Teko CLASSES ..... the support

➤➤➤ CIRCLES ⮜⮜⮜

9.1 CIRCLE

A circle is the locus of a points which moves in a plane in such a way that its distance from a fixed point remains constant.

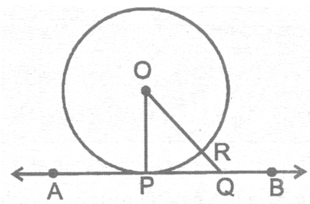
9.2 SECANT AND TANGENT :

⇒ Secant to a circle is a line which intersects the circle in two distinct points.

⇒ A tangent to a circle is a line that intersects the circle in exactly one point.

9.3 THEOREM :

Statement : A tangent to a circle i perpendicular to the radius through the point of contact.



Given : A circle C (O, r) and a tangent AB at a point P.

To prove : OP $\perp$ AB

Construction : Take any points Q, other than P on the tangent AB. Join OQ. Suppose OQ meets the circle at R.

Proof: Among all line segments joining the point O to a point on AB, the shorted one is perpendicular to AB. So, to prove that OP $\perp$ AB, it is sufficient to prove that OP is shorter than any other segment joining O to any point of AB.

Clearly OP = OR Now, OQ OR + RQ

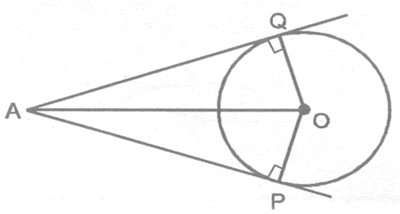
⇒ OQ > OR ⇒ OQ > OP ($\because$ OP = OR)

Thus, OP is shorter than any other segment joining O to any point of AB.

Hence, OP $\perp$ AB.

9.4 THEORM :

Statement : Lengths of two tangents drawn from an external point to a circle are equal.



Given: AP and AQ are two tangents drawn from a point A to a circle C (O, r).

To prove : AP = AQ

Construction : Join OP, OQ and OA.

Proof : In ΔAOQ and ΔAPO

$\angle$OQA = $\angle$OPA [Tangent at any point of a circle is perp. to radius through the point of contact]

AO = AO [Common] OQ = OP [Radius]

So, by R.H.S. criterion of congruency $\Delta AOQ \cong \Delta AOP$

$\therefore$ AQ = AP [By CPCT] Hence Proved.

Result :

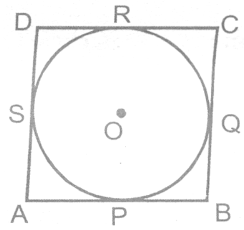
(i) If two tangents are drawn to a circle from an external point, then they subtend equal angles at the centre. $\angle OAQ = \angle OAP$ [By CPCT]

(ii) If two tangents are drawn to a circle from an external point, they are equally inclined to the segment, joining the centre to that point $\angle OAQ = \angle OAP$ [By CPCT]

Ex. 1 If all the sides of a parallelogram touches a circle, show that the parallelogram is a rhombus.

Sol. Given : Sides AB, BC, CD and DA of a $\|^{gm}$ ABCD touch a circle at P,Q,R and S respectively.

To prove $\|^{gm}$ ABCD is a rhombus.



Proof : AP = AS ....(i)

BP = BQ ....(ii)

CR = CQ ....(iii)

DR = DS ....(iv)

[Tangents drawn from an external point to a circle are equal]

Adding (1), (2), (3) and (4), we get

$\Rightarrow$ AP + BP + CR + DR = AS + BQ + CQ + DS

$\Rightarrow$ (AP + BP) + (CR + DR) = (AS + DS_ + (BQ + CQ)

$\Rightarrow$ AB + CD = AD + BC

$\Rightarrow$ AB + AB = AD + AD [In a $\|^{gm}$ ABCD, opposite side are equal]

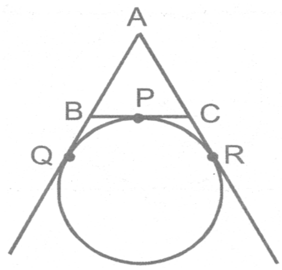
$\Rightarrow$ 2AB = 2AD or AB = AD

But AB = CD AND AD = BC [Opposite sides of a $\|$ gem]

$\therefore$ AB = BC = CD = DA Hence, $\|^{gm}$ ABCD is a rhombus.

Ex.2 A circle touches the BC of a Δ ABC at P and touches AB and AC when produced at Q and R respectively as shown in figure, Show that $= \frac{1}{2}$ (Perimeter of Δ ABC).

So. **Given :** A circle is touching side BC of Δ ABC at P and touching AB and AC when produced at Q and R respectively.



To prove : $AQ = \frac{1}{2}$ (perimeter of Δ ABC)

Proof : AQ = AR ....(i)

BQ = BP ....(ii)

CP = CR ....(iii)

[Tangents drawn from and external point to a circle are equal]

Now, perimeter of Δ ABC = AB + BC + CA

= AB + BP + PC + CA

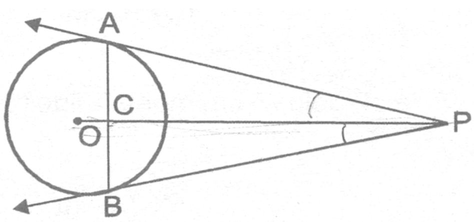
= (AB + BQ) + (CR + CA) [From (ii) and (iii)]

= AQ + AR = AQ + AQ [From (i)]

$AQ = \frac{1}{2}$ (perimeter of Δ ABC).

Ex.3 Prove that the tangents at the extremities of any chord make equal angles with the chord.

Sol. Let AB be a chord of a circle with centre O, and let AP and BP be the tangents at A and B respectively. Suppose, the tangents meet at point P. Join OP. Suppose OP meets AB at C.



We have to prove that

$\angle PAC = \angle PBC$

In triangles PCA and PCB

PA = PB [$\therefore$ Tangent from an external point are equal]

$\angle APC = \angle BPC$ [$\therefore$ PA and PB are equally inclined to OP]

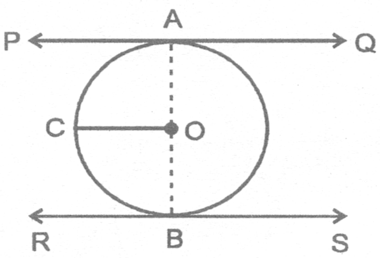
And PC = PC [Common]

So, by SAS criteria of congruence

$\Delta PAC \cong \Delta BPC$ $\Rightarrow$ $\angle PAC = \angle PBC$ [By CPCT]

Ex.4 Prove that the segment joining the points of contact of two parallel tangents passes through the centre.

Sol. Let PAQ and RBS be two parallel tangents to a circle with centre O. Join OA and OB. Draw $OC \| PQ$ Now, $PA \| CO$



$\Rightarrow$ $\angle PAO + \angle COA = 180^0$ [Sum of co-interior angle is 180^0]

$\Rightarrow$ $90^0 + \angle COA = 180^0$ [$\therefore$ $\angle PAO = 90$]

$\Rightarrow$ $\angle COA = 90^0$

Similarly, $\angle CON = 90^0$

$\therefore$ $\angle COA + \angle COB = 90^0 + 90^0 = 180^0$ Hence, AOB is a straight line passing through O.

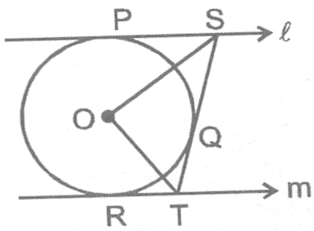
DAILY PRACTICE PROBLEMS # 9

OBJECTIVE DPP - 9.1

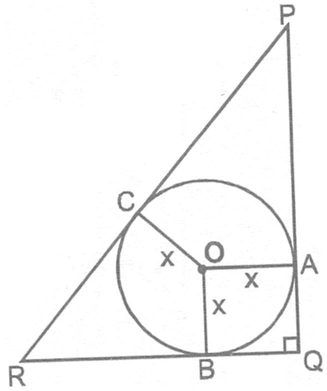
1. The length of the tangent drawn from a point 8 cm away from the centre of a circle of radius 6 cm is
(A) $\sqrt{7}$ cm (B) $\sqrt[2]{7}$ cm (C) 10 cm (D) 5 cm
2. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q, so that OQ = 12 cm. Length of PQ is :
(A) 12 cm (B) 13 cm (C) 8.5 cm (D) $\sqrt{119}$ cm
3. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at an angle of 80^0 then $\angle POA$ is equal to
(A) 50^0 (B) 60^0 (C) 70^0 (D) 80^0
4. Two circle touch each other externally at C and AB is a common tangent to the circle. Then $\angle ACB =$
(A) 60^0 (B) 45^0 (C) 30^0 (D) 90^0
5. ABC is a right angled triangle, right angled at B such that BC = 6 am and AB = 8 cm. A circle with centre O is inscribed in ΔABC. The radius of the circle is
(A) 1 cm (B) 2 cm (C) 3 cm (D) 4 cm

SUBJECTIVE DPP - 9.2

1. ABCD is a quadrilateral such than $\angle D = 90^0$. A circle C (O, r) touches the sides AB, BC, CD and DA at P, Q, R and S respectively. If BC = 38 cm, CD = 25 cm and BP = 27 cm, find r.
2. Two concentric circles are of radius 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.
3. In a circle of radius 5 cm, AB and AC are two chords, such that AB = AC = 6 cm. Find the length of chord BC.
4. The radius of the incircle of a triangle is 4 cm and the segments into which one side is divided by the point of contact are 6 cm and 8 cm. Determine the other two sides of the triangle.
5. In **figure,** ℓ and m are two parallel tangents at P and R. The tangent at Q makes an intercept ST between ℓ and m. Prove that $\angle SOT = 90^0$



6. PQR is a right angled triangle with PQ = 12 cm and QR = 5 cm. A circle with centre O and radius x is inscribed in Δ PQR. Find the value of x.

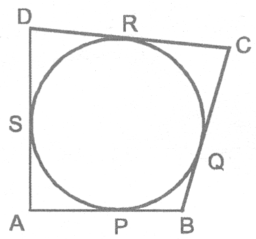


7. From an external point P, two tangents PA and PB are drawn to the circle with centre O. Prove that OP is the perpendicular dissector of AB.

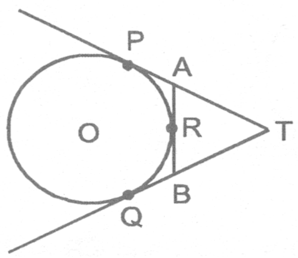
8. Two tangent TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.

9. A circle touches the sides of a quadrilateral ABCD at P, Q, R, S respectively. Show that the angles subtended at the centre by a pair of opposite sides are supplementary.

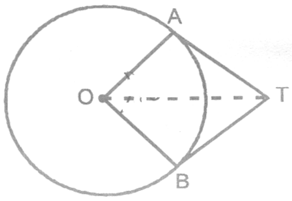
10. In **figure**, a circle touches all the four sides of a quadrilateral ABCD with AB= 6 cm, BC = 7 cm and CD = 4 cm. Find AD. **[CBSE - 2002]**



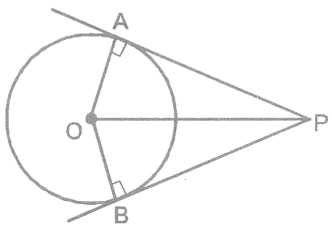
11. Prove that the lengths of the tangents drawn from an external point to a circle are equal.
Using the above, do the following :
In figure, TP and TQ are tangents from T to the circle with centre O and R is any point on the circle. If AB is a tangent to the circle at R, prove that
TA + AR = TB + BR. **[CBSE - 208]**



12. In figure, if $\angle ATO = 40^0$, find $\angle AOB$ **[CBSE - 2008]**



13. In figure OP is equal to diameter of the circle. Prove that ABP is an equilateral triangle. **[CBSE - 2008]**



ANSWERS

(Objective DPP 9.1)

Qus.	1	2	3	4	5
Ans.	B	D	A	D	B

(Subjective DPP 9.2)

1. 14 cm 2. 8 cm 3. 9.6 cm 4. 13 cm and 15 cm

6. 2 cm 10. 3 cm 12. 100^0

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CONSTRUCTION

10.1 DIVISION OF A LINE SEGENT :

In order to divide a line segment internally is a given ratio m: n, where both m and n are positive integers, we follow the following steps:

Step of construction :

(i) Draw a line segment AB of given length by using a ruler.

(ii) Draw and ray AX making an acute angle with AB.

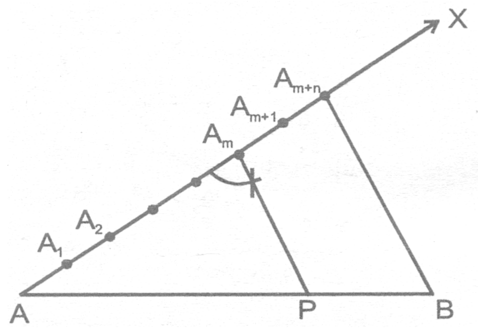
(iii) Along AX mark off (m + n) points A_1, A_2,..., A_{m+n} such that $AA_1 = A_1A_2 = ...=A_{m+n+}A_{m+n}$.

(iv) Join B A_{m+n}

(v) Through the point A_m draw a line parallel to A_{m+n} B by making an angle equal to $\angle AA_{m+n}B$ at A_m.

Suppose this line meets AB at a point P.

The point P so obtained is the required point which divides AB internally in the ratio m : n.



Ex.1 Divide a line segment of length 12 cm internally in the ratio 3 : 2.

Sol. Following are the steps of construction.

Step of construction :

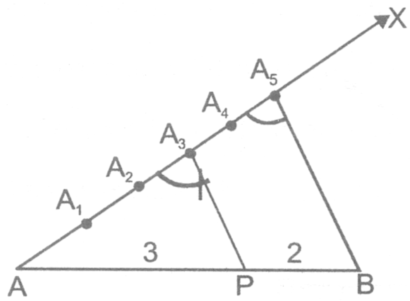
(i) Draw a line segment AB = 12 cm by using a ruler.

(ii) Draw any ray making an acute angle $\angle BAX$ with AB.

(iii) Along AX, mark-off 5 (=3 + 2) points A_1,A_2,A_3,A_4 and A_5 such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$.

(iv) Join BA_5

(v) Through A_3 draw a line A_3P parallel to A_5B by making an angle equal to $\angle AA_5$ B at A_3 intersecting
AB at a point P.



The point P so obtained is the required point, which divides AB internally in the ratio 3 : 2.

10.2 ALTERNATIVE METHOD FOR DIVISION OF A LINE SEGMENT INTERNALLY IN A GIVEN RATIO :

Use the following steps to divide a given line segment AB internally in a given ration m : n, where m and natural members.

Steps of Construction :

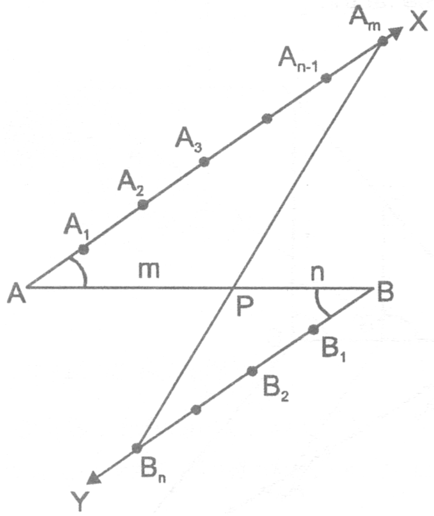
(i) Draw a line segment AB of given length.

(ii) Draw any ray AZ making an acute angle $\angle BAX$ with AB.

(iii) Draw a ray BY, on opposite side of AX, parallel to AX making an angle $\angle ABY$ equal to $\angle BAX$.

(iv) Mark off a points $A_1, A_2, \ldots A_{m'}$ on AX and n points $B_1, B_2, \ldots B_n$ on BY such that $AA_1 = A_1A_2 = \ldots\ldots = A_{m-1}A_m = B_1B_2 = \ldots B_{n-1}B_n$.

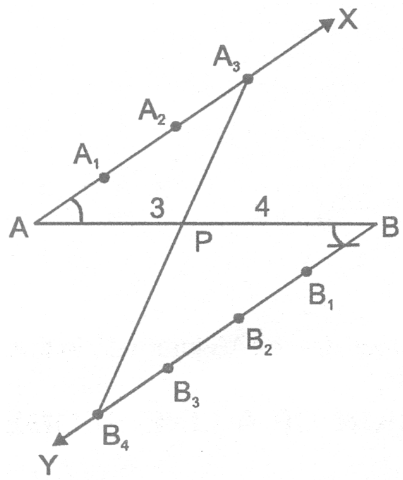
(v) Join A_mB_n. Suppose it intersect AB at P.



The point P is the required point dividing AB in the ratio m : n.

Ex.2 Decide a line segment of length 6 cm internally in the ratio 3:4.

Sol. Follow the following steps :



Steps of Construction :

(i) Draw a line segment AB of length 6 cm.

(ii) Draw any ray AX making an acute angle $\angle BAX$ with AB.

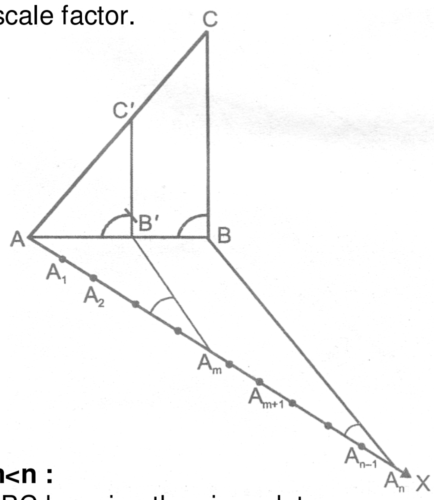
(iii) Draw a ray BY parallel to AX by making $\angle ABY$ equal to $\angle BAX$.

(iv) Mark of three point A_1, A_2, A_3 on AX and 4 points B_1, B_2m B_3, B_4 on BY such that $AA_1 = A_1A_2 = A_2A_3 = BB_1 = B_1B_2 = B_2B_3 = B_2B_4$.

(v) Join A_3B_4. Suppose it intersects AB at a point P.
Then, P is the point dividing AB internally in the ratio 3:4.

10.3 CONTRUCTION OF A TRIANGLE SIMILAR TO A GIVEN TRIANGLE :

Scale Factor : The ratio of the sides of the triangle to be constructed with the corresponding sides of the given triangle is known as their scale factor.



Steps of Construction when m<n :

(i) Construct the given triangle ABC by using the given data.

(ii) Take any one of the three side of the given triangle as base. Let AB be the base of the given triangle.

(iii) At one end, say A, of base AB. Construct an acute angle $\angle BAX$ below the base AB.

(iv) Along AX mark of n points $A_1, A_2, A_3, \ldots\ldots A_n$ such that $AA_1 = A_1A_2 = \ldots\ldots = A_{n-1}A_n$.

(v) Join A_n B.

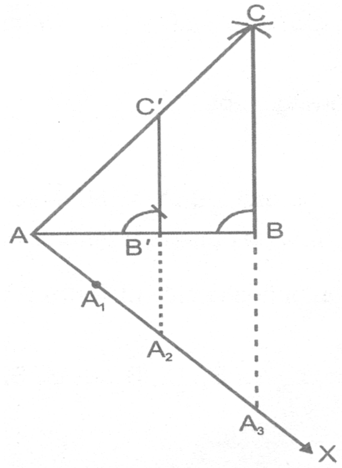
(vi) Draw A_mB' parallel to A_n B which meets AB at B'. (vii) From B' draw B' C' $\|$ CB meeting AC at C'.

Triangle AB'C' is the required triangle each of whose side is $\left(\frac{m}{n}\right)^{th}$ of the corresponding side of Δ ABC.

Ex.3 Construction a ΔABC in which AB = 5 cm, BC = 6 cm and AC = 7 cm. Now, construct a triangle similar to ΔABC such that each of its side is two-third of the corresponding side of ΔABC.

Sol. Steps of Construction

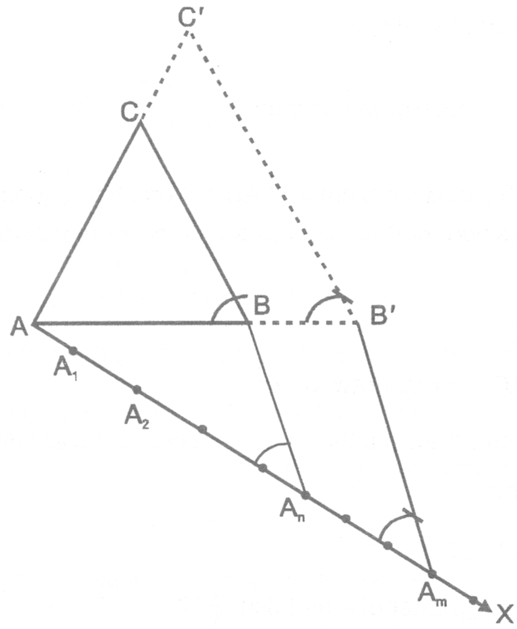
(i) Draw a line segment AB = 5 cm.
(ii) With A as centre and radius AC = 7 cm, draw an arc.
(iii) With B as centre and BC= 6 cm, draw another arc, intersecting the arc draw in step (ii) at C.
(iv) Join AC and BC to obtain ΔABC.
(v) Below AB, make an acute angle $\angle$BAX .
(vi) Along AX, mark off three points (greater of 2 and 3 in $\frac{2}{3}$) A_1, A_2, A_3 such that $AA_1 = A_1A_2 = A_2A_3$.
(vi) Join A_3B.



(viii) Draw $A_2B' \parallel A_3B$, meeting AB at B'.
(iv) From B', draw $B'C' \parallel BC$, meeting AC at C'.

AB'C' is the required triangle, each of the whose sides is two-third of the corresponding sides of ΔABC.

Steps of Construction when m > n: (i) Construct the given triangle by using the given data.
(ii) Take any of the three sides of the given triangle and consider it as the base. Let AB be the base of the given triangle.
(iii) At one end, say A, of base AB construct an acute angle $\angle$BAX below base AB i.e. on the composite side of the vertex C.
(iv) Along AX, mark-off m (large of m and n) points $A_1, A_2, ...A_m$ on AX such that $AA_1 = A_1A_2 = ... A_{m-1}A_m$.
(v) Join A_n to B and draw a line through A_m parallel to A_n B, intersecting the extended line segment AB at B'.
(vi) Draw a line through B' parallel to BC intersecting the extended line segment AC at C'.
(vii) ΔAB'C' so obtained is the required triangle.



Ex.4 Draw a triangle ABC with side BC = 7 cm, $\angle B = 45^0$, $\angle A = 150^0$ Construct a triangle whose side are (4/3) times the corresponding side of ΔABC.

Sol. In order to construct ΔABC, follow the following steps : (i) Draw BC = 7 cm.

