\sum f_i = 30 + f_1 + f_2$		$\sum f_i u_i = 58 + 2f_2$

Given $30 + f_1 + f_2 = 50$

$f_1 + f_2 = 20$ ....(i)

$$\bar{x} = A + \left(\frac{\sum f_i u_i}{\sum f_i} \times h\right)$$

$$62.8 = 30 = \left(\frac{58 + 2f_2}{50} \times 20\right)$$

$$62.8 = (58 + 2f_2) \times \frac{2}{5}$$

$32.8 \times 5 = 116 + 4f_2$

$164 = 116 + 4f_2$

$4f_2 = 164 - 116$

$4f_2 = 48$

$f_2 = 12$

Now, $f_1 = f_2 = 20$ $\quad f_1 + 12 = 20$ $\quad f_1 = 8$

So, the missing frequencies are $f_1 = 8$ and $f_2 = 12$.

Ex.7 Find the mean marks from the following data :

Marks	No. of Students
Below 10	**5**
Below 20	**9**
Below 30	**17**
Below 40	**29**
Below 50	**45**
Below 60	**60**
Below 70	**70**
Below 80	**78**
Below 90	**83**
Below 100	**85**

Sol. Charging less than type frequency distribution in general frequency distribution.

Marks	x_i	f_i	A = 45, h =10 $u_i = \frac{x_i - A}{h}$	f_iu_i
0-10	5	5	-4	-20
10-20	15	4	-3	-12
20-30	25	8	-2	-16
30-40	35	12	-1	-12
40-50	45	16	0	0
50-60	55	15	+1	15
60-70	65	10	+2	20
70-80	75	8	+3	24
80-90	85	5	+4	20
90-100	95	2	+5	10
		$\sum f_i = 85$		$\sum f_iu_i = 29$

According to step deviation formula for mean

$\overline{x} = A + \left(\frac{\sum f_iu_i}{\sum f_i} \times h\right)$ $\overline{x} = 45 + \left(\frac{29}{85} \times 10\right)$

$\overline{x} = 45 + 3.41$ $\overline{x} = 48.41$ So, the mean marks is 48.41

14.4 PROPERTIES OF MEAN :

(i) Sum of deviations from mean is zero. i.e. $\sum_{i=1}^{n}(x_i - \overline{x}) = 0$

(ii) If a constant real number 'a' is added to each of the observation than new mean will be $\overline{x} + a$.

(iii) If a constant real number 'a' is subtracted from each of the observation then new mean will be $\overline{x} - a$.

(iv) If constant real number 'a' is multiplied with each of the observation then new mean will be $a\overline{x}$.

(v) If each of the observation is divided by a constant no 'a', then new mean will be $\frac{\overline{x}}{a}$.

14.5 MERITS OF ARITHETIC MEAN :

(i) It is rigidly defined, simple, easy to understand and easy to calculate.
(ii) It is based upon all the observations.
(iii) Its value being unique, we can use it to compare different sets of data.
(iv) It is least affected by sampling fluctuations.
(v) Mathematical analysis of mean is possible. So, It is relatively reliable.

14.6 DEMERITS OF ARITHMETCI MEAN :

(i) It can not be determined by inspection nor it can be located graphically.
(ii) Arithmetic mean cannot be used for qualities characteristics such as intelligence, honesty, beauty etc.
(iii) It cannot be obtained if a single observation is missing.
(iv) It is affected very much by extreme values. In case of extreme items, A.M. gives a distorted picture of the distribution and no longer remains representative of the distribution.
(v) It may lead to wrong conclusions if the details of the data from which it is computed are not given.
(vi) It can not be calculated if the extreme class is open, e.g. below 10 or above 90.
(vii) It cannot be used in the study of rations, rates etc.

14.7 USES OF ARITHMETIC MEAN :

(i) It is used for calculating average marks obtained by a student.
(ii) It is extensively used in practical statistics and to obtain estimates.
(iii) It is used by businessman to find out profit per unit article, output per machine, average monthly income and expenditure etc.

14.8 MEDIAN :

Median is the middle value of the distribution. It is the value of variable such that the number of observations above it is equal to the number of observations below it.

Median of raw data

(i) Arrange the data in ascending order.
(ii) Count the no. of observation (Let there be 'n' observation)

(A) if n be odd then median = value of $\left(\frac{n+1}{2}\right)^{th}$ observation.

(B) if n is even then median is the arithmetic mean of $\left(\frac{n}{2}\right)^{th}$ observation and $\left(\frac{n}{2}+1\right)^{th}$ observation.

