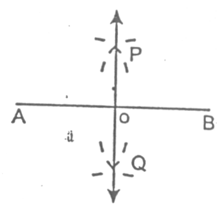
➤➤➤ CONSTRCUTIONS ⮜⮜⮜

ML - 16

TO CONSTRUCT THE BISECTOR OF A LINE SEGMENT

Ex.1 Draw a line segment of length 7.8 cm draw the perpendicular bisector of this line segment.



Sol. Given the given the segment be AB = 7.8 cm.

STEPS :

(i) Draw the line segment AB = 7.8 cm.

(ii) With point A as centre and a suitable radius, more than half the length of AB, draw arcs on both the sides of AB.

(iii) With point B as centre and with the same radius draw arcs on both the sides of AB. Let these arc cut at points P & Q as shown on in the figure.

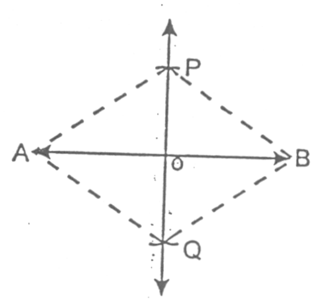
(iv) Draw a line through the points P and Q. The line so obtained is the required perpendicular bisector of given line segment AB.

Line PQ is perpendicular bisector of AB.

(A) PQ bisects AB i.e., OA = OB.

(B) PQ is perpendicular to AB i.e., $\angle PAO = \angle POB = 90^0$.

Proof : In ΔAPQ and ΔBPQ :



AP = BP [By construction]

AQ = BQ [By construction]

PQ = PQ [Common]

$\Rightarrow$ $\Delta APQ = \angle BPQ$ [By SSS]

$\Rightarrow$ $\angle APQ = \angle BPQ$ [By cpctc]

Now, in Δ APO & ΔBPO

AP = BP [By construction]

OP = OP [Common side]

$\angle APO = \angle BPO$ [Proved above]

$\Rightarrow$ $\Delta APO \cong \Delta BPO$ [By SAS]

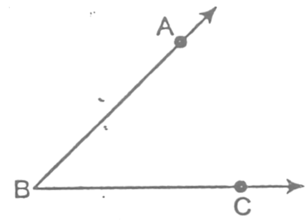
And, $\angle POA = \angle POB$

$= \frac{180^0}{2} = 90^0$ [$\because \angle POA + \angle POB = 180^0$]

$\Rightarrow$ PQ is perpendicular bisector of AB.

TO CONSTRCUT THE BISECTOR OF A GIVEN ANGLE

Let ABC be the given angle to be bisected.



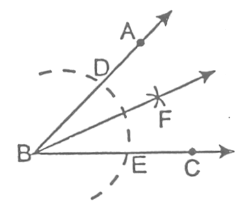
STEPS :

(i) With B as centre and a suitable radius, draw an arc which cuts ray BA at point D and ray BC at point E.

(ii) Taking D and E as centres and with equal radii draw arcs which intersect each other at point F. In this step, each equal radius must be more than half the length DE.

(iii) Join B and F and produce to get the ray BF.

Ray BF is the required bisector of the given angle ABC.



Proof : Join DF and EF.

In Δ BDF and Δ BEF :

	BD = BE	[Radii of the same arc]
	DF = EF	[Radii of the equal arcs]
	BF = BF	[Common]
$\Rightarrow$	$\Delta BDF \cong \Delta BEF$	[By SSS]
$\Rightarrow$	$\angle DBF = \angle EBF$	[By cpctc]
i.e.,	$\angle ABF = \angle CBF$	
$\Rightarrow$	BF bisects $\angle ABC$.	

Hence Proved.

TO CONSTRUCT THE REQUIRED ANGLE

(a) To Construct the Required Angle of 60^0 :

STEPS :

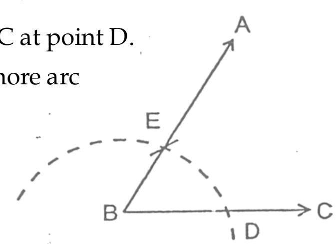
(i) Draw a line BC of any suitable length.

(ii) With B as centre and any suitable radius, draw an arc which cuts BC at point D.

(iii) With D as centre and radius same, as taken in step (ii), draw one more arc which cuts previous arc at point E.

(iv) Join BE and produce upto any point A.

Then, $\angle ABC = 60^0$



(b) To Construct an Angle of 120^0 :

STEPS

(i) Draw a line BC of any suitable length.

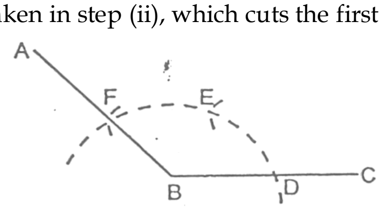
(ii) Taking B as centre and with any suitable radius, draw an arc which cuts BC at point D.

(iii) Taking D as centre, draw an arc of the same radius, as taken in step (ii), which cuts the first arc at point E.

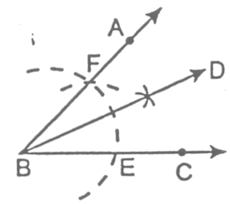
(iv) Taking E as centre and radius same, as taken in step (ii), draw one more arc which cuts the first arc at point F.

(v) Join BF and produce upto any suitable point A.

Then, $\angle ABC = 120^0$



(c) To Construct and Angle of 30^0 :



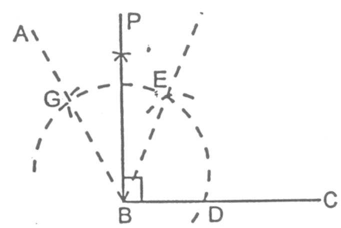
STEPS :

(i) Construct angle ABC = 60^0 by compass.

(ii) Draw BD, the bisector of angle ABC.

The, $\angle DBC = 30^0$

(d) To Construct an Angle of 90^0 :



STEPS

(i) Construct angle ABC = 120.0 by using compass.

(ii) Draw PB, the bisector of angle EBG.

Then, $\angle PBC = 90^0$

Alternative Method :

(i) Draw a line segment BC of any suitable length.

(ii) Produce CB upto a arbitrary point O.

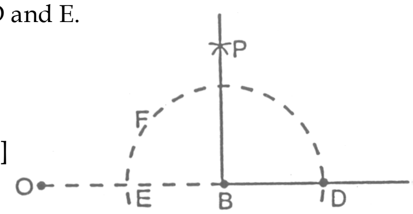
(iii) Taking B as centre, draw as arc which cuts OC at points D and E.

(iv) Taking D and E as centres and with equal radii draw arcs with cut each other at point P.

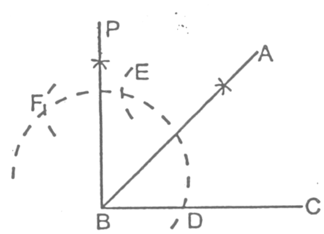
[The radii in this step must be of length more than half of DE.]

(v) Join BP and produce.

Then, $\angle PBC = 90^0$



(d) To Construct an Angle of 45^0



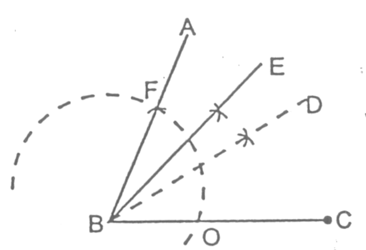
STEPS

(i) Draw $\angle PBC = 90^0$

(ii) Draw AB which bisects angle PBC,

Then, $\angle ABC = 45^0$

Alternative Method :



STEPS :

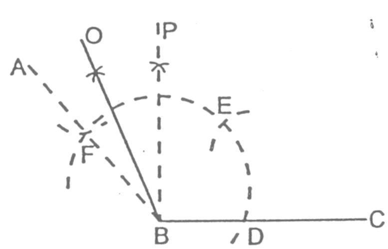
(i) Construct $\angle ABC = 60^0$

(ii) Draw BD, the bisector of angle ABC.