(ii) At B construct $\angle CBX = 45^0$ and at C construct $\angle BCY = 180^0 - (45^0 + 105^0) = 30^0$
Suppose BC and CY intersect at A. Δ ABC so obtained is the given triangle.

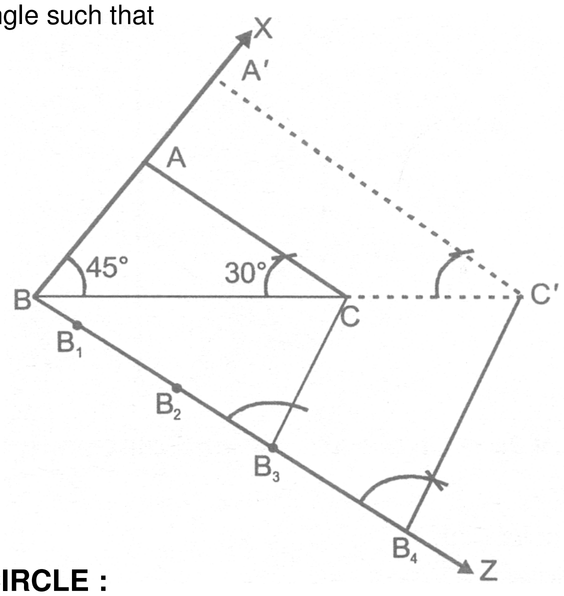
(iii) Construct an acute angle $\angle CBZ$ at B on opposite side of vertex A of Δ ABC.

(iv) Mark-off four (greater of 4 and 3 in $\frac{4}{3}$) points, B_1, B_2, B_3, B_4 on BZ such that $BB_2 - B_1B_2 = V_2B_3 = B_3B_4$.

(v) Join B_3 (the third point) to C and draw a line through B_4 parallel to B_3C, intersecting the extended line segment BC at C'.

(vi) Draw a line through C' parallel to CA intersecting the extended line segment BA at A' Triangle A'BC' so obtained is the required triangle such that

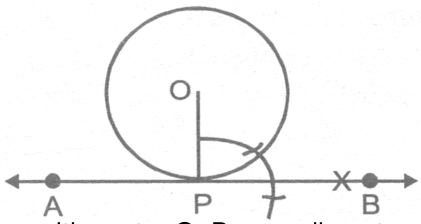
$$\frac{A'B'}{AB} = \frac{BC'}{BC} = \frac{A'C'}{AC} = \frac{4}{3}$$



10.4 CONSTRCUTION OF TANGENT TO A CIRCLE :

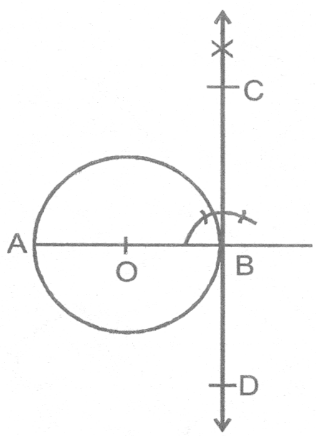
10.4 (a)To Draw the Tangent to a Circle at a Given Point on it, When the Centre of the Circle is Known :

Given : A circle with centre O and a point P and it.
Required : To draw the tangent to the circle at P.
Steps of Construction.
(i) Join OP.
(ii) Draw a line AB perpendicular to OP at the point P. APB is the required tangent at P.



Ex.5 Draw a circle of diameter 6 cm with centre O. Draw a diameter AOB. Through A or B draw tangent to the circle.

Sol. **Given :** A circle with centre O and a point P on it.
Required : To draw tangent to the circle at B or A.
Steps of Construction.
(i) With O as centre and radius equal to 3 cm (6 ÷ 2) draw a circle.
(ii) Draw a diameter AOB.
(iii) Draw CD $\perp$ AB.
(iv) So. CD is the required tangent.



10.4 (b)To Draw the Tangent to a Circle at a Given Point on it, When the Centre of the Circle is not Known :

Given : A circle and a point P on it.

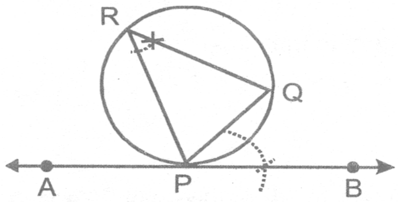
Required : To draw the tangent to the circle at P.

Steps of Construction

(i) Draw any chord PQ and Joint P and Q to a point R in major arc PQ (or minor arc PQ).

(ii) Draw $\angle QPB$ equal to $\angle PRQ$ and on opposite side of chord PQ.

The line BPA will be a tangent to the circle at P.



Ex.6 Draw a circle of radius 4.5 cm. Take a point P on it. Construct a tangent at the point P without using the centre of the circle. Write the steps of construction.

Sol. Given : To draw a tangent to a circle at P.

Steps of Construction

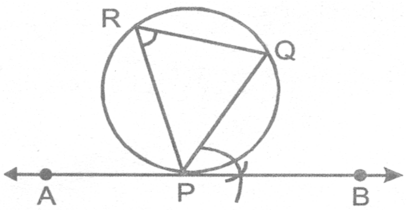
(i) Draw a circle of radius = 4.5 cm.

(ii) Draw a chord PQ, from the given point P on the circle.

(iii) Take a point R on the circle and joint PR and QR.

(iv) Draw $\angle QPB = \angle PRQ$ on the opposite side of the chord PQ.

(v) Produce BP to A. Thus, APB is the required tangent.



10.4 (c) To Draw the Tangent to a Circle from a Point Outside it (External Point) When its Centre is known :

Given : A circle with centre O and a point P outside it.

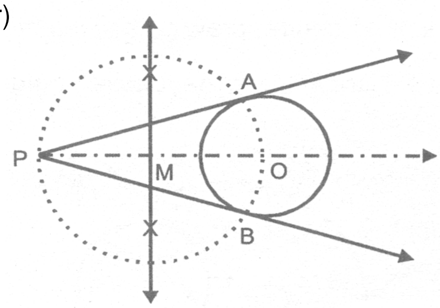
Required : To construct the tangents to the circle from P.

Steps of Construction :

(i) Join OP and bisect it. Let M be the mid point of OP.

(ii) Taking M as centre and MO as radius, draw a circle to intersect C (O, r) in two points, say A and B

(iii) Join PA and PB. These are the required tangents from P to C(O,r)



Ex.7 Draw a circle of radius 2.5 cm. From a point P, 6 cm apart from the centre of a circle, draw two tangents to the circle.

Sol. **Given :** A point P is at a distance of 6 cm from the centre of a circle of radius 2.5 cm

Required : To draw two tangents to the circle from the given point P.

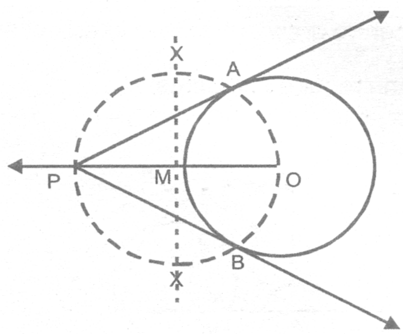
Steps of Construction :

(i) Draw a circle of radius 2.5 cm. Let it centre be O.

(ii) Join OP and bisect it. Let M be mid-point of OP.

(iii) Taking M as centre and MO as radius draw a circle to intersect C in two points, say A and B.

(iv) Join PA and PB. These are the required tangents from P to C.



10.4 (d) To Draw Tangents to a Circle From a Point Outside it (When its Centre is not Known):

Given : P is a point outside the circle.

Required : To draw tangents from a point P outside the circle.

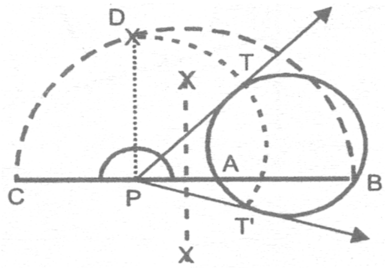
Steps of Construction :

(i) Draw a secant PAB to intersect the circle at A and B.
(ii) Produce AP to a point C, such that PA = PC.
(iii) With BC as a diameter, draw a semicircle.

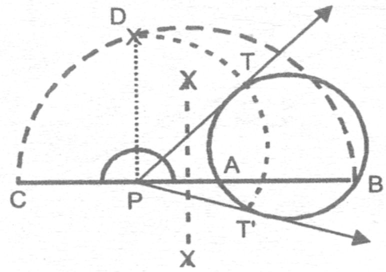
(iv) Draw PD $\perp$ CB, intersecting the semicircle at D.

(v) Taking PD as radius and P as centre, draw arcs to intersect the circle at T and T'.

(iv) Join PT and PT'. Then, PT and PT' are the required tangents.



Ex.8 Draw a circle of radius 3 cm. From a point P, outside the circle draw two tangents to the circle without using
the centre of the circle.
Given : A point P is outside the circle of radius 3 cm.
Required : To draw two tangents to the circle from the point P, without the use of centre.
Steps of constructing
(i) Draw a circle of radius 3 cm.
(ii) Take a point P outside the circle and draw a secant PAB, intersecting the circle at A and B.
(iii) Produce AP to C such that AP = CP.
(iv) Draw a semicircle, wit CB as a diameter.
(v) Draw PD $\perp$ AB, intersecting the semi-circle AT D.
(vi) With PD as radius and P as centre draw two arcs to intersect the given circle at T and T'.
(vii) Joint PT and PT'. Which are the required tangents.



DAILY PRATICE PROBLEMS # 10

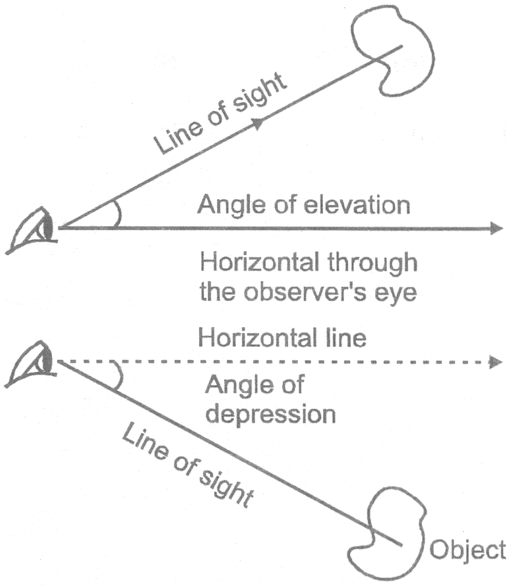
SUBEJCTIVE DPP -10.1

1. Draw a circle of radius 2.5 cm. Take a point P on it. Draw a tangent to the circle at the point P.
2. From a point P on the circle of radius 4 cm, draw a tangent to the circle without using the centre. Also, write steps of construction.
3. Draw a circle of radius 3.5 cm. Take a point P on it. Draw a tangent to the circle at the point P, without using the centre of the circle.
4. Draw a circle of radius 3 cm. Take a point P at a distance of 5.6 cm from the centre of the circle. From the point P, draw two tangents to the circle.
5. Draw a circle of radius 4.5 cm. Take point P outside the circle. Without using the centre of the circle, draw two tangents to the circle from the point P.
6. Construct a triangle ABC, similar to a given equilateral triangle PQR with side 5 cm. such that each of its side is 6/7th of the corresponding side of the Δ PQR.
7. Construct a triangle ABC. similar to a given isosceles triangle PQR with QR = 5 cm, PR = PQ = cm, such that each of its side is 5/3 of the corresponding sides of the Δ PQR.
8. Draw a line segment AB = 7 cm. Divide it externally in the ratio of
   (i) 3 : 5 (ii) 5 : 3
9. Draw a Δ ABC with side BC = 6 cm, AB = 5cm and $\angle$ABC = 60^0. Construct a Δ AB'C' similar to Δ ABC such that sides of Δ AB'C' are $\frac{3}{4}$ of the corresponding sides of Δ ABC. **[CBSE - 2008]**

HEIGHTS & DISTANCES

12.1 ANGLE OF ELEVATION :

In order to see an object which is at a higher level compared to the ground level we are to look up. The line joining the object and the eye of the observer is known as the line sight and the angle which this line of sight makes with the horizontal drawn through the eye of the observer is known as the angle of elevation. Therefore, the angle of elevation of an object helps in finding out its height (**figure)**



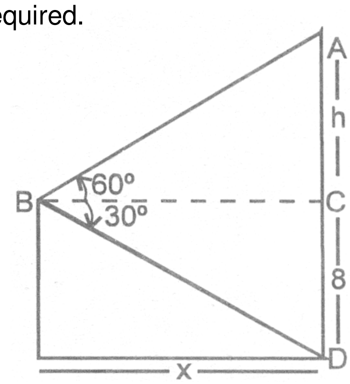
12.2 ANGLE OF DEPRESSION :

When the object is at a lower level tan the observer's eyes, he has to look downwards to have a view of the object. It that case, the angle which the line of sight makes with the horizontal thought the observer's eye is known as the **angle of depression (Figure).**

ILLUSTRACTIONS :

EX.1 A man is standing on the deck of a ship, which is 8 m above water level. He observes the angle of elevations of the top of a hill as 60^0 and the angle of depression of the base of the hill as 30^0. Calculation the distance of the hill from the ship and the height of the hill. **[CBSE = 2005]**

Sol. Let x be distance of hill from man and h + 8 be height of hill which is required.
is right triangle ACB.



$\Rightarrow \quad \tan 60^0 = \frac{AC}{BC} = \frac{h}{x} \quad \Rightarrow \quad \sqrt{3} = \frac{h}{x}$

In right triangle BCD.

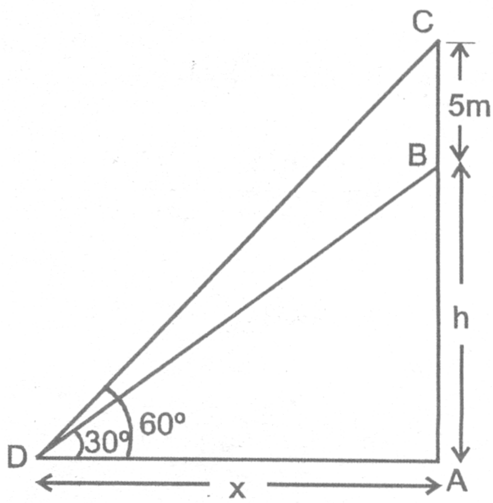
$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{8}{x} \quad \Rightarrow \quad x = 8\sqrt{3}$

$\therefore$ Height of hill = h + 8 = $\sqrt{3}.x + 8 = (\sqrt{3})(8\sqrt{3}) + 8 = 32$ m.

Distance of ship from hill = x = $8\sqrt{3}$ m.

Ex.2 A vertical tower stands on a horizontal plane and is surmounted by vertical flag staff of height 5 meters. At a point on the plane, the angle of elevation of the bottom and the top of the flag staff are respectively 30^0 and 60^0 find the height of tower. **[CBSE-2006]**

Sol. Let AB be the tower of height h metre and BC be the height of flag staff surmounted on the tower, Let the point of the place be D at a distance x meter from the foot of the tower in Δ ABD



$\tan 30^0 = \frac{AB}{AD}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x}$

$\Rightarrow \quad x = \sqrt{3}h \quad$ .....(i)

In Δ ABD $\qquad \tan 60^0 = \frac{AC}{AD}$

$\Rightarrow \qquad \sqrt{3} = \frac{5+h}{x} \qquad \Rightarrow \qquad x = \frac{5+h}{\sqrt{3}} \qquad$ ....(ii)

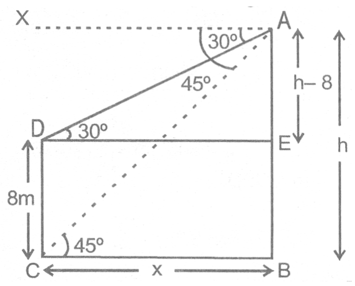
From (i) and (ii)

$\Rightarrow \qquad \sqrt{3}\ h\frac{5+h}{\sqrt{3}} \qquad \Rightarrow \qquad 3h = 5+h$

$\Rightarrow \qquad 2h = 5 \qquad \Rightarrow \qquad h = \frac{5}{2} = 2.5m$ So, the height of tower = 2.5 m

Ex.3 The angles of depressions of the top and bottom of 8m tall building from the top of a multistoried building are 30^0 and 45^0 respectively. Find the height of multistoried building and the distance between the two buildings.

Sol. Let AB be the multistoried building of height h and let the distance between two buildings be x meters.



$\angle XAC = \angle ACB = 45^0$ [Alternate angles $\because$ AX$\|$ DE]

$XAD = ADE = 30^0$ [Alternate angles $\because$ AX$\|$ BC]

In Δ ADE

$\tan 30^0 = \frac{AE}{ED}$

$\Rightarrow \qquad \frac{1}{\sqrt{3}} = \frac{h-8}{x} \qquad (\because CB = DE = x)$

$\Rightarrow \qquad x = \sqrt{3}(h-8) \qquad$ ....(i)

In Δ ACB

$\tan\ 45^0 = \frac{h}{x}$

$\Rightarrow \qquad 1 = \frac{h}{x} \qquad \Rightarrow \qquad x = h \qquad$ ....(ii)

Form (i) and (ii)

$\sqrt{3}(h-8) = h \qquad \Rightarrow \qquad \sqrt{3}h - 8\sqrt{3} = h$

$\Rightarrow \qquad \sqrt{3}h - h = 8\sqrt{3} \qquad \Rightarrow \qquad h(\sqrt{3}-1) = 8\sqrt{3}$

$\Rightarrow \qquad h = \frac{8\sqrt{3}}{\sqrt{3}-1}\times\frac{(\sqrt{3}+1)}{\sqrt{3}+1} \qquad \Rightarrow \qquad h = \frac{8\sqrt{3}(\sqrt{3}+1)}{2} \qquad \Rightarrow \qquad h = 4\sqrt{3}(\sqrt{3}+1)$

$\Rightarrow \qquad h = 4(3+\sqrt{3})$ metres

Form (ii) x = h

So, x = 4(3 + $\sqrt{3}$) metres $\qquad$ Hence, height of multistoried building = 4(3 + $\sqrt{3}$) metres

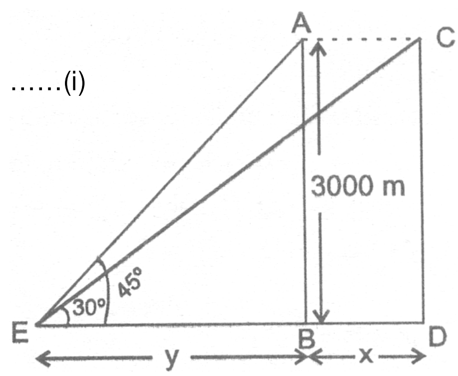
Distance between two building = 4(3 + $\sqrt{3}$) metres

Ex.4 The angle of elevation of an aeroplane from a point on the ground is 45^0. After a flight of 15 sec, the elevation changes to 30^0. If the aeroplane is flying at a height of 3000 metres, find the speed of the aeroplane.

Sol. Let the point on the ground is E which is y metres from point B and let after 15 sec flight it covers x metres distance.

In Δ AEB.

$\tan 45^0 = \frac{AB}{EB} \qquad \Rightarrow \qquad 1 = \frac{3000}{y} \qquad \Rightarrow \qquad y = 3000\,m \qquad$ ......(i)



In Δ CED

$\Rightarrow \qquad \tan 30^0 = \frac{CD}{ED}$

$\Rightarrow \qquad \frac{1}{\sqrt{3}} = \frac{3000}{x+y} \qquad (\because AB = CD)$

$\Rightarrow \qquad x + y = 3000\ \sqrt{3} \qquad$ ......(ii)

From equation (i) and (ii)

$\Rightarrow \qquad x + 3000 = 3000\sqrt{3} \qquad \Rightarrow \qquad x = 3000\sqrt{3} - 3000 \qquad \Rightarrow \qquad x = 3000(\sqrt{3}-1)$

$\Rightarrow \quad x = 3000\times(1.732-1) \quad \Rightarrow \quad x = 2196\,m$

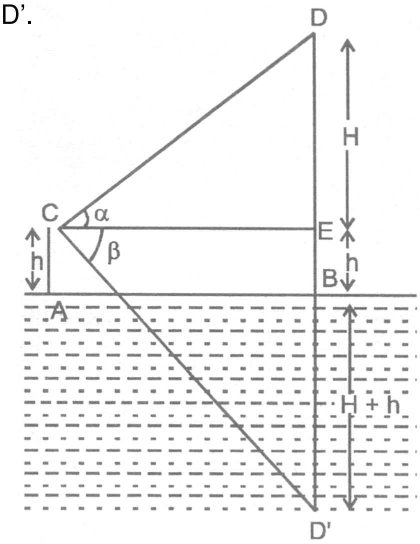
Speed of Aeroplane $= \dfrac{\text{Dis tan ce cov ered}}{\text{Tiem taken}} \qquad = \dfrac{2196}{15}\,m/sec. = 146.4\ m/sec.$

$= \dfrac{2196}{15}\times\dfrac{18}{5}\,Km/hr \qquad = 527.04\ Km/hr$

Hence, the speed of aeroplane is 527.04 Km/hr.

Ex.5 If the angle of elevation of cloud from a point h metres above a lake is α and the angle of depression of its reflection in the lake is β, prove that the distance of the cloud from the point of observation is $\dfrac{2h\ \sec\ \alpha}{\tan\beta-\tan\alpha}$.

Sol. Let AB be the surface of the lake and let C be a point of observation such that AC- h metres. Let D be the position of the cloud and D' be its reflection in the lake. Then BD = BD'.



In Δ DCE

$\tan\alpha = \dfrac{DE}{CE} \quad \Rightarrow \quad CE = \dfrac{H}{\tan\alpha} \quad$ ......(i)

In Δ CED'

$\tan\beta = \dfrac{ED'}{EC}$

$\Rightarrow \quad CE = \dfrac{h+H+h}{\tan\beta}$

$\Rightarrow \quad CE = \dfrac{2h+H}{\tan\beta} \quad$ ......(ii)

From (i) & (ii)

$\Rightarrow \quad \dfrac{H}{\tan\alpha} = \dfrac{2h+H}{\tan\beta} \quad \Rightarrow \quad H\tan\beta = 2h\ \tan\alpha + H\tan\alpha$

$\Rightarrow \quad H\tan\beta - H\ \tan\alpha + 2h\tan\alpha \quad \Rightarrow \quad H(\tan\beta-\tan\alpha) = 2h\ \tan\alpha$

$\Rightarrow \quad H = \dfrac{2h\ \tan\alpha}{\tan\beta-\tan\alpha} \quad$ ....(iii)

In ΔDCE

$\text{Sin}\ \alpha = \dfrac{DE}{CD}$

$\Rightarrow \quad CD = \dfrac{DE}{\sin\alpha} \quad \Rightarrow \quad CD = \dfrac{H}{\sin\alpha}$

Substituting the value of H from (iii)

$CD = \dfrac{2h\ \tan\alpha}{(\tan\beta-\tan\alpha)\sin\alpha} \quad \Rightarrow \quad CD = \dfrac{2h\dfrac{\sin\alpha}{\cos\alpha}}{(\tan\beta-\tan\alpha)\sin\alpha}$

$CD = \dfrac{2h\ \tan\alpha}{\tan\beta-\tan\alpha}$

Hence, the distance of the cloud from the point of observation is $\dfrac{2h\ \sec\alpha}{\tan\beta-\tan\alpha}$ **Hence Proved.**

Ex.6 A boy is standing on the ground and flying a kite with 100 m of string at an elevation of 30^0. Another boy is standing on the roof of a 10 m high building and is flying his kite at an elevation of 45^0. Both the boys are on opposite sides of both the kites. Find the length of the string that the second boy must have so that the two kites meet.