Median of class - interval data (Grouped)

$$\text{Median} = \ell + \frac{\frac{N}{2} - C}{f} \times h$$

ℓ = lower limit of median class, N = total no of observation
C = cumulative frequency of the class preceding the median class
h = size of the median class
f = frequency of the median class.

What is median class :

The class in which $\left(\frac{N}{2}\right)^{th}$ item lie is median class.

Ex.8. Following are the lives in hours of 15 pieces of the components of air craft engine. Fin the median :
715, 724, 725, 710, 729, 745, 649, 699, 696, 712, 734, 728, 716, 705, 719

Sol. Arranging the data in ascending order
644. 696, 705, 710, 712, 715, 716, 719, 724, 725, 728, 729, 734, 745
N = 15

So, Median $= \left(\frac{N+1}{2}\right)^{th}$ observation $= \left(\frac{15+1}{2}\right)^{th}$ observation

= 716.

Ex. 9 The daily wages (in rupees) of 100 workers in a factory are given below :

Daily wages (in Rs.)	125	130	135	140	145	150	160	180
No. of workers	6	20	24	28	15	4	2	1

Find the median wage of a worker for the above date.

Sol.

Daily wages (in Rs.)	No. of workers	Cumulative frequency
125	6	6
130	20	26
135	24	50
140	28	78
145	15	93
150	4	97
160	2	99
180	1	100

N = 100 (even)

$\therefore$ Median $= \dfrac{\left(\frac{N}{2}\right)^{th} \text{observation} + \left(\frac{N}{2}+1\right)^{th} \text{observation}}{2}$

Median $= \dfrac{50^{th} \text{ observation} + 51^{th} \text{ observation}}{2}$

$= \dfrac{135+140}{2}$ = 137.50

$\therefore$ Median wage of a workers in the factory is Rs 137.50.

Ex.10 Calculate the median for the following distribution class :

Class	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	5	10	20	7	8	5

Sol. (i) First we find $\left(\frac{N}{2}\right)^{th}$ value i.e. $\left(\frac{55}{2}\right)^{th}$ = 27.5th

Class	f	c.f.
0-10	5	5
10-20	10	15
20-30	20	35
30-40	7	42
40-50	8	50
50-60	5	55

which lies in 20-30.

∴ 20-30 class in median class
here ℓ = 20

$\frac{N}{2}$ = 27.5, C = 15, f = 20, h = 10

∴ Median = 20 + $\frac{275-15}{20}\times 10$

Median = 26.25

Ex. 11 in the median of the following frequency distribution is 46, find the missing frequencies :

Variable	10-20	20-30	30-40	40-50	50-60	60-70	70-80	Total
Frequency	12	13	?	65	?	25	18	229

Sol.

Class Interval	Frequency	C.F
10-20	12	12
20-30	30	42
30-40	f_1	$42 + f_1$
40-50	65	$107 + f_1$
50-60	f_2	$107 + f_1 + f_2$
60-70	25	$132 + f_1 + f_2$
70-80	18	$150 + f_1 + f_2$

Let the frequency of the class 30 - 40 be f_1 and that of the class 50 - 60 be f_2. The total frequency is 229

$12 + 30 + f_1 + 65 + f_2 + 25 + 18 = 229$

⇒ $f_1 + f_2 = 79$

It is given that median is 46., clearly, 46 lies in the class 40 - 50. So, 40 - 50 is the median class

∴ ℓ = 40, h = 10, f = 65 and C = 42 + f_1, N = 229

Median = $\ell + \frac{\frac{N}{2}-C}{f}\times h$

$46 = 40 + \frac{\frac{229}{2}-(42+f_1)}{65}\times 10$

$46 = 40 + \frac{145-2f_1}{13}$

⇒ $6 = \frac{145-2f_1}{13}$ ⇒ $2f_1 = 67$ ⇒ $f_1 = 33.5$ or 34

Since, $f_1 + f_2 = 79$ ∴ $f_1 = 45$ Hence, $f_1 = 34$ and $f_2 = 45$.

Merits of Median :
(i) It is rigidly defined, easily, understood and calculate.
(ii) It is not all affected by extreme values.
(iii) It can be located graphically, even if the class - intervals are unequal.

(iv) It can be determined even by inspection is some cases.

Demerits of Median :

(i) In case of even numbers of observations median cannot be determined exactly.
(ii) It is not based on all the observations.
(iii) It is not subject to algebraic treatment.
(iv) It is much affected by fluctuations of sampling.

Uses of Median :

(i) Median is the only average to be used while dealing with qualitative data which cannot be measured quantitatively but can be arranged in ascending or descending order of magnitude.
(ii) It is used for determining the typical value in problems concerning wages, distribution of wealth etc.