(iii) Draw BE, the bisector of angle ABD.

Then, $\angle EBC = 45^0$

(e) To Construct an Angle of 105^0 :



STEPS :

(i) Construct $\angle ABC = 120^0$ and $\angle PBC = 90^0$

(ii) Draw BO, the bisector of $\angle ABP$.

Then, $\angle OBC = 105^0$

(f) To Construct an Angle of 150^0.

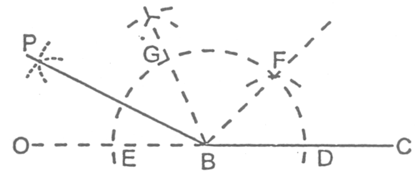
STEPS :

(i) Draw line segment BC of any suitable length. Produce CB upto any point O.

(ii) With B as centre, draw an arc (with any suitable radius) which buts OC at points D and E.

(iii) With D as centre, draw an arc of the same radius, as taken in step 2, which cuts the first arc at point F.

(iv) With F as centre, draw one more arc of the same radius, staken in step 2, which cuts the first arc at point G.



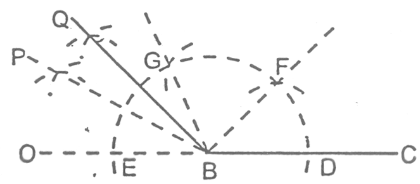
(v) Draw PB, the bisector of angle EBG.

Now $\angle FBD = \angle GBF = \angle EBG = 60^0$

Then, $\angle PBC = 150^0$

(g) To Construct an Angle of 135^0.

STEPS :



(i) Construct $\angle PBC = 150^0$ and $\angle GBC = 120^0$

(ii) Construct BQ, the bisector of angle PBG.

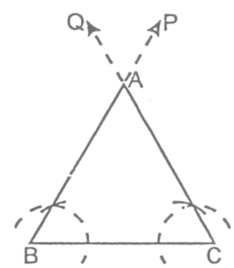
Then, $\angle QBC = 135^0$

TO CONSTRUCT A TRIANGLE

Case (i) To construct an equilateral triangle when its one side is given.

Ex.2 Draw an equilateral triangle having each side of 2.5 cm.

Sol. Given one side of the equilateral triangle be 2.5 cm.



STEPS :

(i) Draw a line segment BC = 2.5 cm.

(ii) Through B, construct ray BP making angle 60^0 with BC.

i.e. $\angle PBC = 60^0$

(iii) Through C, construct CQ making angle 60^0 with BC

i.e., $\angle QCB = 60^0$

(iv) Let BP and CQ intersect each other at point A.

The n, ΔABC is the require equilateral triangle.

Proof : Since, $\angle ABC = \angle ACB = 60^0$

$\therefore$ $\angle BAC = 180^0 - (60^0 + 60^0) = 60^0$

$\Rightarrow$ All the angles of the ΔABC drawn are equal.

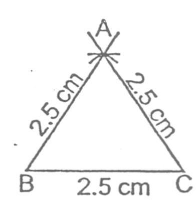
$\Rightarrow$ All the sides of the ΔABC drawn are equal.

$\Rightarrow$ ΔABC is the required equilateral triangle. **Hence Proved.**

Alternate method :

If one side is 2.5 cm, then each side of the required equilateral triangle is 2.5 cm.

STEPS :



(i) Draw BC = 2.5 cm

(ii) With B as centre, draw an arc of radius 2.5 cm

(iii) With C as centre, draw an arc of radius 2.5 cm

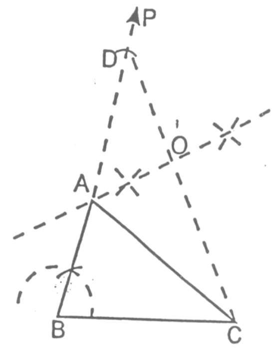
(iv) Let the two arc intersect each other at point A. Join AB and AC.

Then, ABC is the required equilateral triangle.

Case (ii) When the base of the triangle, one base angle and the sum of other two sides are given.

Ex.3 Construct a triangle with 3 cm base and sum of other two sides is 8 cm and one base angle is 60^0.

Sol. Given the base BC of the triangle ABC be 3 cm, one base angle $\angle B = 60^0$ and the sum of the other two sides be 8 cm i.e, AB + AC = 8 cm.



STEPS :

(i) Draw BC = 3 cm

(ii) At point B, draw PB so that $\angle PBC = 60^0$

(iii) From BP, cut BC = 8 cm.

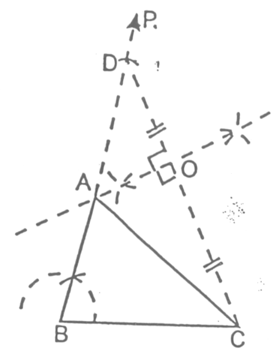
(iv) Join D and C.

(v) Draw perpendicular bisector of CD, which meets BD at point A.

(vi) Join A and C.

Thus, ABC is the required triangle.

Proof : Since, OA is perpendicular bisector of CD



$\Rightarrow$ OC = OD

$\angle AOC = \angle AOD = 90^0$

Also, OA = OA [Common]

$\therefore$ $\Delta AOC \cong \Delta AOD$ [By SAS]

$\Rightarrow$ AC = AD

$\therefore$ BD = BA + AD

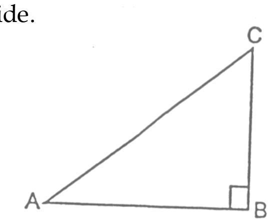
= BA + AC

= Given sum of the other two sides

Thus, base BC and $\angle B$ are draw as given and BD = AC. **Hence Proved.**

Ex.4 Construct a right triangle, when one side is 3.8 cm and the sum of the other side and hypotenuse is 6 cm.

Sol. Here, if we consider the required triangle to be ΔABC, as shown alongside.



Clearly, AB = 3.8 cm, $\angle B = 90^0$ and BC + AC = 6 cm.

STEPS :

(i) Draw AB = 3.8 cm

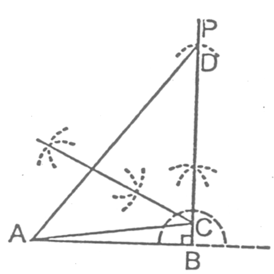
(ii) Through B, draw line BP so that $\angle ABP = 90^0$

(iii) From BP, cut BD = 6 cm

(iv) Join A and D.

(v) Draw perpendicular bisector of AD, which meets BD at point C.

Thus, **ABC is the required triangle.**



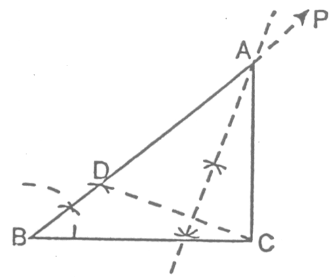
Case (iii) When the base of the triangle, one base angle and the difference of the other two sides are given.

Ex.5 Construct a triangle with base of 8 cm and difference between the length of other two sides is 3 cm and one base angle is 60^0

Sol. Given the base BC of the required triangle ABC be 8 cm i.e., BC = 8 m, base angle B = 60^0 ant the difference between the lengths of other two sides AB and AC be 3 cm.

i.e., AB - AC = 3 cm or AC - AB = 3 cm.