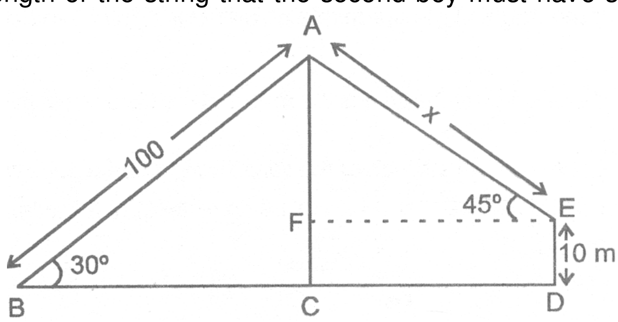
Sol. Let the length of second string b x m.

In Δ ABC

$\text{Sin}\ 30^0 = \dfrac{AC}{AB}$

$\dfrac{1}{2} = \dfrac{AC}{100} \Rightarrow AC = 50\,m$

In ΔAEF



$\text{Sin } 30^0 = \frac{AF}{AE}$

$\frac{1}{\sqrt{2}} = \frac{AC - FC}{x}$

$\frac{1}{\sqrt{2}} = \frac{50-10}{x}$ [$\therefore$ AC = 50 m, FC = ED = 10 m] $\frac{1}{\sqrt{2}} = \frac{40}{x}$

$x = 40\sqrt{2}$m (So the length of string that the second boy must have so that the two kites meet = $40\sqrt{2}$ m.)

DAILY PRACTICE PROBLEMS # 12

OBJECTIVE DPP - 12.1

1. Upper part of a vertical tree which is broken over by the winds just touches the ground and makes an angle of 30^0 with the ground. If the length of the broken part is 20 metres, then the remaining part of the trees is of length (A) 20 metres (B) $10\sqrt{3}$ metres (C) 10 metres (D) $10\sqrt{2}$ metres

2. The angle of elevation of the top of a tower as observed from a point on the horizontal ground is 'x'. If we move a distance 'd' towards the foot of the tower, the angle of elevation increases to 'y', then the height of the tower is
(A) $\frac{d \tan x \tan y}{\tan y - \tan x}$ (B) d(tan y + tan x) (C) d(tan y – tan x) (D) $\frac{d \tan x \tan y}{\tan y + \tan x}$

3. The angle of elevation of the top of a tower, as seen from two points A & B situated in he same line and at distances 'p' and 'q' respectively from the foot of the tower, are complementary, then the height of the tower is (A) pq (B) $\frac{p}{q}$ (C) $\sqrt{pq}$ (D) noen of these

4. The angle of elevation of the top of a tower at a distance of $\frac{50\sqrt{3}}{3}$ metres from the foot is 60^0. Find the height of the tower
(A) $50\sqrt{3}$ metres (B) $\frac{20}{\sqrt{3}}$ metres (C) -50 metres (D) 50 metres

5. The Shadow of a tower, when the angle of elevation of the sun is 30^0, is found to be 5 m longer than when its was 45^0, then the height of tower in metre is
(A) $\frac{5}{\sqrt{3}+1}$ (B) $\frac{5}{2}(\sqrt{3}-1)$ (C) $\frac{5}{2}(\sqrt{3}+1)$ (D) None of these.

SUBJECTIVE DPP - 12.2

1. From the top a light house, the angles of depression of two ships of the opposite sides of it are observed to be α and β. If the height of the light house be h meters and the line joining the ships passes thought the foot of the light house. Show that the distance between the ships is $\frac{h(\tan\alpha + \tan\beta)}{\tan\alpha\tan\beta}$ meters.

2. A ladder rests against a wall at angle α to the horizontal. Its foot is pulled away from the previous point through a distance 'a', so that is slides down a distance 'b' on the wall making an angle β. With the horizontal show that $\frac{a}{b} = \frac{\cos\alpha - \cos\beta}{\sin\beta - \sin\alpha}$

3. From an aeroplanne vertically above a straight horizontal road, the angle of depression of two consecutive kilometer stone on opposite side of aeroplane are observed to be α and β. Show that the height of aeroplane above the road is $\frac{\tan\alpha\tan\beta}{\tan\alpha + \tan\beta}$ kilometer.

4. A round balloon of radius 'r' subtends an angle θ at the eye of an observer while the angle of elevation of its centre is ϕ. Prove that the height of the centre of the balloon is r sin ϕ cosec $\frac{\theta}{2}$.

5. A window in a building is at a height of 10 m from the ground. The angle of depression of a point P on the ground from the window is 30^0. The angle of elevation of the top of the building from the point P is 60^0. Find the height of the building.

6. A man on a cliff observers a boat at an angle of depression of 30^0 which is approaching the shore to the point immediately beneath the observer with a uniform speed. Six minutes later, the angle of depression

of the boat is found to be 60^0. Find the total time taken by the boat from the initial point to reach the shore.

7. The angles of elevation of the top of a tower two points 'P' and 'Q' at distances of 'a' and 'b' respectively from the base and in the same straight line with it, are complementary. Prove that the height of the tower is $\sqrt{ab}$. **[CBSE - 2004]**

8 Two pillars of equal height are on either side of a road, which s 100m wide. The angles of elevation of the top the pillars are 60^0 and 30^0 at a point on the road between the pillar. Find the position of the pint between the pillars. Also find the height of each pillar, **[CBSE - 2005]**

9 At a point, the angle of elevation of a tower is such that its tangent is $\frac{5}{12}$,On walking 240mnearer the tower, the tangent to the angle of elevation becomes $\frac{3}{4}$, Find the height of the tower. **[CBSE - 2006]**

10 From a window 'x'mtres high above the ground in a street, the angles of elevation and depression of the top and foot of the other hose on the opposite side of the street are α and β respectively, Show that the opposite house is x $(1 + \tan\alpha\cot\beta)$ metres. **[CBSE - 2006]**

11 A pole 5m high is fixed on the top of a towel, the angle of elevation of the top of the pole observed from a point 'A' on the ground is 60^0 an the angle of depression the point ;A; from the top of the tower is 45^0 Find the height of the tower. **[CBSE - 2007]**

12 The angle of elevation of a jet fighter from a point A on the ground is 60^0 After a flight of 15 seconds, the angle o elevation changes to 30^0 If the jet is flying at a spies of 720km/fr, find the constant height at which the jet is flying. [use $\sqrt{3}$ = 1.732] **[CBSE - 2008]**

ANSWERS

(Objective DPP 12 .1)

Q	1	2	3	4	5
A.	C	A	C	D	C

(Subjective DPP 12.2)

5. 30 m 6. 9 min.

8. Height = 43.3 m, Position – point is 25 mfrom 1^{st} end and 75 m from 2^{nd} end.

9. 225 m 11. 6.82 m 12. 2598 m.

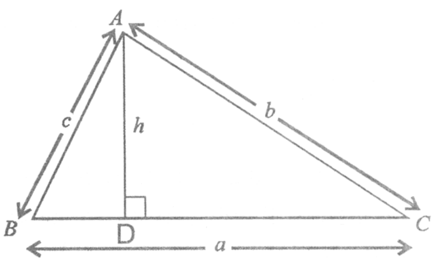
Teko CLASSES ..... the support

➤➤➤MENSURATION⮜⮜⮜

13.1 MENSRTION:

Figure lying in a plane is called a plane figure. A plane figure made up of lines or curve or both, is said to be a closed figure if it has on free ends. Closed figure in a plane covers some part of the plane, then magnitude o that part of the plane is called the area of that closed figure. the unit of measurement of that part of the plane is called the area of that closed figure. the unit o measurement of area is square unit (i.e. square centimeter, square metre etc.)

13.1 (a)Mensuration of a Triangle:



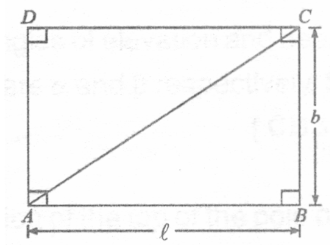
perimeter = a + b + c

$$\text{Area} = \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$= \frac{1}{2}ah$$

Heron's formula: **Area** $= \sqrt{s(s-a)(s-b)(s-c)}$

Where's = semi – perimeter $= \dfrac{a+b+c}{2}$

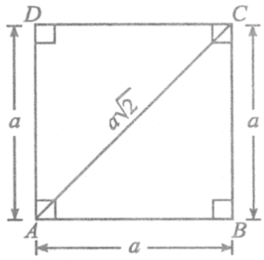


13.1(b) Menstruation of a Rectangle:

Perimeter $= 2(\ell + b)$

Area $= \ell \times b$

Length of diagonal $= \sqrt{\ell^2 + b^2}$



13.1(c) Menstruation of a Square:

Perimeter = 4 a

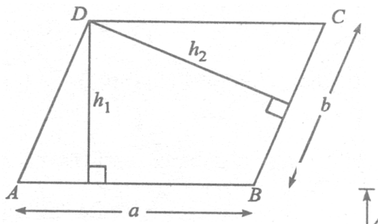
Area = a^2

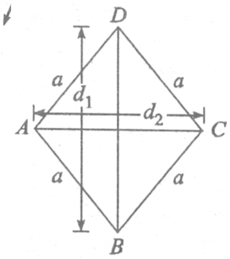
Length of diagonal = $a\sqrt{2}$

13.1(d) Menstruation of a parallelogram:

Perimeter = 2 (a + b)

Area $= ah_1 = bh_2$

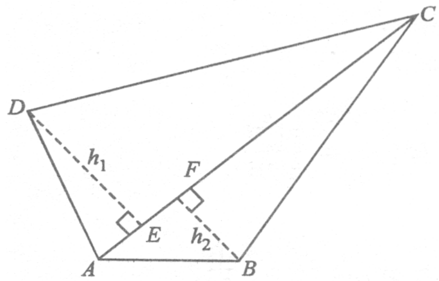




13.1(e)Mensuration of a Rhombus:

Perimeter $= 4a = 2\sqrt{d_1^2 + d_2^2}$

Area $= \dfrac{1}{2} d_1 d_2$



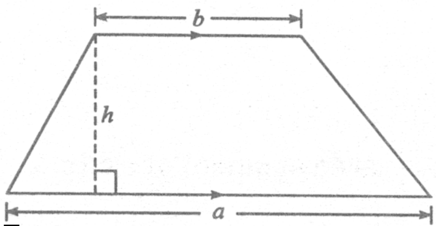
13.1 (f) Mensuration of a Quadrilateral:

Let AC = d

Area $= \dfrac{1}{2} d(h_1 + h_2)$

13.1(g)Menstruation of a Trapezium:

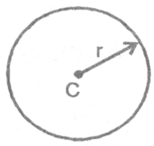
Area $= \dfrac{1}{2} h(a + b)$



13.2 AREA RELTED TO CIRCLE:

Circle: Circle is a point, which moves so such a manner that its distance from a fixed point id always equal. The fixed point is called center of the circle of the circle and the fixed distance is called radius of the circle.

Area of circle (A) = πr^2 **Circumference (C) =** $2\pi r$ **Diameter (D) = 2r**



RESULTS:

(i) If two circles touch internally. then the distance between their centers is equal to the difference of their radii,

(ii) If two circles touch externally, then the distance between their centers is equal to the sum of their radii.

(iii) Distance moved by a rotating wheel in one revolution is the circumference of the wheel.

(iv) Number of revolutions completed by a rotating wheel in one minute

$$= \frac{\text{Distance moved in one minute}}{\text{Circumference}}$$

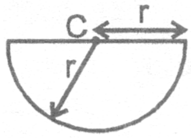
(v) Angle described by minute hand is one minute = 6^0.

(vi) Angle described by hour hand in one hour =30^0.

13.5(a) Semicircle:

Perimeter $= \pi r + 2r = (\pi + 2)r$

Area (A) $= \dfrac{\pi r^2}{2}$



Semi-Circle

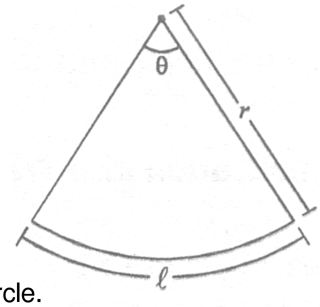
13.2(b)Sector:

Area (A) $= \dfrac{\pi r^2 \theta}{360^0}$

Length of arc $(\ell) = \dfrac{\pi r \theta}{180^0}$

Area(A) $= \dfrac{1}{2} \times \ell \times r$

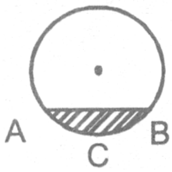
Perimeter $= \ell + 2r$



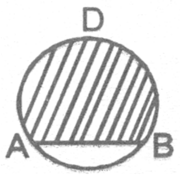
Sector of a Circle

13.2(c)Segment :

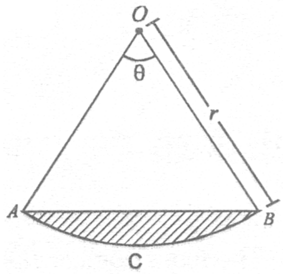
Shaded portion in the figure id called segment of a circle.



Minor segment



Major segment



Minor Segment

Area of minor segment =Area of the sector -Area of triangle OAB

$A = \dfrac{\pi r^2 \theta}{360^0} - r^2 \sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}$ **OR** $A = \dfrac{\pi r^2 \theta}{360^0} - \dfrac{r^2}{2}\sin\theta$

Here, segment ACB is called manor segment while ADB is called major segment.

13.3 MENSURATION (SOLID FIGURES) :

If any figure such as cuboids, which has three dimensions length, width and height are height are known as three dimensional figures. Where as rectangle has only two dimensional i.e., length and width. Three dimensional figures have volume in addition to areas of surface from which these soils figures are formed.

Some of the main solid figures are:

13.3 (a) Cuboid:

Total Surface Area (T.S.A.) : The area of surface from which cuboid is formed. There are six faces (rectangular), eight vertices and twelve edges n a cuboid.

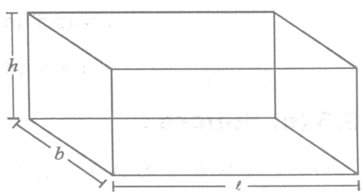
(i)Total Surface Area (T.S.A.) $= 2[\ell 0215b + b \times h + h \times \ell]$

(ii) Lateral Surface Area (L.A.A.) $= 2[b \times h + h \times \ell]$

(or Area of 4 walls) $= 2h[\ell + b]$

(iii) Volume of Cuboid = (Area of base) × height

(iv) Length of diagonal $= \sqrt{\ell^2 + b^2 + h^2}$



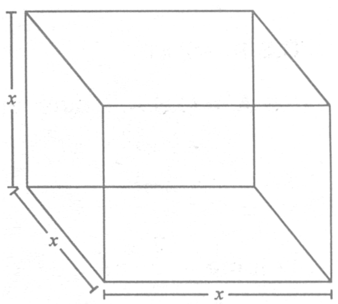
13.3 (b) Cube :

Cube has six faces. Each face is a square.

(i) **T.S,A** = 2[. x + x. x + x. x]
$= 2\ 2x^2 + x^2 + x^2] = 2(3x^2) = 6x^2$

(ii) **L.S.A.** $= 2[x^2 + x^2] = 4x^2$

(iii) **Volume** = (Area of base) × Height)
$= (x^2) . x = x^3$



(iv) **Length of altitude** $= x\sqrt{3}$

13.3 (c) Cylinder :

Curved surface area of cylinder (C.S.A.) : It is the area of surface from which the cylinder is formed. When we cut this cylinder, we will find a rectangle with length $2\pi r$ and height h units.

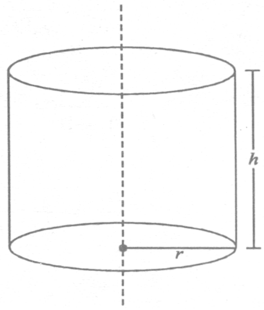
(i) C.S.A. of cylinder $= (2\pi r)\times h = 2\pi rh$.

(ii) Total Surface Area (T.S.A.) :

T.S.A. = C.S.A. + circular top & bottom

$= 2\pi rh + 2\pi r^2$

$= 2\pi r(h + r)$ sq. units.



(iii) Volume of cylinder :

Volume = Area of base × height

$= (\pi r^2) \times h$

$= \pi r^2 h$ cubic units

13.3 (d) Cone :

(i) C.S.A. $= \pi r\ell$

(II) T.S.A. = C.S.A. + Other area

$= \pi r\ell$

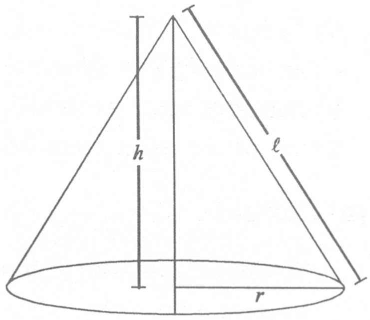
$= \pi r(\ell + r)$

(iii) Volume $= \frac{1}{3}\pi r^2 h$

Where, h = height

r = radius of base

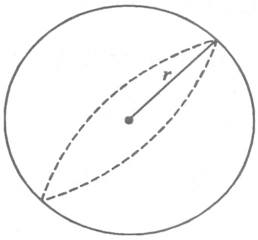
ℓ = slant height



13.3 (e) Sphere :

T.S.A. = S.A. $= 4\pi r^2$

Volume $= \frac{4}{3}\pi r^3$



13.3 (f) Hemisphere :

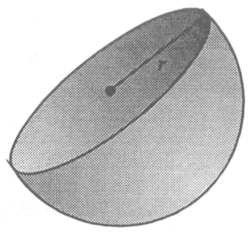
C.S.A $= 2\pi r^2$

T.S.A = C.S.A. + other area

$= 2\pi r^2 + \pi r^2$

$= 3\pi r^2$

Volume = $\frac{2}{3}\pi r^3$



13.3 (g) Frustum of a Cone :

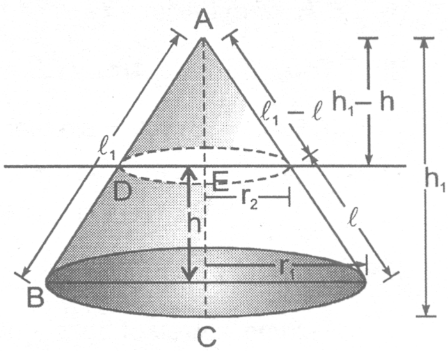
When a cone is cut by a plane parallel to base, a small cone is obtained at top and other part is obtained at bottom. This is known as **'Frustum of Cone'.**

Δ ABC ~ Δ ADE

$\therefore \quad \frac{AC}{AE} = \frac{AB}{AD} = \frac{BC}{DE}$

$\frac{h_1}{h_1 - h} = \frac{\ell}{\ell_1 - \ell} = \frac{r_1}{r_2}$

Or $\frac{h_1}{h} = \frac{\ell_1}{\ell} = \frac{r_1}{r_1 - r_2}$



Volume of Frustum $= \frac{1}{3}\pi r_1^2 h_1 - \frac{1}{3}\pi r_2^2 (h_1 - h)$

$= \frac{1}{3}\pi[r_1^2 h_1 - r_2^2(h_1 - h)]$

$= \frac{1}{3}\pi\left[r_1^2\left(\frac{r_1 h}{r_1 - r_2}\right) - r_2^2\left(\frac{r_1 h}{r_1 - r_2} - h\right)\right] = \frac{1}{3}\pi h\left[\frac{r_1^3 - r_2^3}{r_1 - r_2}\right]$

$= \frac{1}{3}\pi h\left[r_1^2 + r_2^2 + r_1 r_2\right]$

Curved Surface Area of Frustum $= \pi r_1 \ell_1 - \pi r_2(\ell_1 - \ell)$

$= \pi\left[r_1\left(\frac{r_1\ell}{r_1 - r_2}\right) - r_2\left(\frac{r_1\ell}{r_{1-r_2}} - \ell\right)\right] = \pi\ell\left[\frac{r_1^2}{r_1 - r_2} - \frac{r_2^2}{r_1 - r_2}\right]$

$= \pi\ell(r_1 + r_2)$

Total Surface Area of Frustum = CSA of frustum $+ \pi r_1^2 + \pi r_1^2 + \pi r_2^2$

$= \pi\ell(r_1 - r_2) + \pi r_1^2 + \ell r_2^2$

Slant height of a Frustum = $\sqrt{h^2 + (r_1 - r_2)^2}$

where,

h - height of the frustum

r_1 = radius of larger circular end

r_2 = radius of smaller circular end

ILLUSTRACTION :

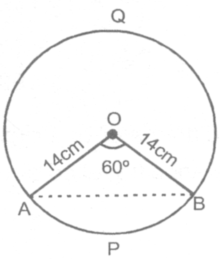
Ex.1 A chord of circle 14 cm makes an angle of 60^0 at the center of the circle. Find :

(i) area of minor sector (ii) area of the minor segment

(iii) area of the major sector (iv) area of the major segment

Sol. **Given,** r = 14 cm, $\theta = 60^0$

(i) Area of minor sector OAPB = $\frac{\theta}{360^0}\pi r^2$



$= \frac{60^0}{360^0} \times 3.14 \times 14 \times 14$

= 102.57 cm^2

(ii) Area of minor segment APB = $\frac{\pi r^2\theta}{360^0} - \frac{r^2}{2}\sin\theta$

= 102.57 - $\frac{14\times14}{2}\sin 60^0$

= 102.57 – 98 × $\frac{\sqrt{3}}{2}$

= 17.80 cm^2

(iii) Area of major sector = Area of circle - Area of minor sector OAPB

= π $(14)^2$ - 102.57

= 615.44 - 102.57 = 512.87 cm^2

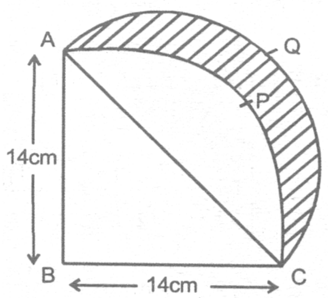
(iv) Area of major segment AQB

= Area of circle - Area of minor segment APB

= 615.44 - 17.80

= 597.64 cm^2.

Ex.2 ABCP is a quadrant of a circle of radius 14 cm. With AC as diameter, a semicircle is drawn. Find the area of the shaded portion **(figure).**



Sol. In right angled triangle ABC,
we have.

