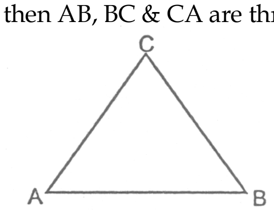
TRIANGLES

ML - 10

TRIANGLE

A plane figure bounded by three lines in a plane is called a triangle. Every triangle have three sides and three angels. If ABC is any triangle then AB, BC & CA are three sides and ∠A, ∠B and ∠C are three angles.



(a) Types of Triangles :

(i) On the basis of sides we have three types of triangles:

(A) Scalene triangle : A triangle whose no two sides are equal is called a scalene triangle.

(B) Isosceles triangle - A triangle having two sides equal is called an isosceles triangle.

(C) Equilateral triangle - A triangle in which all sides are equal is called an equilateral triangle.

(ii) On the basis of angles we have three types of triangles :

(A) Right triangle - A triangle in which any one angle is right angle ($=90^0$) is called right triangle.

(B) Acute triangle - A triangle in which all angles are acute ($>90^0$) is called an acute triangle.

(C) Obtuse triangle - A triangle in which any one angle is obtuse ($<90^0$) is called an obtuse triangle.

CONGRUENT FIGURES

The figures are called congruent if they have same shape and same size. In order words, two figures are called congruent if they are having equal length, width and height.

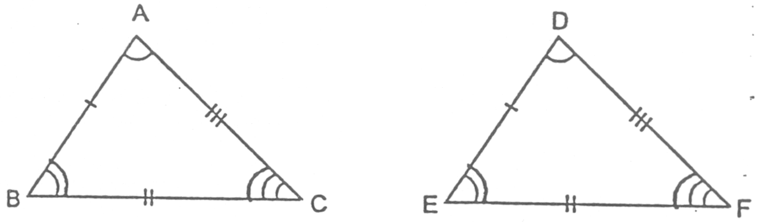
Fig. (i)

Fig. (ii)

In the above figures {fig. (i) and fig. (ii)} both are equal in length, width and height, so these are congruent figures.

(a) Congruent Triangles :

Two triangles are congruent if and only if one of them can be made to superimposed on the other, so an to cover it exactly.



If two triangles ΔABC and ΔDEF are congruent then there exist a one to one correspondence between their vertices and sides. i.e. we get following six equalities.

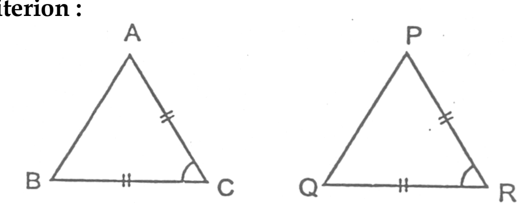
$\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$ and AB = DE, BC = EF, AC = DF.

If two ΔABC & ΔDEF are congruent under $A \leftrightarrow D$, $B \leftrightarrow E$, $C \leftrightarrow F$ one to one correspondence then we write $\Delta ABC \cong \Delta DEF$ we can not write as $\Delta ABC \cong \Delta DFE$ of $\Delta ABC \cong \Delta EDF$ or in other forms because $\Delta ABC \cong \Delta DFE$ have following one-one correspondence $A \leftrightarrow D$, $B \leftrightarrow F$, $C \leftrightarrow E$.

Hence we can say that "two triangles are congruent if and only if there exists a one-one correspondence between their vertices such that the corresponding sides and the corresponding angles of the two triangles are equal.

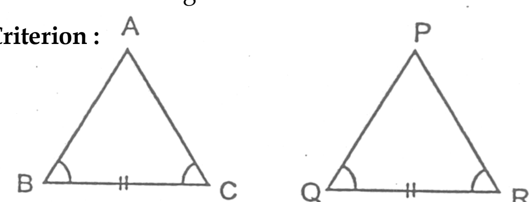
(b) Sufficient Conditions for Congruence of two Triangles :

(i) SAS Congruence Criterion :



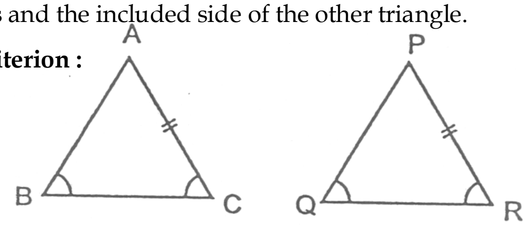
Two triangles are congruent if two sides and the included angle of one are equal to the corresponding sides and the included angle of the other triangle.

(ii) ASA Congruence Criterion :



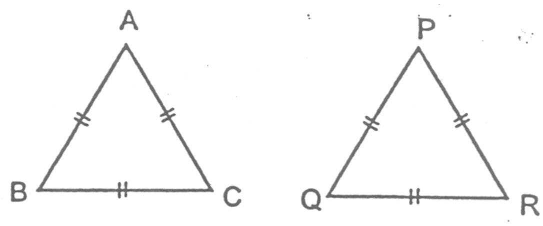
Two triangles are congruent if two angles and the included side of one triangle are equal to the corresponding two angles and the included side of the other triangle.

(iii) AAS Congruence Criterion :



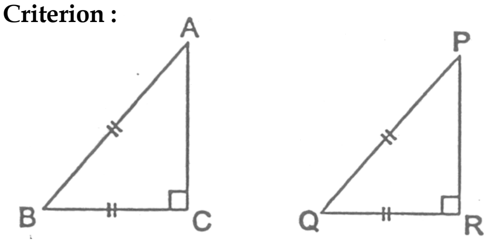
If any two angles and a non included side of one triangle are equal to the corresponding angles and side of another triangle, then the two triangles are congruent.

(iv) SSS Congruence Criterion :



Two triangles are congruent if the three sides of one triangle are equal to the corresponding three sides of the other triangle.

(v) RHS Congruence Criterion :



Two right triangles are congruent if the hypotenuse and one side of one triangle are respectively equal to the hypotenuse and one side of the other triangle.

(c) Congruence Relation in the Set of all Triangles :

By the definition of congruence of two triangles, we have following results.

(I) Every triangle is congruent to itself i.e. $\Delta ABC \cong \Delta ABC$

(II) If $\Delta ABC \cong \Delta DEF$ then $\Delta DEF \cong \Delta ABC$

(III) If $\Delta ABC \cong \Delta DEF$ and $\Delta DEF \cong \Delta PQR$ then $\Delta ABC \cong \Delta PQR$

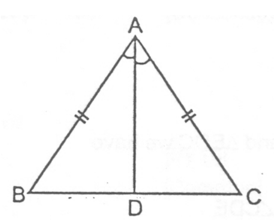
NOTE : If two triangles are congruent then their corresponding sides and angles are also congruent by cpctc (corresponding parts of congruent triangles are also congruent).

Theorem-1 : Angles opposite to equal sides of an isosceles triangle are equal.

Given : ΔABC in which AB = AC

To Prove : $\angle B = \angle C$

Construction : We draw the bisector AD of $\angle A$ which meets BC in D.



Proof : In ΔABD and ΔACD we have

	AB = AC	[Given]
	$\angle BAD = \angle CAD$	[$\because$ AD is bisector of $\angle A$]
And,	AD = AD	[Common side]
$\therefore$	By SAS criterion of congruence, we have	
	$\Delta ABD \cong \Delta ACD$	

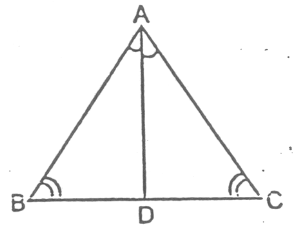
$\Rightarrow$ $\angle B = \angle C$ by cpctc **Hence Proved.**

Theorem - 2: if two angles of a triangle are equal, then sides opposite to them are also equal.

Given : ΔABC in which ∠B = ∠C

To Prove : AB = AC

Construction: We draw the bisector of ∠A which meets BC in D.



Proof : In ΔABD and ΔACD we have

∠B = ∠C	[Given]
∠BAD = ∠CAD	[∵ AD is bisector of ∠A]
AD = AD	[Common side]

∴ By AAS criterion of congruence, we get

ΔABD ≅ ΔACD

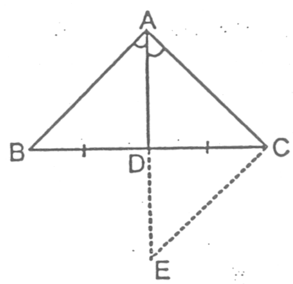
⇒ AB = AC [By cpctc] **Hence, Proved.**

Theorem-3 : if the bisector of the vertical angle bisects the base of the triangle, then the triangle is isosceles.

Given : A ΔABC in which AD is the bisector of ∠A meeting BC in D such that BD = CD

To Prove : ΔACD is an isosceles triangle.

Construction : We produce AD to E such that AD = DE and join EC.



Proof : In ΔADB and ΔEDC we have

AD = DE	[By construction]
∠ADB = ∠CDE	[Vertically opposite angles]
BD = DC	[Given]

∴ By SAS criterion of congruence, we get

ΔADB ≅ ΔEDC ⇒ AB = EC ...(i)

And, ∠BAD = ∠CED [By cpctc]

But, ∠BAD = ∠CAD

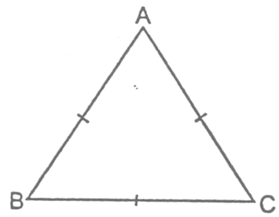
∴ ∠CAD = ∠CED

⇒ AC = EC [Sides opposite to equal angles are equal]

⇒ AC = AB [By eg. (i)] **Hence Proved.**

Ex.1 Prove that measure of each angle of an equilateral triangle is 60^0.

Sol. Let ΔABC be an equilateral triangle, then we have



$AB = BC = CA$ ...(i)

$\therefore$ $AB = BC$

$\therefore$ $\angle C = \angle A$ ...(ii) [Angles opposite to equal sides are equal]

Also, $BC = CA$

$\therefore$ $\angle A = \angle B$ ...(iii) [Angles opposite to equal sides]

By (ii) & (iii) we get $\angle A = \angle B = \angle C$

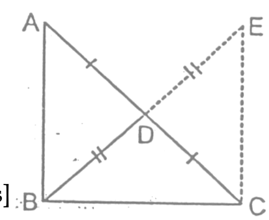
Now in ΔABC $\angle A + \angle B + \angle C = 180^0$

$\Rightarrow$ $3\angle A = 180^0$ [$\therefore \angle A = \angle B = \angle C$]

$\Rightarrow$ $\angle A = 60^0 = \angle B = \angle C$ **Hence Proved.**

Ex.2 If D is the mid-point of the hypotenuse AC of a right triangle ABC, prove that $BD = \frac{1}{2}AC$.

Sol. Let ΔABC is a right triangle such that $\angle B = 90^0$ and D is mid point of AC then we have to prove that $BD = \frac{1}{2}AC$ we produce BD to E such that BD = AC and EC.



Now is ΔADB and ΔCDE we have

$AD = DC$ [Given]

$BD = DE$ [By construction]

And, $\angle ADB = \angle CDE$ [Vertically opposite angles]

$\therefore$ By SAS criterion of congruence we have

$\Delta ADB \cong \Delta CDE$

$\Rightarrow$ $EC = AB$ and $\angle CED = \angle ABD$ ....(i) [By cpctc]

But $\angle CED$ & $\angle ABD$ are alternate interior angles

$\therefore$ $CE \parallel AB \Rightarrow \angle ABC + \angle ECB = 180^0$ [Consecutive interior angles]

$\Rightarrow$ $90 + \angle ECB = 180^0$

$\Rightarrow$ $\angle ECB = 90^0$

Now, In ΔABC & ΔECB we have

$AB = EC$ [By (i)]

$BC = BC$ [Common]

And, $\angle ABC = \angle ECB = 90^0$

$\therefore$ BY SAS criterion of congruence

$\Delta ABC \cong \Delta ECB$

$\Rightarrow$ $AC = EB$ [By cpctc]

$\Rightarrow$ $\frac{1}{2}AC = \frac{1}{2}EB$

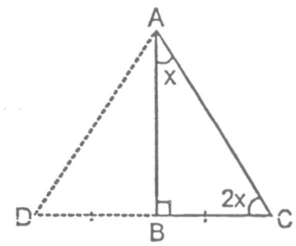
$\Rightarrow$ $BD = \frac{1}{2}AC$ **Hence Proved.**

Ex.3 In a right angled triangle, one acute angle is double the other. Prove that the hypotenuse is double the smallest side.

Sol. Let ΔABC is a right triangle such that $\angle B = 90^0$ and $\angle ACB = 2\angle CAB$, then we have to prove AC = 2BC.

we produce CB to D such that BD = CB and join AD.

Proof : In ΔABD and ΔABC we have



BD = BC [By construction]

AB = AB [Common]

$\angle ABD = \angle ABC = 90^0$

$\therefore$ By SAS criterion of congruence we get

$\Delta ABD \cong \Delta ABC$

$\Rightarrow$ AD = AC and $\angle DAB = \angle CAB$ [By cpctc]

$\Rightarrow$ AD = AC and $\angle DAB = x$ [$\therefore \angle CAB = x$]

Now, $\angle DAC = \angle DAB + \angle CAB = x + x = 2x$

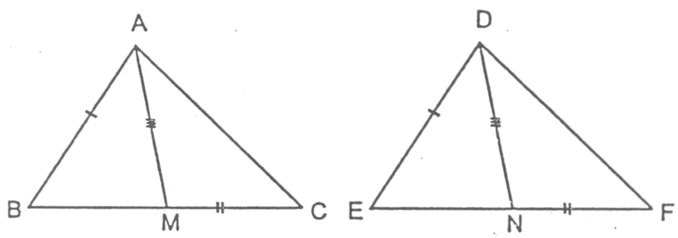
$\therefore$ $\angle DAC = \angle ACD$

$\Rightarrow$ DC = AD [Side Opposite to equal angles]

$\Rightarrow$ 2BC = AD [$\because$ DC = 2BC

$\Rightarrow$ 2BC = AC [AD = AC] **Hence Proved.**

Ex.4 In figure, two sides AB and BC and the median AM of a ΔABC are respectively equal to sides DE and EF and the median DN of ΔDEF. Prove that $\Delta ABC \cong \Delta DEF$.



Sol. $\therefore$ AM and DN are medians of ΔABC & ΔDEF respectively

$\therefore$ BM = MC & EN = NF

$\Rightarrow$ $BM = \frac{1}{2}BC$ & $EN = \frac{1}{2}EF$

But, BC = EF $\therefore$ BM = EN ...(i)

In ΔABM & ΔDEN we have

AB = DE [Given]

AM = DN [Given]

BM = EN [By (i)]

$\therefore$ By SSS criterion of congruence we have

$\Delta ABM \cong \Delta DEN \Rightarrow \angle B = \angle E$ ...(ii) [By cpctc]

Now, In ΔABC & ΔDEF

AB = DE [Given]

$\angle B = \angle E$ [By (ii)]

BC = EF [Given]

$\therefore$ By SAS criterion of congruence we get

$\Delta ABC \cong \Delta DEF$

Hence Proved.

SOME INEQUALITY RELATIONS IN A TRIANGLE

(i) If two sides of triangle are unequal, then the longer side has greater angle opposite to it. i.e. if in any $\Delta ABC AB > AC$ then $\angle C > \angle B$.

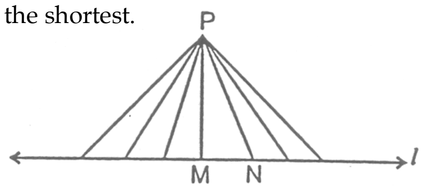
(ii) In a triangle the greater angle has the longer side opposite to it.

i.e. if in any ΔABC $\angle A > \angle B$ then $BC > AC$.

(iii) The sum of any two sides of a triangle is greater than the third side.

i.e. if in any ΔABC, $AB + BC > AC$, $BC + CA > AB$ and $AC + AB > BC$.

(iv) Of all the line segments that can be drawn to a given line, from a point, not lying on it, the perpendicular line segment is the shortest.

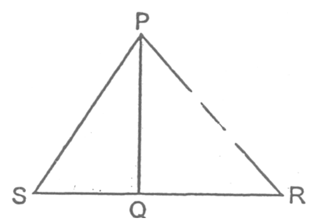


P is any point not lying online ℓ, PM $\perp$ then $PM < PN$.

(v) The difference of any two sides of a triangle is less than the third side.

i.e. In any ΔABC, $AB - BC < AC$, $BC - CA < AB$ and $AC < AB < BC$.

Ex.5 In figure, PQ = PR, show that PS > PQ



Sol. In ΔPQR

$\therefore$ $PQ = PR$

$\Rightarrow$ $\angle PRQ = \angle PQR$ ....(i) [Angles opposite to equal sides]

In ΔPSQ, SQ is produced to R

$\therefore$ Ext. $\angle PQR > \angle PSQ$ ....(ii)

$\angle PRQ > \angle PSQ$ [By eq. (i) and (ii)]

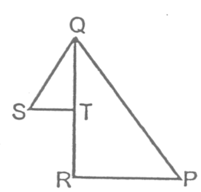
$\Rightarrow$ $\angle PRS > \angle PSR$

$\Rightarrow$ $PS > PR$ [Sides opposite to greater angles is larger]

But, $PR = PQ$

$\therefore$ $PS > PQ$ **Hence Proved.**

Ex.6 In figure, T is a point on side QR of ΔPQR and S is a point such that RT = ST. Prove that PQ + PR > QS



Sol. In ΔPQR we have

$PQ + PR > QR$

$\Rightarrow$ $PQ + PR > QT + TR$

$\Rightarrow$ $PQ + PR > QT + ST$ $\therefore RT = ST$

In ΔQST $QT + ST > SQ$

$\therefore$ $PQ + PR > SQ$ **Hence Proved.**

EXERCISE

OBJECTIVE DPP # 10.1

1. In the three altitudes of a Δ are equal then triangle is :

(A) isosceles (B) equilateral (C) right angled (D) none

2. ABCD is a square and P, Q, R are points on AB, BC and CD respectively such that AP = BQ = CR and ∠PQR = 90^0, then ∠QPR

(A) 45^0 (B) 50^0 (C) 60^0 (D) LM

3. In a ΔXYZ, LM ∥ YZ and bisectors YN and ZN of ∠Y & ∠Z respectively meet at N on LM then YL + ZM =

(A) YZ (B) XY (C) XZ (D) LM

4. In a ΔPQR, PS is bisector of ∠P and ∠Q = 70^0 ∠R = 30^0, then

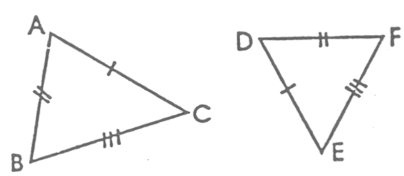
(A) QS > PQ > PR (B) QS < PQ < PR (C) PQ > QS > SR (D) PQ < QS < SR

5. If D is any point on the side BC of a ΔABC, then :

(A) AB + BC + CA > 2AD (B) AB + BC + CA < 2AD

(C) AB + BC + CA > 3AD (D) None

6. For given figure, which one is correct :



(A) ΔABC ≅ ΔDEF (B) ΔABC ≅ ΔFED (C) ΔABC ≅ ΔDFE (D) ΔABC ≅ ΔEDF

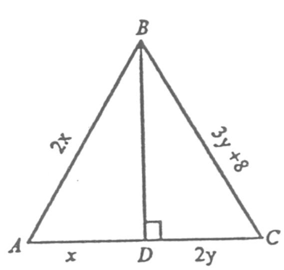
7. In a right angled triangle. One acute angle is double the other then the hypotenuse is :

(A) Equal to smallest side (B) Double the smallest side

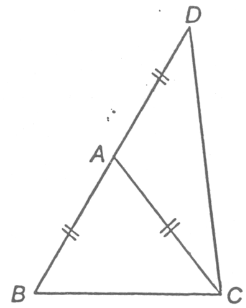
(C) Triple the smallest side (D) None of these

SUBJECTIVE DPP - 10.2

1. In the ΔABC given below, BD bisects ∠B and is perpendicular to AC. If the lengths of the sides of the triangle are expressed in terms of x and y as shown, find the values of x and y.