(a) When AB - AC = 3 cm i.e., AB > AC :



STEPS :

(i) Draw BC = 8 cm

(ii) Through point B, draw BP so that $\angle PBC = 60^0$

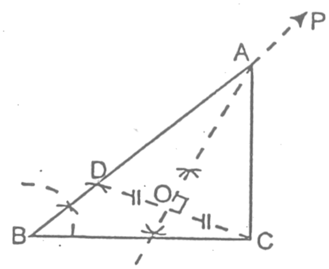
(iii) From BP cut BD = 3 cm.

(iv) Join D and C.

(v) Draw perpendicular bisector of DC ; which meets BP at point A.

(vi) Join A and C.

Thus, ΔBC is the required triangle.



Proof : Since OA is perpendicular bisector of CD

$\Rightarrow$ OD = OC

$\angle AOD = \angle AOC = 90^0$

And,	OA = OA	[Common]
$\therefore$	$\Delta AOD \cong \Delta AOC$	[By SAS]
$\Rightarrow$	AD = AC	[By cpctc]

Now, BD = BA - AD

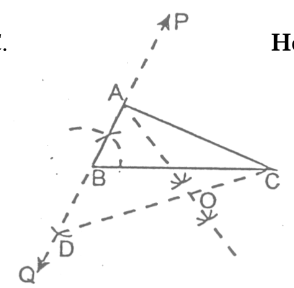
= BA - AD

= BA - AC

= Given difference of other two sides.

Thus, the base BC and $\angle B$ are drawn as given and BD = BA - AC. **Hence Proved.**

(b) When AC - AB = 3 cm i.e, AB < AC :



STEPS :

(i) Draw BC = 8 cm

(ii) Through B, draw line BP so that angle PBC = 60^0.

(iii) Produce BP backward upto a suitable point Q.

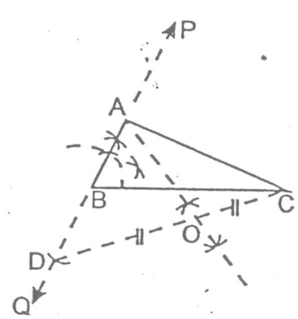
(iv) Fro BQ, cut BD = 3 cm.

(v) Join D and C.

(vi) Draw perpendicular bisector of DC, which meets BP at point A.

(vii) Join A and C.

Thus, ΔABC is the required triangle.



Proof : Since, OA is perpendicular bisector of CD

$\Rightarrow$ OD = OC

$\angle AOD = \angle AOC = 90^0$

And	OA = OA	[Common]
$\therefore$	$\Delta AOD \cong \Delta AOC$	[By SAS]
$\Rightarrow$	AD = AC	[By cpctc]

Now, BD = AD - AB

= AC - AB [$\because$ AD = AC]

= Given difference of other two sides.

Thus, the base BC and $\angle B$ are drawn as given and BD = AC - AB. **Hence Proved.**

Case (iv) When the perimeter of the triangle and both the base angles are given :

Ex.6 Contruct a triangle ABC with AB + BC + CA = 12 cm $\angle B = 45^0$ and $\angle C = 60^0$

Sol. Given the perimeter of the triangle ABC be 12 cm i.e., AB + BC + CA = 12 cm and both the base angles be 45^0 and 60^0 i.e., $\angle B = 45^0$ and $\angle C = 60^0$

STEPS :

(i) Draw a line segment PQ = 12 cm

(ii) At P, construct line PR so that $\angle RPO = 45^0$ and at Q, construct a line QS so that $\angle SQP = 60^0$

(iii) Draw bisector of angles RPQ and SQP which meet each other at point A.

(iv) Draw perpendicular bisector of AP, which meets PQ at point B.

(v) Draw perpendicular bisector of AQ, which meets PQ at point C.

(vi) Join AB and AC.

Thus, ABC is the required triangle.

Proof : Since, MB is perpendicular bisector of AP

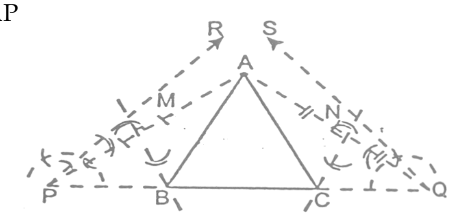
$\Rightarrow$ $\Delta QNC \cong \Delta ANC$ [By SAS]

PB = AC

Similarly, NC is perpendicular bisector of AQ.

$\Rightarrow$ $\Delta QNC \cong \Delta ANC$ [By SAS]

$\Rightarrow$ CQ = AC [By cpctc]



Now, PQ = PB + BC + CQ

= AB + BC + AC

= Given perimeter of the ΔABC drawn.

Also, $\angle BPA = \angle BAP$ [As Δ PMB $\cong \Delta$ A MB]

$\therefore$ $\angle ABC = \angle BPA + \angle BAP$ [Ext. angle of a triangle = sum of two interior opposite angles]

$\angle ABC = \angle BPA + \angle BAP = 2\,\angle BPA = \angle RPB = \angle ACB$ [Given]

$\angle ACB = \angle CQA + \angle CQA$

$= 2\,\angle CQA$ [$\because \Delta QNC \cong \Delta ANC \therefore \angle CQA = \angle CAQ$]

$= \angle SQC$ = Given base angle ACB.

Thus, given perimeter = perimeter of ΔABC.

given one base angle = angle ABC

and, given other base angle = angle ACB.

Ex.7 Construct and equilateral triangle if its altitude is 3.2 cm.

Sol. Given In an equilateral ΔABC an altitude AD = 3.2 cm

Required to Construct an equilateral triangle ABC from the given data.

STEPS :

(i) Draw a line PQ and mark and point D on it.

(ii) Construct a ray DE perpendicular to PQ.

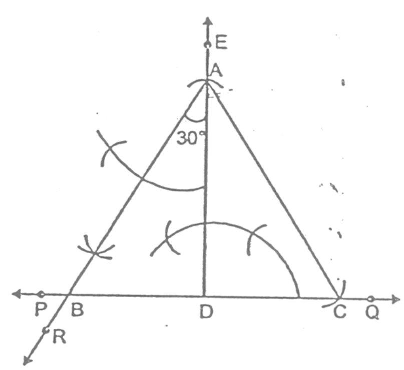
(iii) Cut off DA = 3.2 cm from DE.

(iv) Construct $\angle DAR = 30^0 = \left(\frac{1}{2} \times 60^0\right)$.

The ray AR intersects PQ at B.

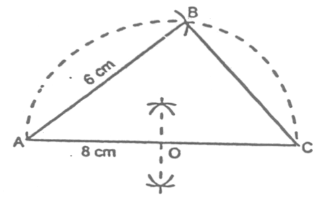
(v) Cut off line segment DC = BD.

(vi) Join A and C. We get the required ΔABC.



Ex.7 Construct a right angled triangle whose hypotenuse measures 8 cm and one side is 6 cm.

Sol. **Given Hypotenuse AC of a** ΔABC = 8 cm and one side AB = 6 cm.



Required To construct a right angled ΔABC from the given data.

STEP.

(i) Draw a line segment AC = 8 cm.

(ii) Mark the mid point O of AC.

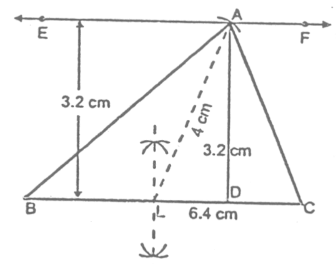
(iii) With O as centre and radius OA, draw a semicircle on AC.

(iv) With A as centre and radius equal to 6 cm, draw an arc, cutting the semicircle a B.

(v) Jon A and B ; B and C. We get the required right angled triangle ABC.

Ex.8 Construct a ΔABC in which BC = 6.4 cm, altitude from A is 3.2 cm and the median bisecting BC is 4 cm.