14.9 MODE:

Mode or modal value of the distribution is that value of variable for which the frequency is maximum.

Mode of ungrouped data : - (By inspection only)

Arrange the data in an array and then count the frequencies of each variate.
The variant having maximum frequency is the mode.

Mode of continuous frequency distribution

Mode = $\ell + \frac{f_1 + f_0}{2f_1 - f_0 - f_2} \times h$

Where ℓ = lower limit of the modal class
f_1 = frequency of the class i.e. the largest frequency.
f_0 = frequency of the class preceding the modal class.
f_2 = frequency of the class succeeding the modal class.
h = width of the modal class

Ex.12. Fin the mode of the following data :
25, 16, 19, 48, 19, 20, 34, 15, 19, 20, 21, 24, 19, 16, 22, 16, 18, 20, 16, 19.

Sol. Frequency table for the given data as given below :

Value x_i	15	16	18	19	20	21	22	24	25	34	48
Frequency f_i	1	4	1	5	3	1	1	1	1	1	1

19 has the maximum frequency of 5. So, Mode = 19.

Ex.13. The following table shows the age distribution of cases of a certain disease admitted during a year in a particular hospital.

Age (in Years)	5-14	15-24	25-34	35-44	45-54	55-64
No. of Cases	6	11	21	23	14	5

Sol. Here class intervals are not is inclusive form. So, Converting the above frequency table in inclusive form.

Age (in Years)	4.5-14.5	14.5-24.5	24.5-34.5	34.5-44.5	44.5-54.5	54.5-64.5
No. of Cases	6	11	21	23	14	5

Class 34.5 - 44.5 has maximum frequency. So it is the modal class.
ℓ 34.5, h = 10, f_0 = 21, f_1 = 23 and f_2 = 14.

$\therefore$ Mode = $\ell + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

Mode = 34.5 + $\frac{23-21}{46-21-14} \times 10$ = 34.5 + $\frac{2}{11} \times 10$ = 36.31 Ans.

Ex.14 Find the mode of following distribution :

Daily Wages	31-36	37-42	43-48	49-54	55-60	61-66
No. of workers	6	12	20	15	9	4

Sol.

Daily Wages	No. of workers	Daily wages	No of workers
31-36	6	30.5-36.5	6
37-43	12	36.5-42.5	12
43-48	20	42.5-48.5	20
49-54	15	48.5-54.5	15
55-60	9	54.5-60.6	9
61-66	4	60.5-66.5	4

Modal class frequency is 42.5 - 48.5.
ℓ = 42.5
f_1 = 20 f_0 = 12, f_2 = 15, h = 6

$\therefore$ Mode = 42.5 + $\frac{20-12}{2(20)-12-15} \times 6$

$\therefore$ Mode = 46.2

Merits of Mode

(i) It can be easily understood and is easy to calculate.
(ii) It is not affected by extreme values and can be found by inspection is some cases.
(iii) It can be measured even if open - end classes and can be represented graphically.

Demerits of Mode :

(i) It is ill - fined. It is not always possible to find a clearly defined mode.
(ii) It is not based upon all the observation.
(iii) It is not capable of further mathematical treatment. it is after indeterminate.
(iv) It is affected to a greater extent by fluctuations of sampling.

Uses of Mode :

Mode is the average to be used to find the ideal size, e.g., in business forecasting, in manufacture of ready- made garments, shoes etc.

Relation between Mode, Median & Mean : Mode = 3 median - 2 mean.

14.10 CUMULATIVE FREQUENCY CURVE OR OGIVE :

In a cumulative frequency polygon or curves, the cumulative frequencies are plotted against the lower and upper limits of class intervals depending upon the manner in which the series has been cumulated. There are two methods of constructing a frequency polygon or an Ogive.
(i) Less than method (ii) More than method

In ungrouped frequency distribution :

Ex.15 The marks obtained by 400 students in medical entrance exam are given in the following table.

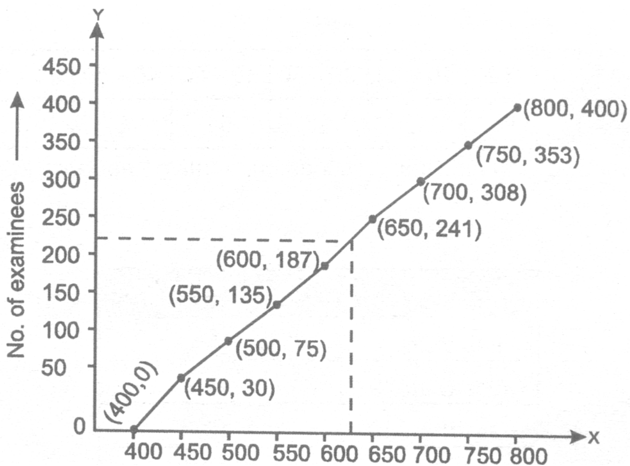
Marks Obtained	400-450	450-500	500-550	550-600	600-650	650-700	700-750	750-80
No. of Examinees	30	45	60	52	54	67	45	47

(i) Draw Ogive by less than method. (ii) Draw Ogive by more than method.
(iii) Find the number of examinees, who have obtained the marks less than 625.
(iv) Find the number of examinees, who have obtained 625 and more than marks.