2. In the figure, AB = AD prove that ∠BCD is a right angle.

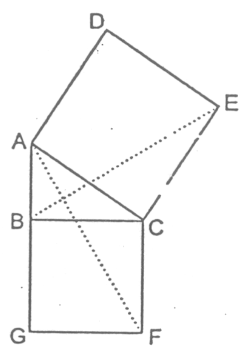


3. If the bisector of an angle of a triangle also bisects the opposite side, prove that the triangle is isosceles.

4. AD is meadian of ΔABC. Prove that AB + AC > 2 AD.

5. O is any point in the interior of a triangle ABC. Prove that OB + OC < AB + AC.

6. In figure, ΔABC is a right angled triangle at B. ADEC and BCFG are square Prove that AF = BE.

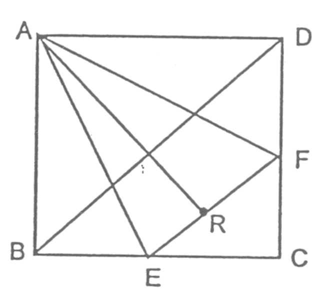


7. In figure CD is the diameter perpendicular to the chord AB of a circle with centre O. Prove that

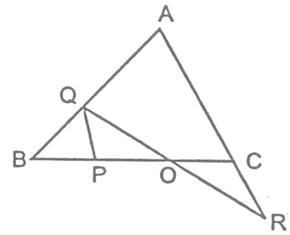
(a) ∠CAO = ∠CBO (b) ∠AOB = 2∠ACB

8. ABCD is a square and EF ∥ BD. E and F are the mid point of BC and DC respectively. Prove that

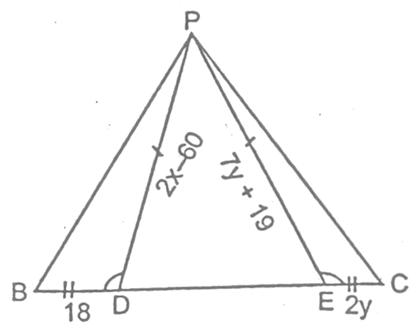
(a) BE = DF (b) AR bisects ∠BAD



9. In figure, ΔABC is an equilateral triangle $PQ \parallel AC$ and AC is produced to R such that CR = PQ. Prove that QR bisects PC.



10. In figure, the congruent parts of triangles have been indicated by line markings. Find the values of x & y.



ANSWER KEY

(Objective DPP # 10.1)

Qus.	1	2	3	4	5	6	7
Ans.	B	A	D	B	A	C	B

(Subjective DPP # 10.2)

1. 16, 8

10. 71, 9

QUADRILATERIAL

ML - 11

QUADRILATERL

A quadrilateral is a closed figure obtained by joining four points (with no three points collinear) In an order.

(I) Since, **'quad'** means **'four'** and **'lateral'** is for **'sides'** therefore **'quadrilateral'** means **'a figure bounded by four sides'.**

(II) Every quadrilateral has :

(A) Four vertices,

(B) Four sides

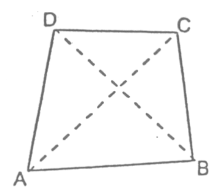
(C) Four angles and

(D) Two diagonals.

(III) A diagonals is a line segment obtained on joining the opposite vertices.

(a) Sum of the Angles of a Quadrilateral :

Consider a quadrilateral ABCD as shown alongside. Join A and C to get the diagonal AC which divides the quadrilateral ABCD into two triangles ABC and ADC.



We know the sum of the angles of each triangle is 180^0(2 right angles).

$\therefore$ In ΔABC ; $\angle CAB + \angle B + \angle BCA = 180^0$ and

In ΔADC ; $\angle DAC + \angle D + \angle DCA = 180^0$

On adding, we get : $(\angle CAB + \angle DAC) + \angle B + \angle D + (\angle BCA + \angle DCA) = 180^0 + 180^0$

$\Rightarrow \angle A + \angle B + \angle D + \angle C = 360^0$

Thus, the sum of the angles of a quadrilateral is 360^0 (4-right angles).

Ex.1 The angles of a quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilateral.

Sol. Given the ratio between the angles of the quadrilateral = 3 : 5 : 9 : 13 and 3 + 5 + 9 + 13 = 30

Since, the sum of the angles of the quadrilateral = 360^0

$\therefore$ First angle of it = $\frac{3}{30} \times 360^0 = 36^0$,

Second angle = $\frac{5}{30} \times 360^0 = 60^0$,

Third angle = $\frac{9}{30} \times 360^0 = 108^0$,

And, Fourth angle = $\frac{13}{30} \times 360^0 = 156^0$

$\therefore$ The angles of quadrilateral are 360^0, 60^0, 108^0 and 156^0.

ALTERNATE SOLUTION :

Let the angles be 3x, 5x, 9x and 13.

$\therefore$ $3x + 5x + 9x + 13x = 360^0$

$\Rightarrow$ $30x = 360^0$ and $x = \frac{360^0}{30} = 12^0$

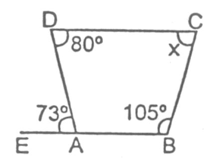
$\therefore$ 1st angle $= 3x = 2 \times 12^0 = 360^0$

2nd angle $= 5x = \ \times 12^0 = 60^0$

3rd angle $= 9x = 9 \times 12^0 = 108^0$

And, 4th angle $= 13 \times 12^0 = 156^0$.

Ex.2 Use the informations given in adjoining figure to calculate the value of x.



Sol. Since, EAB is a straight line.

$\therefore$ $\angle DAE + \angle DAB = 180^0$

$\Rightarrow$ $73^0 + \angle DAB = 180^0$

i.e., $\angle DAB = 180^0 - 73^0 = 107^0$

Since, the sum of the angles of quadrilateral ABCD is 360^0

$\therefore$ $107^0 + 105^0 + x + 80^0 = 360^0$

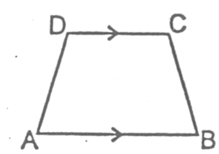
$\Rightarrow$ $292^0 + x = 360^0$

$\Rightarrow$ $x = 360^0 - 292^0$

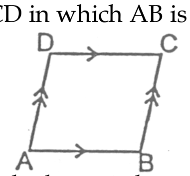
$\Rightarrow$ $x = 68^0$ **Ans.**

(b) Types of Quadrilaterals :

(i) Trapezium : It is a quadrilateral in which one pair of opposite sides are parallel. In the quadrilateral ABCD, drawn alongside, sides AB and DC are parallel, therefore it is a trapezium.



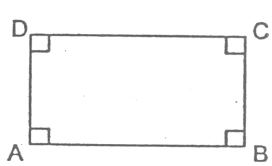
(ii) Parallelogram : It is a quadrilateral in which both the pairs of opposite sides are parallel. The adjoining figure shows a quadrilateral ABCD in which AB is parallel to DC and AD is parallel to BC, therefore ABCD is a parallelogram.



(iii) Rectangle : it is a quadrilateral whose each angle is 90^0

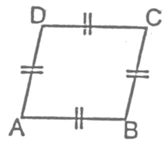
(A) $\angle A + \angle B = 90^0 + 90^0 = 180^0 \Rightarrow AD \parallel BC$

(B) $\angle B + \angle C = 90^0 + 90^0 = 180^0 \Rightarrow AB \parallel DC$

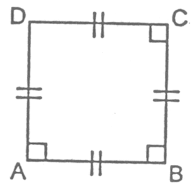


Rectangle ABCD is a parallelogram Also.

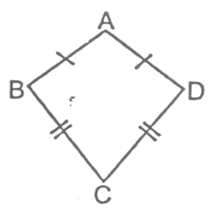
(iv) Rhombus : It is a quadrilateral whose all the sides are equal. The adjoining figure shows a quadrilateral ABCD in which AB = BC = CD = DA ; therefore it is a rhombus.



(v) Square : It is a quadrilateral whose all the sides are equal and each angle is 90^0. The adjoining figure shows a quadrilateral ABCD in which AB = BC = CD = DA and $\angle A = \angle B = \angle C = \angle D = 90^0$, therefore ABCD is a square.



(vi) Kite : It is a quadrilateral in which two pairs of adjacent sides are equal. The adjoining figure shows a quadrilateral ABCD in which adjacent sides AB and AD are equal i.e., AB = AD and also the other pair of adjacent sides are equal i.e., BC = CD; therefore it is a kite or kite shaped figure.



REMARK :

(i) Square, rectangle and rhombus are all parallelograms.

(ii) Kite and trapezium are not parallelograms.

(iii) A square is a rectangle.

(iv) A square is a rhombus.

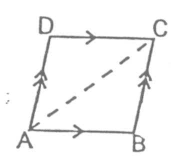
(v) A parallelogram is a trapezium.

PARALLELOGRAM

A parallelogram is a quadrilateral in which both the pairs of opposite sides are parallel.

Theorem 1 : A diagonal of a parallelogram divides the parallelogram into two congruent triangles.

Given : A parallelogram ABCD.



To Prove : A diagonal divides the parallelogram into two congruent triangles

i.e., if diagonal AC is drawn then $\Delta ABC \cong \Delta CDA$

and if diagonal BD is drawn then $\Delta ABD \cong \Delta CDB$

Construction : Join A and C

Proof : Sine, ABCD is a parallelogram

$AB \parallel DC$ and $AD \parallel BC$

In ΔABC and ΔCDA

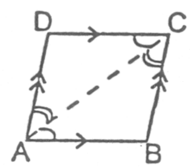
∠BAC = ∠DCA	[Alternate angles]
∠BCA = ∠DAC	[Alternate angles]
And, AC = AC	[Common side]
∴ ΔABC ≅ ΔCDA	[By ASA]

Similarly, we can prove that

ΔABD ≅ ΔCDB

Theorem 2 : In a parallelogram, opposite sides are equal.

Given : A parallelogram ABCD in which AB ∥ DC and AD ∥ BC.



To Prove : Opposite sides are equal i.e., AB = DC and AD = BC

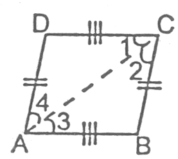
Construction : Join A and C

Proof : In ΔABC and ΔCDA

∠BAC = ∠DCA	[Alternate angles]	
∠BCA = ∠DAC	[Alternate angles]	
AC = AC	[Common]	
∴ ΔABC ≅ ΔCDA	[By ASA]	
⇒ AB = DC and AD = BC	[By cpctc]	**Hence Proved.**

Theorem 3: If each pair of opposite sides of a quadrilateral is equal, then it is a parallelogram

Given : A quadrilateral ABCD in which



To Prove: ABCD is a parallelogram i.e., AB ∥ DC and AD ∥ BC

Construction : Join A and C

Proof : In ΔABC and ΔCDA

	AB = DC	[Given]
	AD = BC	[Given]
And	AC = AC	[Common]
∴	ΔABC ≅ ΔCDA	[By SSS]
⇒	∠1 = ∠3	[By cpctc]
And	∠2 = ∠4	[By cpctc]

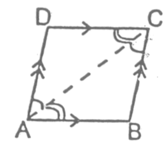
But these are alternate angles and whenever alternate angles are equal, the lines are parallel.

∴ AB ∥ DC and AD ∥ BC

⇒ ABCD is a parallelogram. **Hence Proved.**

Theorem 4 : In a parallelogram, opposite angles are equal.

Given : A parallelogram ABCD in which AB ∥ DC and AD ∥ BC.



To Prove : Opposite angles are equal

i.e. $\angle A = \angle C$ and $\angle B = \angle D$

Construction : Draw diagonal AC

Proof : In ΔABC and ΔCDA :

$\angle BAC = \angle DCA$ [Alternate angles]

$\angle BCA = \angle DAC$ [Alternate angles]

$AC = AC$ [Common]

$\therefore$ $\Delta ABC \cong \Delta CDA$ [By ASA]

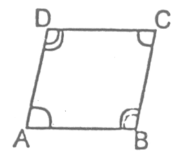
$\Rightarrow$ $\angle B = \angle D$ [By cpctc]

And, $\angle BAD = \angle DCB$ i.e., $\angle A = \angle C$

Similarly, we can prove that $\angle B = \angle D$ **Hence Proved.**

Theorem 5: If in a quadrilateral, each pair of opposite angles is equal, then it is a parallelogram.

Given : A quadrilateral ABCD in which opposite angles are equal.



i.e., $\angle A = \angle C$ ad $\angle B = \angle D$

To prove : ABCD is a parallelogram i.e, AB ∥ DC and AD ∥ BC.

Proof : Since, the sum of the angles of quadrilateral is 360^0

$\Rightarrow$ $\angle A + \angle B + \angle C + \angle D = 360^0$

$\Rightarrow$ $\angle A + \angle D + \angle A + \angle D = 360..$ [$\angle A = \angle C$ and $\angle B = \angle D$]

$\Rightarrow$ $2\angle A = 2\angle D = 360^0$

$\Rightarrow$ $\angle A + \angle D = 180^0$ [Co-interior angle]

$\Rightarrow$ AB ∥ DC

Similarly,

$\angle A + \angle B + \angle C + \angle D = 360^0$

$\Rightarrow$ $\angle A + \angle B + \angle A + \angle B = 360^0$ [$\angle A = \angle C$ and $\angle B = \angle D$]

$\Rightarrow$ $2\angle A + 2\angle V = 360^0$

$\Rightarrow$ $\angle A + \angle B = 180^0$ [$\because$ This is sum of interior angles on the same side of transversal AB]

$\therefore$ AD ∥ BC

So, AB ∥ DC and AD ∥ BC

$\Rightarrow$ ABCD is a parallelogram. **Hence Proved.**

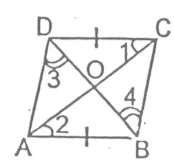
Theorem 6 : The diagonal of a parallelogram bisect each other.

Given : A parallelogram ABCD. Its diagonals AC and BD intersect each other at point O.

To Prove : Diagonals AC and BD bisect each other i.e., OA = OC and OB = OD.

Proof : In ΔAOB and ΔCOD

$\because$ AB ∥ DC and BD is a transversal line.



$\therefore$ $\angle ABO = \angle DCO$ [Alternate angles]

$\therefore$ AB ∥ DC and AC is a transversal line.

$\therefore$ $\angle BAO = \angle DCO$ [Alternate angles]

And, $AB = DC$

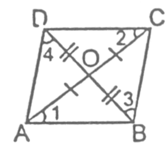
$\Rightarrow$ $\Delta AOB \cong \Delta COD$ [By ASA]

$\Rightarrow$ $OA = OC$ and $OB = OD$ [By cpctc] **Hence Proved.**

Theorem 7 : If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.

Given : A quadrilateral ABCD whose diagonals AC and BD bisect each other at point O.

i.e., OA = OC and OB = OD



To prove : ABCD is a parallelogram

i.e., AB ∥ DC and AD ∥ BC.

Proof : In ΔAOB and ΔCOD

OA = OC [Given]

OB = OD [Given}

And, ∠AOB = ∠COD [Vertically opposite angles]

⇒ ΔAOB ≅ ΔCOD [By SAS]

⇒ ∠1 = ∠2 [By cpctc]

But these are alternate angles and whenever alternate angles are equal, the lines are parallel.

∴ AB is parallel to DC i.e., AB ∥ DC

Similarly,

ΔAOD ≅ ΔCOB [By SAS]

⇒ ∠3 = ∠4

But these are also alternate angles ⇒ AD ∥ BC

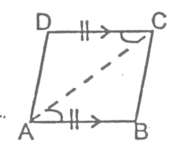
AB ∥ DC and AD ∥ BC ⇒ ABCD is parallelogram. **Hence Proved.**

Theorem 8 : A quadrilateral is a parallelogram, if a pair of opposite sides is equal and parallel.

Given : A quadrilateral ABCD in which AB ∥ DC and AB = DC.

To Prove : ABCD is a parallelogram

i.e., AB ∥ DC and AD ∥ BC.



Construction : Join A and C.

Proof : Since AB is parallel to DC and AC is transversal

∠BAC = ∠DCA [Alternate angles]

AB = DC [Given]

And AC = AC [Common side]

⇒ ΔBAC ≅ ΔDCA [By SAS]

⇒ ∠BCA = ∠DAC [By cpctc]

But these are alternate angles and whenever alternate angles are equal, the lines are parallel.

⇒ AD ∥ BC

Now, AB ∥ DC (given) and AD ∥ BC [Proved above]

⇒ ABCD is a parallelogram **Hence Proved.**

REMARKS :

In order to prove that given quadrilateral is parallelogram, we have to prove that :

(i) Opposite angles of the quadrilateral are equal, or

(ii) Diagonals of the quadrilateral bisect each other, or

(iii) A pair of opposite sides is parallel and is of equal length, or

(iv) Opposite sides are equal.

(v) Every diagonal divides the parallelogram into two congruent triangles.

EXERCISE

OBJECTIVE DPP # 11.1

1. In a parallelogram ABCD, $\angle D = 105^0$, then the $\angle A$ and $\angle B$ will be.

(A) $105^0, 75^0$ (B) $75^0, 105^0$ (C) $105^0, 105^0$ (D) $75^0, 75^0$

2. In a parallelogram ABCD diagonals AC and BD intersects at O and AC = 12.8 cm and BD = 7.6 cm, then the measure of OC and OD respectively equal to :

(A) 1.9 cm, 6.4 cm (B) 3.8 cm, 3.2 cm (C) 3.8 cm, 3.2 cm (D) 6.4 cm, 3.8 cm

3. Two opposite angles of a parallelogram are $(3x - 2)^0$ and $(50 - x)^0$ then the value of x will be :

(A) 17^0 (B) 16^0 (C) 15^0 (D) 13^0

4. When the diagonals of a parallelogram are perpendicular to each other then it is called.

(A) Square (B) Rectangle (C) Rhombus (D) Parallelogram

5. In a parallelogram ABCD, E is the mid-point of side BC. If DE and AB when produced meet at F then :

(A) $AF = \frac{1}{2}AB$ (B)AF = 2AB (C) AF = 4AB (D) Data Insufficient

6. ABCD is a rhombus with $\angle ABC = 56^0$, then the $\angle ACD$ will be.

(A) 56^0 (B) 62^0 (C) 124^0 (D) 34^0

7. In a triangle, P,Q, and R are the mid-points of the sides BC, CA and AB respectively. If AC = 16 cm, BC = 20 cm and AB = 24 cm then the perimeter of the quadrilateral ARPQ will be.