Sol. **Given :** one side BC = 6.4 cm, Altitude AD = 3.2 am and the median AL = 4 cm.



Required : To construct a ΔABC form the given data.

STEP :

(i) Draw BC = 6.4 cm

(ii) Bisect BC at L.

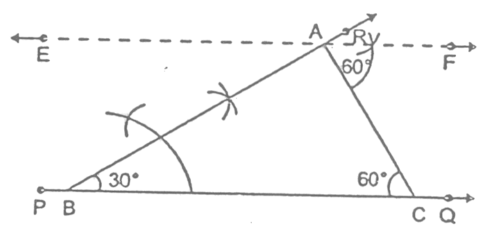
(iii) Draw EF ‖ BC at a distance 3.2 cm for BC.

(iv) With L as centre and radius equal to 4 cm, draw an arc, cutting EF at A.

(v) Join A and B ; A and C. We get the required triangle ABC.

Ex.9 Construct a ΔABC in which ∠B = 30^0 and ∠C = 60^0 and the perpendicular from the vertex A to the base BC is 4.8 cm.

Sol. **Given :** ∠B = 30^0 ∠C = 60^0, length of perpendicular from vertex A to be base BC = 4.8 cm.



Required : To construct a ΔABC from the given data.

STEP :

(i) Draw any ray line PQ.

(ii) Take a point B on line PQ and construct ∠QBR = 30^0

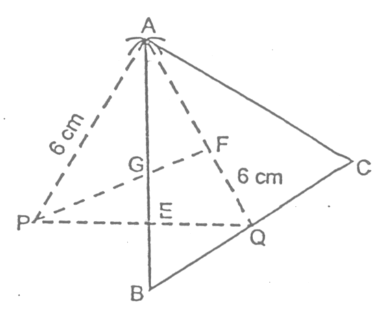
(iii) Draw a line EF ‖ PQ a distance of 4.8 cm from PQ, cutting BR at A.

(iv) Construct an angle ∠FAC = 60^0, cutting PQ C.

(v) Join A and C. We get the required triangle ABC.

Ex.10 Construct a triangle ABC, the lengths of whose medians are 6 cm, 7cm and 6 cm.

Sol. **Given :** Median AD = 6 cm Median BE = 7 cm Median CF = 6 cm.



Required : To construct a ΔABC from the given data.

STEP :

(i) Construct a ΔAPQ with AP = 6 cm, PQ = 7 cm and AQ = 6 cm.

(ii) Draw the medians AE and PF of ΔAPQ intersecting each other at G.

(iii) Produce AE to B such that GE = EB

(iv) Join B and Q and produce it to C, such that BQ = QC

(v) Join A and C. We get the required triangle ABC.

EXERCISE

SUBJECTIVE DPP # 16.1

For each angle, given below, make a separate construction. Draw a ray BC and an another ray BA so that the $\angle ABC$ is equal to :

1. 15^0 **2.** $22\frac{1}{2}^0$ **3.** 75^0 **4.** $52\frac{1}{2}^0$

5. $67\frac{1}{2}^0$ **6.** 165^0 **7.** 135^0

8. Construct and equilateral triangle with side :
(i) 5 cm (ii) 5.4 cm (iii) 6.2 cm

9. Construct a triangle ABC, in which :
(i) base AB = 5.4 cm, $\angle B = 45^0$ and AC + BC = 9 cm.
(ii) base BC = 6 cm, $\angle B = 60^0$ and AB + AC = 9.6 cm.
(iii) base AC = 5 cm, $\angle C = 90^0$ and AB + BC = 10.6 cm.

10. Construct a right triangle, with base = 4 cm and the sum of the other side and hypotenuse = 9.4 cm.

11. Construct a triangle ABC, in which :
(i) BC = 4.8 cm, $\angle B = 45^0$ and AB - AC = 2.4 cm.
(ii) BC = 4.8 cm, $\angle B = 45^0$ and AC - AB = 2.4 cm.
(iii) AB = 5.3 cm, $\angle A = 60^0$ and AC - BC = 2 cm.
(iv) AB = 5.3 cm, $\angle A = 60^0$ and BC - AC = 2 cm.

12. Construct a triangle ABC, with :
(i) perimeter = 12 cm, $\angle B = 45^0$ and $\angle C = 60^0$.
(ii) perimeter = 11.6 cm, $\angle B = 60^0$ and $\angle C = 90^0$
(iii) perimeter = 11 cm, $\angle A = 60^0$ and $\angle C = 45^0$.
(iv) perimeter = 10 cm, $\angle B = \angle C = 60^0$

13. Construct as equilateral triangle with perimeter 15.6 cm.
14. Without finding the length of each side of the equilateral triangle construct it. If its perimeter is 16 cm.
15. Construct an equilateral triangle whose altitude is 4.8 cm.
16. Construct a Δ PQR in which base QR = 4 cm, $\angle R = 30^0$ and PR - PQ = 1.1 cm.
17. Construct a Δ XYZ with perimeter 9.6 cm and base angle 30^0 and 60^0
18. Construct a Δ PQR in which PQ = 3.7 cm, QR = 3.6 cm and median PA = 3.1 cm.
19. Construct a Δ DEF, the lengths of whose medians are 6 cm, 7 cm and 8 cm.
20. Construct on equilateral triangle, one of whose altitudes measures 6.4 cm.

HERON'S FORMULA

ML - 17

MENSURTION

A branch of mathematics which concerns itself measurement of lengths, areas and volumes of plane and solid figure is called Monsuration.

(a) Perimeter :

The perimeter of a plane figure is the length of its boundary. In case of a triangle or a polygon, the perimeter is the sum of the lengths of its sides.

(b) Units of Perimeter :

The unit of perimeter is the same as the unit of length i.e. centimetre (cm), metre(m), kilometere (k m) etc.

1 centimetre (cm) = 10 milimetre (mm)
1 decimetre (dm) = 10 centrimetre
1 metre (m) = 10 decimetre
= 100 centimetre
= 1000 milimetre
1 decamete (dam) = 10 metre
= 1000 centimetre
1 hectometre (hm) = 10 decametre
= 100 metre
1 kilometre (km) = 1000 metre
= 100 decametre
= 10 hectometre

AREA

The area of a plane figure is the measure of the surface enclosed by its boundary.

The area of a triangle or a polygon is the measure of the surface enclosed by its sides.

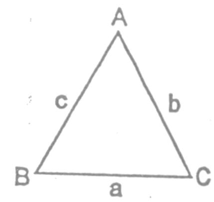
(a) Units of Area :

The various units of measuring area are, squire centimetre (cm^2), square metre (m^2), 1 hectare etc.

1 square centrimetre (cm^2) =1 cm × 1 cm.
= 10 mm × 10 mm = 100 sq. mm.
1 square decimetre (cm^2) = 1 dm × dm
= 10 cm × 10 cm = 100 sq. cm.
1 square metre (m^2) = 1 m × 1 m
= 10 dm × 10 dm = 100 sq. dm.
1 square decametre (dam^2) = 1 dam × 1 dam
= 10 m × 10 m = 100 sq. m.

1 square hectometre (hm^2)	= 10 dam × 10 dam = 100 sq. dam
(or 1 hectare)	= 1000 sq. m.
1 square kilometre (km^2)	= 1 km × 1 km
	= 10 hm × 10 hm = 100 sq. hm.