Sol. (i) Cumulative frequency table for less than Ogive method is as following.

Marks Obtained	No. of Examinees
Less than 450	30
Less than 500	75
Less than 550	135
Less than 600	187
Less than 650	241
Less than 700	308
Less than 750	353
Less than 800	400

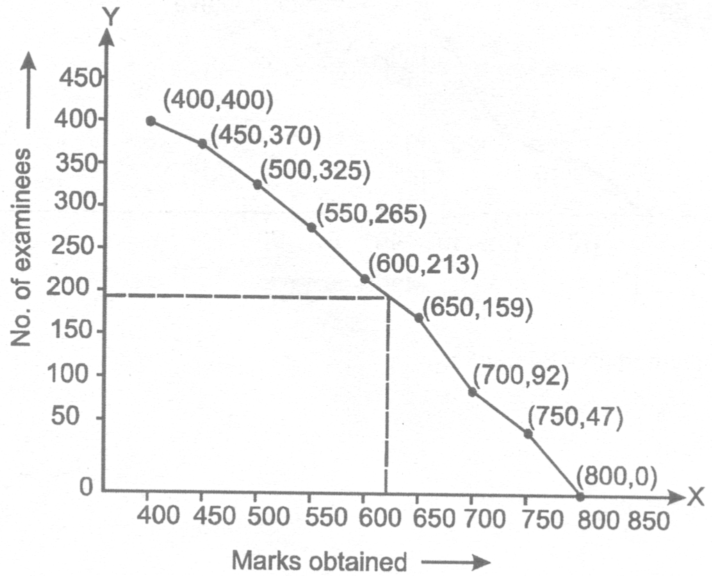
Following are the Ogive for the above cumulative frequency table by applying the given method and the assumed scale.



(ii) Cumulative frequency table for more than Ogive method is as following : -

Marks Obtained	No. of Examinees
400 and more	400
450 and more	370
500 and more	325
550 and more	265
600 and more	213
650 and more	159
700 and more	92
750 and more	47

Following are the Ogive for the above cumulative frequency table.



(iii) So, the number of examinees, scoring marks less than 625 are approximately 220.
(iv) So, the number of examinees, scoring marks 625 and more will be approximately 190.

Ex.16 Draw on O-give for the following frequency distribution by less than method and also find its median from the graph.

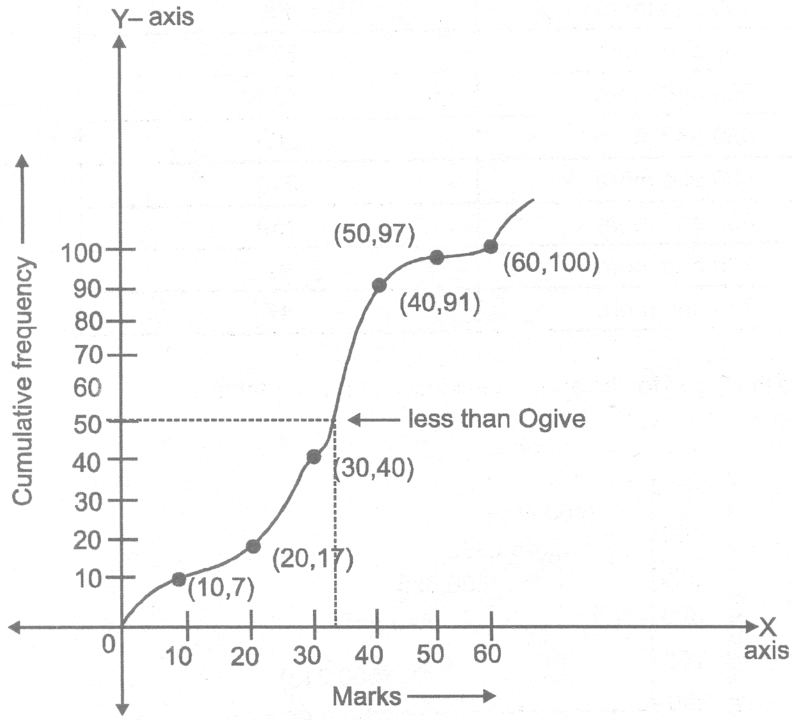
Marks	0-10	10-20	20-30	30-40	40-50	50-60
Number of students	7	10	23	51	6	3

Sol. Converting the frequency distribution into less than cumulative frequency distribution.