(A) 60 cm (B) 30 cm (C) 40 cm (D) None

8. LMNO is a trapezium with LM $\parallel$ NO. If P and Q are the mid-points of LO and MN respectively and LM = 5 cm and ON = 10 cm then PQ =

(A) 2.5 m (B) 5 cm (C) 7.5 cm (D) 15 cm

9. In a Isosceles trapezium ABCD if $\angle A = 45^0$ then $\angle C$ will be.

(A) 90^0 (B) 135^0 (C) 90^0 (D) None

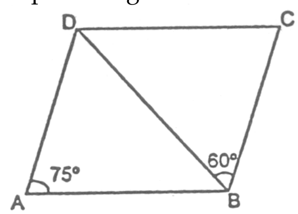
10. In a right angle triangle ABC is right angled at B. Given that AB = 9 cm, AC = 15 cm and D, E are the mid-points of the sides AB and AC respectively, then the area of ΔADE=

(A) 67.5 cm^2 (B) 13.5 cm^2 (C) 27 cm^2 (D) Data insufficient

SUBJECTIVE DPP - 11.2

1. Find the measures of all the angles of a parallelogram , if one angle is 24^0 less than twice the smallest angle.

2. In the following figure, ABCD is a parallelogram in which $\angle DAB = 75^0$ and $\angle DBC = 60^0$. Find $\angle COB$ and $\angle ADB$.



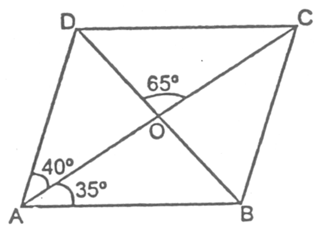
3. In the following figure, ABCD is a parallelogram $\angle DAO = 40^0$, $\angle BAO = 35^0$ and $\angle COD = 65^0$. Find

(i) $\angle ABO$

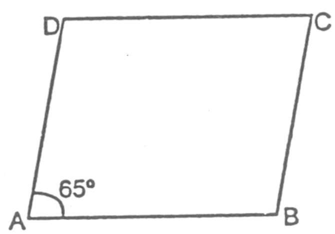
(ii) $\angle ODC$

(iii) $\angle ACB$

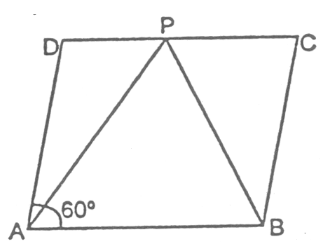
(iv) $\angle CBD$



4. In the following figure, ABCD is a parallelogram in which $\angle A = 65^0$. Find $\angle B$, $\angle C$ and $\angle D$.



5. In the following figure, ABCD is a parallelogram in which $\angle A = 60^0$. If the bisectors of $\angle A$ and $\angle B$ meet at P, prove that $\angle APB = 90^0$. Also, prove that AD = DP, PC = BC and DC = 2AD.

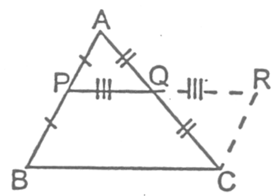


QUADRILATERAL

ML - 12

MID-POINT THEOREM

Statement : In a triangle, the line segment joining the mid-points of any two sides is parallel to the third side and is half of it.



Given : A triangle ABC is which P is the mid-point of side AB and Q is the mid-point of side AC.

To Prove : P is parallel to BC and is half of it i.e., PQ ∥ BC and PQ = $\frac{1}{2}$BC

Construction : Produce PQ upto point R such that PQ = QR. Join T and C.

Proof : In ΔAPQ and ΔCRQ : -

PQ = QR	[By construction]
AQ = QC	[Given]
And, ∠AQP = ∠CQR	[Vertically opposite angles]
⇒ ΔAPQ ≅ ΔCRQ	[By SAS]
⇒ AP = CR	[By cpctc]
And, ∠APQ = ∠CRQ	[By cpctc]

But, ∠APQ and ∠CRQ are alternate angles and we know, whenever the alternate angles are equal, the lines are parallel.

⇒ AP ∥ CR

⇒ AB ∥ CR

⇒ BP ∥ CR

Given, P is mid-point of AB

⇒ AP = BP

⇒ CR = BP [As, AP = CR]

Now, BP = CR and BP ∥ CR

⇒ BCRP is a parallelogram.

[When any pair of opposite sides are equal and parallel, the quadrilateral is a parallelogram]

BCRP is a parallelogram and opposite sides of a parallelogram are equal and parallel.

∴ PR = BC and PR ∥ BC

Since, PQ = QR

⇒ PQ = $\frac{1}{2}$PR

$= \frac{1}{2}BC$ [As, PR = BC]

Also, PQ ∥ BC [As, PR ∥ BC]

∴ PQ ∥ BC and $P + \frac{1}{2}BC$ **Hence Proved.**

ALTERNATIVE METHOD :

Construction : Draw CR parallel to BA intersecting PQ produced at point R.

Proof : In ΔAPQ and ΔCRQ

AQ = CQ [Given]

∠AQP = ∠RQC [Vertically opposite angles]

And ∠PAQ = ∠RCQ [Alternate angles, as AB∥CR]

ΔAPQ ≅ ΔCRQ [By ASA]

⇒ CR = AP and QR = PQ [By cpctc]

Since, CR = AP and AP = PB

⇒ CR = PB

Also, CR ∥ PB [By construction]

∴ PBCR is a parallelogram [As, opposite sides PB and CR are equal and parallel]

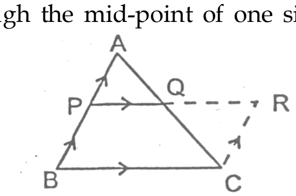
⇒ BC ∥ PR and BC = PR

⇒ BC ∥ PQ and BC = 2PQ [∵ PQ = QR]

⇒ PQ ∥ BC and $PQ = \frac{1}{2}BC$ **Hence Proved.**

(a) Converse of the Mid-Point Theorem

Statement : The line drawn through the mid-point of one side of a triangle parallel to the another side bisects the third side.



Given : A triangle ABC in which P is the mid-point of side AB nd PQ is parallel to BC.

To prove: PQ bisects the third side AB i.e., AQ = QC.

Construction : Through C, draw CR parallel to BA, which meets PQ produced at point R.

Proof : Since, PQ ∥ BC i.e., PR ∥ BC [Given] and CR ∥ BA i.e., CR ∥ BP [By construction]

∴ Opposite sides of quadrilateral PBCR are parallel.

⇒ PBCR is a parallelogram

⇒ BP = CR

Also, BP = AP [As, P is mid-point of AB]

∴ CR = AP

∴ AB ∥ CR and AC is transversal, ∠PAQ = ∠RCQ [Alternate angles]

∴ AB ∥ CR and PR is transversal, ∠APQ = ∠CRQ [Alternate angles]

In ΔAPQ and ΔCRQ

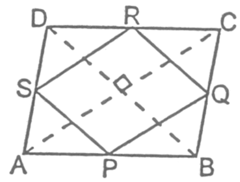
CR = AP, ∠PAQ = ∠RCQ and ∠APQ = ∠CRQ

⇒ ΔAPQ ≅ ΔCRQ [By ASA]

⇒ AAQ = QC **Hence Proved.**

Ex.1 ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Prove that the quadrilateral PQRS is a rectangle.

Sol. According to the given statement, the figure will be a shown alongside; using mid-point theorem :-



In ΔABC, PQ ∥ AC and PQ = $\frac{1}{2}$AC ....(i)

In ΔADC, SR ∥ AC and SR = $\frac{1}{2}$AC ....(ii)

∴ P = SR and PQ ∥ SR [From (i) and (ii)]

⇒ PQRS is a parallelogram.

Now, PQRS will be a rectangle if any angle of the parallelogram PWRS is 90^0

PQ ∥ AC [By mid-point theorem]

QR = BD [By mid-point theorem]

But, AC ⊥ BD [Diagonals of a rhombus are perpendicular to each other]

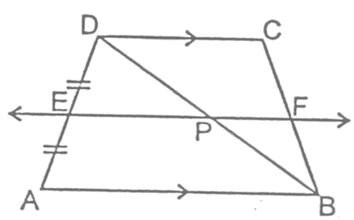
∴ PQ ⊥ QR [Angle between two lines = angles between their parallels]

⇒ **PQRS is a rectangle** **Hence Proved.**

Ex.2 ABCD is a trapezium in which AB ∥ DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (as shown). Prove that F is the mid-point of BC.

Sol. Given line EF is parallel to AB and AB ∥ DC

∴ EF ∥ AB ∥ DC.



According to the converse of the mid-point theorem, is ΔABD, E is the mid-point of AD.

EP is parallel to AB [As EF ∥ AB]

∴ P is mid-point of side BD

[The line through the mid-point of a side of a triangle and parallel to the other side, bisects the third side]

Now, in ΔBCD, P is mid-point of BD [Proved above]

And, PF is parallel to DC [As EF ∥ DC]

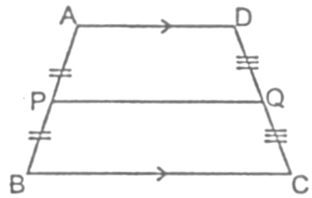
∴ F is mid-point of BC

[The line through the mid-point of a side of a triangle and parallel to the other side, bisects the third side]

Hence Proved.

REMARK :

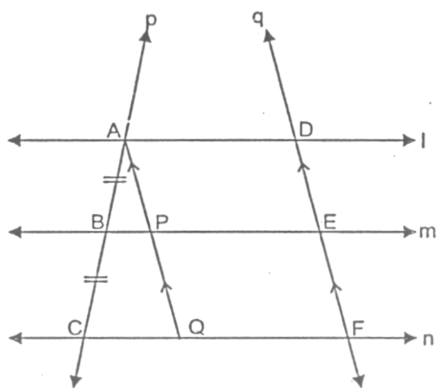
In quadrilateral ABCD, if side AD is parallel to side BC; ABCD is a trapezium.



Now, P and Q are the mid-points of the non-parallel sides of the trapezium; then PQ= $\frac{1}{2}$ (AD + BC). i.e. The length of the line segment joining the mid-points of the two non-parallel sides of a trapezium is always equal to half of the sum of the lengths of its two parallel sides.

Theorem.3: If there are three or more parallel lines and the intercepts made by them on a transversal are equal, then the corresponding intercepts on any other transversal are also equal.

Given : Three parallel lines I, m and n i.e., I ∥ m ∥ n. A transversal p meets these parallel lines are points A, B and C respectively such that AB = BC. Another transversal q also meets parallel lines I, m and n at points D, E and F respectively.



To Prove : DE = EF

Construction : Through point A, draw a line parallel to DEF; which meets BE at point P and CF and point Q.

Proof : In ΔACQ, B is mid-point of AC and BP is parallel to CQ and we know that the line through the mid-point of one side of the triangle and parallel to another sides bisects the third side.

∴ AP = PQ ...(i)

When the opposite sides of a quadrilateral are parallel, it is a parallelogram and so its opposite sides are equal.

∴ AP ∥ DE and AD ∥ PE ⇒ APED is a parallelogram.

⇒ AP = DE ....(ii)

And PQ ∥ EF and PE ∥ QF ⇒ PQFE is a parallelogram

⇒ PQ = EF ....(iii)

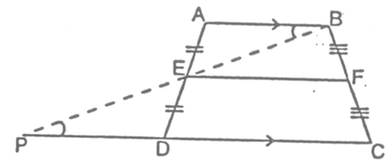
From above equations, we get

DE = EF **Hence Proved.**

Ex.3 **In the given figure, E and F are respectively, the mid-points of non-parallel sides of a trapezium ABCD. Prove that**

(i) EF ∥ AB

(ii) EF = $\frac{1}{2}$(AB + DC).



Sol. Join BE and produce it to intersect CD produced at point P. In ΔAEB and ΔDEP, AB ∥ PC and BP is transversal

⇒ ∠ABE = ∠DPE [Alternate interior angles]

∠AEB = ∠DEP [Vertically opposite angles]

And AE = DE [E is mid - point of AD]

⇒ ΔAEB ≅ ΔDEP [By ASA]

⇒ BE = PE [By cpctc]

And AB = DP [By cpctc]

Since, the line joining the mind-points of any two sides of a triangle is parallel and half of the third side, therefore, is ΔBPC,

E is mid-point of BP [As, BE = PE]

and F is mid-point of BC [Given]

⇒ EF ∥ PC and EF = $\frac{1}{2}$PC

⇒ EF ∥ DC and EF = $\frac{1}{2}$(PD + DC)

⇒ EF ∥ AB and EF = $\frac{1}{2}$(AB + DC) [As, DC ∥ AB and PD = AB] **Hence Proved.**

EXERCISE

OBJECTIVE DPP # 12.1

1. When the opposite sides of quadrilateral are parallel to each other then it is called.

(A) Square (B) Parallelogram (C) Trapezium (D) Rhombus

2. In a ΔABC, D, E and F are respectively, the mid-points of BC, CA and AB. If the lengths of side AB, BC and CA are 17 cm, 18 cm and 19 cm respectively, then the perimeter of ΔDEF equal to :

(A) 54 cm (B) 18 cm (C) 27 cm (D) 13.5 cm

3. When only one pair of opposite sides of a quadrilateral parallel to each other it is called.

(A) Square (B) Rhombus (C) Parallelogram (D) Trapezium

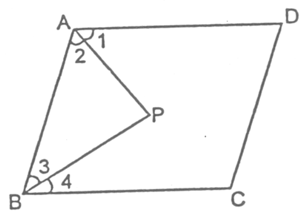
4. When the diagonals of a parallelogram are equal but not perpendicular to each other it is called a.

(A) Square (B) Rectangle (C) Rhombus (D) Parallelogram

5. When each angle of a rhombus equal to 90.0, it is called a.

(A) Square (B) Rectangle (C) Trapezium (D) Parallelogram

6. In the adjoining figure, AP and BP are angle bisectors of $\angle A$ and $\angle B$ which meets at P on the parallelogram ABCD. Then $2\angle APB$ =



(A) $\angle C + \angle D$ (B) $\angle A + \angle C$ (C) $\angle B + \angle D$ (D) $2\angle C$

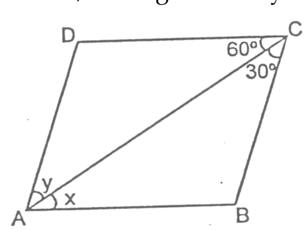
7. In a quadrilateral ABCD, AO & DO are angle bisectors of $\angle A$ and $\angle D$ and given that $\angle C = 105^0$, $\angle B = 70^0$ then the $\angle AOD$ is :

(A) 67.5^0 (B) 77.5^0 (C) 87.5^0 (D) 99.75^0

8. In a parallelogram the sum of the angle bisectors of two adjacent angle is :

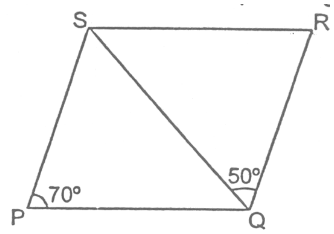
(A) 30^0 (B) 45^0 (C) 60^0 (D) 90^0

9. In the adjoining parallelogram ABCD, the angles x and y are :



(A) $60^0, 30^0$ (B) $30^0, 60^0$ (C) $45^0, 45^0$ (D) $90^0, 90^0$

10. From the figure find the value of ∠SQP and ∠QSP of parallelogram PQRS.



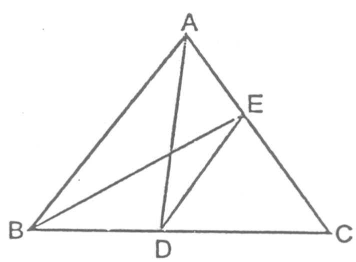
(A) $60^0, 50^0$ (B) $60^0, 45^0$ (C) $70^0, 35^0$ (D) $35^0, 70^0$

SUBJECTIVE DPP 12.2

1. Prove that the line joining the mid-points of the diagonals of a trapezium is parallel to each to the parallel sides and is equal to half of the difference of these sides.

2. ABCD is a parallelogram. P is a point on AD such that AP = $\frac{1}{3}$ AD. Q is a point on BC such that CQ = $\frac{1}{3}$ BC. Prove that AQCP is a parallelogram.

3. In the following figure, AD is a median and DE ∥ AB. Prove that BE is a median.



4. Prove that "If a diagonal of a parallelogram bisects one of the angles of the parallelogram, it also bisects the second angle and then the two diagonals are perpendicular to each other.

5. Prove that the figure formed by joining the mid-points of the consecutive sides of a quadrilateral is a parallelogram.

6. In a parallelogram ABCD, the bisector of ∠A also bisects BC at P. Prove that AD = 2AB.

7. The diagonals of parallelogram ABCD intersect at O. A line through O intersects AB at X and DC at Y. Prove that OX = OY.

8. Show that the quadrilateral formed by joining the mid points of the sides of square is also a square.

9. ABCD is a trapezium in which side AB is parallel to side DC and E is the mid-point of side AD. If F is a point on side BC such that segment EF is parallel to side DC. Prove that EF = $\frac{1}{2}$(AB + DC).

10. In ΔABC, AD is the median through A and E is the mid-point of AD. BE produced meets AC in F. Prove that AF= $\frac{1}{3}$ AC.

ANSWER KEY

(Objective DPP # 11.1)

Qus.	1	2	3	4	5	6	7	8	9	10
Ans.	B	D	D	C	B	B	C	C	B	C

(Subjective DPP # 11.2)

1. 68^0, 12^0, 68^0, 112^0

2. 45^0 & 60^0

3. (i) 80^0 (ii) 80^0 (iii) 40^0 (iv) 25^0

4. 115^0, 65^0 and 115^0

(Objective DPP # 12.1)

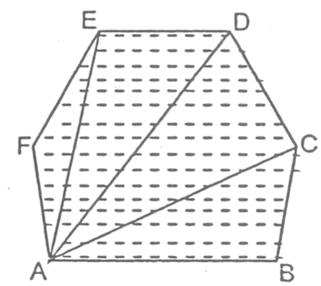
Qus.	1	2	3	4	5	6	7	8	9	10
Ans.	B	C	D	B	A	A	C	D	A	A

AREA OF PARALLELOGRAMS AND TRIANGLE

ML - 13

POLYGONAL REGION

Polygon region can be expressed as the union of a finite number of triangular regions in a plane such that if two of these intersect, their intersection is either a point or a line segment. It is the shaded portion including its sides as shown in the figure.



(a) Area Axioms :

Every polygonal region R has an area, measure in square unit and denoted by ar(R).

(i) **Congruent area axiom :** if R_1 and R_2 be two regions such that $R_1 \cong R_2$ then $ar(R_1) = ar(R_2)$.

(ii) **Area monotone axiom :** If $R_1 \subset R_2$, then are $(R_1) \leq ar(R_2)$.

(iii) **Area addition axiom :** If R_1 are two polygonal regions, whose intersection is a finite number of points and line segments and $R = R_1 \cup R_2$, then $ar(R) = ar(R_1) + ar(R_2)$.

(iv) **Rectangular area axiom :** If AB = a metre and AD = b metre then,
ar (Rectangular region ABCD) = ab sq. m.

(b) Unit of Area :

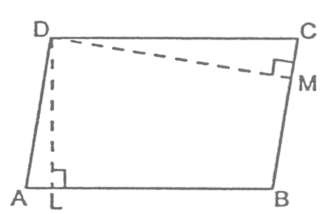
There is a standard square region of side 1 metre, called a square metre, which is the unit of area measure. The area of a polygonal region is square metres (sq. m or m^2) is a positive real number

AREA OF A PARALLELOGRAM

(a) Base and Altitude of a Parallelogram :

(i) Base : Any side of parallelogram can be called its base.