$AC^2 = AB^2 + BC^2$

$AC^2 = 14^2 + 14^2$

$AC = \sqrt{2 \times 14^2} = 14\sqrt{2}$ cm

Now required Area = Area APCQA
= Area ACQA - Area ACPA
= Area ACQA - (Area ABCPA - Area of ΔABC)

$= \frac{1}{2} \times \pi \times \left(\frac{14\sqrt{2}}{2}\right)^2 - \left[\frac{1}{4} \times \pi(14)^2 + \frac{1}{2} \times 14 \times 14\right]$

$= \frac{1}{2} \times \frac{22}{7} \times 7\sqrt{2} \times 7\sqrt{2} - \frac{1}{4} \times \frac{22}{7} \times 14 \times 14 + 7 \times 14$

= 154 – 154 ÷ 98 = 98 cm^2

Ex.3 The diameter of cycle wheel is 28 cm. How many revolution will it make in moving 13.2 km ?

Sol. Distance traveled by the wheel is one revolution $= 2\pi r$

$= 2 \times \frac{22}{7} \times \frac{28}{2} = 88$ cm

and the total distance covered by the wheel = 13.2 × 1000 × 100 cm
= 1320000 cm

∴ Number of revolution made by the wheel $= \frac{1320000}{88}$ = 15000.

Ex. 4 How many balls, each of radius 1 cm, can be made from a solid sphere of lead of radius 8 cm ?

Sol. Volume of the spherical ball of radius 8 cm = $\frac{4}{3}\pi \times 8^3$ cm^3

Also, volume of each smaller spherical ball of radius 1 cm = $\frac{4}{3}\pi \times 1^3 cm^3$.

Let n be the number of smaller balls that can be made. Then, the volume of the larger ball is equal to the sum of all the volumes of n smaller balls.

Hence, $\frac{4}{3}\pi \times n = \frac{4}{3}\pi \times 8^3$ $\Rightarrow$ $n = 8^3 = 512$

Hence, the required number of balls = 512.

Ex.5 An iron of length 1 m and diameter 4 cm is melted and cast into thin wires of length 20 cm each. If the number of such wires be 2000, find the radius of each thin wire.

Sol. Let the radius of each thin wire be r cm. The, the sum of the volumes of 2000 thin wire will be equal to the volume of the iron rod. Now, the shape of the iron rod and each thin wire is cylindrical.

Hence, the volume of the iron rod of radius $\frac{4}{2}$ cm = 2 cm is $\pi \times 2^2 \times 100 cm^3$

Again, the volume of each thin wire = $\pi r^2 \times 20$

Hence, we have $\pi \times 2^2 \times 100 = 2000 \times \pi r^2 \times 20$

$\Rightarrow$ $40r^2 = 4 \Rightarrow r^2 = \frac{1}{100}$ $\Rightarrow r = \frac{1}{10}$ [Taking positive square root only]

Hence, the required radius of each thin wire is $\frac{1}{10}$ cm. of 0.1 cm.

Ex.6 By melting a solid cylindrical metal, a few conical materials are to be made. If three times the radius of the cone is equal to twice the radius of the cylinder and the ratio of the height of the cylinder and the height of the cone is 4 : 3 find the number of cones which can be made.

Sol. Let R be the radius and H be the height of the cylinder and let r and h be the radius and height of the cone respectively. Then.

3r = 2R

and H : h = 4 : 3 .....(i)

$\Rightarrow \frac{H}{h} = \frac{4}{3}$

$\Rightarrow$ 3H = 4h .....(ii)

Let be the required number of cones which can be made from the material of the cylinder. The, the volume of the cylinder will be equal to the sum of the volumes of n cones. Hence, we have

$\pi R^2 H = \frac{n}{3}\pi r^2 h \Rightarrow 3R^2H = nr^2h$

$\Rightarrow n = \frac{3R^2H}{r^2h} = \frac{3\times\frac{9r^2}{4}\times\frac{4h}{3}}{r^2h}$ [$\therefore$ From (i) and (ii), $R = \frac{3r}{2}$ and $H = \frac{4h}{3}$]

$\Rightarrow n = \frac{3\times 9\times 4}{3\times 4}$

$\Rightarrow n = 9$

Hence, the required number of cones is 9.

Ex.7 The base diameter of solid in the form of a cone is 6 cm and the height of the cone is 10 cm. It is melted and recast into spherical balls of diameter 1 cm. Find the number of balls, thus obtained.

Sol. Let the number of spherical balls be n. Then, the volume of the cone will be equal to the sum of the volumes

of the spherical balls. The radius of the base of the cone = $\frac{6}{2}$ cm = 3 cm

and the radius of the sphere $= \frac{1}{2}$cm

Now, the volume of the cone $= \frac{1}{3}\pi\times 3^2\times 10cm^3 = 30\pi cm^3$

and, the volume of each sphere $= \frac{4}{3}\pi\left(\frac{1}{2}\right)^3 cm^3 = \frac{\pi}{6}cm^3$

Hence, we have

$n\frac{\pi}{6} = 30\pi \Rightarrow n = 6 \times 30 = 180$

Hence, the required number of balls = 180.

Ex.8 A conical empty vessel is to be filled up completely by pouring water into it successively with the help of a cylindrical can of diameter 6 cm and height 12 cm. The radius of the conical vessel if 9 cm and its height is 72 cm. How many times will it required to pour water into the conical vessel to fill it completely, if, in each time, the cylindrical can is filled with water completely ?

Sol. Let n be the required number of times. Then, the volume of the conical vessel will be equal to n times the volume of the cylindrical can.

Now, the volume of the conical vessel $= \frac{1}{3}\pi\times 9^2\times 72cm^3 = 24\times 81\pi\ cm^3$

Add the volume of the cylindrical can = $\pi \times 3^2 \times 12\ cm^3 = 9\times 12\ \pi\ cm^3$

Hence , $24 \times 81\pi = 9 \times 12\pi \times n$

$\Rightarrow n = \frac{24\times 81}{9\times 12} = 18$ Hence, the required number of times = 18.

Ex.9 The height of a right circular cylinder is equal to its diameter. It is melted and recast into a sphere of radius equal to the radius of the cylinder, find the part of the material that remained unused.

Sol. Let n be height of the cylinder. Then, its diameter is h and so its radius is $\frac{h}{2}$. Hence, its volume is

$V_1 = \pi\left(\frac{h}{2}\right)^2 h = \frac{\pi h^3}{4}$

Again, the radius of the sphere = $\frac{h}{2}$

Hence, the volume of the sphere is $V_2 = \frac{4}{3}\pi\left(\frac{h}{2}\right)^3 = \frac{\pi h^3}{6}$

$\therefore$ The volume of the unused material = $V_1 - V_2 = \frac{\pi h^3}{4} - \frac{\pi h^3}{6} = \frac{\pi h^3(3-2)}{12} = \frac{\pi h^3}{12} = \frac{1}{3} = \times\frac{\pi h^3}{4} = \frac{1}{3}V_1$

Hence, the required volume of the unused material is equal to $\frac{1}{3}$ of the volume of the cylinder.

Ex.10 Water flows at the rate of 10 m per minute through a cylindrical pipe having its diameter as 5 mm. How much time till it take to fill a conical vessel whose diameter of the base is 40 cm and depth 24 cm ?

Sol. Diameter of the pipe = 5 mm $\frac{5}{10}$ cm = $\frac{1}{2}$ cm.

$\therefore$ Radius of the pipe = $\frac{1}{2}\times\frac{1}{2}$ cm = $\frac{1}{4}$ cm.

In 1 minute, the length of the water column in the cylindrical pipe = 10 m = 1000 cm.

$\therefore$ Volume, of water that flows out of the pipe in 1 minute = $\pi\times\frac{1}{4}\times\frac{1}{4}\times 1000$ cm^3.

Also, volume of the cone = $\frac{1}{3}\times\pi\times 20\times 20\times 24$ cm^3.

Hence, the time needed to fill up this conical vessel = $\left(\frac{1}{3}\pi\times 20\times 20\times 24\div\pi\times\frac{1}{4}\times\frac{1}{4}\times 1000\right)$ minutes

$= \left(\frac{20\times 20\times 24}{3}\times\frac{4\times 4}{1000}\right) = \frac{4\times 24\times 16}{30}$ minutes

$= \frac{256}{5}$ minutes = 51.2 minutes.

Hence, the required time of 51.2 minutes.

Ex.11 A hemispherical tank of radius $1\frac{3}{4}$ is full of water. It is connected with a pipe which empties it at the rate of 7 liters per second. How much time will it take to empty the tank completely ?

Sol. Radius of the hemisphere = $\frac{7}{4}$ m = $\frac{7}{4}\times 100$ cm $= 175$ cm

$\therefore$ Volume of the hemisphere = $\frac{2}{3}\times\pi\times 175\times 175\times 175$ cm^3

The cylindrical pipe empties it at the rate of 7 liters i.e., 7000 cm^3 of water per second.

Hence, the required time to empty the tank = $\left(\frac{2}{3}\times\frac{22}{7}\times 175\times 175\times 175\div 7000\right)$s

$= \frac{2}{3}\times\frac{22}{7}\times\frac{175\times 175\times 175}{7000\times 60}$ min $= \frac{11\times 25\times 7}{3\times 2\times 12}$ min $= \frac{1925}{72}$ min

$\cong$ 26.75 min, nearly.

Ex.12 A well of diameter 2 m is dug 14 m deep. The earth taken out of its is spread evenly all around it to a width of 5 m to from an embankment. Find the height of the embankment.

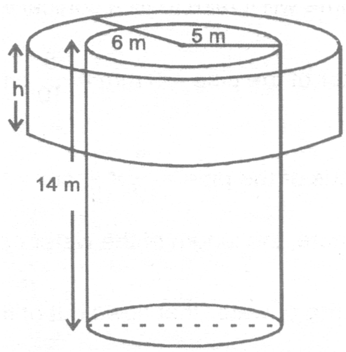
Sol. Let n be the required height of the embankment.

The shape of the embankment will be like the shape of a cylinder of internal radius 1 m and external radius (5 + 1) m = 6 m [figure].

The volume of the embankment will be equal to the volume of the earth dug out from the well. Now, the volume of the earth = volume of the cylindrical well

$= \pi\times 1^2\times 14\,m^3$

= 14 π m^3



Also, the volume of the embankment

= π (6^2 - 1^2) h cm^3 = 35 π h m^3

Hence, we have

35 πh = 14 π

$\Rightarrow$ h = $\frac{14}{35} = \frac{2}{5} = 0.4$

Hence, the required height of the embankment = 0.4 m

Ex.13 Water in a canal, 30 dm wide and 12 dm deep, is flowing with a speed of 10 km/hr. How much area will it irrigate in 30 minutes if 8 cm of standing water is required from irrigation.

Sol. Speed of water in the canal = 10 km. h = 10000 m.60 min = $\frac{500}{3}$ m/min.

$\therefore$ The volume of the water flowing out of the canal in 1 minute = $\left(\frac{500}{3}\times\frac{30}{10}\times\frac{12}{10}\right)$ m^2 = 600 m^3

$\therefore$ In 30 min, the amount of water flowing out of the canal = (600 × 30) m^3 = 600 m^3

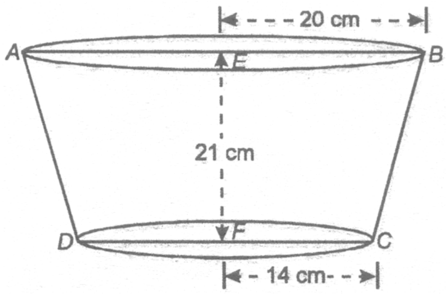
If the required area of the irrigated land is × m^2, then the volume of water to be needed to irrigate the land

$= \left(x\times\frac{8}{100}\right)m^3 \qquad = \frac{2x}{25}m^3 \qquad$ Hence, $\frac{2x}{25} = 18000 \qquad \Rightarrow \qquad x = 18000\times\frac{25}{2} = 225000$

Hence, the required area is 225000 m^2.

Ex.14 A bucket is 40 cm in diameter at the top and 28 cm in diameter at the bottom. Find the capacity of the bucket in litters, if it is 21 cm deep. Also, find the cost of tin sheet used in making the bucket, if the cost of tin is Rs. 1.50 per sq dm.

Sol. Given : r_1 = 20 cm r_2 = 14 cm and h = 21 cm



Now, the required capacity (i.e. volume) of bucket = $\frac{\pi h}{3}(r_1^2 + r_1r_2 + r_2^2)$

$\cong \frac{22\times21}{7\times3}(20^2 + 20\times14 + 14^2)$ cm^3 = 22 × 876 cm^3 = 19272 cm^3 = $\frac{19272}{1000}$ liters = 19.272 liters.

Now, l = $\sqrt{(r_1 - r_2)^2 + h^2} = \sqrt{(20-14)^2 + 21^2}$ cm $= \sqrt{6^6 + 21^2}$ cm = $\sqrt{36+441}$ cm = $\sqrt{477}$ cm $\cong 21.84$ **cm.**

$\therefore$ Total surface area of the bucket (which is open at the top)

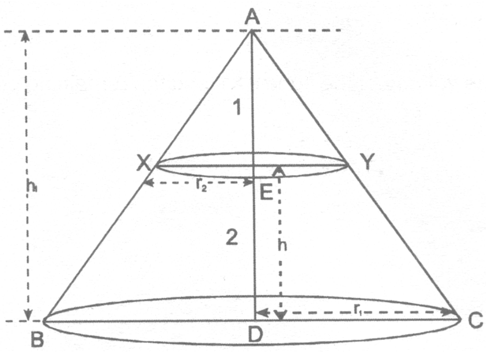
$= \pi\ell(r_1 + r_2) + \pi r_2^2 \qquad = \pi[(r_1 + r_2)\ell + r_2^2] \qquad = \frac{22}{7}\left[(20+14)21.84 + 14^2\right]$

= 2949.76 cm^3 $\therefore$ Required cost of the tin sheet at the rate of Rs. 1.50 per dm^2 i.e., per 100 cm^2

= Rs $\frac{1.50\times2949.76}{100}$ $\cong$ Rs 44.25

Ex.15 A cone is divided into two parts by drawing a plane through a point which divides its height in the ratio 1 : 2 starting from the vertex and the place is parallel to the base. Compare the volume of the two parts.

Sol. Let the plane XY divide the cone ABC in the ratio AE : ED = 1 : 2, where AED is the axis of the cone. Let r_2 and r_2 be the radii of the circular section XY and the base BC of the cone respectively and let h_1 - h and h_1 be their heights **[figure].**



Then, $\frac{h_1}{h} = \frac{3}{2} \quad \Rightarrow \quad h = \frac{3}{2}h$

And $\frac{r_1}{r_2} = \frac{h_1}{h_1 - h} = \frac{\frac{3}{2}h}{\frac{1}{2}h} = 3$

$\therefore$ $r_1 = 3r_2$

Volume of cone AXY

$= \frac{1}{32}\pi r_2^2(h_1 - h)$

$= \frac{1}{3}\pi r_2^2(\frac{3}{2}h - h) \qquad = \frac{1}{6}\pi r_2^2 h$

Volume of frustum XYBC

$= \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1r_2) \qquad = \frac{1}{3}\pi h(9r_2^2 + r_2^2 + 3r_2^2) \qquad = \frac{1}{3}\pi h(13r_2^2)$

So, $\frac{\text{Volume of cone AXY}}{\text{Volume of frustum XYBC}} = \frac{\frac{1}{6}\pi r_2^2 h}{\frac{13}{3}\pi r_2^2 h}$

$$\frac{\text{Volume of cone AXY}}{\text{Volume of frustum XYBC}} = \frac{1}{26}.$$

i.e. the ratio between the volume of the cone AXY and the remaining portion BCYX is 1 : 26.

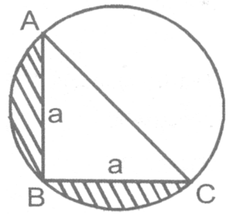
DAILY PRACTIVE PROBLEMS # 13

OBJECTIVE DPP - 13.1

1. If BC passed through the centre of the circle, then the area of the shaded region in the given figure is

(A) $\frac{a^2}{2}(3-\pi)$ (B) $a^2\left(\frac{\pi}{2}-1\right)$

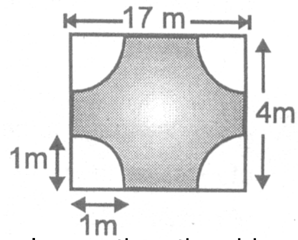
(C) $2a^2(\pi-1)$ (D) $\frac{a^2}{2}\left(\frac{\pi}{2}-1\right)$



2. The perimeter of the following shaded portion of the figure is:

(A) 40 m (B) 40.07 m

(C) 40.28 m (D) 35 m

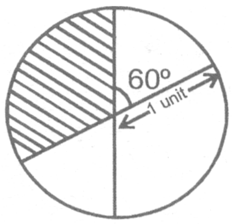


3. If a rectangle of sides 5 cm and 15 cm is be divided into three squared of equal area, then the sides of the squares will be :
(A) 4 cm (B) 6 cm (C) 7 cm (D) None

4. The area of the shaded region in the given figure is :

(A) $\frac{\pi}{3}$ sq. units (B) $\frac{\pi}{2}$ units

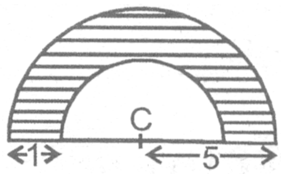
(B) $\frac{\pi}{4}$ sq. units (D) π^2 sq. units



5. The area of the shaded portion in the given figure is :

(A) 7.5 π sq. units (B) 6.5 π sq. units

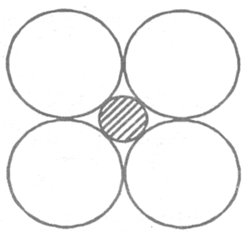
(C) 5.5 π sq. units (D) 4.5 π sq. units



6. In the adjoining figure, the radius of the inner circle, if other circles are of radii 1 m, is :

(A) $(\sqrt{2}-1)$m (B) $\sqrt{2}$ m

(C) $\frac{1}{\sqrt{2}}$ m (D) $\frac{2}{\sqrt{2}}$ m



7. The height of a conical tent of the centre is 5cm. The distance of any point on its circular base from the top of the tent is 13m. The area of the slant surface is :
(A) 144 π sq m (B) 130 π sq m (C) 156 π sq m (D) 169 π sq m

8. The radius of circle is increased by 1 cm, then the ratio of the new circumference to the new diameter is :

(A) π + 2 (B) π + 1 (C) π (D) $\pi-\frac{1}{2}$

9. A hemispherical bowl of internal diameter 36 cm is full of some liquid. This liquid is to be filled in cylindrical bottles of radius 3 cm and height 6 cm., Then no of bottles needed to empty the bowl.
(A) 36 (B) 75 (C) 18 (D) 144

10. There is a cylinder circumscribing the hemisphere such that their bases are common. The ratio of their volume is
(A) 1 : 3 (B) 1 : 2 (C) 2 : 3 (D) 3 : 4

11. A sphere of radius 3 cms is dropped into a cylindrical vessel of radius 4 cms. If the sphere is submerged completely, then the height (in cm) to which the water rises, is
(A) 2.35 (B) 2.30 (C) 2.25 (D) 2.15

12. If a rectangular sheet of paper 44 cm × 22 cm is rolled along its length of form a cylinder, then the volume of cylinder in cm^3 is
(A) 1694 (B) 3080 (C) 3388 (D) none of these

13. Two cones have their heights in the ratio 1 : 3 and the radii of their bases are in the ratio 3 : 1 , then the ratio of their volumes is
(A) 1 : 3 (B) 27 :1 (C) 3 : 1 (D) 1 : 27

14. The total surface area of a cube is numerically equal to the surface area of a sphere then the ratio of their volume is
(A) $\frac{\pi}{6}$ (B) $\sqrt{\frac{\pi}{6}}$ (C) $\frac{\pi}{216}$ (D) $\sqrt{\frac{6}{\pi}}$

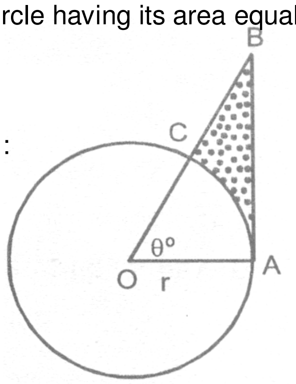
15. A cone is dived into two parts by drawing a plane through the mid point of its axis parallel to its base then the ratio of the volume of two parts is
(A) 1 : 3 (B) 1 : 7 (C) 1 : 8 (D) 1 : 9

SUBJECTIVE DPP - 13.2

1. The area of a circle inscribed in an equilateral triangle is 154 cm^2. Find the perimeter of the triangle.

2. The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having its area equal to the sum of the areas of the two circles.

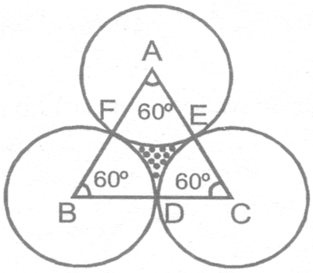
3. **Figure,** shows a sector of a circle, centre O, containing an angle θ^0. Prove that :



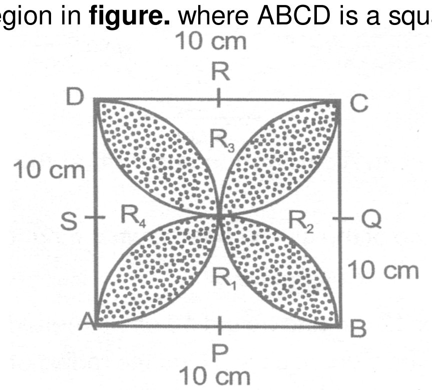
(i) Perimeter of the shaded region is $r\left(\tan\theta + \sec\theta + \frac{\pi\theta}{180} - 1\right)$

(ii) Area of the shaded region is $\frac{r^2}{2}\left(\tan\theta - \frac{\pi\theta}{180}\right)$

4. The area of an equilateral triangle is $49\sqrt{3}$ cm^2. Taking each angular point as centre, a circle is described with radius equal to half the length of the side of the triangle as shown in **figure.** Find the area of the triangle not included in the circle.



5. Find the area of the shaded region in **figure.** where ABCD is a square of side 10 cm. (use π = 3.14)



6. A hollow cone is cut by a plane parallel to the base and the upper portion is removed. If the curved surface of the remainder is $\frac{8}{9}$ of the curved surface of whole cone, find the ratio of the line - segment into which the cone's altitude is divided by the plane.

7. A right - angled triangle whose sides are 15 cm and 20 cm, is made to revolve about its hypotenuse. Find the volume and the surface area of the double cone so formed. [Take $\pi \cong$ 3.14]

8. 50 persons took dip in a rectangular tank which is 80 m long and 50 m broad. What is the rise in the level of water in the tank, if the average displacement of water by a person is 0.04 m^3?