(b) Heron's formula :



In ΔABC if sides of triangle BC, CA, & AB are a, b, c respectively then

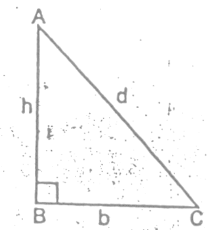
perimeter = 2s = a + b + c

Area = $\sqrt{s(s-a)(s-b)(s-c)}$

(c) Perimeter and Area of a Triangle :

(i) Right - angled triangle :

For an right-angled triangle, let b be the base, h be the perpendicular and d be the hypotenuse. Then



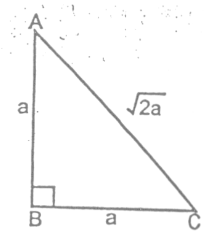
(A) Perimeter = b + h + d

(B) Area = $\frac{1}{2}$(Base × Height) = $\frac{1}{2}$bh

(C) Hypotenuse, d = $\sqrt{b^2+h^2}$ [Pythagoras theorem]

(ii) Isosceles right-angled triangle

For an isosceles right-angled triangle, let a bet the equal sides, then



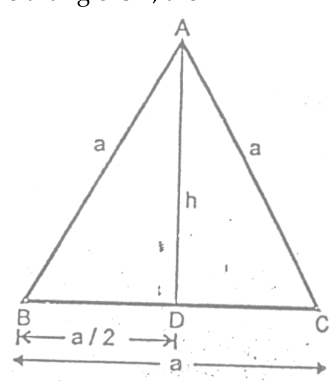
(A) Hypotenuse = $\sqrt{a^2+a^2} = \sqrt{2}a$

(B) Perimeter = 2a + $\sqrt{2}$a

(C) Area = $\frac{1}{2}$(Base × Height) = $\frac{1}{2}$(a × a) = $\frac{1}{2}a^2$.

(iii) Equilateral triangle

For an equilateral triangle, let each side be a, and the height of the triangle is h, then



(A) ∠A = ∠B = ∠C = 60^0

(B) ∠BAD = ∠CAD = 30^0

(C) AB = BC = AC = a(say)

(D) BD = DC = $\frac{a}{2}$

(E) $\left(\frac{a}{2}\right)^2 + h^2 = a^2 \Rightarrow h^2 = \frac{3a^2}{4}$

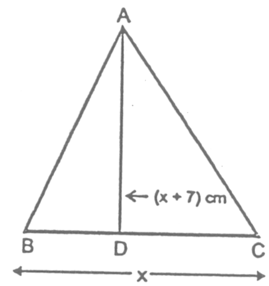
∴ Height (h) = $\frac{\sqrt{3}}{2}a$

(F) Area = $\frac{1}{2}$(Base × Height) = $\frac{1}{2} \times a \times \frac{\sqrt{3}}{2}a = \frac{\sqrt{3}}{4}a^2$

(G) Perimeter = a + a + a = 3a.

Ex.1 The area of a triangle is 30 cm^2. Find the base if the altitude exceeds the base by 7 cm.

Sol. Let base BC = x cm then altitude = (x + 7) cm



Area of $\Delta ABC = \frac{1}{2} \times \text{base} \times \text{height}$

$\Rightarrow$ $30 = \frac{1}{2}(x)(x+7)$

$\Rightarrow$ $60 = x^2 + 7x$

$\Rightarrow$ $x^2 + 7x - 60 = 0$

$\Rightarrow$ $x^2 + 12x - 5x - 60 = 0$

$\Rightarrow$ $x(x + 12) - 5(x + 12) = 0$

$\Rightarrow$ $(x - 5)(x + 12) = 0$

$\Rightarrow$ $x = 5$ or $x = -12$

$\Rightarrow$ $x = 5$ $[\because x \neq -12$

$\therefore$ Base (x) = 5 cm and Altitude = x + 7 = 5 + 7 = 12 cm. **Ans.**

Ex.2 The cost of turfing a triangle field at the rate of Rs. 45 per 100 m^2 is Rs. 900. Find the height, if double the base of the triangle is 5 times the height.

Sol. Let the height of triangular field be h metres.

It is given that 2 x (base) = 5 × (Height)

$\therefore$ Base $= \frac{5}{2}h$

Area $= \frac{1}{2} \times \text{Base} \times \text{Height}$

Area $= \frac{1}{2} \times \frac{5}{2}h \times h = \frac{5}{4}h^2$ ....(i)

$\therefore$ Cost of turfing the field is Rs. 45 per 100 m^2

$\therefore$ Area $= \frac{\text{Total cost}}{\text{Rate per sq.}}$

$= \frac{900}{45/100}$

$= \frac{9000}{45}$

$= 2000\ m^2$ ....(ii)

From (i) and (ii)

$\frac{5}{4}h^2 = 2000$

$\Rightarrow$ $5h^2 = 8000$

$\Rightarrow$ $h^2 = 1600$

$\Rightarrow$ h = 40 m

$\therefore$ Height of the triangular field is 40 cm. **Ans.**

Ex.3 From a point in the interior of an equilateral triangle, perpendicular drawn to the three sides are 8 cm, 10 cm and 11 cm respectively. Find the area of the triangle.

Sol. Let each side of the equilateral $\Delta ABC = x$ cm,

From an interior point O, OD, OE and OF be drawn perpendicular to BC, AC and AB respectively. It is given that OD = 11 cm, OE = 8 cm and OF = 10 cm. Join OA, OB and OC.

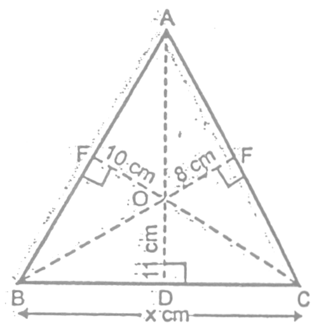
Area of ΔABC = Area of ΔOBC + Area of ΔOCA + Area of ΔOAB

$$= \frac{1}{2}.x.11 + \frac{1}{2}.x.8 + \frac{1}{2}.x.10$$

$$= \frac{29}{2}\ x\ cm^2$$

But, area of an equilateral triangle, whose ease side is x

$$= \frac{\sqrt{3}}{4}\ x^2\ cm^2$$



Therefore, $\frac{\sqrt{3}}{4}\ x^2 = \frac{29}{2}\ x$

$\therefore\quad x = \frac{4\times 29}{2\times\sqrt{3}} = \frac{58}{\sqrt{3}}$ cm

$\therefore\quad$ Area of $\Delta ABC = \frac{29}{2}\times\frac{58}{\sqrt{3}} cm^2 = \frac{841}{1.73} cm^2$

$\therefore\quad$ Area of $\Delta ABC = 486.1\ cm^2$ **Ans.**

Ex.4 The difference between the sides at right angles in a right-angled triangle is 14 cm. The area of the triangle is 120 cm^2. Calculate the perimeter of the triangle.

Sol. Let the sides containing the right angle be x cm and (x - 14) cm.

The, its area = $\left[\frac{1}{2}.x.(x-14)\right]cm^2$.

But, area = 120 cm^2 [Given]

$\therefore\quad \frac{1}{2}x\,(x - 14) = 120$

$\Rightarrow\quad x^2 - 14x - 240 = 0$

$\Rightarrow\quad x^2 - 24x + 10x - 240 = 0$

$\Rightarrow\quad x(x - 24) + 10\,(x - 24) = 0$

$\Rightarrow\quad (x - 24)\,(x + 10) = 0$

$\Rightarrow\quad x = 24$ [Neglecting x = -10]

$\therefore\quad$ one side = 24 cm, other side = (24 - 14) cm = 10 cm

Hypotenuse $= \sqrt{(24)^2 + (10)^2}$ cm

$= \sqrt{576 + 100}$ cm

$= \sqrt{576}$ cm

= 26 cm.

$\therefore\quad$ Perimeter of the triangle = (24 + 10 + 26) c m = 60 cm. **Ans.**

Ex.5 Find the percentage increase in the area of a triangle if its each side is doubled.