(ii) Altitude : The length of the line segment which is perpendicular to the base from the opposite side is called the altitude or height of the parallelogram corresponding to the given base.



In the Adjoining Figure

(i) DL is the altitude of $\|^{gm}$ ABCD, corresponding to the base AB.

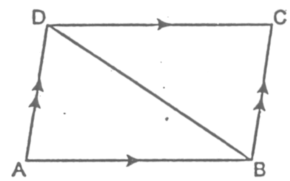
(ii) DM is the altitude of $\|^{gm}$ ABCD, corresponding to the base BC.

Theorem -1 A diagonal of parallelogram divides it into two triangles of equal area.

Given : A parallelogram ABCD whose one of the diagonals is BD.

To prove : ar (ΔABD) = ar (ΔCDB).

Proof : In ΔABD and ΔCDB.



AB = DC [Opp. sides of a $\|^{gm}$]

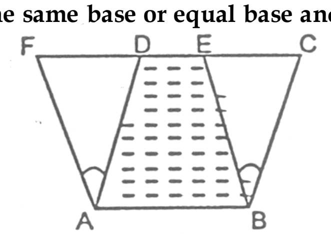
AD = BC [Opp. sides of a $\|^{gm}$]

BD = BD [Common side]

∴ ΔABD ≅ ΔCDB [By SSS]

∴ ar (ΔABD) = ar(ΔCDB) [Congruent area axiom] **Hence Proved.**

Theorem -2: Parallelograms on the same base or equal base and between the same parallels are equal in area.



Given : Two $\|^{gm}$ ABCD and ABEF on the same base AB and between the same parallels AB and FC.

To Prove : ar($\|^{gm}$ ABCD) = ar($\|^{gm}$ ABEF)

Proof : In ΔADF and ΔBCE, we have

AD = BC [Opposite sides of a $\|^{gm}$]

AF = BE [Opposite sides of a $\|^{gm}$]

∠DAF = ∠CBE [∵ AD ∥ BC and AF ∥ BE]

[Angle between AD and AF = angle between BC and BE]

∴ ΔADF ≅ ΔBCE [By SAS]

∴ ar(ΔADF) = ar(ΔBCE) ....(i)

∴ ar($\|^{gm}$ ABCD) = ar(ABED) + ar(ΔBCE)

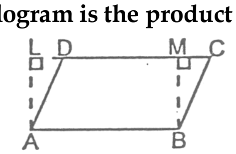
= ar(ABED) + ar(ΔADF) [Using (i)]

= ar($\|^{gm}$ ABEF).

Hence, ar($\|^{gm}$ ABCD) = ar($\|^{gm}$ ABEF). **Hence Proved.**

NOTE : A rectangle is also parallelogram.

Theorem -3: The are of parallelogram is the product of its base and the corresponding altitude.



Given : A $\|^{gm}$ ABCD in which AB is the base and AL is the corresponding height.

To prove : Area ($\|^{gm}$ ABCD) = AB × AL.

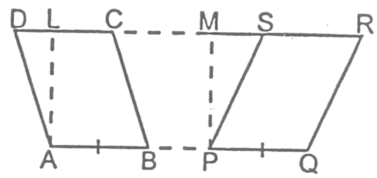
Construction : Draw BM $\perp$ DC so that rectangle ABML is formed.

Proof : $\|^{gm}$ ABCD and rectangle ABML are on the same base AB and between the same parallel lines AB and LC.

$\therefore$ ar($\|^{gm}$ ABCD) = ar(rectangle ABML) = AB × AL.

$\therefore$ area of a $\|^{gm}$ = base × height. **Hence Proved.**

Theorem-4 : Parallelograms on equal bases and between the same parallels are equal in area.



Given : Two $\|^{gm}$ ABCD and PQRS with equal base AB and PQ and between the same parallels, AQ and DR.

To prove: ar($\|^{gm}$ ABCD) = ar($\|^{gm}$ PQRS).

Construction : Draw AL $\perp$ DR and PM $\perp$ DR.

Proof : AB $\|$ DR, AL $\perp$ DR and PM $\perp$ Dr

$\therefore$ AL = PM.

$\therefore$ ar($\|^{gm}$ ABCD) = AB × AL

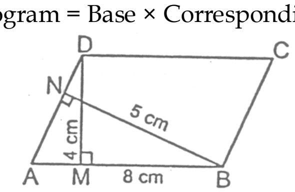
= PQ × PM [$\because$ AB = PQ and AL = PM]

= a($\|^{gm}$ PQRS). **Hence Proved.**

ILLUSTRATIONS :

Ex.1 In a parallelogram ABCD, AB = 8 cm. The altitudes corresponding to sides AB and AD are respectively 4 m and 5 cm. Find AD.

Sol. We know that, Area of a parallelogram = Base × Corresponding altitude



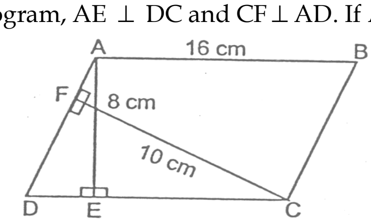
$\therefore$ Area of parallelogram ABCD = AD × BN = AB × DM

$\Rightarrow$ AD × 5 = 8 × 4

$\Rightarrow$ AD = $\frac{8\times4}{5}$

= 6.4 cm. **Ans.**

Ex.2 **In figure,** ABCD is a parallelogram, AE $\perp$ DC and CF $\perp$ AD. If AB = 16 cm, AE = 8 cm and CF = 10 cm find AD.



Sol. We have AB = 16 cm, AE = 8 cm CF = 10 cm.

We know that are of parallelogram = Base × Height [Base = CD, height = AE]

ABCD = CD × AE = 16 × 8 = 128 cm^2

Again, Area of parallelogram = Base × Height = AD × CF [Base = AD, height = CF]

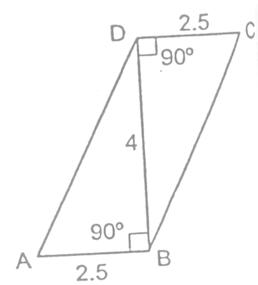
128 = AD × 10

$\Rightarrow$ AD = $\frac{128}{10}$ =12.8 cm **Ans.**

Ex.3 ABCD is a quadrilateral and BD is one of its diagonal as shown in the figure. Show that the quadrilateral ABCD is a parallelogram and find its area.

Sol. From figure, the transversal DB is intersecting a pair of lines DC and AB such that

$\angle CDB = \angle ABD = 90^0$.



Hence these angles from a pair of alternate equal angles.

$\therefore$ DC ∥ AB.

Also DC = AB = 2.5 units.

$\therefore$ Quadrilateral ABCD is a parallelogram.

Now, area of parallelogram ABCD

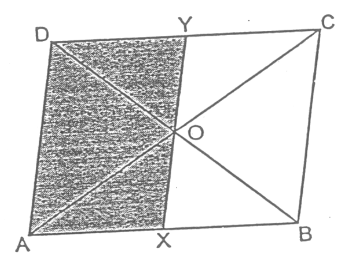
= Base × Corresponding altitude

= 2.5 × 4

= 10 sq. units **Ans.**

Ex.4 The diagonals of a parallelogram ABCD intersect in O. A line through O meets AB is X and the opposite side CD in Y. Show that ar (quadrilateral AXYD) = $\frac{1}{2}$ far(parallelogram ABCD).

Sol. $\therefore$ AC is a diagonal of the parallelogram ABCD.



ar(ΔACD) = $\frac{1}{2}$ ar(ABCD) ...(i)

Now, in Δs AOX and COY,

AO = CO

$\because$ Diagonals of parallelogram bisect each other.

$\angle AOX = \angle COY$ [Vert. opp. $\angle$s]

$\angle OAX = \angle OCY$ [Alt. Int. $\angle$s]

$\therefore$ AB ∥ DC and transversal AC intersects them

$\therefore$ $\Delta AOX \cong \Delta COY$ [ASA]

$\therefore$ ar(ΔAOX) = ar(ΔCOY) ....(ii)

Adding ar(quad. AOYD) to both sides of (ii), we get

ar(quad. AOYD) + ar(ΔAOX) = ar(quad. AOYD) + ar(ΔCOY)

$\Rightarrow$ ar(quad. AXYD) = ar(ΔACD) = $\frac{1}{2}$ ar(∥gm ABCD) (using (i)) **Hence Proved.**

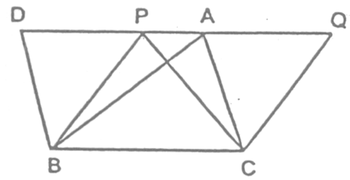
AREA OF A TRIANGLE

Theorem-5 : Two triangles on the same base (or equal bases) and between the same parallels are equal in area.

Given : Two triangles ABC and PCs on the same base BC and between the same parallel lines BC and AP.

To prove : ar(ΔABC) = ar(ΔPBC)

Construction : Through B, draw BD ∥ CA intersecting PA produced in D and through C, draw CQ ∥ BP, intersecting line AP in Q.



Proof : We have,

BD ∥ CA [By construction]

And, BC ∥ DA [Given]

∴ Quad. BCAD is a parallelogram.

Similarly, Quad. BCQP is a parallelogram.

Now, parallelogram BCQP and BCAD are on the same base BC, and between the same parallels.

∴ $ar(\|^{gm}\ BCQP) = ar(\|^{gm}\ BCAD)$ ....(i)

We know that the diagonals of a parallelogram divides it into two triangles of equal area.

∴ $ar(\Delta PBC = \frac{1}{2} ar(\|^{gm}\ BCQP)$ ....(ii)

And $ar(\Delta ABC) = \frac{1}{2} ar(\|^{gm}\ BCAD)$ ....(iii)

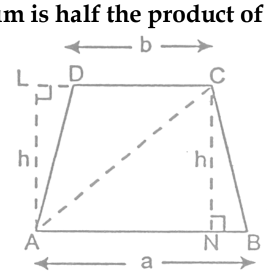
Now, $ar(\|^{gm}\ BCQP) = ar(\|^{gm}\ BCAD)$ [From (i)]

⇒ $\frac{1}{2} ar(\|^{gm}\ BCAD) = \frac{1}{2} ar(\|^{gm}\ BCQP)$

Hence, $ar(\Delta ABC) = ar(\Delta PBC)$ [Using (ii) and (iii)]

Hence Proved.

Theorem-6 : The area of a trapezium is half the product of its height and the sum of the parallel sides.



Given : Trapezium ABCD in which AB ∥ DC, AL ⊥ DC, CN ⊥ AB and AL = CN = h (say)
AB = a, DC = b.

To prove : $ar(trap.\ ABCD) = \frac{1}{2} h \times (a + b)$.

Construction : Join AC.

Proof : AC is a diagonal of quad. ABCD.

∴ $ar(trap.\ ABCD) = ar(\Delta ABC) + ar(\Delta ACD) = \frac{1}{2} h \times a + \frac{1}{2} h \times b = \frac{1}{2} h(a + b)$. **Hence Proved.**

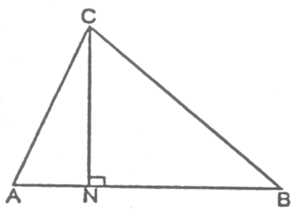
Theorem -7: Triangles having equal areas and having one side of the triangle equal to corresponding side of the other, have their corresponding altitudes equal/

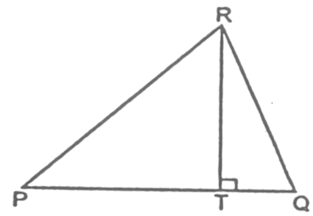
Given : Two triangles ABC and PQR such that (i) $ar(\Delta ABC) = ar(\Delta PQR)$ and (ii) AB = PQ.
CN and RT and the altitude corresponding to AB and PQ respectively of the two triangles.

To prove : CR = RT.

Proof : In ΔABC, CN is the altitude corresponding to the side AB.

ar(ΔABC) = $\frac{1}{2}$ AB × CN ....(i)





Similarly, ar(ΔPQR) = $\frac{1}{2}$ PQ × RT ....(ii)

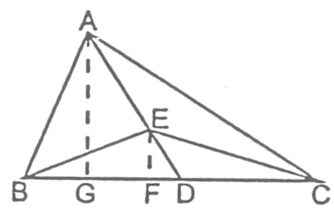
Since ar(ΔABC) =ar(ΔPQR) [Given]

∴ $\frac{1}{2}$ AB × CN = $\frac{1}{2}$ PQ × RT

Also, AB = PQ [Given]

CN = RT **Hence Proved.**

Ex.5 In figure, E is any point on median AD of a ΔABC. Show that ar(ABE) = ar(ACE).



Sol. **Construction :** From A draw AG ⊥ BC and from E draw EF ⊥ BC.

Proof : ar(ΔABD) = $\frac{BD \times AG}{2}$

ar(ΔADC) = $\frac{DC \times G}{2}$

But, BD = DC [∴ D is the mid-point of BC, AD being the median]

ar(ΔABD) = ar(ΔADC) ...(i)

Again, ar(ΔEBD)= $\frac{BD \times EF}{2}$

ar(ΔEDC) = $\frac{DC \times EF}{2}$

But, BD = DC

∴ ar(ΔEBD) = ar(ΔEDC) ...(ii)

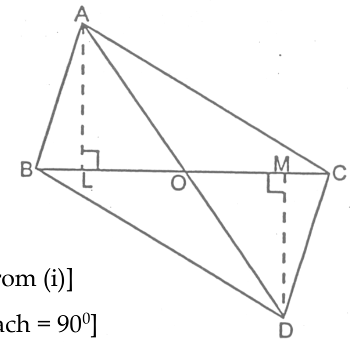
Subtracting (ii) from (i), we get

ar(ΔABD) - ar(ΔEBD) = ar(ΔADC) - ar(ΔEDC)

⇒ ar(ΔABE) = ar(ΔACE). **Hence Proved.**

Ex.6 Triangles ABC and DBC are on the same base BC; with A, D on opposite sides of the line BC, such that ar(ΔABC) = ar(ΔDBC). Show that BC bisects AD.

Sol. **Construction :** Draw AL $\perp$ BC and DM $\perp$ BC.



Proof : ar(ΔABC) = ar(ΔDBC) [Given]

$\Rightarrow \frac{BC \times AL}{2} = \frac{BC \times DM}{2}$

$\Rightarrow$ AL = DM ....(i)

Now in Δs OAL and OMD

AL = DM [From (i)]

$\Rightarrow \angle ALO = \angle DMO$ [Each = 90^0]

$\Rightarrow \angle AOL = \angle MOD$ [Vert. opp. ∠s]

$\Rightarrow \angle OAL = \angle ODM$ [Third angles of the triangles]

$\therefore$ ΔOAL ≅ ΔOMD [By ASA]

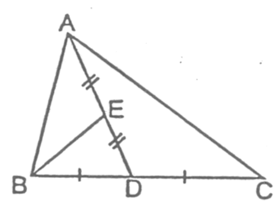
$\therefore$ OA = OD [By cpctc]

i.e., BC bisects AD. **Hence Proved.**

Ex.7 ABC is a triangle in which D is the mid-point of BC and E is the mid-point of AD. Prove that the area of ΔBED= $\frac{1}{4}$ area of ΔABC.

Sol. **Given : A ΔABC** in which D is the mid-point of BC and E is the mid-point of AD.

To prove: ar(ΔBED) = $\frac{1}{4}$ ar(ΔABC).



Proof : $\because$ AD is a median of ΔABC.

$\therefore$ ar(ΔABD) = ar(ΔADC) = $\frac{1}{2}$ ar(ΔABC) .....(i)

[$\therefore$ Median of a triangle divides it into two triangles of equal area) = $\frac{1}{2}$ ar(ΔABC)

Again,

$\because$ BE is a median of ΔABD,

$\therefore$ ar(ΔBEA) = ar(ΔBED) = $\frac{1}{2}$ ar(ΔABD)

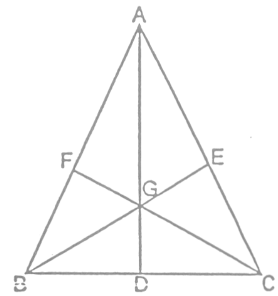
[$\therefore$ Median of a triangle divides it into two triangles of equal area]

And $\frac{1}{2}$ ar(ΔABD) = $\frac{1}{2} \times \frac{1}{2}$ ar(ΔABC) [From (i)]

$\therefore$ ar(ΔBED) = $\frac{1}{4}$ ar(ΔABC). **Hence Proved.**

Ex.8 if the medians of a ΔABC intersect at G, show that ar(ΔAGB) = ar(ΔBGC) = $\frac{1}{3}$ ar(ΔABC).

Sol. **Given :** A ΔABC its medians AD, BE and CF intersect at G.



To prove : ar(ΔAGB) = ar(ΔAGC) = ar(ΔBGC) = $\frac{1}{3}$ ar(ΔABC).

Proof : A median of triangle divides it into two triangles of equal area.

In ΔABC, AD is the median.

$\therefore$ ar(ΔABD) = ar(ΔACD) ...(i)

In ΔGBC, GD is the median.

$\therefore$ ar(ΔGBD) = ar(ΔGCD) ...(ii)

From (i) and (ii), we get

ar(ΔABD) - ar(ΔGBD) = ar(ΔACD) -ar(ΔGCD)

$\therefore$ a(ΔAGB) = ar(ΔAGC).

Similarly,

ar(ΔAGB) =ar(ΔAGC) =ar(ΔBGC) ....(iii)

But, ar(ABC) = ar(ΔAGB) + ar(ΔAGC) + ar(ΔBGC)

= 3 ar(ΔAGB) [Using (iii)]

$\therefore$ ar(ΔAGB) = $\frac{1}{3}$ ar(ΔABC).

Hence, ar(ΔAGB) = ar(ΔAGC) = arΔ(BGC) = $\frac{1}{3}$ ar(ΔABC). **Hence proved.**

Ex.9 **D,E** and F are respectively the mid points of the sides BC, CA and AB of a ΔABC. Show that

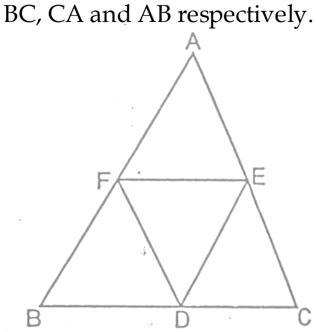
(i) BDEF is parallelogram

(ii) ar($\|^{gm}$ BDEF) = $\frac{1}{2}$ ar(ΔABC)

(iii) ar(ΔDEF) = $\frac{1}{4}$ ar(ΔABC)

Sol. **Given :** A ΔABC in which D,E,F are the mid-point of the side BC, CA and AB respectively.

To prove:



(i) Quadrilateral BDEF is parallelogram.

(ii) ar($\|^{gm}$ BDEF) = $\frac{1}{2}$ ar(ΔABC).

(iii) ar(ΔDEF) = $\frac{1}{4}$ ar(ΔABC).

Proof:

(i) In ΔABC,

$\therefore$ F is the mid-point of side AB and E is the mid point of side AC.

$\therefore$ EF $\|$ BD

[$\because$ Line joining the mid-points of any two sides of a Δ is parallel to the third side.]

Similarly,

ED $\|$ FB.

Hence, BDEF is a parallelogram. **Hence Proved.**

(ii) Similarly, we can prove that AFDE and FDCE are parallelograms.