9. Water is flowing at the rate of 5 km per hour through a pipe of diameter 14 cm into a rectangular tank, which is 50 m long and 44 m wide. Find the time in which the level of water in the tank will rise by 7 cm.

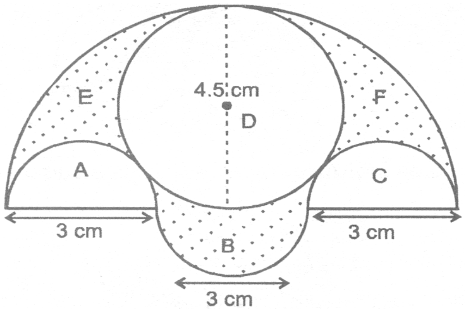
10. A circus tent is cylindrical to a height of 3 m and conical above it. If its base radius is 52.5 m and slant height of the conical portion is 53 m, find the area of the canvas needed to make the tent.

11. The diameters external and internal surfaces of a hollow spherical shell are 10 cm and 6 cm respectively. If it is melted and recast into a solid cylinder of length of $2\frac{2}{3}$ cm, find the diameter of the cylinder.

12. A cylindrical container of radius 6 cm and height 15 cm is fulled with ice-cream. The whole ice-cream has to be distributed to 10 children in equal cones with hemispherical tops. If the height of the conical portion is four times the radius of its base, find the radius of the ice-cream cone.

13. A hemi-spherical depression is cutout from one face of the cubical wooden block such that the diameter ℓ of the hemisphere is equal to the edge of the cube., Determine the surface are of the remaining solid.

14. In **figure** there are three semicircles, A,B and C having diameter 3 cm each, and another semicircle E having a circle D with diameter 4.5 cm are shown. Calculate. (i) the area of the shaded region
(ii) the cost of painting the shaded region of the 25 paisa per cm^2, to the nearest rupee.



15. The height of a cone is 30 cm. A small cone is cut off at the top by a plane parallel to the base. If its volume be $\frac{1}{27}$ of the volume of the given cone, at what above the vase is the section made ?

16. A solid cylinder of diameter 15 cm and height 15 cm is melted and recast into 12 toys in the shape of a right circular cone mounted on a hemisphere. Find the radius of the hemisphere and the total height of the to if height of the conical par is 3 times its radius. **[CBSE - 2005]**

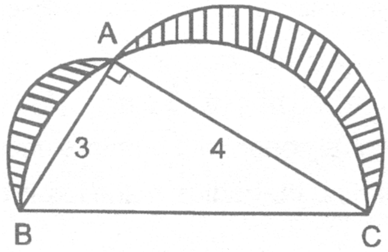
17. if the rail of the ends of bucket, 45 cm high are 28 cm and 7cm, determine the capacity and total surface area of the bucket. **[CBSE - 2006]**

18. A tent is in the form of cylinder of diameter 4.2 m and height 4 m, surmounted by a cone of equal base and height 2.8 m. Find the capacity of the tent and the cost of canvas for making the tent at Rs. 100 per sq. m. ? **[CBSE - 2006]**

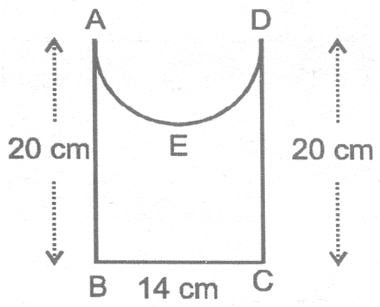
19. Water flows out through a circular pipe whose internal radius is 1 cm, at the rate of 80 cm/second into an empty cylindrical tank, the radius of whose base is 40 cm. By how much will the level of water rise in the tank in half an hour ? **[CBSE - 2007]**

20 A hemispherical bowl of internal radius 36 cm is full of liquid. The liquid is to be filled into cylindrical shaped small bottles each of diameter 3 cm and height 6 cm. How many bottles are need to empty the bowl ?

21 In figure ABC is a right - angled triangle right-angled at A. Semicircles are drawn on AB, AC and BC as diameters. Find the area of the shaded region. **[CBSE - 2008]**



22. Find the permetre of figure , where $\overset{\frown}{AED}$ is a semi-circle and ABCD is a rectangle. **[CBSE - 2008]**



23. A tent consists of a frustum of a cone, surmounted by a cone. If the diameters of the upper and lower circular ends of the frustum b 14 m and 26 m respectively, the height of the frustum be 8 m and the slant height of the surmounted conical portion be 12 m, find the area of canvas required to make the tent. (Assume that the radii of the upper circular end of the frustum and the base of surmounted conical portion are equal) **[CBSE - 2008]**

ANSWERS

(Objective DPP - 13.1)

Qus.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	D	C	D	A	D	A	C	C	B	C	C	C	C	B	B

(Subjective DPP - 13.2)

1. 72.7 cm
2. 10 cm
4. 7.77 cm^2
5. 57 cm^2
6. 1 : 2
7. 3768 cm^3, 1318.8 cm^2
8. 0.5 cm
9. 2 hrs.
10. 9735 cm^2
11. 14 cm
12. 3 cm
13. $\frac{\ell^2}{4}(24+\pi)$
14. 12.375 cm^2, Rs. 3
15. 20 cm
16. radius = 3 cm and height = 9 cm
17. 48510cm^3, 5621 cm^3
18. 68.376 m^3, Rs. 7590
19. 90 cm
20. 2304
21. 6 sq.
22. 76 cm
23. 892.57 m^2

➤➤➤PROBABILITY◄◄◄

15.1 EXPERIMENT :

The word experiment means an operation, which can produce well defined outcomes. The are two types of experiment :

(i) Deterministic experiment (ii) Probabilistic or Random experiment

(i) **Deterministic Experiment :** Those experiment which when repeated under identical conditions, produced the same results or outcome are known as deterministic experiment. For example, Physics or Chemistry experiments performed under identical conditions.

(ii) Probabilistic or Random Experiment :- In an experiment, when repeated under identical conditions donot produce the same outcomes every time. For example, in tossing a coin, one is not sure that if a head or tail will be obtained. So it is a random experiment.

Sample space : The set of all possible out comes of a random experiment is called a sample space associated with it and is generally denoted by S. For example, When a dice is tossed then
$S = \{1, 2, 3, 4, 5, 6\}$.

Even : A subset of sample space associated with a random experiment is called an event. For example, In tossing a dive getting an even no is an event.

Favorable Event : Let S be a sample space associated with a random experiment and A be event associated with the random experiment. The elementary events belonging to A are know as favorable events to the event A. For example, in throwing a pair of dive, A is defined by "Getting 8 as the sum". Then following elementary events are as out comes : (2, 6), (3, 5), (4, 4) (5, 3) (6, 2). So, there are 5 elementary events favorable to event A.

15.2 PROBABILITY :

If there are n elementary events associated with a random experiment and m of them are favorable to an event A, then the probability of happening or occurrence of event A is denoted by P(A)

Thus, $P(A) = \dfrac{\text{Total number of favourable outcomes}}{\text{Total number of possible outcomes}} = \dfrac{m}{n}$

And $0 \le P(A) \le 1$

If, $P(A) = 0$, then A is called impossible event

If, $P(A) = 1$, then A is called sure event

$P(A) + P(\overline{A}) = 1$

Where $P(A)$ = probability of occurrence of A.

$P(\overline{A})$ = probability of non - occurrence of A.

ILLUSTRATIONS :

Ex.1 A box contains 5 red balls, 4 green balls and 7 white balls. A ball is drawn at random from the box. Find the probability that the ball drawn is

(i) white (ii) neither red nor white

Sol. Total number of balls in the bag = 5 + 4 + 7 = 16

∴ Total number of elementary events =16

(i) There are 7 white balls in the bag.

∴ Favorable number of elementary events = 7

Hence, P (Getting a white ball) = $\dfrac{\text{Total No. favourable elementary events}}{\text{Total No. of elementary events}} = \dfrac{7}{16}$

(ii) There are 4 balls that are neither red nor white

∴ Favorable number of elementary events = 4

Hence, P (Getting neither red not white ball) = $\dfrac{4}{16} = \dfrac{1}{4}$

Ex.2 All the three face cards of spades are removed from a well-shuffled pack of 52 cards. A card is then drawn at random from the remaining pack. Find the probability of getting **[CBSE - 2007]**

(i) black face card (ii) a queen (iii) a black card.

Sol. After removing three face cards of spades (king, queen, jack) from a deck of 52 playing cards, there are 49 cards left in the pack. Out of these 49 cards one card can be chosen in 49 ways.

∴ Total number of elementary events = 49

(i) There are 6 black face cards out of which 3 face cards of spades are already removed. So, out of remaining 3 black face cards one black face card ban be chosen in 3 ways.

∴ Favorable number of elementary events = 3

Hence, P (Getting a black face card) = $\frac{3}{49}$

(ii) There are 3 queens in the remaining 49 cards. So, out of these three queens, on queen can be chosen in 3 ways

∴ Favorable number of elementary events = 3

Hence P (Getting a queen) = $\frac{3}{49}$

(iii) There are 23 black cards in the remaining 49 cards, So, out to these 23 black card, one black card can be chosen in 23 ways

∴ Favorable number of elementary events = 23

Hence, P (Getting a black card) = $\frac{23}{49}$

Ex.3 A die is thrown, Find the probability of

(i) prime number (ii) multiple of 2 or 3 (iii) a number greater than 3

Sol. In a single throw of die any one of six numbers 1,2,3,4,5,6 can be obtained. Therefore, the tome number of elementary events associated with the random experiment of throwing a die is 6.

(i) Let A denote the event "Getting a prime no". Clearly, event A occurs if any one of 2,3,5 comes as out come.

∴ Favorable number of elementary events = 3

Hence, P (Getting a prime no.) = $\frac{3}{6} = \frac{1}{2}$

(ii) An multiple of 2 or 3 is obtained if we obtain one of the numbers 2,3,4,6 as out comes

∴ Favorable number of elementary events = 4

Hence, P (Getting multiple of 2 or 3) = $\frac{4}{6} = \frac{2}{3}$

(iii) The event "Getting a number greater than 3" will occur, if we obtain one of number 4,5,6 as an out come.

∴ Favorable number of out comes = 3

Hence, required probability = $\frac{3}{6} = \frac{1}{2}$

Ex.4 Two unbiased coins are tossed simultaneously. Find the probability of getting

(i) two heads (ii) at least one head (iii) at most one head.

Sol. If two unbiased coins are tossed simultaneously, we obtain any one of the following as an out come :

HH, HT, TH, TT

∴ Total number of elementary events = 4

(i) Two heads are obtained if elementary event HH occurs.

∴ Favorable number of events = 1

Hence, P (Two heads) = $\frac{1}{4}$

(ii) At least one head is obtained if any one of the following elementary events happen :

HH, HT, TH

∴ favorable number of events = 3

Hence P (At least one head) = $\frac{3}{4}$

(iii) If one of the elementary events HT, TH, TT occurs, than at most one head is obtained

∴ favorable number of events = 3 Hence, P (At most one head) = $\frac{3}{4}$

Ex.5 A box contains 20 balls bearing numbers, 1,2,3,4......20. A ball is drawn at random from the box. What is the probability that the number of the ball is

(i) an odd number (ii) divisible by 2 or 3 (iii) prime number

Sol. Here, total numbers are 20.

∴ Total number of elementary events = 20

(i) The number selected will be odd number, if it is elected from 1,3,5,7,9,11,13,15,17,19

∴ Favorable number of elementary events = 10

Hence, P (An odd number) = $\frac{10}{20} = \frac{1}{2}$

(ii) Number divisible by 2 or 3 are 2,3,4,6,8,9,10,12,14,15,16,18,20

$\therefore$ Favorable number of elementary events = 13

P (Number divisible by 2 or 3) = $\frac{13}{20}$

(iii) There are 8 prime number from 1 to 20 i.e., 2,3,5,7,11,13,17,19

$\therefore$ Favorable number of elementary events = 8

P (prime number) = $\frac{8}{20} = \frac{2}{5}$

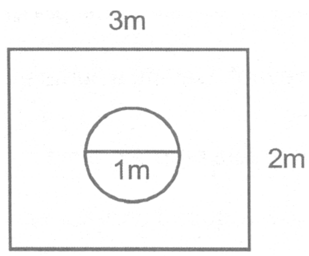
Ex.6 A die is drop at random on the rectangular region as shown in figure. What is the probability that it will land
inside the circle with diameter 1m ?

Sol. Area of rectangular region = 3m × 2m = 6m^2

Area of circle $= \pi r^2$

$= \pi \times \left(\frac{1}{2}\right)^2$

$= \frac{\pi}{4} m^2$



$\therefore$ Probability that die will land inside the circle

$= \frac{\pi/4}{6}$

$= \frac{\pi}{24}$

DAILY PRACTICE PROBLEMS # 15

OBJECTIVE DPP - 15.1

1. If there coins are tossed simultaneously, then the probability of getting at least two heads, is

(A) $\frac{1}{4}$ (B) $\frac{3}{8}$ (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

2. A bag contains three green marbles four blue marbles, and two orange marbles. If marble is picked at random, then the probability that it is not a orange marble is

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{4}{9}$ (D) $\frac{7}{9}$

3. A number is selected from number 1 to 27. The probability that it is prime is

(A) $\frac{2}{3}$ (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) $\frac{2}{9}$

4. IF (P(E) = 0.05, then P (not E) =

(A) -0.05 (B) 0.5 (C) 0.9 (D) 0.95

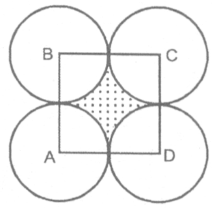
5. A bulb is taken out at random from a box of 600 electric bulbs that contains 12 defective bulbs. Then the probability of a non-defective bulb is

(A) 0.02 (B) 0.98 (C) 0.50 (D) None

SUBJECTIVE DPP - 15.2

1. To dice are thrown simultaneously. Find the probability of getting :
(i) An even number of the sum
(ii) The sum as a prime number
(iii) A total of at least 10
(iv) A multiple of 2 on one dice and a multiple of 3 on the other.
2. Find the probability that a leap year selected at random will contain 53 Tuesdays.
3. A bag contains 12 balls out of which x are white.
(i) If one ball is drawn at random, what is the probability it will be a white ball ?
(ii) If 6 more white balls are put in the box. The probability of drawing a white ball will be double than that is (i). Find x.
4. In a class, there are 18 girls and 16 boys. The class teacher wants to choose one pupil for class monitor. What she does, she writes the name of each pupil a card and puts them into a basket and mixes thoroughly. A child is asked to pick one card from the basket. What is the probability that the name written on the card is :
(i) The name of a girl (ii) The name of boy ?
5. The probability of selecting a green marble at random from a jar that contains only green, white and yellow marbles is 1/4. The probability of selecting a white marble from the same jar is 1/3. If this jar contains 10 yellow marbles. What is the total number of marbles in the jar ?

6. A card is drawn at random from a well suffled desk of playing cards. Find the probability that the card drawn is
(i) A card of spade or an ace (ii) A red king
(iii) Neither a king nor a queen (iv) Either a king or a queen
7. There are 30 cards of same size in a bag on which number 1 to 30 are written. One card is taken out of the bag at random. Find the probability that the number of the selected card is not divisible by 3.
8. In **figure** points A,B,C and D are the centers of four circles that each have a radius of length on unit. If a point is selected at random from the interior of square ABCD. What is the probability that the point will be chosen from the shaded region ?



9. A bag contains 5 white balls, 6 red balls, 6 black balls and 8 green balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is
(i) White (ii) Red or black
(iii) Note green (iv) Neither white nor black **[CBSE - 2006]**
10. A bag contains 4 red and 6 black balls. A ball is taken out of the bag at random. Find the probability of getting a black ball. **[CBSE - 2008]**
11. Cards. marked with number 5 to 50, are placed in a box and mixed thoroughly. A card is drawn from the box at random. Find the probability that the number on the taken out card is
(i) a prime number less than 10.
(ii) a number which is a perfect square. **[CBSE - 2008]**

ANSWERS

(Objective DPP 15.1)

Q	1	2	3	4	5
A	C	D	C	D	B

(Subjective DPP 15.2)

1. **(i)** $\frac{1}{2}$ **(ii)** $\frac{15}{36}$ **(iii)** $\frac{1}{6}$ **(iv)** $\frac{11}{36}$

2. $\frac{2}{7}$ **3.** **(i)** $\frac{x}{12}$ **(ii) 3**

4. **(i)** $\frac{9}{17}$ **(ii)** $\frac{8}{17}$ **5.** **24** **6.** **(i)** $\frac{4}{13}$ **(ii)** $\frac{1}{26}$ **(iii)** $\frac{11}{13}$ **(iv)** $\frac{2}{13}$ **7.** $\frac{2}{3}$

8. $\left(1-\frac{\pi}{4}\right)$ **9.** **(i)** $\frac{1}{5}$ **(ii)** $\frac{12}{25}$ **(iii)** $\frac{17}{25}$ **(iv)** $\frac{14}{25}$ **10.** $\frac{3}{5}$

11. **(i)** $\frac{1}{23}$ **(ii)** $\frac{5}{46}$

Teko CLASSES ..... the support

QUADRATIC EQUATIONS

5.1 QUADRATIC EQUATION :

If P(x) is quadratic expression in variable x, then P(x) = 0 is known as a quadratic equation.

5.1 (a) General form of a Quadratic Equation :

The general form of quadratic equation is $ax^2 + bx + c = 0$, where a,b,c are real numbers and $a \neq 0$ Since $a \neq 0$, quadratic equations, in general are of the following types :-

(i) $b = 0,\ c \neq 0$ i.e., of he type $ax^2 + c=0$. (ii) $b \neq 0, c = 0$, i.e. of the type $ax^2 + bx = 0$.

(iii) $b = 0, c = 0$, i.e. of the type $ax^2 = 0$. (iv) $b \neq 0,\ c \neq 0$, i.e., of the type $ax^2 + bx + c = 0$.

5.2 ROOTS OF A QUADRATIC EQUATION :

The value of x which satisfies the given quadratic equation is known as its root. The roots of the given equation are known as its solution.

General form of a quadratic equation is :

$ax^2 + bx + c = 0$

or $4a^2x^2 + 4abx + 4ac = - 4ac$ [Multiplying by 4a]

or $4a^2x^2 + 4abx = - 4ac$ [By adding b^2 both sides]

or $4a^2x^2 + 4abc + b^2 = b^2 - 4ac$ or $(2ax + b)^2 = b^2 - 4ac$

Taking square root of both the sides

$2ax + b = \pm\sqrt{b^2 - 4ac}$ or $x = \dfrac{-b \pm \sqrt{b2 - 4ac}}{2a}$

Hence, roots of the quadratic equation $ax^2 + bx + c = 0$ are $\dfrac{-b+\sqrt{b^2-4ac}}{2a}$ and $\dfrac{-b-\sqrt{b^2-4ac}}{2a}$

REMARK :

A quadratic equation is satisfied by exactly two values of 'a' which may be real or imaginary. The equation, $ax^2 + bx + c = 0$ is :

- ★ A quadratic equation if $a \neq 0$ Two roots
- ★ A linear equation if $a = 0,\ b \neq 0$ One root
- ★ A contradiction if $a = b = 0,\ c \neq 0$ No root
- ★ An identify if $a = b = c = 0$ Infinite roots
- ★ A quadratic equation cannot have more than two roots.
- ★ If follows from the above statement that if a quadratic equation is satisfied by more than two values of x, then it is satisfied by every value of x and so it is an identity.

5.3 NATURE OF ROOTS :

Consider the quadratic equation, $ax^2 + bx + c = 0$ having $\alpha\beta$ as its roots and $b^2 - 4ac$ is called discriminate of roots of quadratic equation. It is denoted by D or Δ.

Roots of the given quadratic equation may be

(i) Real and unequal (ii) Real and equal (iii) Imaginary and unequal.

Let the roots of the quadratic equation $ax^2 + bx + c = 0$ (where $a \neq 0, b, c \in R$) be α and β then

$\alpha = \dfrac{-b+\sqrt{b^2-4ac}}{2a}$ .....(i) and $\beta = \dfrac{-b-\sqrt{b^2-4ac}}{2a}$ ....(ii)

The nature of roots depends upon the value of expression **'b^2 - 4ac'** with in the square root sign. This is known as discriminate of the given quadratic equation.

Consider the Following Cases :

Case-1 When $b^2 - 4ac > 0$, (D > 0)

In this case roots of the given equation are real and distinct and are as follows

$\alpha = \dfrac{-b+\sqrt{b^2-4ac}}{2a}$ and $\beta = \dfrac{-b-\sqrt{b^2-4ac}}{2a}$

(i) When $a(\neq 0), b, c \in Q$ and b^2 - 4ac is a perfect square

In this case both the roots are rational and distinct.

(ii) When $a(\neq 0), b, c \in Q$ **and b^2 - 4ac is not a perfect square**

In this case both the roots are irrational and distinct. **[See remarks also]**

Case-2 When b^2 - 4ac = 0, (D = 0) In this case both the roots are real and equal to $-\frac{b}{2a}$.

Case-3 When b^2 - 4ac < 0, (D < 0) In this case b^2 - 4ac < 0, then 4ac - b^2 > 0

$\therefore \quad \alpha = \frac{-b+\sqrt{-(4ac-b^2)}}{2a}$ and $\beta = \frac{-b-\sqrt{(4ac-b^2)}}{2a}$

or $\quad \alpha = \frac{-b+i\sqrt{4ac-b^2}}{2a}$ and $\beta = \frac{-b-i\sqrt{4ac-b^2}}{2a}$ $\quad \lfloor \because \sqrt{-1} = i \rfloor$

i.e. in this case both the root are imaginary and distinct.

REMARKS :

★ If **a,b,c** $\in$ **Q** and **b^2 - 4ac** is positive **(D > 0)** but not a perfect square, then the roots are irrational and they always occur in conjugate pairs like $2+\sqrt{3}$ and $2-\sqrt{3}$. However, if **a,b,c** are irrational number and **b^2 - 4ac** is positive but not a perfect square, then the roots may not occur in conjugate pairs.

★ If **b^2 - 4ac** is negative **(D > 0)**, then the roots are complex conjugate of each other. In fact, complex roots of an equation with real coefficients always occur in conjugate pairs like 2 + 3i and 2 - 3i. However, this may not be true in case of equations with complex coefficients. For example, x^2 - 2ix - 1 = 0 has both roots equal to i.

★ If **a** and **c** are of the same sign and b has a sign opposite to that of a as well as c, then both the roots are positive, the sum as well as the product of roots is positive $(D \geq 0)$.

★ If **a,b,** are of the same sign then both the roots are negative, the sum of the roots is negative but the product of roots is positive $(D \geq 0)$.

5.4 METHODS OF SOLVING QUADRATIC EQUATION :

5.4 (a) By Factorisation :

ALGORITHM :

Step (i) Factorise the constant term of the given quadratic equation.

Step (ii) Express the coefficient of middle term as the sum or difference of the factors obtained in step 1. Clearly, the product of these two factors will be equal to the product of the coefficient of x^2 and constant term.