Sol. Let a,b,c be the sides of the given triangle and s be its semi-perimeter

$\therefore \quad s = \frac{1}{2}(a + b + c)$ ....(i)

The sides of the new triangle are 2a, 2b and 2c.

Let s′ be its semi-perimeter.

$\therefore \quad s' = \frac{1}{2}(2a + 2b + 2c) = a + b + c = 2s$ [Using (i)]

Let Δ = Area of given triangle

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$ .....(ii)

And, Δ' = Area of new triangle

$\Delta' = \sqrt{s'(s'-2a)(s'-2b(s'-2c}$

$= \sqrt{2s(2s-2)(2s-2b)(2s-2c)}$ [Using (i)]

$= \sqrt{16s(s-a)(s-b)(s-c}$

$\Delta' = 4\Delta$

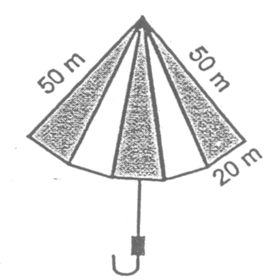
$\therefore$ Increase in the area of the triangle = $\Delta' - \Delta = 4\Delta - \Delta = 3\Delta$

$\therefore$ % increase in area = $\left(\frac{3\Delta}{\Delta}\times 100\right)\% = 300\%$ % **Ans.**

Ex.6 An umbrella is made by stitching 10 triangular pieces of cloth of two different colours (see figure), each piece measuring 20 cm, 50 cm and 50 cm. How much cloth of each colour is required for the umbrella ?

Sol. The sides of a triangular piece are

20 cm, 50 cm and 50 cm



$s = \frac{a+b+c}{2} = \frac{20+50+50}{2} = 60\text{cm} = 60$ cm

Area of one triangular piece

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{60(60-20)(60-50)(60-50)}$

$= \sqrt{60\times 40\times 10\times 10} = \sqrt{24000}$

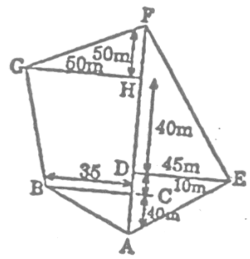
$= 200\sqrt{6}$ cm^2

Area of cloth of each colour for five triangular pieces = $5 \times 200\sqrt{6} = 1000\sqrt{6}$ cm^2 **Ans.**

EXERCISE

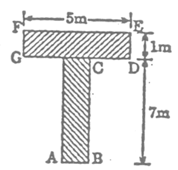
OBJECTIVE DPP 17.1

1. The area of the field ABGFEA is :



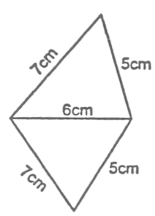
(A) 7225 m^2 (B) 7230 m^2 (C) 7235 m^2 (D) 7240 m^2

2. Area of shaded portion as shown in the figure :



(A) 12 m^2 (B) 13 m^2 (C) 14 m^2 (D) 15 m^2

3. The lengths of four sides and a diagonal of the given quadrilateral are indicated in the diagram. If a denotes the area of quadrilateral, then A is



(A) $12\sqrt{6}$ (B) $\sqrt{6}$ (C) $6\sqrt{6}$ (D) $\sqrt{6}$'

4. In the sides of a triangle are doubled, then its area :

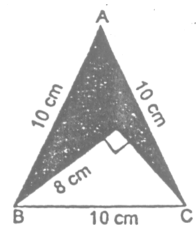
(A) Remains the same (B) Becomes doubled (C) Becomes three times (D) Becomes four times

5. Inside a triangular garden there is a flower bed in the form of a similar triangle. Around the flower bed runs a uniform path of such a width that the side of the garden are double of the corresponding sides of the flower bed. The areas of the path and the flower bed are in the ratio :

(A) 1 : 1 (B) 1 : 2 (C) 1 : 2 (D) 3 : 1

SUBJECTIVE DPP - 17.2

1. In the given figure, ΔABC is a equilateral triangle the length of whose side is equal to 10 cm and ΔDBC is right-angled at D and BD = 8 cm. Find the area of the shaded region. Take $\sqrt{3}$ = 1.732.



2. Calculate the area of the triangle whose sides are 18 cm, 24 cm and 30 cm in length. Also, find the length of the altitude corresponding to the smallest side of the triangle.
3. The sides of a triangle are 10 cm, 24 cm and 26 cm. Find its area and the longest altitude.
4. Two sides of a triangular field are 85 m and 154 m in length, and its perimeter is 324 cm. Find (i) the area of the field, and (ii) the length of the perpendicular from the opposite vertex on the side measuring 154 cm.
5. The sides of a triangular field are 165 cm, 143 cm and 154 cm. Find the cost of ploughing it at 12 paise per sq. m.
6. The base of an isosceles triangle measures 80 cm and its area is 360 cm^2. Find the perimeter of the triangle.
7. The perimeter of an isosceles triangle is 42 cm and its base is $1\frac{1}{2}$ times each of the equal sides. Find (i) the length of each side of the triangle, (ii) the area of the triangle, and (iii) the height of the triangle.
8. The perimeter of a right angle triangle is 40 cm. Its hypotenuse is 17 cm. Find the sides containing the right angle. Also find the area of the triangle.
9. Find the area and perimeter of an isosceles right-angled triangle, each of whose equal sides measures 10 cm. Take $\sqrt{2}$ = 1.414.
10. The area of a square field in 8 hectares. How long would a man take to cross its diagonal by walking at the rate of 4 km per hour ?
11. A rhombus shaped field has green for 18 cows to graze. If each side of the rhombus is 30 m and its longer diagonal is 48 m, how much area of grass field will each cow be getting ?

EXERCISE

(Objective DPP # 17.1)

Qus.	1	2	3	4	5
Ans.	A	A	A	D	D

(Subjective DPP # 17.2)

1. 19.3 cm^2

2. 216 cm, 24 cm

3. 120 cm^2, 24 cm

4. () 2772 cm^2 (ii) 36 cm^2

5. Rs. 1219.68

6. 162 cm

7. (i) 12 cm, 12 cm, 18 cm (ii) 71.42 cm^2 (iii) 7.94 cm

8. 8 cm, 15 cm & 60 cm^2

9. 50 cm^2, 34.14 cm

10. 6 minutes

11. 48 m^2

SURFACE AREA AND VOLUME

ML - 18

SOLID FIGURES

If any figure such as cuboids, which has three dimensions length, width and height are known as three dimensional figures. Where are rectangle has only two dimensions i.e. length and width. Three dimensional figures have volume in addition to areas of surface from which these solid figures are formed.

(a) Cuboids :

There are six faces (rectangular), eight vertices and twelve edges in a cuboids.

Total Surface Area (T.S.A.) : The area of surface from which cuboids is formed.

(i) Total Surface Area (T.S.A.) $= 2[\ell \times b + b \times h + h \times \ell]$

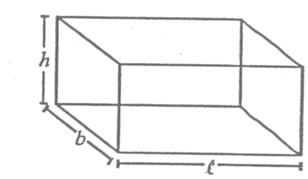
(ii) Lateral Surface Area (L.S.A.) $= 2[b \times h + h \times \ell]$

(or Area of 4 walls) $= 2h[\ell + b]$

(iii) Volume of Cuboids = (Area of base) × height

$= (\ell \times b) \times h$

(iv) Length of diagonal $= \sqrt{\ell^2 + b^2 + h^2}$



(b) Cube :

Cube ahs six faces. Each face is a square.