$\therefore$ FD is diagonals of parallelogram BDEF.

$\therefore$ ar(ΔFBD) = ar(ΔDEF) ...(i)

Similarly,

ar(ΔFAE) = ar(ΔDEF) . ..(ii)

And ar(ΔDCE) = ar(ΔDEF) ...(iii)

From above equations, we have

ar(ΔFBD) = ar(ΔFAE) = ar(ΔDCE) = ar(ΔDEF)

And ar(ΔFBD) + ar(ΔDCE) + ar(ΔDEF) + ar(ΔFAE) = ar(ΔABC)

$\Rightarrow$ 2[ar(ΔFBD) + ar(ΔDEF)] = ar(ΔAC) [By using (i), (ii) and (iii)]

$\Rightarrow$ 2[ar(‖gmBDEF)] = ar(ΔABC)

$\Rightarrow$ ar(‖gm BDEF) = $\frac{1}{2}$ar(ABC)

(iii) Since, ΔABC is divided into four non-overlapping triangles FBD, FAE, DCE and DEF.

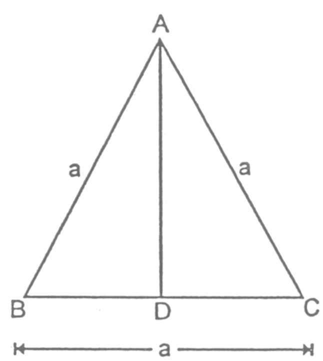
$\therefore$ ar(ΔABC) = ar(ΔFBD) + ar(ΔFAE) + ar(ΔDCE) + ar(ΔDEF)

$\Rightarrow$ ar(ΔABC) = 4 ar(ΔDEF) [Using (i), (ii) and (iii)]

$\Rightarrow$ ar(ΔDEF) = $\frac{1}{2}$ar(ΔABC). **Hence Proved.**

Ex.10 Prove that the area of an equilateral triangle is equal to $\frac{\sqrt{3}}{4}a^2$, where a is the side of the triangle.

Sol. Draw AD ⊥ BC



$\Rightarrow$ ΔABD ≅ ΔACD [Br R.H.S.]

$\therefore$ BD = DC [By cpctc]

$\therefore$ BC = a

$\therefore$ BD = DC = $\frac{a}{2}$

In right angled ΔABD

$$AD^2 = AB^2 - BD^2 = a^2 - \left(\frac{a}{2}\right)^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4}$$

$\Rightarrow$ AD = $\frac{\sqrt{3}a}{2}$

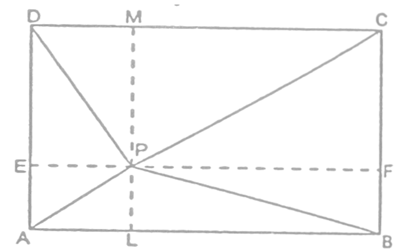
Area of ΔABC = $\frac{1}{2}$BC × AD = $\frac{1}{2}a\frac{\sqrt{3}a}{2} = \frac{\sqrt{3}a^2}{4}$. **Hence Proved.**

Ex.11 In figure, P is a point in the interior of rectangle ABCD. Show that

(i) ar(ΔAPB) + ar(ΔPCD) = $\frac{1}{2}$ar(rect. ABCD)

(ii) ar(APD) + ar(PBC) = ar(APB) + ar(PCD)

Sol. **Given :** A rect. ABCD and P is a point inside it. PA, PB, PC and PD have been joined.



To prove :

(i) ar(ΔAPB) + ar(ΔPCD) = $\frac{1}{2}$ar(rect. ABCD)

(ii) ar(ΔAPD) + ar(ΔBPC) = ar(ΔAPB) + ar(ΔCPD).

Construction : Draw EPF ∥ AB and LPM ∥ AD.

Proof : (i) EPF ∥ AB and DA cuts them,

∴ $\angle DEP = \angle EAB = 90^0$ [Corresponding angles]

∴ PE ⊥ AD.

Similarly, PR ⊥ BC; PL ⊥ AB and PM ⊥ DC.

∴ ar(ΔAPD) + ar(ΔBPC)

$= \left(\frac{1}{2}\times AD\times PE\right)+ar\left(\frac{1}{2}\times BC\times PF\right)=\frac{1}{2}AD\times(PE+PF)$ [∴BC = AD]

$= \frac{1}{2}\times AD\times EF = \frac{1}{2}\times AD\times AB$ [∴ EF = AB]

$= \frac{1}{2}\times$ (rectangle ABCD).

(ii) ar (ΔAPB) + ar(PCD)

$= \left(\frac{1}{2}\times AB\times PL\right)+\left(\frac{1}{2}\times DC\times PM\right)=\frac{1}{2}\times AB\times(PL+PM)$ [∴EF = AB]

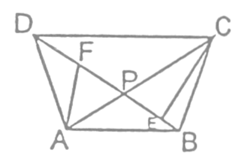
$= \frac{1}{2}\times AB\times LM = \frac{1}{2}\times AB\times AD$ [∵ LM = AD]

$= \frac{1}{2}\times$ ar(rect. ABCD).

ar(ΔAPD) + ar(PBC) = ar(ΔAPB) + ar(PCD) **Hence Proved.**

Ex.12 Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that :

ar(APB) + ar(CPD) = ar(APD) × ar(BPC)



Sol. Draw perpendiculars AF and CE on BD.

$ar(APB)\times ar(CPD) = \left(\frac{1}{2}\times PB\times AF\right)\times\left(\frac{1}{2}\times PD\times CE\right)$ ....(i)

$ar(APD)\times ar(BPC) = \left(\frac{1}{2}\times PD\times AF\right)\times\left(\frac{1}{2}\times BP\times CE\right)$ ....(ii)

From above equations, we get

ar(APB) × ar(CPD) = ar(APD) × ar(BPC) **Hence Proved.**

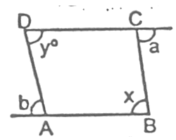
EXERCISE

OBJECTIVE DPP - 13.1

1. The sides BA and DC of the parallelogram ABCD are produced as shown in the figure then

(A) a + x = b + y (B) a + y = b + a

(C) a + b = x + y (D) a - b = x - y



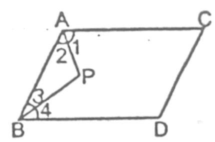
2. The sum of the interior angles of polygon is three times the sum of its exterior angles. Then numbers of sides in polygon is

(A) 6 (B) 7 (C) 8 (D) 9

3. In the adjoining figure, AP and BP are angle bisector of ∠A and ∠B which meet at a point P of the parallelogram ABCD. Then 2∠APB =

(A) ∠A + ∠B (B) ∠A + ∠C

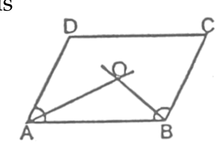
(C) ∠B + ∠D (D) ∠C + ∠D



4. In a parallelogram the sum of the angle bisector of two adjacent angles is

(A) 30^0 (B) 45^0

(C) 60^0 (D) 90^0



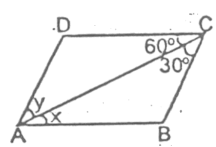
5. In a parallelogram ABCD ∠D = 60^0 then the measurement of ∠A

(A) 120^0 (B) 65^0 (C) 90^0 (D) 75^0

6. In the adjoining figure ABCD, the angles x and y are

(A) 60^0, 30^0 (B) 30^0, 60^0

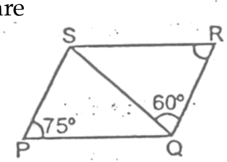
(C) 45^0, 45^0 (D) 90^0 , 90^0



7. From the figure parallelogram PQRS, the values of ∠SQP and ∠QSP are are

(A) 45^0, 60^0 (B) 60^0, 45^0

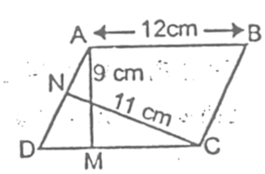
(C) 70^0, 35^0 (D) 35^0, 70



8. In parallelogram ABCD, AB = 12 cm. The altitudes corresponding to the sides AB and AD are respectively 9 cm and 11 cm. Find AD.

(A) $\frac{108}{11}$cm (B) $\frac{108}{10}$cm

(C) $\frac{99}{10}$cm (D) $\frac{108}{17}$cm



9. In ΔABC, AD is a median and P is a point is AD such that AP : PD = 1 : 2 then the area of ΔABP =

(A) $\frac{1}{2}$ × Area of ΔABC (B) $\frac{2}{3}$ × Area of ΔABC (C) $\frac{1}{3}$ × Area of ΔABC (D) $\frac{1}{6}$ × Area of ΔABC

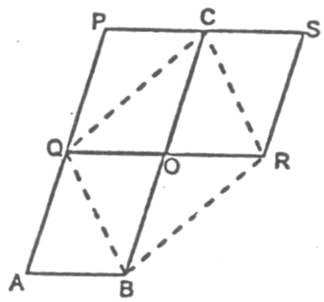
10. In ΔABC if D is a point in BC and divides it the ratio 3 : 5 i.e., if BD : DC = 3 : 5 then, ar (ΔADC) : ar(ΔABC) = ?

(A) 3 : 5 (B) 3 : 8 (C) 5 : 8 (D) 8 : 3

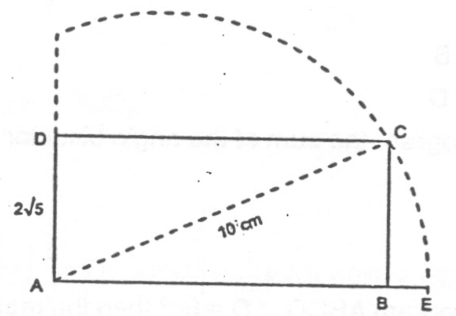
SUBJECTIVE DPP - 13.2

1. If each diagonal of a quadrilateral separates into two triangles of equal area, then show that the quadrilateral is a parallelogram.

2. In the adjoining figure, PQRS and PABC are two parallelograms of equal area. Prove that QC ∥ BR.



3. In the figure ABCD is rectangle inscribed in a quadrant of a circle of radius 10 cm. If AD = $2\sqrt{5}$ cm. Find the area of the rectangle.

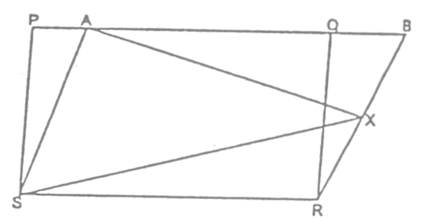


4. P and Q are any two points lying on the sides DC and AD respectively of parallelogram ABCD. Prove that : ar (ΔAPB) = ar(ΔBQC).

5. In the figure, given alongside, PQRS and ABRS are parallelograms and X is any point on side BR. Prove that :

(i) ar(PQRS) = ar(ABRS)

(ii) ar(AXS) = $\frac{1}{2}$ ar(PQRS)

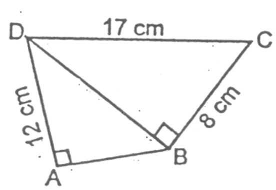


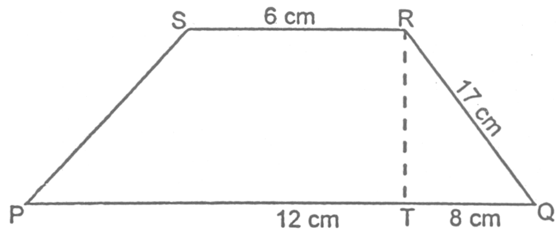
6. Find the area a rhombus, the lengths of whose diagonals are 16 cm and 24 cm respectively.

7. Find the area of trapezium whose parallel sides are 8 cm and 6 cm respectively and the distance between these sides is 8 cm.

8. (i) Calculate the area of quad. ABCD, given in fig. (i)

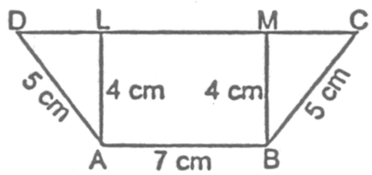
(ii) Calculate the area of trap. PQRS, given in fig. (ii).





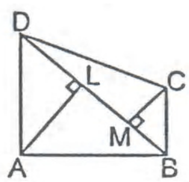
(i) **(ii)**

9. In figure, ABCD is a trapezium in which AB ∥ DC; AB = 7 cm; AD = BC = 5 cm and the distance between AB and DC is 4 cm.

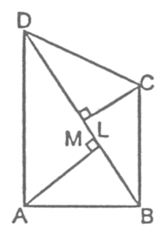


Find the length of DC and hence, find the area of trap. ABCD.

10. BD is one of the diagonals of quadrilateral ABCD. If AL ⊥ BD and CM ⊥ BD, show that : ar(quadrilateral ABCD) = $\frac{1}{2}$ × BC × (AL + CM).



11. In the figure, ABCD is a quadrilateral in which diag. BD = 20 cm. If AL ⊥ BD and CM ⊥ BD, such that : AL = 10 cm and CM = 5 cm, find the area of quadrilateral ABCD.

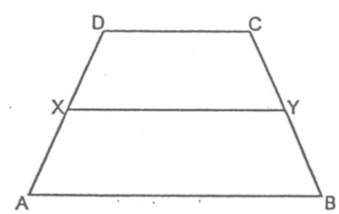


12. In fig. ABCD is a trapezium in which AB ∥ DC and DC = 40 cm and AB = 60 cm. If X and Y are, respectively, the mid - points of AD and BC, prove that

(i) XY = 50 cm

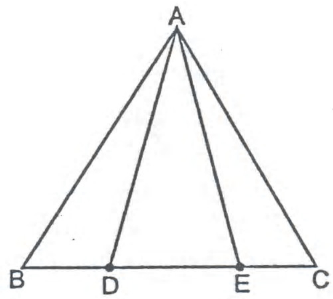
(ii) DCYX is a trapezium

(iii) Area (trapezium DCYX) = $\frac{9}{11}$ Area (trapezium XYBA)



13. Show that a median of a triangle divides it into two triangles of equal area.

14. In the figure, given alongside, D and E are two points on BC such that BD = DE = EC. Prove that : ar(ABD) = ar(ADE) = ar(AEC)



15. In triangle ABC, if a point D divides BC in the ratio 2 : 5, show that : ar(ΔABD) : ar(ΔACD) = 2 : 5.

ANSWER KEY

(Objective DPP # 13.1)

Qus.	1	2	3	4	5	6	7	8	9	10
Ans.	C	C	D	D	A	A	A	A	D	C

(Subjective DPP # 13.2)

3. 40 cm^2

6. 192 cm^2

7. 56 cm^2

8. (i) 114 cm^2 (ii) 195 cm^2

9. 40 cm^2

11. 150 cm^2

CIRCLE

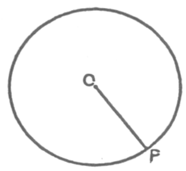
ML - 14

DEFINITIONS

(A) Circle :

The collection of all the points in a plane, which are at a fixed distance from a fixed point in the plane, is called a circle.

The fixed point is called the centre of the circle and the fixed distance is called the radius of the circle.



In figure, O is the centre and the length OP is the radius of the circle. So the line segment joining the centre and any point on the circle is called a radius of the circle.

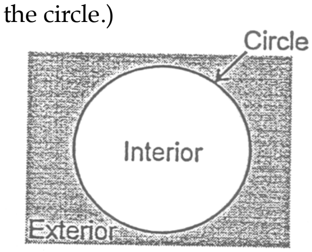
(b) Interior and Exterior of a Circle :

A circle divides the plane on which it lies into three parts. They are

(i) inside the circle (or interior of the circle)

(ii) the circle nd

(iii) outside the circle (or exterior of the circle.)



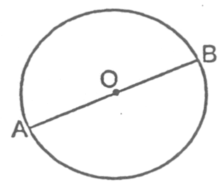
The circle and its interior make up the **circular region.**

(c) Chord :

If we take two points P and Q on a circle, then the line segment PQ is called a chord of the circle.

(d) Diameter:

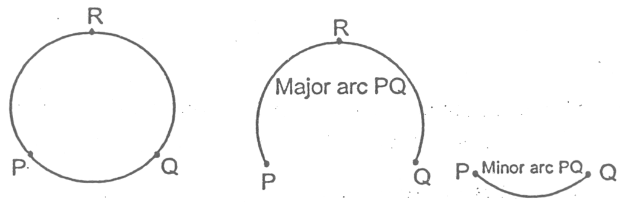
The chord which passes through the centre of the circle, is called **a diameter** of the circle.



A diameter is the longest chord and all diameter have the same length, which is equal to two times the radius. In figure, AOB is a diameter of circle.

(e) Arc :

A piece of a circle between two points is called an arc. If we look at the pieces of the circle between two points P and Q in figure, we find that there are two pieces, one longer and the other smaller. The longer one is called the major arc PQ and the shorter one is called the minor arc PQ. The minor arc PQ is also denoted by PQ and the major arc PQ by PRQ, where R is some point on the arc between P and Q. Unless otherwise states, arc PQ or PQ stands for minor arc PQ. When P and Q are ends of a diameter, then both arcs are equal and each is called a semi circle.

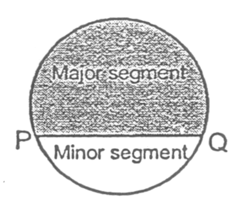


(f) Circumference:

The length of the complete circle is called its **circumference.**

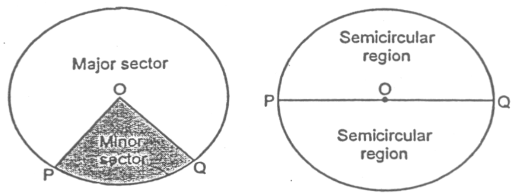
(g) Segment :

The region between a chord and either of its arcs is called a **segment** of the circular region or simply a segment of the circle. There are two types of segments also, which are the major segment and the minor segment (as in figure).

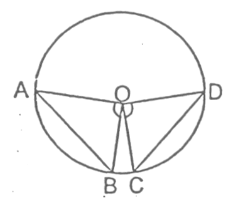


(h) Sector :

The region between an arc and the two radii, joining the centre to the end points of the arc is called a sector. Like segments, we find that the minor arc corresponds to the minor sector and the major arc corresponds to the major sector. In figure, the region OPQ in the minor sector and the remaining part of the circular region is the major sector. When two arcs are equal, then both segments and both sectors become the same and each is known as a semicircular region.



Theorem-1 : Equal chords of a circle subtend equal angles at the centre.



Given : AB and CD are the two equal chords of a circle with centre O.

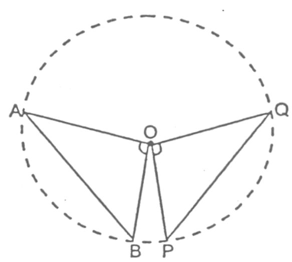
To Prove : ∠AOB = ∠COD.

Proof : In ΔAOB and ΔCOD,

OA = OC	[Radii of a circle]
OB = OD	[Radii of a circle]
AB = CD	[Given]
∴ ΔAOB ≅ ΔCOD	[By SSS]
∴ ∠AO B = ∠COD.	[By cpctc]

Converse of above Theorem :

In the angles subtended by the chords of a circle at the centre are equal, then the chords are equal.



Given : ∠AOB and ∠POQ are two equal angles subtended by chords AB and PQ of a circle at its centre O.