Step (iii) Split the middle term in two parts obtained in step 2.

Step (iv) Factorise the quadratic equation obtained in step 3.

Ex.1 Solve the following quadratic equation by factorisation method: $x^2 - 2ax + a^2 - b^2 = 0$.

Sol. Here, Factors of constant term $(a^2 - b^2)$ are (a - b) and (a + b).

Also, Coefficient of the middle term = - 2a = - [(a - b) + (a + b)]

$\therefore \quad x^2 - 2ax + a^2 - b^2 = 0$

$\Rightarrow \quad x^2 - \{(a - b) + (a + b)\}x + (a - b)(a + b) = 0 \quad \Rightarrow \quad x^2 - (a - b)x - (a + b)x + (a - b)(a + b) =$

$\Rightarrow \quad x\{x - (a - b)\} - (a + b)\{x - (a - b)\} = 0 \quad \Rightarrow \quad \{x - (a - b)\}\{x - (a + b)\} = 0$

$\Rightarrow \quad x - (a - b) = 0$ or, $x - (a + b) = 0 \quad \Rightarrow \quad x = a - b$ or $x = a + b$

Ex.2 Solve $64x^2 - 625 = 0$

Sol. We have $64x^2 - 625 = 0$

or $\quad (8x)^2 - (25)^2 = 0 \quad$ or $\quad (8x + 25)(8x - 25) = 0$

i.e. $\quad 8x + 25 = 0$ o $8x - 25 = 0$. $\quad$ This gives $x = \frac{25}{8}$ or $\frac{25}{8}$.

Thus, $x = -\frac{25}{8}, \frac{25}{8}$ are solutions of the given equations.

Ex.3 Solve the quadratic equation $16x^2 - 24x = 0$.

Sol. The given equation may be written as 8x(2x - 3) = 0

This gives $x = 0$ or $x = \frac{3}{2}$. $x=0, \frac{3}{2}$, are the required solutions.

Ex.4 Solve :- $25x^2 - 30x + 9 = 0$

Sol. $25x^2 - 30x + 9 = 0$ is equivalent to $(5x)^2 - 2(5x) \times 3 + (3)^2 = 0$

or $(5x - 3)^2 =$

This gives $x = \frac{3}{5}, \frac{3}{5}$ or simply $x = \frac{3}{5}$ as the required solution.

Ex.5 Find the solutions of the quadratic equation $x^2 + 6x + 5 = 0$.

Sol. The quadratic polynomial $x^2 + 6x + 5$ can be facorised as follows :-

$x^2 + 6x + 5 = x^2 + 5x + x + 5$

$= x(x + 5) + 1\,(x + 5) \quad = (x + 5)\,(x + 1)$

Therefore the given quadratic equation becomes $(x + 5)\,(x + 1) =$

This gives $x = -5$ or $= -1$

Therefore, $x = -1$ are the required solutions of the given equation.

Ex.6 Solve : $\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0.$

Sol. Obviously, the given equation is valid if $x - 3 \neq 0$ and $2x + 3 \neq 0$.

Multiplying throughout by $(x - 3)\,(2x - 3)$, we get

$2x(2x + 3) + 1(x - 3) + 3x + 9 = 0$

or $4x^2 + 10 + 6 = 0$ or $2x^2 + 5x + 3 = 0$ or $(2x + 3)\,(x + 1) = 0$

But $2x + 3 \neq 0$, so we get $x + 1 = 0$. This gives $x = -1$ as the only solution of the given equation.

5.4 (b) By the Method of Completion of Square :

ALGORITHM :

Step-(i) Obtain the quadratic equation. Let the quadratic equation be $ax^2 + bx + c = 0$, $a \neq 0$.

Step-(ii) Make the coefficient of x^2 unity, if it is not unity. i.e., obtained $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$.

Step-(iii) Shift the constant term $\frac{c}{a}$ on R.H.S. to get $x^2 + \frac{b}{a}x = -\frac{c}{a}$

Step-(iv) Add square of half of the coefficient of x i.e. $\left(\frac{b}{2a}\right)^2$ on both sides to obtain

$$x^2 + 2\left(\frac{b}{2a}\right)x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

Step-(v) Write L.H.S. as the perfect square of a binomial expression and simplify R.H.S. to get

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}.$$

Step-(vi) Take square root of both sides to get $x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$

Step (vii) Obtain the values of x by shifting the constant term $\frac{b}{2a}$ on RHS.

Ex.7 Solve :- $x^2 + 3x + 1 = 0$

Sol. We have $x^2 + 3x + 1 = 0$

Add and subtract $(\frac{1}{2}$coefficient of $x)^2$ in L.H.S. and get

$$x^2 + 3x + 1 + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 = 0$$

$$\Rightarrow \quad x^2 + 2\left(\frac{3}{2}\right)x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 1 = 0 \qquad \Rightarrow \quad \left(x + \frac{3}{2}\right)^2 - \frac{5}{4} = 0$$

$$\Rightarrow \quad \left(x + \frac{3}{2}\right)^2 = \left(\frac{\sqrt{5}}{2}\right)^2 \qquad \Rightarrow \quad x + \frac{3}{2} = \pm\frac{\sqrt{5}}{2}$$

This gives $x = \frac{-(3+\sqrt{5})}{2}$ or $x = \frac{-3+\sqrt{5}}{2}$

Therefore $x = -\frac{3+\sqrt{5}}{2}, \frac{-3+\sqrt{5}}{2}$ are the solutions of the given equation.

Ex.8 By using the method of completing the square, show that the equation $4a^2 + 3x + 5 = 0$ has no real roots.

Sol. We have, $4x^2 + 3x + 5 = 0$

$\Rightarrow \quad x^2 + \frac{3}{4}x + \frac{5}{4} = 0 \Rightarrow \quad x^2 + 2\left(\frac{3}{8}x\right) = -\frac{5}{4} \quad \Rightarrow \quad x^2 + 2\left(\frac{3}{8}\right)x + \left(\frac{3}{8}\right)^2 = \left(\frac{3}{8}\right)^2 - \frac{5}{4}$

$\Rightarrow \quad \left(x + \frac{3}{8}\right)^2 = -\frac{71}{64}$ Clearly, RHS is negative

But, $\left(x + \frac{3}{8}\right)^2$ cannot be negative for any real value of x. Hence, the given equation has no real roots.

5.4 **(c) By Using Quadratic Formula :**

Solve the quadratic equation in general form viz. $ax^2 + bx + c = 0$.

We have, $ax^2 + bx + c = 0$

Step (i) By comparison with general quadratic equation, find the value of a,b and c.

Step (ii) Find the discriminate of the quadratic equation. $D = b^2 - 4ac$

Step (iii) Now find the roots of the equation by given equation $x = \frac{-b+\sqrt{D}}{2a}, \frac{-b-\sqrt{D}}{2a}$

REMARK :

★ If $b^2 - 4ac < 0$ i.e. negative, then $\sqrt{b^2 - 4ac}$ is not real and therefore, the equation does not have any real roots.

Ex.9 Solve the quadratic equation $x^2 - 7x - 5 = 0$.

Sol. Comparing the given equation with $ax^2 + bx + c = 0$, we find that a = 1, b = - 7 and c = -5.

Therefore, $D = (-7)^2 - 4 \times 1 \times (-5) = 49 + 20 = 69 > 0$

Since D is positive, the equation has two roots given by $\frac{7+\sqrt{69}}{2}, \frac{7-\sqrt{69}}{2}$

$\Rightarrow \quad x = \frac{7+\sqrt{69}}{2}, \frac{7-\sqrt{69}}{2}$ are the required solutions.

Ex.10 For what value of k, $(4 - k)x^2 + (2k + 4)x + (8k + 1)$ is a perfect square.

Sol. The given equation is a perfect square, if its discriminate is zero i.e. $(2k + 4)^2 - 4(4 - k)(8k + 1) = 0$

$\Rightarrow \quad 4(k + 2)^2 - 4(4 - k)(8k + 1) = 0 \Rightarrow 4[4(k + 2)^2 - (4 - k)(8k + 1)] = 0$

$\Rightarrow \quad [(k^2 + 4k + 4) - (-8k^2 + 31k + 4)] = 0 \Rightarrow 9k^2 - 27k = 0$

$\Rightarrow \quad 9k(k - 3) = 0 \Rightarrow k = 0$ or $k = 3$

Hence, the given equation is a perfect square, if k = 0 or k = 3.

Ex.11 If the roots of the equation $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ are equal, show that $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$.

Sol. Since the roots of the given equations are equal, so discriminant will be equal to zero.

$\Rightarrow \quad b^2(c - a)^2 - 4a(b - c) . c(a - b) = 0$

$\Rightarrow \quad b^2(c^2 + a^2 - 2ac) - 4ac(ba - ca - b^2 + bc) = 0,$

$\Rightarrow \quad a^2b^2 + b^2c^2 + 4a^2c^2 + 2b^2ac - 4ac^2bc - 4abc^2 = 0 \Rightarrow (ab + bc - 2ac)^2 = 0$

$\Rightarrow \quad ab + bc - 2ac = 0 \quad \Rightarrow \quad ab + bc = 2ac$

$\Rightarrow \quad \frac{1}{c} + \frac{1}{a} = \frac{2}{b} \quad \Rightarrow \quad \frac{2}{b} = \frac{1}{a} + \frac{1}{c}$. **Hence Proved.**

Ex.12 If the roots of the equation $(b - c)x^2 + (c - a)x + (a - b) = 0$ are equal, then prove that 2b = a + c.

Sol. If the roots of the given equation are equal, then discriminant is zero i.e.

$(c - a)^2 - 4(b - c)(a - b) = 0 \Rightarrow c^2 + a^2 - 2ac + 4b^2 - 4ab + 4ac - 4bc = 0$

$\Rightarrow \quad c^2 + a^2 + 4b^2 + 2ac - 4ab - 4bc = 0 \Rightarrow (c + a - 2b)^2 = 0 \Rightarrow c + a = 2b$ **Hence Proved.**

Ex.13 If the roots of the equation $x^2 - 8x + a^2 - 6a = 0$ are real and distinct, then find all possible values of a.

Sol. Since the roots of the given equation are real and distinct, we must have D > 0

$\Rightarrow$ $64 - 4(a^2 - 6a) > 0$ $\Rightarrow$ $4[16 - a^2 + 6a] > 0$ $\Rightarrow$ $-4(a^2 - 6a - 16) > 0$

$\Rightarrow$ $a^2 - 6a - 16 < 0$ $\Rightarrow$ $(a - 8)(a + 2) < 0 \Rightarrow$ $-2 < a < 8$

Hence, the roots of the given equation are real if **'a'** lies between -2 and 8.

5.5 APPLICATIONS OF QUADRATIC EQUATIONS :

ALGORITHM : The method of problem solving consist of the following three steps :

Step (i) Translating the word problem into symbolic language (mathematical statement) which means identifying relationship existing in the problem and then forming the quadratic equation.

Step (ii) Solving the quadratic equation thus formed.

Step (iii) Interpreting the solution of the equation, which means translating the result of mathematical statement into verbal language.

REMARKS :

★ Two consecutive odd natural numbers be $2x - 1, 2x + 1$ where $x \in N$

★ Two consecutive even natural numbers be $2x, 2x + 2$ where $x \in N$

★ Two consecutive even positive integers be $2x, 2x + 2$ where $x \in Z^+$

★ Consecutive multiples of 5 be $5x, 5x + 5, 5x + 10$ .....

Ex.14 The sum of the squares of two consecutive positive integers is 545. Find the integers.

Sol. Let x be one of the positive integers. Then the other integer is $x + 1$, $x \in Z^+$

Since the sum of the squares of the integers is 545, we get

$x^2 + (x + 1)^2 = 545$

or $2x^2 + 2x - 544 = 0$

or $x^2 + x - 272 = 0$

$x^2 + 17x - 16x - 272 = 0$

or $x(x + 17) - 16(x + 17) = 0$

or $(x - 16)(x + 17) = 0$

Here, x = 16 or x = -17 But, x is a positive integer. Therefore, reject x = - 17 and take **x = 16.** Hence, two consecutive positive integers are 16 and (16 + 1), i.e., 16 and 17.

Ex.15 The length of a hall is 5 m more than its breath. If the area of the floor of the hall is 84 m^2, what are the length and the breadth of the hall ?

Sol. Let the breadth of the hall be x metres. Then the length of the ball is (x + 5) metres.

The area of the floor = $x(x + 5)$ m^2

Therefore, $x(x + 5) = 84$ or $x^2 + 5x - 84 = 0$

or $(x + 12)(x - 7) = 0$ This given x = 7 or x = - 12.

Since, the breadth of the hall cannot be negative, we reject x = - 12 and take x = - only.

Thus, breadth of the hall = 7 metres, and length of the hall = (7 + 5), i.e., 12 metres.

Ex.16 Out of group of swans $\frac{7}{2}$ times the square root of the total number are playing on the shore of a tank. The two remaining ones are playing, in deep water. What is the total number of swans ?

Sol. Let us denote the number of swans by x.

Then, the number of swans playing on the shore of the tank $= \frac{7}{2}\sqrt{x}$.

There are two remaining swans. Therefore, $x = \frac{7}{2}\sqrt{x} + 2$

or $x - 2 = \frac{7}{2}\sqrt{x}$ or $(x - 2)^2 = \left(\frac{7}{2}\right)^2 x$

or $4(x^2 - 4x + 4) = 49x$ or $4x^2 - 65x + 16 = 0$

or $4x^2 - 64x - x + 16 = 0$ or $4x(x - 16) - 1(x - 16) = 0$

or $(x - 16)(4x - 1) = 0$ This gives x = 16 or $x = \frac{1}{4}$

We reject $x = \frac{1}{4}$ and take x = 16. Hence, the total number of swans is 16.

Ex.17 The hypotenuse of a right triangle is 25 cm. The difference between the lengths of the other two sides of the triangle is 5 cm. Find the lengths of these sides.

Sol. Let the length of the shorter side b x cm. Then, the length of the longer side = (x + 5) cm.

Since the triangle is right-angled, the sum of the squares of the sides must be equal to the square of the hypotenuse (Pythagoras Theorem).

$x^2 + (x + 5)^2 = 235^2$

or $x^2 + x^2 + 10x + 25 = 625$ or $2x^2 + 10x - 600 = 0$

or $x^2 + 5x - 300 = 0$ or $(x + 20)(x - 15) = 0$

This gives x = 15 or x = - 20 We reject x = - 20 and take x = 15.

Thus, length of shorter side = 15 cm. Length of longer side = (15 + 5) cm, i.e., 20 cm.

Ex.18 Swati can row her boat at a speed of 5 km/h in still water. If it takes her 1 hour more to row the boat 5.25 km upstream than to return downstream, find the speed of the stream.

Sol. Let the speed of the stream be x km/h

$\therefore$ Speed of the boat in upstream = (5 - x)km/h

Speed of the boat in downstream = (5 + x)km/h

Time, say t_1 (in hours), for going 5.25 km upstream $= \frac{5.25}{5-x}$

Time, say t_2 (in hours), for returning 5.25 km downstream $= \frac{5.25}{5+x}$

Obviously $t_1 > t_2$

Therefore, according to the given condition of the problem,

$t_1 = t_2 + 1$

i.e., $\frac{5.25}{5-x} = \frac{5.25}{5+x} + 1$

or $\frac{21}{4}\left(\frac{1}{5-x} - \frac{1}{5+x}\right) = 1$ or $21\left(\frac{5+x-5+x}{25-x^2}\right) = 4$

or $42x = 100 - 4x^2$ or $4x^2 + 42x - 100 = 0$

or $2x^2 + 21x - 50 = 0$ or $(2x + 25)(x - 2) = 0$

This gives x = 2, since we reject $x = \frac{-25}{2}$. Thus, the speed of the stream is 2 km/h.

Ex.19 The sum of the square of two positive integers is 208. If the square of the larger number is 18 times the smaller number, find the numbers. **[CBSE - 2007]**

Sol Let x be the smaller number.

Then, square of the larger number will be 18x.

Therefore, $x^2 + 18x = 208$

or $x^2 + 18x - 208 = 0$ or $(x - 8)(x + 26) = 0$

This gives x = 8 or x = - 26

Since the numbers are positive integers, we reject x = - 26 and take x = 8.

Therefore, square of larger number = 18 × 8 = 144.

So, larger number $= \sqrt{144} = 12$

Hence, the larger number is 12 and the smaller is 8.

Ex.20 The sum 'S' of first n natural number is given by the relation $S = \frac{n(n+1)}{2}$. Find n, if the sum is 276.

Sol. We have

$S = \frac{n(n+1)}{2} = 276$ or $n^2 + n - 552 = 0$

This gives $n = \frac{-1+\sqrt{1+2208}}{2}, \frac{-1-\sqrt{1+2208}}{2}$ or $n = \frac{-1+\sqrt{2209}}{2}, \frac{-1-\sqrt{2209}}{2}$

or $n = \frac{-1+47}{2}, \frac{-1-47}{2}$ or n = 23, -24 We reject n = - 24, since -24 is not a natural number.

Therefore, n = 23.

DAILY PRACTIVE PROBLEMS # 5

OBJECTIVE DPP - 5.1

1. If one root of $5x^2 + 13x + k = 0$ is reciprocal of the other then k =

(A) 0 (B) 5 (C) $\frac{1}{6}$ (D) 6

2. The roots of the equation $x^2 - x - 3 = 0$ are
(A) Imaginary (B) Rational (C) Irrational (D) None of these

3. The difference between two numbers is 5 different in their squares is 65. The larger number is
(A) 9 (B) 10 (C) 11 (D) 12

4. The sum of ages of a father and son is 45 years. Five years ago, the product of their ages was 4 times the age of the father at that time. The present age of the father is
(A) 30 yrs (B) 31 yrs (C) 36 yrs (D) 41 yrs

5. If one of the roots of the quadratic equation is $2+\sqrt{3}$ then find the quadratic equation.
(A) $x^2 - (2+\sqrt{3})x + 1 = 0$ (B) $x^2 + (2+\sqrt{3})x + 1 = 0$
(C) $x^2 - 4x + 1 = 0$ (D) $x^2 + 4x - 1 = 0$

SUBJECTIVE DPP - 5.2

1. If x = - and $x = \frac{1}{5}$ are solutions of the equations $x^2 + kx + \lambda = 0$. Find the value of k and λ.

2. Find the value of k for which quadratic equation $(k - 2)x^2 + 2(2k - 3)x + 5k - 6 = 0$ has equal roots.

3. The sum of the squares of two consecutive positive integers is 545. Find the integers.

4. A man is five times as old as his son and the sum of the squares of their ages is 2106. Find their ages.

5. The sides (in cm) of a right triangle containing the right angles are 5x and 3x - 1. If the area of the triangle is 60 cm^2. Find its perimeter.

6. The lengths of the sides of right triangle are 5x + 2, 5x and 3x - 1. If x > 0 find the length of each sides.

7. A two digit number is four times the sum and three times the product of its digits, find the number **[CBSE – 2000]**

8. The number of a fraction is 1 less than its denominator. If 3 is added to each of the numerator and denominator, the fraction is increased by $\frac{3}{28}$. Find the fraction **[CBSE - 2007]**

9. Solve the quadratic equation $\frac{x-1}{x-2} - \frac{x-2}{x-3} = \frac{x-5}{x-6} - \frac{x-6}{x-7}$

10. An aeroplane left 30 minutes later then its scheduled time and in order to reach its destination 1500 km away in time. it has to increase its speed by 250 km/h from its usual speed. Determine its usual speed. **[CBSE-2005]**

11. A motor boat whose speed is 18 km/h in still water takes 1 hours more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream. **[CBSE-2008]**

12. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less that the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank. **[CBSE-2008]**

ANSWERS

(Objective DPP # 5.1)

Que.	1	2	3	4	5
Ans.	B	C	A	C	C

(Subjective DPP # 5.2)

1.	k = 9 λ = -2	**2.**	k = 3 or 1	**3.**	16, 17
4.	9 years & years	**5.**	40 cm	**6.**	17, 15, 8
7.	24	**8.**	$\frac{3}{4}$	**9.**	$\frac{9}{2}$
10.	75 km/h	**11.**	6 km/hr		
12.	Smaller tap = hr, Larger tap = 15 hr				

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ARITHMATIC PROGRESSIONS

6.1 PROGRESSIONS : Those sequence whose terms follow certain patterns are called progression. Generally there are three types of progression.

(i) Arithmetic Progression **(A.P.)** **(ii)** Geometric Progression **(G.P.)** (iii) Harmonic Progression **(H.P.)**

6.2 ARTHMETIC PROGRESSION :

A sequence is called an **A.P.,** if the difference of a term and the previous term is always same. **i.e.** $d = t_{n+1} - t_n$ **= Constant** for all $n \in N$. The constant difference, generally denoted by **'d'** is called the common difference.

Ex.1 Find the common difference of the following A.P. : 1,4,7,10,13,16 ......

Sol. 4 - 1 = 7 - 4 = 10 - 7 = 13 - 10 = 16 - 13 = 3 (constant). $\therefore$ Common difference (d) = 3.

6.3 GENERAL FORM OF AN A.P. :

If we denote the starting number i.e. the 1st number by **'a'** and a fixed number to the added is **'d'** then **a, a + d, a + 2d, a + 3d, a + 4d, ...** forms an **A.P.**

Ex.2 Find the A.P. whose 1st term is 10 & common difference is 5.

Sol. Given : First term (a) = 10 & Common difference (d) = 5. $\therefore$ A.P. is 10, 15, 20, 25, 30, ...

6.4 n^{th} TERM OF AN A.P. :

Let A.P. be a, a + d, a + 2d, a + 3d, ...

Then, First term **(a_1)** = a + 0.d

Second term **(a_2)** = a + 1.d

Third term **(a_3)** = a + 2.d

. .

. .

. .

n^{th} term **(a_n)** = a + (n - 1) d

$\therefore$ **$a_n = a + (n - 1) d$ is called the n^{th} term.**

Ex.3 Determine the A.P. whose their term is 16 and the difference of 5th term from 7th term is 12.

Sol. Given : $a_3 = a + (3 - 1) d = a + 2d = 16$ .....(i)

$a_7 - a_5 = 12$ ....(ii)

$(a + 6d) - (a + 4d) = 12$

$a + 6d - a - 4d = 12$ $2d = 12$

$d = 6$ Put d = 6 in equation (i)

$a = 16 - 12$ $a = 4$ $\therefore$ A.P. is 4, 10, 16, 22, 28, ...

Ex.4 Which term of the sequence 72, 70, 68, 66, ... is 40 ?

Sol. Here 1st term x = 72 and common difference d = 70 - 72 = - 2

$\therefore$ For finding the value of n

$a_n = a + (n - 1)d$

$\Rightarrow$ $40 = 72 + (n - 1) (-2)$ $\Rightarrow$ $40 - 72 = - 2n + 2$

$\Rightarrow$ $-32 = - 2n + 2$ $\Rightarrow$ $-34 = - 2n$

$\Rightarrow$ $n = 17$ $\therefore$ 17th term is 40.