Marks	No. of Students
Less than 10	7
Less than 20	17
Less than 30	40
Less than 40	91
Less than 50	97
Less than 60	100

According to graph median = 34 marks.

.



DAILY PRACTIVE PROBLEMS # 14

OBJECTIVE DPP - 14.1

1. The median of following series if 520, 20, 340, 190, 35, 800, 1210, 50, 80
   (A) 1210 (B) 520 (C) 190 (D) 35
2. If the arithmetic mean of 5, 7, 9 x is9 then the value of x is
   (A) 11 (B) 15 (C) 18 (D) 16
3. The mode of the distribution 3, 5, 7, 4, 2, 1, 4, 3, 4 is
   (A) 7 (B) 4 (C) 3 (D) 1
4. If the first five elements of the set x_1, x_2,...x_{10} are replaced by $x_i + 5$, i = 1, 2, 3, 4, 5 and next five elements are repla
   (A) 0 (B) $\frac{n+1}{2}$ (C) 10 (D) 25

Class	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	5	f_1	10	f_2	7	8

5. If the mean and median of a set of numbers are 8.9 and 9 respectively, then the mode will be
   (A) 7.2 (B) 8.2 (C) 9.2 (D) 10.2

SUBJECTIVE DPP - 14.2

1. Find the value of p, if the mean of the following distribution whose mean is 20

x	15	17	19	20 + p	23
f	2	3	4	5p	6

2. Find the mean of following distribution by step deviation method : -

Class interval	50-70	70-90	90-110	110-130	130-150	150-170
No. of workers	18	12	13	27	8	22

3. The mean of the following frequency distribution is 62.8 and the sum of all the frequencies is 50. Compute the missi
4. Calculate the median from the following data :

Rent (in Rs.)	15-25	25-35	35-45	45-55	55-65	65-75	75-85	85-95
No. of House	8	10	15	25	40	20	15	7

5. Find the missing frequencies and the median for the following distribution if the mean is 1.46.

No. of accidents	0	1	2	3	4	5	Total
Frequency (No. of days)	46	f_1	f_2	25	10	5	200

6. If the median of the following frequency distribution is 28.5 find the missing frequencies :

Class interval :	0-10	10-20	20-30	30-40	40-50	50-60	Total
Frequency	5	f_1	20	15	f_2	5	60

7. The marks is science of 80 students of class X are given below : Find the mode of the marks obtained by the students in science.

Class interval :	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	3	5	16	12	13	20	5	4	1	1

8. Find the mode of following distribution :

Class interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	5	8	7	12	28	20	10	10

9. During the medical check - up of 35 students of a class, their weights were recorded as follows :

Weight (in kg)	Number of students
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

Draw a less than type ogive for the given data. Hence, obtain median weight from the graph and verify the result by using the formula.

10. The following table gives the height of trees :

Height	Less than 7	Les than 14	Less than 21	Less than 28	Less than 35	Less than 42	Less than 49	Less than 56
No. of trees	26	57	92	134	216	287	341	360

Draw "less than" ogive and "more than" ogive.

11. If the mean of the following data is 18.75, find the value of p: **[CBSE - 2005]**

x	10	15	P	25	30
f	5	10	7	8	2

12. Find the mean of following frequency distribution **[CBSE - 2006]**

Classes	50-70	70-90	90-110	110-130	130-150	150-170
Frequency	18	12	13	27	8	22

13. Find the median class of the following data : **[CBSE - 2008]**

Marks obtained	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	8	10	12	22	30	18

14. Find the mean, mode and median of the following data : **[CBSE - 2008]**

Classes	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	5	10	18	30	20	12	5

ANSWERS

(Objective DPP- 14.1)

Que.	1	2	3	4	5
Ans.	C	B	B	A	C

(Subjective DPP – 14.2)

]1. p = 1   2. 112.20   3. $f_1 = 8$, $f_2 = 12$

4. 58   5. $f_1 = 76$, $f_2 = 38$, and median = 1   6. $f_1 = 8$, $f_2 = 7$

7. 53.17   8. 46.67   9. 47.5 kg

11 20   12. 20   13. 30-40

14. Mean = 35.6, Median = 35.67 and mode = 35.45

***ALL THE BEST ***