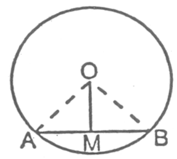
To Prove : AB = PQ

Proof : In ΔAOB and ΔPOQ,

OA = OP	[Radii of a circle]
OB = OQ	[Radii of a circle]
∠AOB = ∠POQ	[Given]
∴ ΔAOB ≅ ΔPOQ	[By SAS]

∴ AB = PQ [By cpctc] **Hence Proved.**

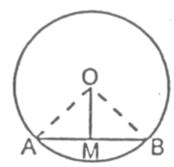
Theorem-2 : The perpendicular from the centre of a circle to a chord bisects the chord.



Given : A circle with centre O. AB is a chord of this circle. OM ⊥ AB.
To Prove : MA = MB.
Construction : Join OA and OB.
Proof : In right triangles OMA and OMB,

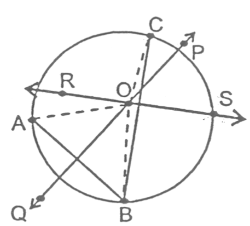
OA = OB	[Radii of a circle]
OM = OM	[Common]
∠OMA = ∠OMB	[90^0 each]
∴ ΔOMA ≅ ΔOMB	[By RHS]
∴ MA = MB	[By cpctc] **Hence Proved.**

Converse of above Theorem :
The line drawn through the centre of a circle to bisect a chord a perpendicular to the chord.
Given : A circle with centre O. AB is a chord of this circle whose mid-point is M.
To Prove : OM ⊥ AB.
Construction : Join OA and OB.
Proof : In ΔOMA and ΔOMB.



MA = MB	[Given]
OM = OM	[Common]
OA = OB	[Radii of a circle]
∴ ΔOMA ≅ ΔOMB	[By SSS]
∴ ∠AMO = ∠BMO	[By cpctc]
But ∠AMO + ∠BMO = 180^0	[Linear pair axiom]
∴ ∠AMO = ∠BMO = 90^0	
⇒ OM ⊥ AB.	

Theorem-3 : There is one and only one circle passing through three given non-collinear points.
Proof : Let us take three points A, B and C, which are not on the same line, or in other words, they are not collinear [as in figure]. Draw perpendicular bisectors of AB and BC say, PQ and RS respectively. Let these perpendicular bistros intersect at one point O. (Note that PQ and RS will intersect because they are not parallel) [as in figure].



∴ O lies on the perpendicular bisector PQ of AB.

∴ OA = OB

[∵ Every point on the perpendicular bisector of a line segment is equidistant from its end points]

Similarly,

∴ O lies on the perpendicular bisector RS of BC.

∴ OB = OC

[∵ Every point on the perpendicular bisector of a line segment is equidistant from its end points]

So, OA = OB = OC

i.e., the points A, B and C are at equal distances from the point O.

So, if we draw a circle with centre O and radius OA it will also pass through B and C. This shows that there is a circle passing through the three points A, B and C. We know that two lines (perpendicular bisectors) can intersect at only one point, so we can draw only one circle with radius OA. In other words, there is a unique circle passing through A, B and C. **Hence Proved.**

REMARK :

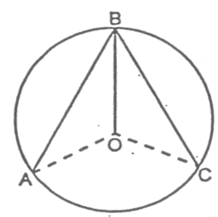
If ABC is a triangle, then by above theorem, there is a unique circle passing through the three vertices A, B and C of the triangle. This circle the circumcircle of the ΔABC. Its centre and radius are called respectively the circumcentre and the circumradius of the triangle.

Ex.1 In figure, AB = CB and O is the centre of the circle. Prove that BO bisects ∠ABC.

Sol. **Given :** In figure, AB = CB and O is the centre of the circle.

To Prove : BO bisects ∠ABC.

Construction : Join OA and OC.



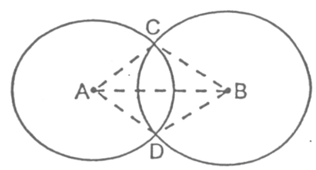
Proof : In ΔOAB and ΔOCB,

OA = OC	[Radii of the same circle]
AB = CB	[Given]
OB = OB	[Common]
∴ ΔOAB ≅ ΔOCB	[By SSS]
∴ ∠ABO = ∠CBO	[By cpctc]

⇒ BO bisects ∠ABC. **Hence Proved.**

Ex.2 Two circles with centres A and B intersect at C and D. Prove that ∠ACB = ∠ADB.

Sol. **Given :** Two circles with centres A and B intersect at C and D.



To Prove : ∠ACB = ∠ADB.

Construction : Join AC, AD, BC, BD and AB.

Proof : In ΔACB an ΔADB,

AC = AD	[Radii of the same circle]
BC = BD	[Radii of the same circle]
AB = AB	[Common]
∴ ΔACB ≅ ΔADB	[By SSS]
∴ ∠ACB = ∠ADB.	[By cpctc]

Hence Proved.

E.3 In figure, AB ≅ ACand O is the centre of the circle. Prove that OA is the perpendicular bisector of BC.

Sol. **Given :** In figure, AB ≅ AC and O is the centre of the circle.

To Prove : OA is the perpendicular bisector of BC.

Construction : Join OB and OC.

Proof :

∴ AB ≅ AC **[Given]**

∴ **chord AB = chord AC.**

[∵ If two arcs of a circle are congruent, then their corresponding chords are equal.]

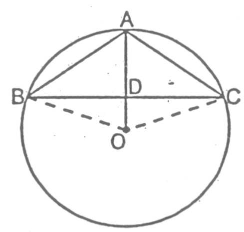
∴ ∠AOB = ∠AOC ....(i) [∵ Equal chords of a circle subtend equal angles at the centre]

In ΔOBC and ΔOCD,

∠DOB = ∠DOC [From (1)]

OB = OC [Radii of the same circle]

OD = OD [Common]



∴ ΔOBD ≅ ΔOCD [By SAS]

∴ ∠ODB = ∠ODC ....(ii) [By cpctc]

And BD = CD ...(ii) [By cpctc]

But ∠BDC = 180^0

∴ ∠ODB + ∠ODC = 180^0

⇒ ∠ODB + ∠ODB = 180^0 [From equation (ii)]

⇒ 2∠ODB = 180^0

⇒ ∠ODB = 90^0

∴ ∠ODB = ∠ODC = 90^0 ....(iv) [From (ii)]

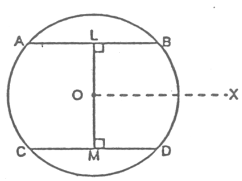
So, by (iii) and (iv), OA is the perpendicular bisector of BC. **Hence Proved.**

Ex.4 Prove that the line joining the mid-points of the two parallel chords of a circle passes through the centre of the circle.

Sol. Let AB and CD be two parallel chords of a circle whose centre is O.

Let l and M be the mid-points of the chords AB and CD respectively. Join PL and OM.

Draw OX ∥ AB or CD.



∴ L is the mid-point of the chord AB and O is the centre of the circle

∴ ∠OLB = 90^0

[∵ The perpendicular drawn from the centre of a circle to chord bisects the chord]

But, OX ∥ AB

∴ ∠LOX = 90^0......(i)

[∵ Sum of the consecutive interior angles on the same side of a transversal is 180^0]

∴ M is the mid-point of the chord CD and O is the centre of the circle.

∴ ∠OMD = 90^0

[∵ The perpendicular drawn from the centre of a circle to a chord bisects the chord]

But OX ∥ CD ....(ii)

[$\because$ Sum of the consecutive interior angles on the same side of a transversal is 180^0]

$\therefore$ $\angle MOX = 90^0$

From above equations, we get

$\angle LOX + \angle MOX = 90^0 + 90^0 = 180^0$

$\Rightarrow$ $\angle LOM = 180^0$

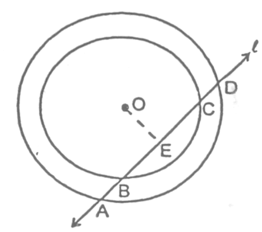
$\Rightarrow$ LM is a straight line passing through the centre of the circle. **Hence Proved.**

Ex.5 ℓ is a line which intersects two concentric circle (i.e., circles with the same centre) with common centre O at A, B, C and D (as in figure). Prove that AB = CD.

Sol. **Given :** ℓ is a line which intersects two concentric circles (i.e., circles with the same centre) with common centre O at A, B, C and D.

To Prove : AB = CD.

Construction : Draw OE $\perp \ell$



Proof :

$\therefore$ The perpendicular drawn from the centre of a circle to a chord bisects the chord

$\therefore$ AE = ED .....(i)

And BE = EC ....(ii)

Subtracting (ii) from (i), we get

AE - BE = ED - EC

$\Rightarrow$ AB = CD. **Hence Proved.**

Ex.6 PQ and RS are two parallel chords of a circle whose centre is O and radius is 10 cm. If PQ = 16 cm and RS = 12 cm, find the distance between PQ and RS, if they lie.

(i) on the same side of the centre O.

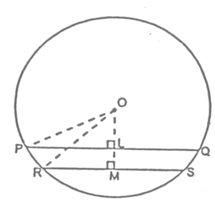
(ii) on opposite sides of the centre O.

Sol. (i) Draw the perpendicular bisectors OL and OM of PQ and RS respectively.

$\therefore$ PQ $\parallel$ RS

$\therefore$ OL and OM are in the same line.

$\Rightarrow$ O, L and M are collinear.



Join OP and OR.

In right triangle OLP,

$OP^2 = OL^2 + PL^2$ [By Pythagoras Theorem]

$\Rightarrow$ $(10)^2 = OL^2 + \left(\frac{1}{2}\times pq\right)^2$

[$\therefore$ The perpendicular drawn from the centre of a circle to a chord bisects the chord]

$\Rightarrow$ $100 = OL^2 + \left(\frac{1}{2}\times 16\right)^2$

$\Rightarrow$ $100 = OL^2 + (8)^2$

$\Rightarrow$ $100 = OL^2 + 64$

$\Rightarrow$ $OL^2 = 100 - 64$

$\Rightarrow$ $OL^2 = 36 = (6)^2$

$\Rightarrow$ OL = 6 cm

In right triangle OMR,

$OR^2 = OM^2 + RM^2$ **[By Pythagoras Theorem]**

$\Rightarrow$ $OR^2 = OM^2 + \left(\frac{1}{2}\times RS\right)^2$

[$\because$ The perpendicular drawn from the centre of a circle to a chord bisects the chord]

$\Rightarrow$ $(10)^2 = OM^2 + \left(\frac{1}{2}\times 12\right)^2$

$\Rightarrow$ $(10)^2 = OM^2 + (6)^2$

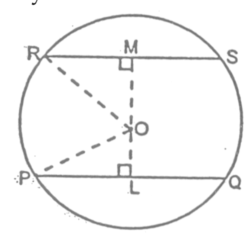
$\Rightarrow$ $OM^2 = (10)^2 - (6)^2 = (10 - 6)(10 + 6) = (4)(16) = 64 = (8)^2$

$\Rightarrow$ OM = 8 cm

$\therefore$ LM = OM - OL = 8 - 6 = 2 cm

Hence, the distance between PQ and RS, if they lie on he same side of the centre O, is 2 cm.

(ii) Draw the perpendicular bisectors OL and OM of PQ and RS respectively.



$\therefore$ PQ $\parallel$ RS

$\therefore$ OL and OM are in the same line

$\Rightarrow$ L, O and M are collinear.

Join OP nd OR.

In right triangle OLP,

$OP^2 = OL^2 + PL^2$ [By Pythagoras Theorem]

$\Rightarrow$ $OP^2 = OL^2 + \left(\frac{1}{2}\times pQ\right)^2$

[$\because$ The perpendicular drawn from the centre of a circle to a chord bisects the chord]

$\Rightarrow$ $(10))^2 = OL^2 + \left(\frac{1}{2}\times 16\right)^2$

$\Rightarrow$ $100 = OL^2 + (8)^2$

$\Rightarrow$ $100 = OL^2 + 64$

$\Rightarrow$ $OL^2 = 100 - 64$

$\Rightarrow$ $OL^2 = 36 = (6)^2$

$\Rightarrow$ OL = 6 cm

In right triangle OMR,

$OR^2 = OM^2 + RM^2$ [By Pythagoras Theorem]

$\Rightarrow$ $OR^2 = OM^2 + \left(\frac{1}{2}\times 12\right)^2$

[$\because$ The perpendicular drawn from the centre of a circle to a chord bisects the chord]

$\Rightarrow$ $(10)^2 = OM^2 + \left(\frac{1}{2}\times RS\right)^2$

$\Rightarrow$ $(10)^2 = OM^2 + (6)^2$

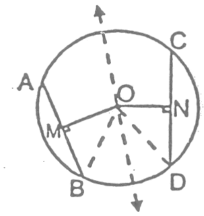
$\Rightarrow$ $OM^2 = (10)^2 - (6)^2 = (10 - 6)(10 + 6) = (4)(16) = 64 = (8)^2$

$\Rightarrow$ OM = 8 cm

$\therefore$ LM = OL + OM = 6 + 8 = 14 cm

Hence, the distance between PQ and RS, if they lie on the opposite side of the centre O, is 14 cm.

Theorem-4 : Equal chords of a circle (or of congruent circles) are equidistant from the centre (or centres).



Given : A circle have two equal chords AB & CD. .e. AB = CD and OM $\perp$ AB, ON $\perp$ CD

To Prove : OM = ON

Construction : Join OB & OD

Proof : AB = CD (Given)

[$\because$ The perpendicular drawn from the centre of a circle to bisect the chord.]

$\therefore$ $\frac{1}{2}AB = \frac{1}{2}CD$

$\Rightarrow$ BM = DN

In ΔOMB & ΔOND

$\angle OMB = \angle OND = 90^0$	[Given]
OB = OD	[Radii of same circle]
Side BM = Side DN	[Proved above]
$\therefore$ $\Delta OMB \cong \Delta OND$	[By R.H.S.]
$\therefore$ OM = ON	[By cpctc] **Hence Proved.**

REMARK :

Chords equidistant from the centre of a circle are equal in length.

Ex.7 AB and CD are equal chords of a circle whose centre is O. When produced, these chords meet at E. Prove that EB = ED.

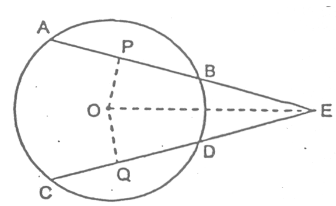
Sol. **Given :** AB and CD are equal chords of a circle whose centre is O. When produced, these chords meet at E.

To Prove : EB = ED.

Construction : From O draw OP $\perp$ AB and OQ $\perp$ CD. Join OE.

Proof : $\therefore$ AB = CD	[Given]
$\therefore$ OP = OQ	[$\therefore$ Equal chords of a circle are equidistant from the centre]

Now in right tingles OPE and OQE,



OE = OE	[Common]
Side OP = Side OQ	[Proved above]
$\therefore$ $\Delta OPE \cong \Delta OQE$	[By RHS]
$\therefore$ OE = QE	[By cpctc]
$\Rightarrow$ PE - $\frac{1}{2}$AB = QE - $\frac{1}{2}$CD	[$\therefore$ AB = CD (Given)]

$\Rightarrow$ PE - PB = QE - QD

$\Rightarrow$ EB = ED. **Hence Proved.**

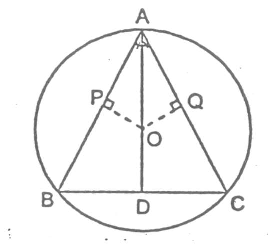
Ex.8 Bisector AD of ∠BAC of ΔABC passed through the centre O of the circumcircle of ΔABC. Prove that AB = AC.

Sol. **Given :** Bisector AD of ∠BAC of ΔABC passed through the centre O of the circumcircle of ΔABC,

To Prove : AB = AC.

Construction : Draw OP ⊥ AB and OQ ⊥ AC.

Proof :



In ΔAPO and ΔAQO,

∠OPA = ∠OQA [Each = 90^0 (by construction)]

∠OAP = ∠OAQ [Given]

OA = OA [Common]

∴ ΔAPO ≅ ΔAQO [By ASS cong. prog.]

∴ OP = OQ [By cpctc]

∴ AB = AC. [∵ Chords equidistant from the centre are equal] **Hence Proved.**

Ex.9 AB and CD are the chords of a circle whose centre is O. They intersect each other at P. If PO be the bisector of ∠APD, prove that AB = CD.

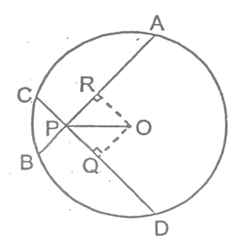
OR

In the given figure, O is the centre of the circle and PO bisect the angle APD. prove that AB = CD.

Sol. **Given :** AB and CD are the chords of a circle whose centre is O. They interest each other at P. PO is the bisector of ∠APD.

To Prove : AB = CD.

Construction : Draw OR ⊥ AB and OQ ⊥ CD.



Proof : In ΔOPR and ΔOPQ,

∠OPR = ∠OPQ [Given]

OP = OP [Common]

And ∠ORP = ∠OQP [Each = 90^0]

∴ ΔORP ≅ ΔOPQ [By AAS]

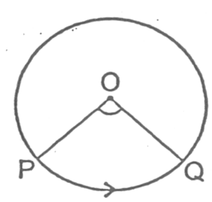
∴ OR = OQ [By cpctc]

∴ AB = CD [∵ Chords of a circle which are equidistant from the centre are equal]

REMARK :

Angle Subtended by an Arc of a Circle :

In figure, the angle subtended by the minor arc PQ at O is ∠POQ and the angle subtended by the major arc PQ at O is reflex angle ∠POQ.



EXERCISE

OBJECTIVE DPP # 14.1

1. If two circular wheels rotate on a horizontal road then locus of their centres will be

(A) Circles (B) Rectangle (C) Two straight line (D) Parallelogram

2. In a plane locus of a centre of circle of radius r, which passes through a fixed point

(A) rectangle (B) A circle (C) A straight line (D) Two straight line

3. In a circle of radius 10 cm, the length of chord whose distance is 6 cm from the centre is

(A) 4 cm (B) 5 cm (C) 8 cm (D) 16 cm

4. If a chord a length 8 cm is situated at a distance of 3 cm form centre, then the diameter of circle is :

(A) 11 cm (B) 10 m (C) 12 cm (D) 15 cm

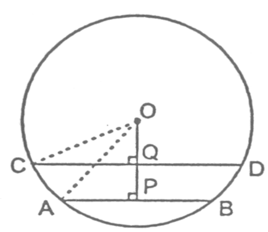
5. In a circle the lengths of chords which are situated at a equal distance from centre are :

(A) double (B) four times (C) equal (D) three times

SUBJECTIVE DPP # 14.2

1. The radius of a circle is 13 cm and the length of one of its chords is 10 cm . Find the distance of the chord from the centre.

2. Show is the figure, O is the centre of the circle of radius 5 cm. OP $\perp$ AB, OQ $\Re$ CD, AB $\parallel$ CD, AB = 6 cm and CD = 8 cm. Determine PQ.



3. AB and CD are two parallel chords of a circle such that AB = 10 cm and CD 24 cm. If the chords are on the opposite side of the centre and the distance between is 17 cm, Find the radius of the circle.

4. In a circle of radius 5 cm, AB and AC are two chords such that AB = AC = 6 cm. Find the length of the chord BC.

5. AB and CD are two parallel chords of a circle whose diameter is AC. Prove that AB = CD.

6. Two circles of radii 10 cm and 8 cm interest and the length of the common chord is 12 cm. Find the distance between their centries.