Ex.5 Is 184, a term of the sequence 3,7,11,..... ?

Sol. Here 1st term (a) = 3 and common difference (d) = 7 - 3 = 4

n^{th} term $(a_n) = a + (n - 1) d$

$\Rightarrow$ $184 = 3 + (n - 1) 4$ $\Rightarrow$ $181 = 4n - 4$ $\Rightarrow$ $185 = 4n$

$\Rightarrow$ $n = \frac{185}{4}$ Since, n is not a natural number.

$\therefore$ 184 is not a term of the given sequence.

Ex.6 Which term of the sequence 20, $19\frac{1}{2}, 18\frac{1}{2}, 17\frac{3}{4}$ is the 1st negative term.

Sol. Here 1st term (a) = 20, common difference (d) = $19\frac{1}{4} - 20 = -\frac{3}{4}$

Let nth term of the given A.P. be 1st negative term $\therefore a_n < 0$

i.e. a + (n - 1) d < 0

$\Rightarrow \quad 20 + (n-1)\left(-\frac{3}{4}\right) < 0 \Rightarrow \frac{83}{4} - \frac{3n}{4} < 0 \quad \Rightarrow \quad 3n > 83 \Rightarrow n > \frac{83}{3} \Rightarrow n > 27\frac{2}{3}$

Since, 28 is the natural number just greater then $27\frac{2}{3}$.

$\therefore$ 1st negative term is 28th.

Ex.7 If pth, qth and rth term of an A.P. are a,b,c respectively, then show than a(q - r) + b(- p) + c(p - q) = 0.

Sol. $a_p = a \Rightarrow A + (p - 1) D = a$ ......(1)

$a_q = b \Rightarrow A + (q - 1) D = b$ .....(2)

$a_r = c \Rightarrow A + (r + 1) D = c$ ......(3)

Now, L.H.S. = a (q - r) + b(r - p) +c (p - q)

= {A + (p - 1)D} (q - r) + {A + (q - 1)D} (r - p) + {A + (r - 1)D} (p - q)

= 0. **R.H.S**

Ex.8 If m times the mth term of an A.P. is equal to n times its nth term. Show that the (m + n)th term of the A.P.

Sol. Let A the 1st term and D be the common difference of the given A.P.

Then, $ma_m = na_n$

$\Rightarrow$ m[A + (m - 1)D] = n[A + (n - 1)D] $\Rightarrow$ A(m - 1) + D[m + n (m - n) - (m - n)] = 0

$\Rightarrow$ A + (m + n - 1)D = 0 $\Rightarrow$ $a_{m+n} = 0$

Ex.9 If the pth term of an A.P. is q and the qth term is p, prove that its nth term is (p + q - n).

Sol. $a_p = q \Rightarrow A + (p - 1) D = q$ ......(i)

& $a_q = p \Rightarrow A + (q - 1) D = p$

Solve (i) & (ii) to get D = - 1 & A = p + q - 1

$\therefore$ $a_n = A + (n - 1) D$

$a_n = (p + q - 1) + (n - 1) (-1)$ $a_n = p + q - n$.

Ex.10 If the mth term of an A.P. $\frac{1}{n}$ and nth term be $\frac{1}{m}$ then show that its (mn) term is 1.

Sol. $a_m = \frac{1}{n} \Rightarrow A + (m-1)D = \frac{1}{n}$ .....(i)

& $a_m = \frac{1}{m} \Rightarrow A + (n-1)D = \frac{1}{m}$ ....(ii)

By solving (i) & (ii) D = $\frac{1}{mn}$ & A = $\frac{1}{mn}$ $\therefore$ $a_{mn} = A + (mn - 1) D = 1$.

6.5 mth TERM OF AN A.P. FROM THE END :

Let **'a'** be the 1st term and **'d'** be the common difference of an **A.P.** having **n** terms. **Then mth** term from the end is **(n - m + 1)th** term from beginning or **{n - (m -)}th** term from beginning.

Ex.11 Find 20 th term from the end of an A.P. 3,7,11..... 407.

Sol. 407 = 3 + (n - 1)4 $\Rightarrow$ n = 102

$\therefore$20th term from end $\Rightarrow$ m = 20

$a_{102-(20-1)} = a_{102-19} = a_{83}$ from the beginning.

$a_{83} = 3 + (83 + 1)4 = 331$.

6.6 SELECTION OF TERMS IN AN A.P. :

Sometimes we require certain number of terms in A.P. The following ways of selecting terms are generally very convenient.

No. of Terms	Terms	Common Difference
For 3 terms	a – d, a, a + d	d
For 4 terms	a – 3d, a – d, a + d, a + 3d	2d
For 5 terms	a – 2d, a – d, a, a + d, a + 2d	d
For 6 terms	a – 5d, a – 3d, a – d, a + d, a + 3d, a + 5d	2d

Ex.12 The sum of three number in A.P. is -3 and their product is 8. Find the numbers.

Sol. Three no. 's in A.P. be a - d, a, a + d

$\therefore$ a - d + a + a + d = - 3

3a = - 3 $\Rightarrow$ a = -1 & (a - d) a (a + d) = 8

$a(a^2 - d^2) = 8$

$(-1)(1 - d^2) = 8$

$1 - d^2 = -8 \quad \Rightarrow \quad d^2 = 9 \quad \Rightarrow \quad d = \pm 3$

If a = 8 & d = 3 numbers are -4, -1, 2. If a = 8 & d = - numbers are 2, -1, -4.

6.7 SUM OF n TERMS OF AN A.P. :

Let A.P. be **a, a + d, a + 1d, a + 3d,...... a + (n - 1)d**

Then, $S_n = a + (a + d) + \; ..... + \{a + (n - 2) d\} + \{a + (n - 1) d\}$ .....(i)

also, $S_n = \{a + (n - 1)d\} + \{a + (n - 2)d\} + ... + (a + d) + a$ .....(ii)

Add (i) & (ii)

$\Rightarrow \quad 2S_n = 2a + (n - 1)d + 2a + (n - 1)d + ... + 2a + (n - 1)d$

$\Rightarrow \quad 2S_n = n [2a + (n - 1) d] \quad \Rightarrow \quad S_n = \frac{n}{2}[2a + (n+1)d]$

$S_n = \frac{n}{2}[a + a + (n-1)d] = \frac{n}{2}[a + \ell] \quad \therefore \quad S_n = \frac{n}{2}[a + \ell]$ where ℓ is the last term.

Ex.13 Find the sum of 20 terms of the A.P. 1,4,7,10.....

Sol. a = 1, d = 3 $\quad S_n = \frac{n}{2}[2a + (n-1)d] \quad S_{20} = \frac{20}{2}[2(1) + (20-1)3]$

Ex.14 Find the sum of all three digit natural numbers. Which are divisible by 7.

Sol. 1st no. is 105 and last no. is 994.

Find n $\quad 994 = 105 + (n + 1)7$

$\therefore \quad n = 128 \quad \therefore$ Sum, $\quad S_{128} = \frac{128}{2}[105 + 994]$

6.8 PROPERTIES OF A.P. :

(A) For any real numbers a and b, the sequence whose n^{th} term is $\mathbf{a_n = an + b}$ is always an **A.P.** with common difference **'a'** (i.e. coefficient of term containing **n**)

(B) If any n^{th} term of sequence is a linear expression in **n** then the given sequence is an **A.P.**

(C) If a constant term is added to or subtracted from each term of an **A.P.** then the resulting sequence is also an **A.P.** with the same common difference.

(D) If each term of a given **A.P.** is multiplied or divided by a non-zero constant **K,** then the resulting sequence is also an **A.P.** with common difference **Kd** or ________ respectively. Where **d** is the common difference of the given **A.P.**

(E) In a finite **A.P.** the sum of the terms equidistant from the beginning and end is always same and is equal to the sum of 1st and **last term.**

(F) If three numbers **a,b,c** are in **A.P.,** then **2b = a + c.**

Ex.15 Check whether $a_n = 2n^2 + 1$ is an **A.p.** or not.

Sol. $a_n = 2n^2 + 1$ Then $a_{n+1} = 2(n + 1)^2 + 1$

$\therefore \quad a_{n+1} - a_n = 2(n^2 + 2n + 1) + 1 - 2n^2 - 1$

$= 2n^2 + 4n + 2 + 1 - 2n^2 - 1$

$= 4n + 2$, which is not constant $\therefore$ The above sequence is not an A.P.

DAILY PRACTIVE PTOBLEMS # 6

OBJECTIVE DPP - 6.1

1. p^{th} term of the series $\left(3 - \frac{1}{n}\right) + \left(3 + \frac{2}{n}\right) + \left(3 - \frac{3}{n}\right) +$ ..... will be

(A) $3 + \frac{p}{n}$ (B) $3 - \frac{p}{n}$ (C) $3 + \frac{n}{p}$ (D) $3 - \frac{n}{p}$

2. 8^{th} term of the series $2\sqrt{2} + \sqrt{2} + 0 +$ ..... will be

(A) $-5\sqrt{2}$ (B) $5\sqrt{2}$ (C) $10\sqrt{2}$ (D) $-10\sqrt{2}$

3. If 9^{th} term of an A.P. be zero then the ratio of its 29^{th} and 19^{th} term is

(A) 1 : 2 (B) 2 : 1 (C) 1 : 3 (D) 3 : 1

4. Which term of the sequence 3,8,13,18,..... is 498

(A) 95th (B) 100 th (C) 102 th (D) 101 th

5. Which of the following sequence is an A.P.

(A) $f(n) = ab + b\, n \in N$ (B) $f(n) = kr^n, n \in N$

(C) $f(n) = (an + b)kr^n, n \in N$ (D) $f(n) = \dfrac{1}{a\left(n + \dfrac{b}{n}\right)}, n \in N$

6. If the n^{th} term of an A.P. be (2n - 1) then the sum of its firs n terms will be

(A) $n^2 - 1$ (B) $(2n - 1)^2$ (C) n^2 (D) $n^2 + 1$

7. The interior angles of polygon are in A.P. if the smallest angles be 120^0 and the common difference be 5, then the number of sides is (A) 8 (B) 10 (C) 9 (D) 6

8. In the first, second and last terms of an A.P. be a,b, 2a respectively, then its sum will

(A) $\dfrac{ab}{-a+b}$ (B) $\dfrac{ab}{2(b-a)}$ (C) $\dfrac{3ab}{2(b-a)}$ (D) $\dfrac{3ab}{4(b-a)}$

SUBJECTIVE DPP - 6.2

1. Is 51 a term of the A.P. 5, 8, 11, 14, ... ?

2. Find the common difference of an A.P. whose first term is 100 and the sum of whose first six terms is five times the sum of the next six terms.

3. Find three number in A.P. whose sum is 21 and their product is 336.

4. A student purchased a pen for Rs. 100. At the end of 8 years, it was valued at Rs. 20. Assuming the yearly depreciation is constant amount, find the annual depreciation./

5. The fourth term of an A.P. is equal to three times the first term and the seventh term exceeds twice the third by one. Find the first term and the common difference.

6. Which term of the sequence $17, 16\frac{1}{5}, 15\frac{2}{5}, 14\frac{3}{5}$...... is the first negative term.

7. If $S_n = n^2p$ and $S_m = m^2p$ $(m \neq n)$ in an A.P. Prove that $S_p = p^3$.

8. Find the sum of all the three digit numbers which leave remainder 2 when divided by 5.

9. Find the sum of all two digit odd positive numbers

10. Find the 10^{th} term from end of the A.P. 4,9,14,....., 254.

11. 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how many rows the 200 logs are placed and how many logs are in the top row ?

12. The sum of the first n term of an A.P. is given by $S_n = 3n^2 - 4n$. Determine the A.P. and its 12^{th} term. **[CBSE - 2004]**

13. Find the sum of the first 25 terms of an A.P. whose n^{th} term is given by $t_n = 2 - 3n$ **[CBSE - 2004]**

14. Find the number of terms of A.P. 54, 54, 48..... so that their sum is 513. **[CBSE - 2005]**

15. In an A.P., the sum of first n terms is $\dfrac{3n^2}{2} + \dfrac{5n}{2}$ Find its 25^{th} term. **[CBSE - 2006]**

16. Which term of the arithmetic progression 8, 14 20, 26, ... will be 72 more than its 41^{st} term ?**[CBSE - 2006]**

17. The first term, common difference and last term of an A.P. are 12, 6 and 252 respectively. Find the sum of all terms of this A.P. **[CBSE - 2007]**

18. Write the next term of the $\sqrt{8}, \sqrt{18}, \sqrt{32}$, ... **[CBSE - 2008]**

19. The sum of the 4^{th} and 8^{th} terms of an A.P. is 24 and the sum of the 6^{th} and 10^{th} terms is 44. Find the first three terms of the A.P **[CBSE - 2008]**

ANSWERS

(Objects DPP # 6.1)

Que.	1	2	3	4	5	6	7	8
Ans.	B	A	B	B	A	C	C	C

(Subjective DPP # 6.2)

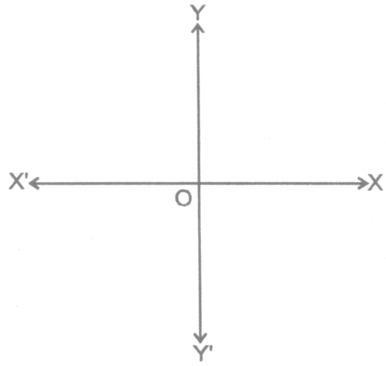
1.	No	**2.**	-10	**3.**	6,7,8	**4.**	10
5.	3,2	**6.**	23^{rd}	**8.**	98910	**9.**	2475
10.	209	**11.**	16 rows, 5 logs	**12.**	-1,5,11,..... & $a_{12} = 65$		
13.	-925	**14.**	18, 19	**15.**	76	**16.**	53^{rd}
17.	5412	**18.**	$\sqrt{50}$	**19.**	-13, -8, -3		

Teko CLASSES ..... the support

CO-ORDINATE GEOMETRY

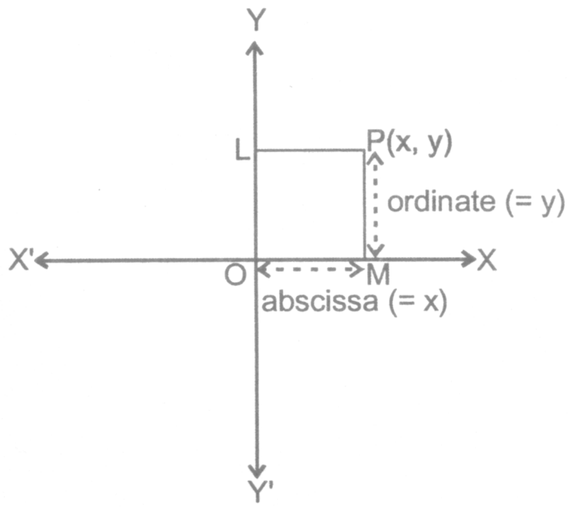
7.1 RECTANGULAR CO-ORDINATES :

Take two perpendicular lines **X'OX** and **Y'OY** intersecting at the point **O. X'OX** and **Y'OY** are called the co-ordinate axes. **X'Ox** is called the **X-axis, Y'OY** is called the **Y-axis** and **O** is called the origin. Lines **X'OX** and **Y'OY** are sometimes also called rectangular axes.



7.1 (a) Co-ordinates of a Point :

Let **P** be any point as shown in figure. Draw **PL** and **PM** perpendiculars on **Y-axis** and **X-axis,** respectively. The length **LP (or OM)** is called the **x - coordinate of the abscissa of point P** and **MP** i called the **y - coordinate or the ordinate of point P.** A point whose abscissa is **x** and ordinate is **y** named as the point **(x,y) or P(x,y).**



The two liens **X'OX** and **Y'OY** divide the plane into four parts called **quadrants. XOY, YOX' X'OY'** and **Y'OX** are, respectively, called the first, second third and fourth quadrants. The following table shows the signs of the coordinates of pins situated in different quadrants :

Quadrant	X-coodrinate	Y-coordinate	Point
First quadrant	+	+	(+, +)
Second quadrant	-	+	(-, +)
Third quadrant	-	-	(-, -)
Fourth quadrant	+	-	(+, -)

REMAKS

(i) Abscissa is the perpendicular distance of a point from **y-axis** (i.e., positive to the right of **y-axis** and negative to the left of **y - axis)**

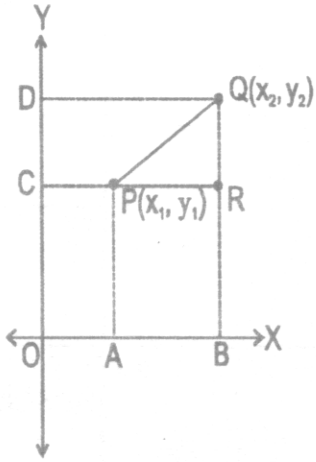
(ii) **Ordinate** is positive above **x - axis** and negative below **x-axis.**

(iii) **Abscissa** of any point on **y-axis** is zero.

(iv) **Ordinate** of any point of **x-axis** is zero.
(v) Co-ordinates of the **origin** are (0,0)

7.2 DISTACE BETWEEN TWO POINTS :

Let two points be **P (x_1, y_1)** and **Q(x_2, y_2)**



Take two mutually perpendicular lines as the coordinate axis with **O** as origin. Mark the points **P(x_1, y_1)** and **Q (x_2, y_2)**. Draw lines **PA, QB** perpendicular to **X-axis** from the points **P** and **Q**, which meet the **X-axis** in points A and B, respectively.

Draw lines PC and QD perpendicular to **Y-axis,** which meet the **Y-axis** in C and D, respectively. Produce CP to meet BQ in R. Now

OA = abscissa of P = x_1

Similarly, OB = x_2, OC = y_1 and OD = y_2

Therefore, we have PR = AB = OB - OA = $x_2 - x_1$

Similarly, QR = QB - RB = QB - PA = $y_2 - y_1$

Now, using Pythagoras Theorem, in right angled triangle PRQ, we have $PQ^2 = Pr^2 + RQ^2$

or $PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

Since the distance or length of the line-segment PQ is always non-negative, on taking the positive square root, we get the distance as

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

This result is known as **distance formula.**

Corollary : The distance of a point **P(x_1, y_1)** from the origin **(0,0)** is given by

$$OP = \sqrt{x_1^2 + y_1^2}$$

Some useful points :

1.In questions relating to geometrical figures, take the given vertices in the given order and proceed as indicated.

(i) For an **isosceles triangle -** We have to prove that at least two sides are equal.
(ii) For an **equilateral triangle -** We have to prove that three sides are equal.
(iii) For a **right -angled triangle** - We have to prove that the sum of the squares of two sides is equal to the square of the third side.
(iv) for a **square** - We have to prove that the four sides are equal, two diagonals are equal.
(v) For a **rhombus** - We have to prove that four sides are equal (and there is no need to establish that two diagonals are unequal as the square is also a rhombus).
(vi) For a **rectangle** - We have to prove that the opposite sides are equal and two diagonals are equal.
(vii) For a **Parallelogram** - We have to prove that the opposite sides are equal (and there is no need to establish that two diagonals are unequal sat the rectangle is also a parallelogram).

2. for three points to be **collinear -** We have to prove that the sum of the distances between two pairs of points is equal to the third pair of points.

Ex.1 Find the distance between the points (8 , -2) and (3, -6).

Sol. Let the points (8, -2) and (3, -6) be denoted by P and Q, respectively.
Then, by distance formula, we obtain the distance PQ as

$$PQ = \sqrt{(3-8)^2 + (-6+2)^2} \qquad = \sqrt{(-5)^2 + (-4)^2} = \sqrt{41} \text{ unit}$$

Ex.2 Prove that the points $(1,-1), \left(-\frac{1}{2}, \frac{1}{2}\right)$ and (1, 2) are the vertices of an isosceles triangle.

Sol. Let the point (1, -1), $\left(-\frac{1}{2}, \frac{1}{2}\right)$ and (1, 2) be denoted by P, Q and R, respectively. Now

$$PQ = \sqrt{\left(-\frac{1}{2}-\right)^2 + \left(\frac{1}{2}+1\right)^2} = \sqrt{\frac{18}{4}} = \frac{3}{2}\sqrt{2}$$

$$QR = \sqrt{\left(1+\frac{1}{2}\right)^2 + \left(2-\frac{1}{2}\right)^2} = \sqrt{\frac{18}{4}} = \frac{3}{2}\sqrt{2}$$

$$PR = \sqrt{(1-1)^2 + (2+1)^2} = \sqrt{9} = 3$$

From the above, we see that PQ = QR $\therefore$ The triangle is isosceles.

Ex.3 Using distance formula, show that the points (-3, 2), (1, -2) and (9, -10) are collinear.

Sol. Let the given points (-3, 2), (1, -2) and (9, -10) be denoted by A, B and C, respectively. Points A, B and C will be collinear, if the sum of the lengths of two line-segments is equal to the third.

Now, $AB = \sqrt{(1+3)^2 + (-2-2)^2} = \sqrt{16+16} = 4\sqrt{2}$

$BC = \sqrt{(9-1)^2 + (-10+2)^2} = \sqrt{64+64} = 8\sqrt{2}$

$AC = \sqrt{(9+3)^2 + (-10-2)^2} = \sqrt{144+144} = 12\sqrt{2}$

Since, AB + BC = $4\sqrt{2} + 8\sqrt{2} = 12\sqrt{2} = AC$, the, points A, B and C are collinear.

Ex.4 Find a point on the X-axis which is equidistant from the points (5, 4) and (-2, 3).

Sol. Since the required point (say P) is on the X-axis, its ordinate will be zero. Let the abscissa of the point be x.
Therefore, coordinates of the point P are (x, 0).
Let A and B denote the points (5, 4) and (-2, 3), respectively.
Since we are given that AP = BP, we have $AP^2 = BP^2$
i.e., $(x - 5)^2 + (0 - 4)^2 = (x + 2)^2 + (0 - 3)^2$
or $x^2 + 25 - 10x + 16 = x^2 + 4 + 4x + 9$ or $-14x = -28$ or $x = 2$
Thus, the required point is (2, 0).

Ex.5 The vertices of a triangle are (-2, 0) , (2, 3) and (1, -3). Is the triangle equilateral, isosceles or scalene?

Sol. Let the points (-2, 0), (2, 3) and (1, -3) be denoted by A, B and C respectively. Then,

$AB = \sqrt{(2+2)^2 + (3-0)^2} = 5$ $BC = \sqrt{(1-2)^2 + (-3-3)^2} = \sqrt{37}$

and $AC = \sqrt{(1+2)^2 + (-0-0)^2} = 3\sqrt{2}$ Clearly, $AB \neq BC \neq AC$.

Therefore, ABC is a scalene triangle.

Ex.6 The length of a line-segments is 10. If one end is at (2, -3) and the abscissa of the second end is 10, show that its ordinate is either 3 or -9.