(i) T.S.A. $= 2[x . x + x . x + x . x]$

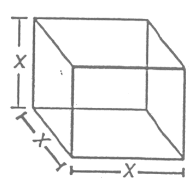
$= 2[x^2 + x^2 + x^2] = 2(33^2) = 6x^2$

(ii) L.S.A. $= 2[x^2 + x^2] = 4x^2$

(iii) Volume = (Area of base) × Height

$= (x^2) \times x = x^3$

(iv) Length of diagonal $= x\sqrt{3}$



(c) Cylinder :

Curved surface area of cylinder (C.S.A.) : It is the area of surface from which the cylinder is formed. When we cut this cylinder, we will find a rectangle with length $2\pi r$ are height h units.

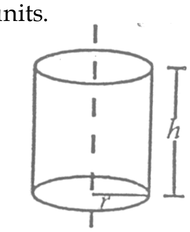
(i) C.S.A. of cylinder $= (2\pi r) \times h = 2\pi rh$.

(ii) T.S.A = C.S.A. + circular top & bottom

$= 2\pi rh + (\pi r^2) + (\pi r^2)$

$= 2\pi rh + 2\pi r^2$

$= 2\pi r(h + r)$ sq. units



(iii) Volume of cylinder = Area of base × height

$= (\pi r^2) \times h$

$= \pi r^2 h$ cubic units

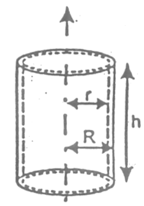
Hollow cylinder :

(i) C.S.A. of hollow cylinder = $2\pi\ (R + r)h$ sq. units

(ii) T.S.A. of hollow cylinder = $2\pi (R + r)h + \pi (R^2 - r^2)$

$= \pi (R + r)\ [2h + R - r]$ sq. units

(iii) Volume of hollow cylinder = $\pi (R^2 - r^2)h$ cubic units



Where, r = inner radius of cylinder

R = outer radius of cylinder

h = height of the cylinder

(d) Cone :

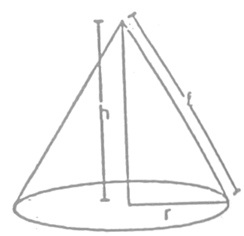
(i) C.S.A. of cone = $\pi\ell$

(ii) T.S.A . of cone = C.S.A. + Base area

$= \pi r\ell + \pi r^2$

$= \pi r(\ell + r)$

(iii) Volume of cone = $\frac{1}{3}\pi r^2 h$



Where, h = height

r = radius of base

ℓ = slant height

(e) Sphere :

(i) T.S.A. of sphere = $4\pi r^2$

(ii) Volume of sphere = $\frac{4}{3}\pi r^3$

(f) Hemisphere :

(i) C.S.A. = $2\pi r^2$

(ii) T.S.A. = C.S.A. + other area

$= 2\pi r^2 + \pi r^2$

$= 3\pi r^2$

(iii) Volume = $\frac{2}{3}\pi r^3$

(g) Hollow Hemisphere :

(i) C.S.A. = $2\pi (R^2 + r^2)$

(ii) T.S.A = $2\pi (R^2 + r^2) + \pi (R^2 - r^2)$

(iii) Volume = $\frac{2}{3}\pi (R^3 - r^3)$

ILLUSTRATIONS :

Ex.1 Three equal cubes are placed adjacently in a row. Find the ratio of the total surface area of the new cuboids to that of the sum of the surface areas of three cubes.

Sol. Let the side of each of the three equal cubes be a cm.

Then surface area of one cube = $6a^2$ cm^2

$\therefore$ Sum of the surface areas of three cubes = $3 \times 6a^2 = 18a^2$ cm^2

For new cuboids

length (ℓ) = 3a cm

breadth (b) = a cm

height (h) = a cm

$\therefore$ Total surface are of the new cuboids $= 2(\ell \times b + b \times h + h = \ell)$

$= 2[3a \times a + a \times a + a \times 3a]$

$= 2[3a^2 + a^2 + 3a^2] = 14a^2$ cm.2

$\therefore$ Required ratio $= \dfrac{\text{Total surface area of the new cuboid}}{\text{Sum of the surace areas of three cubes}}$

$= \dfrac{14a^2}{18a^2} = \dfrac{7}{9} = 7 : 9$ **Ans.**

Ex.2 A class room is 7 m long, 6.5 m wide and 4 m high. It has one door 3 m × 1.4 m and three windows each measuring 2 m × 1 m The interior walls are to be colour-washed. The contractor charges Rs. 15 per sq. m. Find the cost of colour washing.

Sol. $\ell = 7$m, n = 6.5 m and h = 4 m

$\therefore$ Area of the room $= 2(\ell + b)h = 2(7 + 6.5)\,4 = 108\ m^2$

Area of door $= 3 \times 1.4 = 4.2\ m^2$

Area of one window $= 3 \times 2 = 6\ m^2$

$\therefore$ Area of 3 windows $= 3 \times 2 = 6\ m^2$

$\therefore$ Area of the walls of the room to be colour washed $= 108 - (4.2 + 6)$

$= 108 - 10.2 = 97.8\ m^2$

$\therefore$ Cost of colour washing @ Rs. 15 per square metre = Rs. 97.8 × 15 = Rs. 1467. **Ans.**

Ex.3 A cylindrical vessel, without lid, has to be tin coated including both of its sides. If the radius of its base is $\dfrac{1}{2}$ m and its height is 1.4 m, calculate the cost of tin-coating at the rate of Rs. 50 per 1000 cm^2.

Sol. Radius of the base (r) $= \dfrac{1}{2}$ m

$= \dfrac{1}{2} \times 100$ cm = 50 cm

Height (h) = 1.4 m

= 1.4 × 100 cm

= 140 cm.

Surface area of to tin-coated $= 2\,(2\pi r + \pi r^2)$

$= 2[2 \times 3.14 \times 50 \times 140 + 3.14 \times (50)^2]$

$= 2[43960 + 7850] = 2(51810) = 103620\ cm^2$

$\therefore$ Cost of tin-coating at the rate or Rs. 50 per 1000 cm^2

$= \dfrac{50}{1000} \times 103620$ = Rs 5181. **Ans.**

Ex.4 The diameter of a roller 120 cm long is 84 cm. If its takes 500 complete revolutions to level a playground determine the cost of leveling at the rate of Rs. 25 per square metre. (Use $\pi = \frac{22}{7}$)

Sol. $2r = 84$ cm

$\therefore$ $r = \frac{84}{2}$ cm = 42 cm

$h = 120$ cm

Area of the playground leveled in one complete revolution = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 42 \times 120 \times 31680 \text{ cm}^2$$

$\therefore$ Area of the playground = 31680×500 cm^2

$$= \frac{31680 \times 500}{100 \times 100} \text{ m}^2 = 1584 \text{ m}^2$$

$\therefore$ Cost of leveling @ Rs 25 per square metre = Rs $1584 \times 25 = 39600$. **Ans.**

Ex.5 How many metres of cloth of 1.1 m width will be required to make a conical tent whose vertical height is 12 m and base radius is 16 m ? Find also the cost of the cloth used at the rate of Rs 14 per metre.

Sol. $h = 12$ m

$r = 16$ m

$\therefore$ $\ell = \sqrt{r^2 + h^2}$

$= \sqrt{(16)^2 + (12)^2} = \sqrt{256 + 144}$

$= \sqrt{400} = 20\text{m}$

$\therefore$ Curved surface area = $\pi r\ell = \frac{22}{7} \times 16 \times 20 = \frac{7040}{7} \text{m}^2$

Width of cloth = 1.1 m

$\therefore$ Length of cloth = $\frac{7040/7}{1.1} = \frac{70400}{77} = \frac{6400}{7} \text{m}$

$\therefore$ Cost of the cloth used @ Rs 14 per metre = Rs $\frac{6400}{7} \times 14$ = Rs 12800 **Ans.**

Ex.6 The surface area of a sphere of radius 5 cm is five times the area of the curved surface of cone of radius 4 cm. Find the height of the cone.