7. Two circles with centre A and B and of radii 5 cm and 3 cm touch each other internally. If the perpendicular bisector of segment AB meet the bibber circle is P and Q, find the length of PQ.

➤➤➤ CIRCLE ⮜⮜⮜

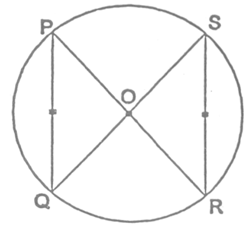
ML - 15

SOME IMPORTANT THEOREMS

Theorem-1 : Equal chords of a circle subtend equal angles at the centre.

Given : A circle with centre O in which chord PQ = chord RS.

To Prove : ∠POQ = ∠ROS.



Proof : In ΔPOQ and ΔROS,

OP = OR [Radii of the same circle]

OQ = OS [Radii of the same circle]

PW = RS [Given]

⇒ ΔPOQ = ΔROS [By SSS]

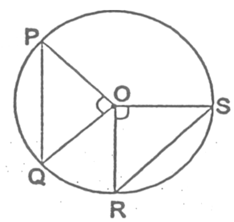
⇒ ∠POQ = ∠ROS [By cpctc] **Hence Proved.**

Theorem-2 : If the angles subtended by the chords at the centre (of a circle) are equal then the chords are equal.

Given : A circle with centre O . Chords PQ and RS subtend equal angles at the enter of the circle.

i.e. ∠POQ = ∠ROS

To Prove : Chord PQ = chord RS.



Proof : In ΔPOQ and ΔROS,

∠POQ = ∠ROS [Given]

OP = OR [Radii of the same circle]

OQ = OS [Radii of the same circle]

⇒ ΔPOQ ≅ ΔROS [By SSS]

⇒ chord PQ = chord RS [By cpctc] **Hence Proved.**

Corollary-1 : Two arc of a circle are congruent, if the angles subtended by them at the centre are equal.

Corollary 2 : If two arcs of a circle are equal, they subtend equal angles at the centre.

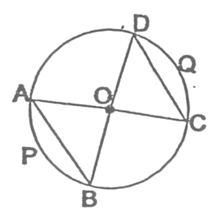
Corollary 3 : If two arc of a circle are congruent (equal), their corresponding chords are equal.

Corollary 4: If two chords of a circle are equal, their corresponding arc are also equal.

∠AOB = ∠COD

∴ Chord AB = Chord CD

∴ Arc APB = Arc COD.

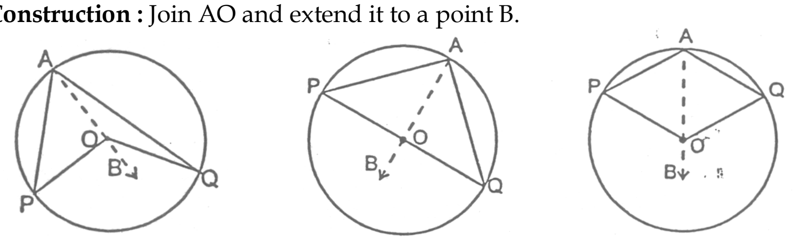


Theorem-3 : The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

Given : An arc PQ of a circle subtending angles POQ at the centre O and PAQ at a point A on the remaining part of the circle.

To Prove : ∠POQ = 2∠PAQ.

Construction : Join AO and extend it to a point B.



(A) (B) (C)

Proof : There arises three cases :

(A) are PQ is minor

(B) arc PQ s a semi - circle

(C) arc PQ is major.

In all the cases,

∠BOQ = ∠OAQ + ∠AQO ....(i)

[∵ An exterior angle of triangle is equal to the sum of the two interior opposite angles]

In OAQ,

OA = OQ [Radii of a circle]

∴ ∠OAQ = ∠OQA ...(ii) [Angles opposite equal of a triangle are equal]

(i) and (ii), give,

∠BOQ = 2∠OAQ ....(iii)

Similarly,

∠BOP = 2∠OAP ....(iv)

Adding (iii) and (iv), we get

∠BOP + ∠BOQ = 2(∠OAP + ∠OAQ)

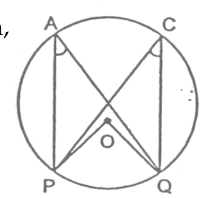
⇒ ∠POQ = 2∠PA. ....(v)

NOTE : For the case (C), where PQ is the major arc, (v) is replaced by reflex angles.

Thus, ∠POQ = 2∠PAQ.

Theorem- 4 : Angles in the same segment of a circle are equal.

Proof : Let P and Q be any two points on a circle to form a chord PQ, A and C any other points on the remaining part of the circle and O be the centre of the circle. Then,



∠POQ = 2∠PAQ ...(i)

And ∠POQ = 2∠PCQ ...(ii)

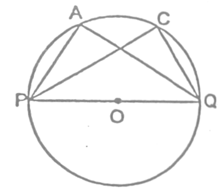
From above equations, we get

2∠PAQ = 2∠PCQ

⇒ ∠PAQ = ∠PCQ **Hence Proved**

Theorem-5 : Angle in the semicircle is a right angle.

Proof : ∠PAQ is an angle in the segment, which is a semicircle.



∴ $\angle PAQ = \frac{1}{2}\angle PAO = \frac{1}{2} \times 180^0 = 90^0$

[∴ ∠PQR is straight line angle or $\angle PQR = 180^0$]

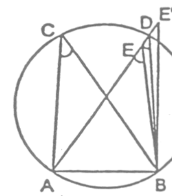
If we take any other point C on the semicircle, then again we get

$\angle PCQ = \frac{1}{2}\angle POQ = \frac{1}{2} \times 180^0 = 90^0$ **Hence Proved.**

Theorem-6: If a line segment joining two points subtend equal angles at two other points lying on the same side of the lien containing the line segment the four points lie on a circle (i.e., they are concyclic).

Given : AB is a line segment, which subtends equal angles at two points C and D. i.e., ∠ACB = ∠ADB.

To Prove : The points A, B, C and D lie on a circle.



Proof : Let us draw a circle through the points A, C and B.

Suppose it does not pass through the point D.

Then it will intersect AD (or extended AD) at a point, say E (or E′).

If points A,C,E and B lie on a circle,

∠ACD = ∠AEB [∴ Angles in the same segment of circle are equal]

But it is given that ∠ACB = ∠ADB

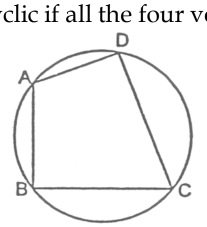
Therefore, ∠AEB = ∠ADB

This is possible only when E coincides with D. [As otherwise ∠AEB > ∠ADB]

Similarly, E′ should also coincide with D. So A, B, C and D are concyclic **Hence Proved.**

CYCLIC QUADRILATERAL

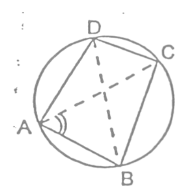
A quadrilateral ABCD is called cyclic if all the four vertices of it lie on a circle.



Theorem-7 : The sum of either pair of opposite angles of a cyclic quadrilateral is 180^0

Given : A cyclic quadrilateral ABCD.

To Prove : $\angle A + \angle C = \angle B + \angle D = 180^0$



Construction : Join AC and BD.

Proof : ∠ACB = ∠ADB [Angles of same segment]

And ∠BAC = ∠BDC [Angles of same segment]

∴ ∠ACB + ∠BAC = ∠ADB + ∠BDC = ∠ADC.

Adding ∠ABC to both sides, we get

∠ACB + ∠BAC + ∠ABC = ∠ADC + ∠ABC.

The left side being the sum of three angles of ΔABC is equal to 180^0.

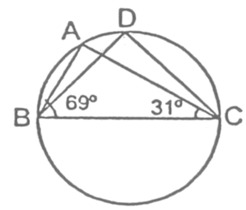
∴ $\angle ADC + \angle ABC = 180^0$

i.e., $\angle D + \angle B = 180^0$

∴ $\angle A + \angle C = 360^0 - (\angle B + \angle D) = 180^0$ [∴ $\angle A + \angle B + \angle C + \angle D = 360^0$] **Hence Proved.**

Corollary : If the sum of a pair of opposite angles of a quadrilateral is 180^0, then quadrilateral is cyclic.

Ex.1 In figure, $\angle ABC = 69^0$, $\angle ACB = 31^0$, find $\angle BDC$.



Sol. In ΔABC.

$\angle BAC + \angle ABC + \angle ACB = 180^0$

[Sum of all the angles of a triangle is 180^0]

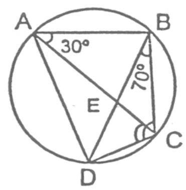
$\Rightarrow \angle BAC + 69^0 + 31^0 = 180^0$

$\Rightarrow \angle BAC + 100^0 = 180^0$

$\Rightarrow \angle BAC = 180^0 - 100^0 = 80^0$

Now, $\angle BDC = \angle BAC = 80^0$. **Ans.** [Angles in the same segment of a circle are equal]

Ex.2 ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If $\angle DBC = 70^0$, $\angle BAC$ is 30^0, find $\angle BCD$. Further, if B = BC, find $\angle ECD$.



Sol. $\angle CDB = \angle BAC = 30^0$ ...(i) [Angles in the same segment of a circle are equal]

$\angle DBC = 70^0$ ....(ii)

In ΔBCD,

$\angle BCD + \angle DBC + \angle CDB = 180^0$ [Sum of all he angles of a triangle is 180^0]

$\Rightarrow \angle BCD + 70^0 + 30.0 = 180^0$ [Using (i) and (ii)

$\Rightarrow \angle BCD + 100^0 = 180^0$

$\Rightarrow \angle BCD = 180^0 - 100^0$

$\Rightarrow \angle BCD = 80^0$ ...(iii)

In ΔABC,

AB = BC

$\therefore \angle BCA = \angle BAC = 30^0$ ...(iv) [Angles opposite to equal sides of a triangle are equal]

Now, $\angle BCD = 80^0$ [From (iii)]

$\Rightarrow \angle BCA + \angle ECD = 80^0$

$\Rightarrow 30^0 + \angle ECD = 80^0$

$\Rightarrow \angle ECD = 80^0 - 30^0$

$\Rightarrow \angle ECD = 50^0$

Ex.3 If the nonparallel side of a trapezium are equal, prove that it is cyclic.

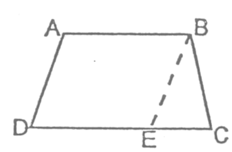
Sol. **Given :** ABCD is a trapezium whose two non-parallel sides AB and BC are equal.

To Prove : Trapezium ABCD is a cyclic.

Construction : Draw BE $\parallel$ AD.

Proof : $\therefore$ AB $\parallel$ DE [Given]

AD $\parallel$ BE [By construction]



$\therefore$ Quadrilateral ABCD is a parallelogram.

$\therefore \angle BAD = \angle BED$ ....(i) [Opp. angles of a $\parallel^{gm}$]

And, AD = BE ....(ii) [Opp. sides of a $\parallel^{gm}$]

But AD = BC ...(iii) [Given]

From (ii) and (iii),

BE = BC

∴ $\angle BEC = \angle BCE$ ....(iv) [Angles opposite to equal sides]

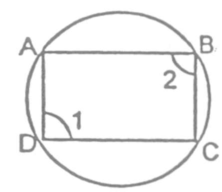
$\angle BEC + \angle BED = 180^0$ [Linear Pair Axiom]

⇒ $\angle BCE + \angle BAD = 180^0$ [From (iv) and (i)]

⇒ Trapezium ABCD is cyclic.

[∴ If a pair of opposite angles of a quadrilateral 180^0, then the quadrilateral is cyclic] **Hence Proved.**

Ex.4 Prove that a cyclic parallelogram is a rectangle.



Sol. **Given :** ABCD is a cyclic parallelogram.

To Prove : ABCD is a rectangle.

Proof : ∴ ABCD is a cyclic quadrilateral

∴ $\angle 1 + \angle 2 = 180^0$ ....(i)

[∴ Opposite angles of a cyclic quadrilateral are supplementary]

∴ ABCD is a parallelogram

∴ $\angle 1 = \angle 2$ ...(ii) [Opp. angles of a ∥ gm]

From (i) and (ii),

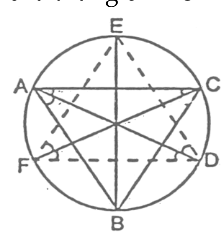
$\angle 1 = \angle 2 = 90^0$

∴ $\|^{gm}$ ABCD is a rectangle. **Hence Proved.**

Ex.5 Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F respectively.

Prove that the angles of the triangle DEF are $90^0 - \frac{1}{2}A, 90^0 - \frac{1}{2}B$ and $90^0 - \frac{\angle C}{2}$.

Sol. **Given :** Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F respectively.



To Prove : The angles of the ΔDEF are $90^0 - \frac{\angle A}{2}, 90^0 - \frac{\angle B}{2}$ and $90^0 - \frac{C}{2}$ respectively.

Construction : Join DE, EF and FD.

Proof : $\angle FDE = \angle FDA + \angle EDA = \angle FCA + \angle EBA$ [∴ Angles in the same segment are equal]

$= \frac{1}{2}\angle C + \frac{1}{2}\angle B$

⇒ $\angle D = \frac{\angle C + \angle B}{2} = \frac{180^0 - \angle A}{2}$ [∴ In ΔABC, $\angle A + \angle B + \angle C = 180^0$]

⇒ $\angle D = 90^0 - \frac{\angle A}{2}$

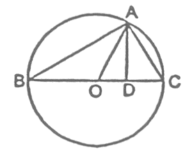
Similarly, we can show that

$\angle E = 90^0 - \frac{\angle B}{2}$

And $\angle F = 90^0 - \frac{\angle C}{2}$. **Hence Proved.**

Ex.6 Find the area of a triangle, the radius of whose circumcircle is 3 cm and the length of the altitude drawn from the opposite vertex to the hypotenuse is 2 cm.

Sol. We know that the hypotenuse of a right angled triangle is the diameter of its circumcircle.



$\therefore$ BC = 2OB = 2 × 3 = 6 cm

Let, AD $\perp$ BC

AD = 2 cm [Given]

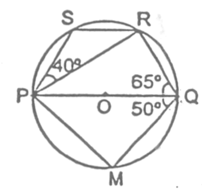
$\therefore$ Area of ΔABC = $\frac{1}{2}$(BC)(AD)

$= \frac{1}{2}(6)(2)$

= 6 cm^2. **Ans.**

Ex.7 In figure, PQ is a diameter of a circle with centre O. IF $\angle$PQR = 65^0, $\angle$SPR = 40^0, $\angle$PQ M = 50^0, find $\angle$QPR, $\angle$PRS and $\angle$QPM.

Sol. (i) $\angle$QPR



$\therefore$ PQ is a diameter

$\therefore$ $\angle$PRQ = 90^0 [Angle in a semi-circle is 90^0]

In ΔPQR,

$\angle$QPR + $\angle$PRQ + $\angle$PQR = 180^0 [Angle Sum Property of a triangle]

$\Rightarrow$ $\angle$QPR + 90^0 + 65^0 = 180^0

$\Rightarrow$ $\angle$QPR + 155^0 = 180^0

$\Rightarrow$ $\angle$QPR = 180^0 - 155^0

$\Rightarrow$ $\angle$QPR = 25^0.

(ii) $\angle$PRS

$\therefore$ PQRS is a cyclic quadrilateral

$\therefore$ $\angle$PSR + $\angle$PQR = 180^0 [$\therefore$ Opposite angles of a cyclic quadrilateral are supplementary]

$\Rightarrow$ $\angle$PSR + 65^0 = 180^0

$\Rightarrow$ $\angle$PSR = 180^0 - 65^0

$\Rightarrow$ $\angle$PSR = 115^0

In ΔPSR,

$\angle$PSR + $\angle$SPR + $\angle$PRS = 180^0 [Angles Sum Property of a triangle]

$\Rightarrow$ 115^0 + 40^0 + $\angle$PRS = 180^0

$\Rightarrow$ 115^0 + $\angle$PRS = 180^0

$\Rightarrow$ $\angle$PRS = 180^0 - 155^0

$\Rightarrow$ $\angle$PRS = 25^0

(iii) $\angle$QPM

$\therefore$ PQ is a diameter

$\therefore$ $\angle$PMQ = 90^0 [$\because$ Angle in a semi - circle is 90^0]

In ΔPMQ,

$\angle$PMQ + $\angle$PQM + $\angle$QPM = 180^0 [Angle sum Property of a triangle]

$\Rightarrow$ 90^0 + 50^0 + $\angle$QPM = 180^0

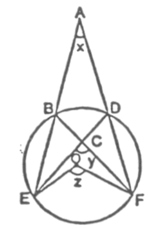
$\Rightarrow$ 140^0 + $\angle$QPM = 180^0

$\Rightarrow$ $\angle$QPM = 180^0 - 140^0

$\Rightarrow$ $\angle$QPM = 40^0.

Ex.8 In figure, O is the centre of the circle. Prove that $\angle x + \angle y = \angle z$.

Sol. $\angle EBF = \frac{1}{2}\angle EOF = \frac{1}{2}\angle z$ [∵ Angle subtended by an arc of a circle at the centre in twice the angle subtended by it at any point of the remaining part of the circle]



∴ $\angle ABF = 180^0 - \frac{1}{2}\angle z$ ...(i) [Linear Pair Axiom]

$\angle EDF = \frac{1}{2}\angle EOF = \frac{1}{2}\angle z$

[∵ Angle subtend by any arc of a circle at the centre is twice the angle subtended by it at any point of the remaining part of the circle]

∴ $\angle ADE = 180^0 - \frac{1}{2}\angle z$ ....(ii) [Linear Pair Axiom]

$\angle BCD = \angle ECF = \angle y$ [Vert. Opp. Angle]

$\angle BAD = \angle x$

In quadrilateral ABCD

$\angle ABC + \angle BCD + \angle CDA + \angle BAD = 2 \times 180^0$ [Angle Sum Property of a quadrilateral]

$\Rightarrow 180^0 - \frac{1}{2}\angle z + \angle y + 180^0 - \frac{1}{2}\angle z + \angle x = 2 \times 180^0$

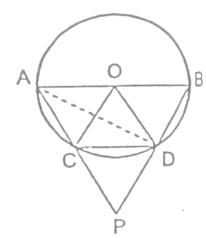
$\Rightarrow \angle x + \angle y = \angle z$ **Hence Proved.**

Ex.9 AB is a diameter of the circle with centre O and chord CD is equal to radius OC, AC and BD produced meet at P. Prove that $\angle CPD = 60^0$.

Sol. **Given :** AB is a diameter of the circle with centre O and chord CD is equal to radius OC. AC and BD produced meet at P.

To Prove : $\angle CPD = 60^0$

Construction : Join AD.