Sol. Let (2, -3) be the point A. let the ordinate of the second end B be *y*. Then its coordinates will be (10, y).

$\therefore$ $AB = \sqrt{(10-2)^2 + (y+3)^2} = 10$ (Given)

or $64 + 9 + y^2 + 6y = 100$ or $y^2 + 6y + 73 - 100 = 0$
or $y^2 + 6y - 27 = 0$ or $(y + 9)(y - 3) = 0$
Therefore, $y = 9$ or $y = 3$.

Ex.7 Show that the points (-2, 5), (3, -4) and (7, 10) are the vertices of a right triangle.

Sol. Let the three points be A(-2, 5), B(3, - 4) and C(7, 10).

Then $AB^2 = (3 + 2)^2 + (-4 - 5)^2 = 106$
$BC^2 = (7 - 3)^2 + (10 + 4)^2 = 212$
$AC^2 = (7 + 2)^2 + (10 - 5)^2 = 106$ We see that $BC^2 = AB^21 + AC^2$
212 = 106 + 106 212 = 212
$\therefore$ $\angle A = 90^0$ Thus, ABC is a right triangle, right angled at A.

Ex.8 If the distance of P (x, y) from A (5, 1) and B(-1, 5) are equal, prove that 3x = 2y.

Sol. P(x, y), A (5, 1) and B (-1, 5) are the given points.
AP = BP (Given)
$\therefore$ $AP^2 = BP^2$ or $AP^2 - BP^2 = 0$
or $\{(x - 5)^2 + (y - 1)\}^2 - \{(x + 1)^2 + (y - 5)^2\} = 0$
or $x^2 + 25 - 10x + y^2 + 1 - 2y - x^2 - 1 - 2x - y^2 - 25 + 10y = 0$
or $-12x + 8y = 0$ or $3xx = 2y$.

7.3 SECTION FORMULAE :

7.3 (a) Formula for Internal Division :

The coordinates of the pint which divided the line segment joining the pints (x_1, y_1) and $x_2, y_2)$ internally in the ratio **m : n** are given by $x = \frac{mx_2 + nx_1}{m+n}, y = \frac{my_2 + my_1}{m+n}$

Proof : Let O be the origin and let OX and OY be the X-axis and Y-axis respectively. Let **A(x_1, y_1)** and **B(x_2, y_2)** bet the given points. Let **(x, y)** be the coordinates of the point **p** which divides **AB** internally in the ratio **m : n** Draw

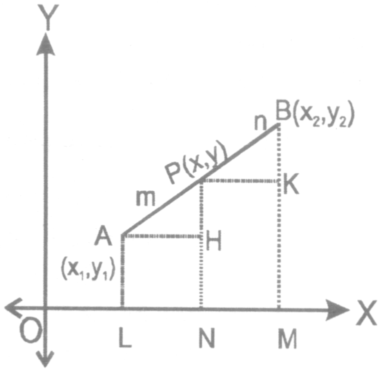
$AL \perp OX$, $BM \perp OX$, $PN \perp Ox$. Also, draw **AH** and **PK** perpendicular from **A** and **P** on **PN** and **BM** respectively. Then

$OL = x_1$, $ON = x$, $OM = x_2$, $AL = y_1$, $PN = y$ and $BM = y_2$.

$\therefore$ $AH = LN = ON - OL = x - x_1$, $PH = PH - HN$

$= PN - AL = y - y_1$, $PK = NM = OM - ON = x_2 - x$

and $BK = BM - MK = BM - PN = y_2 - y$.



Clearly, ΔAHP and ΔPKB are similar.

$\therefore$ $\frac{AP}{BP} = \frac{AH}{PK} = \frac{PH}{BK}$

$\Rightarrow$ $\frac{m}{n} = \frac{x - x_1}{x_2 - x} = \frac{y - y_1}{y_2 - y}$

Now, $\frac{m}{n} = \frac{x - x_1}{x_2 - x}$

$\Rightarrow$ $mx_2 - mx = nx - nx_1$ $\Rightarrow$ $mx + nx = mx_2 + nx_1$

$\Rightarrow$ $x = \frac{mx_2 + nx_1}{m+n}$ and $\frac{m}{n} = \frac{y - y_1}{y_2 - y}$

$\Rightarrow$ $my_2 - my = ny - ny_1$ $\Rightarrow$ $my + ny = my_2 + ny_1$

$\Rightarrow$ $y = \frac{my_2 + ny_1}{m+n}$

Thus, the coordinates of P are $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$

REMARKS

If **P** is the mid-point of **AB,** then it divides **AB** in the ratio **1 : 1,** so its coordinates are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

7.3 (b) Formula for External Division :

The coordinates of the points which divides the line segment joining the points **(x_1, y_1)** and **(x_2, y_2)** externally in the ratio **m : n** are given by

$x = \frac{mx_2 - nx_1}{m-n}, y = \frac{my_2 - ny_1}{m-n}$

Ex.9 Find the coordinates of the point which divides the line segment joining the points (6, 3) and (-4, 5) in the ratio 3 : 2 (i) internally (ii) externally.

Sol. Let P(x, y) be the required point.

(i) For internal division, we have

$x = \frac{3\times -4 + 2\times 6}{3+2}$

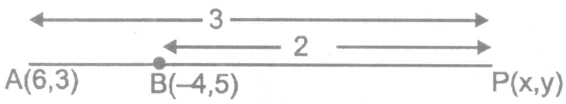
and $y = \frac{3\times 5 + 2\times 3}{3+2}$

$\Rightarrow$ $x = 0$ and $y = \frac{21}{5}$

So the coordinates of P are (0, 21/5)

(ii) For external division, we have

$x = \frac{3\times -4 - 2\times 6}{3-2}$



any $y = \frac{3\times 5 - 2\times 3}{3-2}$

$\Rightarrow$ $x = -24$ and $y = 9$

So the coordinates of P are (-24, 9).

Ex.10 In which ratio does the point (-1, -1) divides the line segment joining the pints (4, 4) and (7, 7) ?

Sol. Suppose the point C(-1, -1) divides the line joining the points A(4, 4) and B(7, 7) in the ratio k : 1 Then, the coordinates of C are $\left(\frac{7k+4}{k+1}, \frac{7k+4}{k+1}\right)$

But, we are given that the coordinates of the points C are (-1, -1). $\therefore$ $\frac{7k+4}{k+1} = -1 \Rightarrow k = -\frac{5}{8}$

Thus, C divides AB externally in the ratio 5 : 8.

Ex.11 In what ratio does the X-axis divide the line segment joining the points (2, -3) and (5, 6) ?

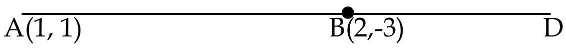
Sol. Let the required ratio be k : 1. Then the coordinates of the point of division are $\left(\frac{5\lambda+2}{k+1}, \frac{6\lambda-3}{k+1}\right)$. But, it is a point on X-axis on which y-coordinate of every point is zero.

$\therefore \quad \frac{6\lambda-3}{k+1} = 0 \quad \Rightarrow \quad k = \frac{1}{2}$ Thus, the required ratio is $\frac{1}{2}$: 1 or 1 : 2.

Ex.12 A (1, 1) and B(2, -3) are two points and D is a point on AB produced such that AD = 3 AB. Find the coordinates of D.

Sol. We have, AD = 3AB. Therefore, BD = 2AB. Thus D divides AB externally in the ratio AD : BD = 3 : 2 Hence, the coordinates of D are

$\therefore \quad \left(\frac{3\times2-2\times1}{3-2}, \frac{3x-3-2\times1}{3-2}\right)$



= (4, -11).

Ex.13 Determine the ratio in which the line 3x + y - 9 = 0 divides the segment joining the pints (1, 3) and (2, 7).

Sol. Suppose the line 3x + y - 9 = 0 divides the line segment joining A(1, 3) and B(2, 7) in the ratio k : 1 at point C. The, the coordinates of C are $\left(\frac{2k+1}{k+1}, \frac{7k+3}{k+1}\right)$ But, C lies on 3x + y - 9 = 0, therefore

$3\left(\frac{2k+1}{k+1}\right) + \frac{7k+3}{k+1} - 9 = 0 \quad \Rightarrow \quad 6k + 3 + 7k + 3 - 9k - 9 = 0 \quad \Rightarrow \quad k = \frac{3}{4}$

So, the required ratio is 3 : 4 internally.

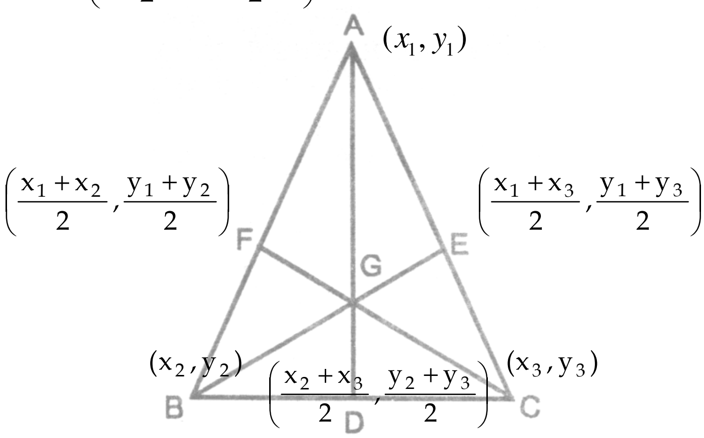
7.4 CENTROID OF A TRIANGLE :

Prove that the coordinates of the triangle whose vertices are (x_1, y_1), (x_2, y_2) and (y_3, y_3) are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$. Also, deduce that the medians of a triangle are concurrent.

Proof :

Let **A**(x_1, y_1), **B**(x_2, y_2) and **C**(x_3, y_3) be the vertices of **ΔABC** whose medians are AD, BE and CF respectively. So. D,E and F are respectively the mid-points of BC, CA and AB.

Coordinates of **D** are $\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$. Coordinates of a point dividing AD in the ratio **2 : 1** are



$$\left(\frac{1.x_1+2\left(\frac{x_2+x_3}{2}\right)}{1+2}, \frac{1.y_1+\left(\frac{y_2+y_3}{2}\right)}{1+2}\right) = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

The coordinates of E are $\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$. The coordinates of a point dividing BE in the ratio 2 : 1 are

$$\left(\frac{1.x_2+\frac{2(x_1+x_3)}{2}}{1+2}, \frac{1.y_2+\frac{2(y_1+y_3)}{2}}{1+2}\right)=\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

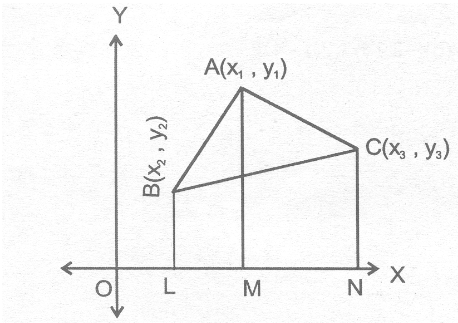
Similarly the coordinates of a point dividing CF in the ratio 2 : 1 are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

Thus, the point having coordinates $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$ is common to AD, BE and CF and divides them in the ratio 1 : 2.

Hence, medians of a triangle are concurrent and the coordinates of the centroid are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$.

7.5 AREA OF A TRIANGLE :

Let **ABC** be any triangle whose vertices are **A(x_1, y_1) B(x_2, y_3).** Draw BL, AM and CN perpendicular from B,A and C respectively, to the X-axis. ABLM, AMNC and BLNC are all trapeziums.



Area of ΔABC = Area of trapezium ABLM + Area of trapezium AMNC - Area of trapezium BLNC We know that, Area of trapezium $=\frac{1}{2}$ (Sum of parallel sides) (distance b/w them)

Therefore

Area of ΔABC $=\frac{1}{2}$(BL + AM) (LM) + $\frac{1}{2}$(AM + CN) MN- $\frac{1}{2}$ (BL + CN) (LN)

Area of ΔABC $=\frac{1}{2}$ $(y_2 + y_1)$ $x_1 - x_2)$ + $\frac{1}{2}$ $(y_1 + y_3)$ $(x_3 - x_1)$ - $\frac{1}{2}$ $(y_2 + y_3)$ $(x_3 - x_2)$

Area of ΔABC $=\frac{1}{2}\left|[x_1(y_2-y_3)+x_2(y_3-y\ \ )+x_3(y_1-y_2)]\right|$

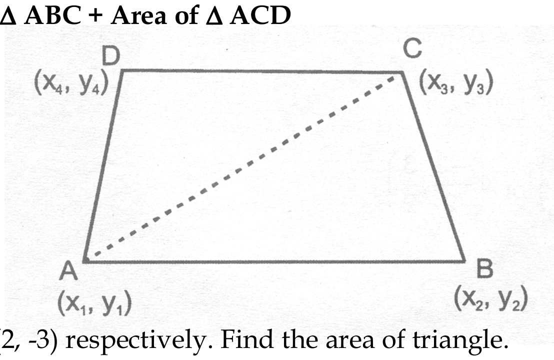
7.5 (a) Condition for collinearity :

Three points **A (x_1, y_1) B(x_2, y_2) and C(x_3, y_3)** are collinear if **Area of ΔABC = 0.**

7.6 AREA OF QUADRILATERAL :

Let the vertices of Quadrilateral ABCD are **A(x_1,y_1), B(x_2,y_2, C(x_3,y_3)** and **D(x_4, y_4)**

So, Area of quadrilateral ABCD = Area of Δ ABC + Area of Δ ACD



Ex.14 The vertices of Δ ABC are (-2, 1), (5, 4) and (2, -3) respectively. Find the area of triangle.

Sol. A(-2, 1), B(-2, 1) and C(2, -3) be the vertices of triangle.

So, x_1 = - 2, y_1 = 1 ; x_2 = 5, y_2 = 4; x_3 = 2 y_3 = -3

Area of Δ ABC $= \frac{1}{2}\left|[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]\right|$

$= \frac{1}{2}\left|[(-2)(4+3) + (5)(-3-1) + 2(1-4)]\right| \quad = \frac{1}{2}\left|[-14 + (-20) + (-6)]\right|$

$= \frac{1}{2}|-40| \quad$ = 20 Sq. unit.

Ex.15 The area of a triangle is 5. Two of its vertices area (2, 1) and (3, -2). The third vertex lies on y = x + 3. Find the third vertex.

Sol. Let the third vertex be (x_3, y_3) area of triangle

$= \frac{1}{2}\left|[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]\right|$

As $x_1 = 2$ $y_1 = 1$; $x_2 = 3$, $y_2 = -2$; Area of Δ = 5 sq. unit

$\Rightarrow \quad 5 = \frac{1}{2}|2(-2 - y_3) + 3(y_3 - 1) + x_3(1+2)| \qquad \Rightarrow \quad 10 = |3x_3 + y_3 - 7|$

$\Rightarrow \quad 3x_3 + y_3 - 7 = \pm 10$

Taking positive sign

$3x_3 + y_3 - 7 = 10 \Rightarrow \quad 3x\therefore + y_3 = 17$ .....(i)

Taking negative sing

$\Rightarrow \quad 3x_3 + y_3 - 7 = -10 \quad \Rightarrow \quad 3x\therefore + y_3 = -3$ .....(ii)

Given that (x_3, y_3) lies on y = x + 3

So, $-x\therefore + y_3 = 3$ ....(iii)

Solving eq. (i) & (iii) $\quad x_3 = \frac{7}{2}, \quad y_3 = \frac{13}{2}$

Solving eq. (ii) & (iii) $\quad x_3 = \frac{-3}{2}, \quad y_3 = \frac{3}{2}$

So the third vertex are $\left(\frac{7}{2}, \frac{13}{2}\right)$ or $\left(\frac{-3}{2}, \frac{3}{2}\right)$

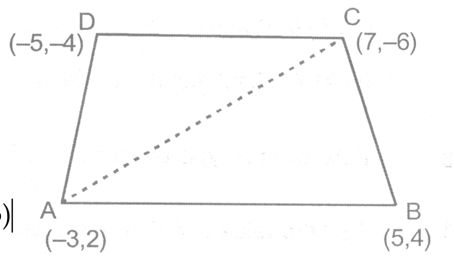
Ex.16 Find the area of quadrilateral whose vertices, taken in order, are (-3, 2), B(5, 4), (7, -6) and D(-5, -4).

Sol. Area of quadrilateral = Area of ΔABC + Area of Δ ACD

So, Area of ΔABC $= \frac{1}{2}|(-3)(4+6) + 5(-6-2) + 7(2-4)|$

$= \frac{1}{2}|-30 - 40 - 14|$

$= \frac{1}{2}|-84| = 42$ Sq. units



Area of Δ ACD $= \frac{1}{2}|-3(-6+4) + 7(-4-2) + (-5)(2+6)|$

$= \frac{1}{2}|+6 - 42 - 40| = \frac{1}{2}|-76| = 38$ Sq. units

So, Area of quadrilateral ABCD = 42 + 38 = 80 Sq. units.

DAILY PRACTIVELY PROBLEMS # 7

OBJECTIVE DPP - 7.1

1. The points (-a, -b), (0, 0), (a, b) and (a^2, ab) are
(A) Collinear (B) Vertices of a parallelogram (C) Vertices of a rectangle (D) None of these
2. If the points (5, 1), (1, p) & (4, 2) are collinear then the value of p will be
(A) 1 (B) 5 (C) 2 (D) -2
3. Length of the median from B on AC where A(-1, 3), B(1, -1), (5, 1) is
(A) $\sqrt{18}$ (B) $\sqrt{10}$ (C) $2\sqrt{3}$ (D) 4
4. The points (0, -1), (-2, 3), (6, 7) and (8, 3) are -
(A) Collinear (B) Vertices of a parallelogram which is not a rectangle
(C) Verticals of a rectangle, which is not a square (D) None of these
5. If (3, -4) and (-6, 5) are the extremities of the diagonal of a parallelogram and (-2, 1) is third vertex, then its fourth vertex is - (A) (-1, 0) (B) (0, -1) (C) (-1, 1) (D) None of these
6. The area of a triangle whose vertices are (a, c + a), (a, c) and (-a, c - a) are

(A) a^2 (B) b^2 (C) c^2 (D) $a^2 + c^2$

7. The are of the quadrilateral's the coordinates of whose verticals are (1, -2,) (6, 2), (5, 3) and (3, 4) are
(A) $\frac{9}{2}$ (B) 5 (C) $\frac{11}{2}$ (D) 11

SUBJECTIVE DPP - 7.2

1. Find the distance between the points :
(i) P (-6, 7) and Q(-1, -5).
(ii) A(at_1^2, $2at_1$) and B(at_2^2, $2at_2$).
2. If the point (x, y) is equidistant from the points (a + b, b - a) and (a - b, a + b), prove that bx = ay.
3. Find the value of x, if the distance between the points (x, -1) and (3, 2) is 5.
4. Show that the points (a, a), (-a, -a) and $-\sqrt{3a}, \sqrt{3a}$) are the vertices of an equilateral triangle.
5. Show that the points (1, 1), (-2, 7) and (3, -3) are collinear.
6. Prove that (2, -2), (-2, 1) and (5, 2) are the vertices of a right angled triangle. Find the area of the triangle and the length of the hypotenuse.
7. If A(-1, 3), B(1, -1) and C(5, 1) are the vertices of a triangle ABC, find the length of the median passing through the vertex A.
8. Show that the points A(1,2), B(5, 4), C(3, 8) and D(-1, 6) are the vertices of a square.
9. The abscissa of a point is twice its ordinate and the sum of the abscissa and the ordinate is -6. What are the coordinates of the point ?
10. If two vertices of triangle are (3, 7) an (-1, 5) and its centroid is (1, 3), find the coordinates of the third vertex.
11. If the mid point of the line-segment joining the points (-7, 14) and (K, 4) is (a, b), where 2a + 3b = 5, find the value of K.
12. Prove hat the points (a, 0), (0, b) and (1, 1) are collinear if $\frac{1}{a}+\frac{1}{b}=1$.
13. The co-ordinates of two points A & B are (3, 4) and (5, -2) respectively. Find the co-ordinate of point P if PA = PB, the area of ΔAPB = 10.
14. Four points A(6, 3), B(-3, 5) C(4, -2) and D(x, 3x) are given in such a way that $\frac{\text{Area }(\Delta DBC)}{\text{Area }(\Delta ABC)}=\frac{1}{2}$ find x.
15. Show that the points A(2, -2), B(14, 10), C(11, 13) and D(-1, 1) are the vertices of a rectangle.**[CBSE-2004]**
16. Determine the ratio in which the point (-6, a) divides the join of A(-3, -1) and B(-8, 9). Also find the value of a. **[CBSE 2004]**
17. Find a pint on X-axis which is equidistant from the points (7, 6) and (-3, 4). **[CBSE - 2005]**
18. The line segment joining the points (3, -4) and (1, 2) is trisected at the pints P and Q. if the coordinates of P and Q are (p, -2) and (5/3,) respectively. Finds the value of p and q. **[CBSE 2005]**
19. If A(-2, -1), B(a, 0), C(4, b) and D(1, 2) are the verities of a parallelogram, find the values of a and b.**[-2006]**
20. The coordinates of one end point of a diameter of a circle are (4, -1) and the coordinates of the centre of the circle are (1, -3). Find the coordinates of the other end of the diameter. **[CBSE-2007]**
21. The pint R divides the line segment AB, where A(-4, 0) and B(0, 6) are such that AR = $\frac{3}{4}$ AB. Find the co-ordinates or R. **(CBSE - 2008)**
22. For what value of k are the pints (1, 1), (3, k) and (-1, 4) collinear ?**[CBSE - 2008]**
23. Find the area of the ΔABC with vertices A(-5, 7), B (-4, -5) and C(4, 5).**[CBSE - 2008]**
24. If the point P(x,y) is equidistant from the points A(3,6) and B(-3,4) prove that 3x + y - 5 = 0.**[CBSE - 2008]**
25. If A(4 -8), B(3,6) and C(5,- 4) are the vertices of a ΔABC, D is the mid-point of BC and is P is point on AD joined such that $\frac{AP}{PD}=2$ find the coordinates of P. **[CBSE - 2008]**

ANSWERS

(Objective DPP # 7.1)

Que.	1	2	3	4	5	6	7
Ans.	A	B	B	C	A	A	C

(Subjective DPP # 7.2)

1. (i) 13 (ii) $a(t_2-t_1)\sqrt{(t_2+t_1)^2+4}$ 3. x = 7 or - 1
6. $\frac{25}{2}$ sq. units, $5\sqrt{2}$

7.	5 units	**9.**	(-4, -2)	**10.**	(1, -3)	**11.**	K = -15
13.	(7, 2) or (1, 0)	**14.**	$\frac{11}{8}, -\frac{3}{8}$	**16.**	3 : 2, a = 5	**17.**	(3, 0)
18.	p = 7/3, q = 0	**19.**	a = 1, b = 3	**20.**	(-2, -5)	**21.**	$(-1, \frac{9}{2})$
22.	k = - 2	**23.**	53 sq. units	**25.**	(4, -2)		