Proof : In ΔOCD,

OC = OD ...(i) [Radii of the same circle]

OC = CD ....(ii) [Given]

From (i) and (ii),

OC = OD = CD

∴ ΔOCD is equilateral

∴ $\angle COD = 60^0$

∴ $\angle CAD = \frac{1}{2}\angle COD = \frac{1}{2}\angle(60^0) = 30^0$

[∵ Angle subtended by any arc of a circle at the centre is twice the angle subtended by it at any point of the reaming part of the circle]

$\Rightarrow \angle PAD = 30^0$ .....(iii)

And, $\angle ADB = 90^0$ .....(iv) [Angle in a semi-circle]

$\Rightarrow \angle ADB + \angle ADP = 180^0$ [Linear Pair Axiom]

$\Rightarrow 90^0 + \angle ADP = 180^0$ [From (iv)]

$\Rightarrow \angle ADP = 90^0$ ....(v)

In ΔDP,

$\angle ADP + \angle PAD + \angle ADP = 180^0$ [∵ The sum of the three angles of a triangles is 180^0]

⇒ $\angle APD + 30^0 + 90^0 = 180^0$ [From (iii) and (v)]

⇒ $\angle APD + 120^0 = 180^0$

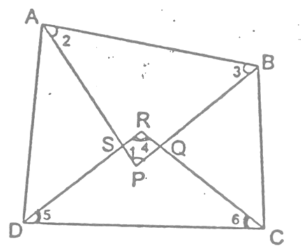
⇒ $\angle APD = 180^0 - 120^0 = 60^0$

⇒ $\angle CPD = 60^0$. **Hence Proved.**

Ex.10 Prove that the quadrilateral formed by angle bisectors of a cyclic quadrilateral is also cyclic.

Sol. **Given :** ABCD is a cyclic quadrilateral. Its angle bisectors from a quadrilateral PQRS.

To Prove : PQRS is a cyclic quadrilateral.



Proof : $\angle 1 + \angle 2 + \angle 3 = 180^0$ ...(i) [∵ Sum of the angles of a Δ is 180^0]

$\angle 4 + \angle 5 + \angle 6 = 180^0$ ...(ii) [∵ Su m of the angles of a Δ is 180^0]

∴ $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 = 360..$ ....(iii) [Adding (i) and (ii)]

But $\angle 2 + \angle 3 + \angle 6 + \angle 5 = \frac{1}{2}[\angle A + \angle B + \angle C + \angle D]$

$= \frac{1}{2}.360^0 = 180^0$ [∵ Sum of the angles of quadrilateral is 360^0]

∴ $\angle 1 + \angle 4 = 360^0 - (\angle 2 + \angle 3 + \angle 6 + \angle 5)$

∴ PQRS is a cyclic quadrilateral.

[∵ If the sum of any pair of opposite angles of a quadrilateral is 180^0, then the quadrilateral is a cyclic]

Hence Proved.

Ex.11 Prove that the angle bisectors of the angles formed by producing opposite sides of a cyclic quadrilateral (Provided they are not parallel) intersect a right angle.

Sol. **Given :** ABCD is a cyclic quadrilateral. Its opposite sides DA and CB are produced to meet at P and opposite sides AB and DC are produced to meet at Q. The bisectors of ∠P and ∠Q meet is F.

To Prove : $\angle PFQ = 90^0$.

Construction : Produce PF to meet DC is G.

Proof : In ΔPEB,

$\angle 5 = \angle 2 + \angle 6$ .....(i)

[∵ Exterior angle of a triangle is equal to the sum of interior opposite angles]

But $\angle 2 = \angle 1$

And, $\angle 6 = \angle D$ [∵ In a cyclic quadrilateral, exterior angle = interior opposite angle]

∴ $\angle 5 = \angle 1 + \angle D$ ....(ii) [From (i)]

Now in ΔPDG,

$\angle 7 = \angle 1 + \angle D$ ...(iii)

[∵ Exterior angle of a triangle is equal to the sum of interior opposite angles]

Frim (ii) and (iii), we have

$\angle 5 = \angle 7$

Now, in Δ QEF and Δ QGF, [Proved above]

$\angle 5 = \angle 7$ [Common side]

QF = QF [Given]

$\angle 3 = \angle 4$ [AAS criterion]

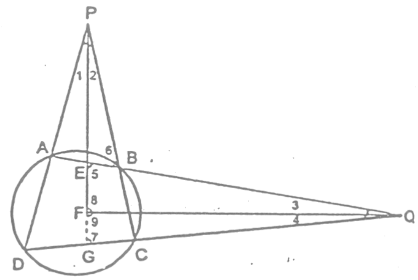
$\therefore$ ΔQEF ≅ ΔQGE [By cpctc]

$\therefore$ $\angle 8 = \angle 9$

But $\angle 8 + \angle 9 = 180^0$

$\therefore$ $\angle 8 = \angle 9 = 90^0$ [Linear Pair Axiom]

$\therefore$ $\angle PFQ = 90^0$



Hence Proved.

Ex.12 Two concentric circles with centre O have A, B, C, D as the points of intersection with the line ℓ as shown in the figure. If AD = 12 cm and BC = 8 cm, find the length of AB, CD, AC and BD.

Sol. Since OM ⊥ BC, a chord of the circle,

$\therefore$ is bisects BC.

$\therefore$ $BM = CM = \frac{1}{2}(BC) = \frac{1}{2}(8) = 4$ cm

Since, OM ⊥ AD, a chord of the circle,

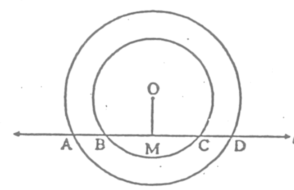
$\therefore$ it bisects AD.

$\therefore$ $AM = AD = \frac{1}{2}AD = \frac{1}{2}(8) = 4$ cm

Since, OM ⊥ CD, a chord of the circle,

$\therefore$ it bisects AD.

$\therefore$ $AM = MD = \frac{1}{2}AD = \frac{1}{2}(12) = 6$ cm



Now, AB = AM - BM = 6 - 4 = 2 cm

CD = MD - MD = 6 - 4 = 2 cm

AC = AM + MC = 6 + 4 = 10 cm

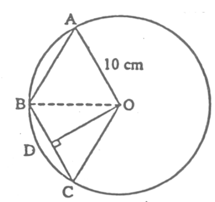
BD = BM + MD = 4 + 6 = 10 cm

Ex.13 OABC is a rhombus whose three vertices, A B and C lie on a circle with centre O. If the radius of the circle is 10 cm. Find the area of the rhombus.

Sol. Since OABC is a rhombus

$\therefore$ OA = AB = BC = OC = 10 cm

Now, OD ⊥ BC $\Rightarrow CD = \frac{1}{2}BC = \frac{1}{2}(10) = 5$ cm



$\therefore$ By Pythagoras theorem,

$OC^2 = OD^2 + DC^2$

$\Rightarrow$ $OD^2 = OC^2 - DC^2 = (10)^2 - (5)^2 = 100 - 25 = 75$

$\Rightarrow$ $OD = \sqrt{75} = 5\sqrt{3}$

$\therefore$ Area (ΔOBC) = $\frac{1}{2}BC \times OD = \frac{1}{2}(10) \times 5\sqrt{3} = 25\sqrt{3}$ sq. cm.

Ex.14 Chords AB and CD of a circle with centre O, intersect at a point E. If OE objects $\angle AED$. Prove that AB = CD.

Sol. In ΔOLE and ΔOME

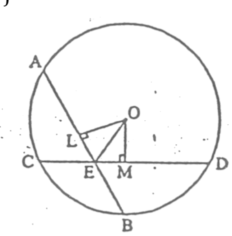
$\angle OLE = \angle OME$ [90^0 each]

$\angle LEO = \angle MEO$ [Given]

And OE = OE [Common]

$\therefore$ ΔOLE ≅ ΔOME [By AAS Criteria]

$\Rightarrow$ OL = OM [By cpctc]

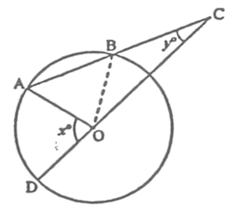


This chords AB and CD are equidistant from centre. But we know that only equal chords are equidistant from centre.

$\Rightarrow$ AB = DC

Ex.15 In the given figure. AB is the chord of a circle with centre O. AB is produced to C such that BC = OB. CO is joined and produced to meet the circle in D. If ∠ACD = y^0 and ∠AOD = x^0, prove that $x^0 = 3y^0$.

Sol. Since BC = OB **[Given]**



∴ ∠OCB = ∠BOC = y^0 [∵ Angles opposite to equal sides are equal]

∠OBA = ∠BOC + ∠OCB = $y^0 + y^0 = 2y^0$.

[∵ Exterior angle of a Δ is equal to the sum of the opposite interior angles]

Also OA = OB [Radii of the same circle]

∠OAB = ∠OBA = $2y^0$ [Angles opposite to equal sides of a triangle are equal]

∠AOD = ∠OAC + ∠OCA

= $2y^0 + y^0 = 3y^0$

[∵ Exterior angle of a Δ is equal to the sum of the opposite interior angles]

Hence $x^0 = 3y^0$ **Hence Proved.**

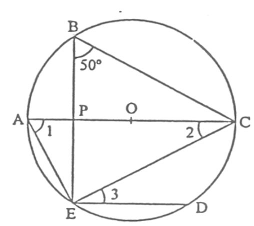
Ex.16 In the given figure, the chord ED is parallel to the diameter AC. Find ∠CED.

Sol. ∠CBE = ∠1 [∠s in the same segment]

∠1 = 50^0 ....(i) [∵ ∠CBE = 50^0]

∠AEC = 90^0 ....(ii) [Semicircle Angle is right angle]

Now, in ΔAEC,



∠1 + ∠AEC + ∠2 = 180^0 [∵ Sum of angles of a Δ = 180^0]

∴ $50^0 + 90^0$ + ∠2 = 180^0

⇒ ∠2 = $180^0 - 140^0 = 40^0$

Thus ∠2 = 40^0 ....(iii)

Also, ED ∥ AC [Given]

∴ ∠@ = ∠3 [Alternate angles]

∴ 40^0 = ∠3 i.e., ∠3 = 40^0

Hence ∠CED = 40^0**Ans.**

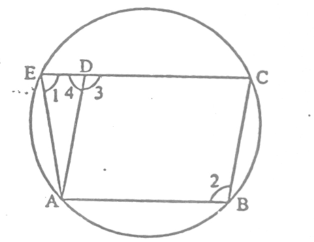
Ex.17 ABCD is a parallelogram. The circle through A, B, C intersects CD (produced if necessary) at E. Prove that AD = AE.

Sol. **Given :** ABCD is a parallelogram. The circle through A, B, C intersects CD, when produced in E.

To prove : AE = AD.

Proof : Since ABCE is a cyclic quadrilateral

$\therefore$ $\angle 1 + \angle 2 = 180^0$ ....(i) [opposite angles of a cyclio quadrilateral are supplementary]



Also $\angle 3 + \angle 4 = 180^0$ [linear pair] ....(ii)

From (i) and (ii), we get $\angle 1 + \angle 2 = \angle 3 + \angle 4$ ....(iii)

But $\angle 2 = \angle 3$ ....(iv)

$\therefore$ From (iii) and (iv), we get $\angle 1 = \angle 4$

Now in ΔADE, since $\angle 1 = \angle 4$

AD = AE [Sides opp. to equal angles of a triangle are equal]

Hence Proved.

EXERCISE

OBJECTIVE DPP # 15.1

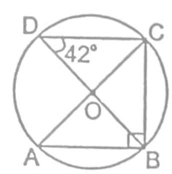
1. I the given circle ABCD, O is the centre and $\angle$BDE = 42^0. The $\angle$ACB is equal to :

(A) 48^0

(B) 45^0

(C) 42^0

(C) 60^0



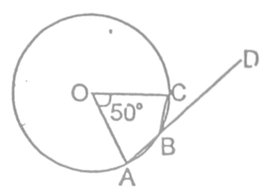
2. In the diagram, O is the centre of the circle. The angles CBD is equal to :

(A) 25^0

(B) 50^0

(C) 40^0

(D) 130^0



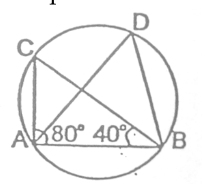
3. In the given figure, $\angle$CAB = 80^0, $\angle$ABC = 40^0. The sum of $\angle$DAB + $\angle$ABD is equal to :

(A) 80^0

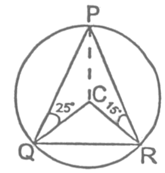
(B) 100^0

(C) 120^0

(D) 140^0



4. In the given figure, if C is the centre of the circle and ∠PC = 25^0 and ∠PRC = 15^0, then ∠QCR is equal to :



(A) 40^0

(B) 60^0

(C) 80^0

(D) 120^0

5. In a cyclic quadrilateral if ∠B - ∠D = 60^0, then the smaller of the angles B and D is :

(A) 30^0 (B) 45^0 (C) 60^0 (D) 75^0

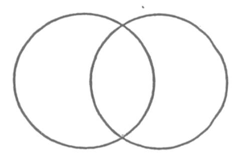
6. Three wires of length ℓ_1, ℓ_2, ℓ_3 from a triangle surmounted by another circular wire, If ℓ_3 is the diameter and $\ell_3 = 2\ell_1$, then the angle between ℓ_1 and ℓ_3 will be

(A) 30^0 (B) 60^0 (C) 45^0 (D) 90^0

7. In a circle with centre O, OD ⊥ chord AB. If BC is the diameter, then :

(A) AC = BC (B) OD = BC (C) AC = 2OD (D) None of these

8. In the diagram two equal circles of radius 4 cm intersect each other such that each passes through the centre of the other. Find the length of the common chord.



(A) $2\sqrt{3}$ cm

(B) $4\sqrt{3}$ cm

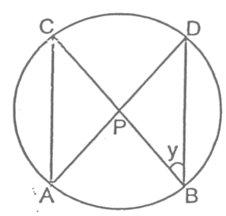
(C) $4\sqrt{2}$ cm

(D) 8 cm

9. The sides AB and DC of cyclic quadrilateral ABCD are produced to meet at P, the sides AD and BC are produced to meet at Q. If ∠ADC = 85^0 and ∠BPC = 40^0, then ∠CQD equals.

(A) 30^0 (B) 45^0 (C) 60^0 (D) 75^0

10. In the given figure, if ∠ACB = 40^0, ∠DPB = 120^0, then will be :



(A) 40^0

(B) 20^0

(C) 0^0

(D) 60^0

11. Any cyclic parallelogram is a.

(A) rectangle (B) rhombus (C) trapezium (D) square

12. The locus of the centre of all circles of given radius r, in the same planes, passing through a fixed point is :

(A) A point (B) A circle (C) A straight line (D) Two straight lines

13. In a cyclic quadrilateral if ∠A - ∠C = 70^0, then the greater of the angles A and C is equal to :

(A) 95^0 (B) 105^0 (C) 125^0 (D) 115^0

14. The length of a chord a circle is equal to the radius of the circle. The angle which this chord subtends on the longer segment of the circle is equal to :

(A) 30^0

(B) 45^0

(C) 60^0

(D) 90^0

15. If a trapezium is cyclic then,

(A) Its parallel sides are equal.

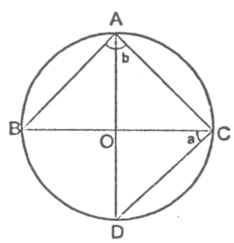
(B) Its non-parallel sides are equal.

(C) Its diagonals are not equal.

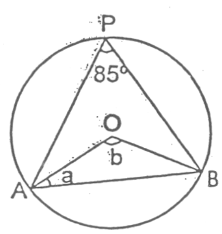
(D) None of these above

SUBJECTIVE DPP - 15.2

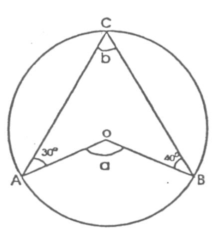
1. In the given figure, BC is diameter bisecting ∠ACD, find the values of a, b (o is centre of circle).



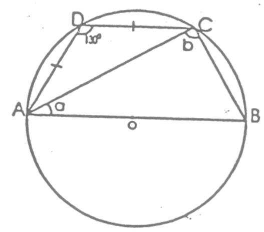
2. In the given figure, find the value of a & b.



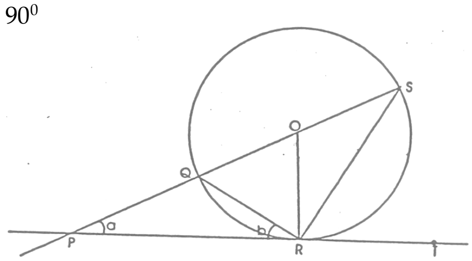
3. Find the value of a & b.



4. Find the value of a & b.

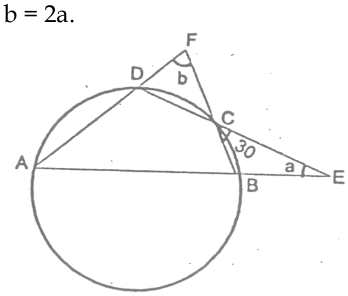


5. Prove that a + 2b = 90^0

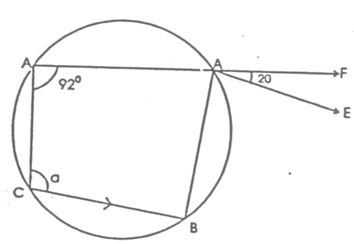


6. ABCD is a cyclic quadrilateral in which $\angle A = (x + y + 10)^0$, $\angle B = (y + 20)^0$, $\angle C = (x + y - 30)^0$ and $\angle D = (x + y)^0$. Find x and y.

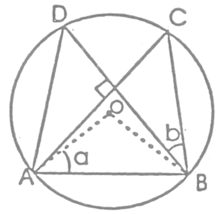
7. Find the value of a and b, if b = 2a.



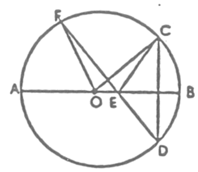
8. Find the value of a if BC ∥ EA



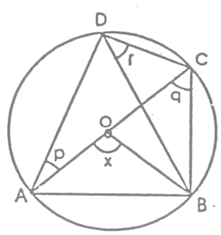
9. In the adjoining fig., O is centre of the circle, chord AC and BD are perpendicular to each other, $\angle OAB = a$ and $\angle DBC = b$. Show that $a = b$.



10. In the fig. given below, AB is diameter of the circle whose centre is O. Given that : $\angle ECD = \angle EDC = 32^0$. Show that $\angle COF = \angle CEF$.



11. In the given fig., AC is the diameter of circle centre O. Chord BD is perpendicular to AB. Write down the angles p,q & r in terms of x.



12. Prove that the line segment joining the mid-point of hypotenuse of a right triangle to its opposite vertex is half of the hypotenuse.

ANSWER KEY

(Objective DPP # 14.1)

Qus.	1	2	3	4	5
Ans.	C	B	D	B	C

(Subjective DPP # 14.2)

1. 12 cm **2.** 7 cm **3.** 13 cm **4.** 9.6 cm

6. 10 cm **7.** 13.29 cm **8.** $4\sqrt{6}$ cm

(Objective DPP # 15.1)

Qus.	1	2	3	4	5	6	7	8	9	10
Ans.	A	A	C	C	C	B	C	B	A	B
A	11	12	13	14	15					
Ans.	A	B	C	A	B					

(Subjective DPP # 15.2)

1. $b = 90^0, a = 45^0$ **2.** $a = 5^0, b = 170^0$ **3.** $a = 140^0, b = 70^0$

4. $a = 40^0, b = 90^0$ **6.** $x = 40, y = 60$

7. $a = 40^0, b = 80^0$ **8.** $a = 108^0$

11. $p = 90^0 - \frac{x}{2}, q = \frac{x}{2}$, and $r = 90 - \frac{x}{2}$