Sol. Surface area of sphere of radius 4 cm = $\pi(4)\ \ell$ cm^2 when ℓ cm is the slant height of the cone.

According to the question,

$4\pi(5)^2 = 5[\pi(4)\ell]$

$\Rightarrow$ $\ell = 5$ cm $\Rightarrow$ $\sqrt{r^2 + h^2} = 5$

$\Rightarrow$ $r^2 + h^2 = 25 \Rightarrow (4)^2 + h^2 = 25$

$\Rightarrow$ $16 + h^2 = 25 \Rightarrow h^2 = 9$

$\Rightarrow$ $h = 3$

Hence the height of the cone is 3 cm. **Ans.**

Ex.7 The dimensions of a cinema hall are 100 m, 50 m and 18m. How many persons can sit in the hall, if each required 150 m^3 of air ?

Sol. $\ell = 100$ m

$b = 50$ m

$h = 18$ m

$\therefore$ Volume of the cinema hall = ℓbh

$= 100 \times 50 \times 18 = 90000 \text{ m}^3$

Volume occupied by 1 person = 150 m^3

$\therefore$ Number of persons who can sit in the hall = $\dfrac{\text{Volume of the ball}}{\text{Volume occupied by 1 person}}$

$$= \frac{90000}{150} = 600$$

Hence 600 persons can sit in the hall. **Ans.**

Ex.8 The outer measurements of a closed wooden box are 42 cm, 30 m and 27 cm. If the box is made of 1 cm thick wood, determine the capacity of the box.

Sol. **Outer dimensions**

$\ell = 42$ cm

$b = 30$ cm

$h = 27$ cm

Thickness of wood = 1 cm

Inner dimensions

$\ell = 42 - (1 + 1) = 40$ cm

$b = 30 - 1(1 + 1) = 28$ cm

$h = 27 - (1 + 1) = 25$ cm

$\therefore$ Capacity of the box $\ell \times b \times h$

$= 40 \times 28 \times 25 = 28000 \text{ cm}^2$. **Ans.**

Ex.9 If v is the volume of a cuboids of dimensions a,b, and c and s is its surface area, then prove that $\dfrac{1}{v} = \dfrac{2}{a}\left(\dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c}\right)$

Sol. L.H.S. = $\dfrac{1}{v} = \dfrac{1}{abc}$ ....(i)

R.H.S. = $\dfrac{2}{s}\left(\dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c}\right)$

$= \dfrac{2}{2(ab + bc + ca)}\left(\dfrac{bc + ca + ab}{abc}\right)$

$= \dfrac{1}{abc}$ ....(ii)

from (i) and (ii) $\dfrac{1}{v} = \dfrac{2}{s}\left(\dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c}\right)$. **Hence Proved.**

Ex.10 The ratio of the volumes of the two cones is 4 : 5 and the ratio of the radii of their bases is 2 : 3. Find the ratio of their vertical heights.

Sol. Let the radii of bases, vertically heights and volumes of the two cones be r_1, h_1, v_1 and r_2, h_2, v_2 respectively.

According to the question,

$$\frac{v_1}{v_2} = \frac{4}{5} \quad ...(i) \qquad \frac{r_1}{r} = \frac{2}{3} \quad ....(i)$$

From (i), we have $\dfrac{\frac{1}{2}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \dfrac{4}{5}$

$\Rightarrow \dfrac{r_1^2 h_1}{r_2^2 h_2} = \dfrac{4}{5}$

$\Rightarrow \left(\dfrac{r_1}{r_2}\right)^2 \dfrac{h_1}{h_2} = \dfrac{4}{5}$

$\Rightarrow \left(\dfrac{2}{3}\right)^2 = \dfrac{h_1}{h_2} = \dfrac{4}{5}$

$\Rightarrow \dfrac{h_1}{h_2} = \dfrac{4}{5}\left(\dfrac{3}{2}\right)^2$ [Using (ii)]

$\Rightarrow \dfrac{h_1}{h_2} = \dfrac{9}{5}$

Hence the ratio of their vertical height is 9 : 5. **Ans.**

Ex.11 If h, c and v be the height, curved surface and volume of a cone, show that $3\pi vh^3 - c^2h^2 + 9v^2 = 0$.

Sol. Let the radius of the base and slant height of the cone be r and ℓ respectively. The n ;

c = curved surface = $\pi r\ell = \pi r\sqrt{r^2 + h^2}$ ....(i)

v = volume = $\dfrac{1}{3}\pi r^2 h$ ...(ii)

$\therefore \quad 3\pi vh^3 - c^2h^2 + 9v^2 = 3\pi(\frac{1}{3}\pi r^2 h)h^2 - \pi^2 r^2 (r^2 + h^2)h^2 + 9\left(\frac{1}{3}\pi r^2 h\right)^2$ [Using (i) and (ii)]

$=^2 r^2h^4 - \pi^2 r^2 h^2 - \pi^2 r^2 h^4 + \pi^2 r^4 h^2 = 0.$ **Hence Proved.**

Ex.12 How many balls, each of radius 1 cm, can be made from a solid sphere of lead of radius 8 cm ?

Sol. Volume of the spherical ball of radius 8 cm = $\dfrac{4}{3}\pi \times 8^3\, c^3$

Also, volume of each smaller spherical ball of radius 1 cm $= \dfrac{4}{3\pi \times 1^3}\text{cm}^3$.

Let n be the number of smaller balls that can be made. Then, the volume of the larger ball is equal to the sum of all the volumes of n smaller balls.

Hence, $\dfrac{4}{3}\pi \times n = \dfrac{4}{3}\pi \times 8^3$

$\Rightarrow \quad n = 8^3 = 512$

Hence, the required number of balls = 512. **Ans.**

Ex.13 By melting a solid cylindrical metal, a few conical materials are to be made. If three times the radius of the cone is equal to twice the radius of the cylinder an the ratio of the height of the cylinder and the height of the cone is 4 : 3, find the number of cones which can be made.

Sol. Let R be the radius and H be the height of the cylinder and let r and h be the radius and height of the cone respectively. Then,

$3r = 2R$

And $H ; h = 4 : 3$ ....(i)

$\Rightarrow \frac{H}{h} = \frac{4}{3}$

$\Rightarrow 3H = 4h$ ....(ii)

Let n be the required number of cones which can be made from the materials of the cylinder. Then, the volume of the cylinder will be equal to the sum of the volumes of n cones. Hence, we have

$$\pi R^2 H = \frac{n}{3}\pi r^2 h$$

$\Rightarrow 3R^2H = nr^2h$

$$\Rightarrow n = \frac{3R^2H}{r^2h} = \frac{3\times\frac{9r^2}{4}\times\frac{4h}{3}}{r^2h} \quad [\because \text{ From (i) and (ii), } R = \frac{3r}{2} \text{ and } H = \frac{4h}{3}]$$

$$= \frac{3\times 9\times 4}{3\times 4} = 9$$

Hence, the required number of cones is 9. **Ans.**

Ex.14 Water flows at the rate of 10 per minute through a cylindrical pipe having its diameter as 5 mm. How much time will it take to fill a conical vessel whose diameter of the base is 40 cm and depth 24 cm ?

Sol. Diameter of the pipe = 5 mm = $\frac{5}{10}$cm = $\frac{1}{2}$cm.

$\therefore$ Radius of the pipe = $\frac{1}{2}\times\frac{1}{2}$cm = $\frac{1}{4}$ cm.

In 1 minute, the length of the water column in the cylindrical pipe = 10 m = 1000 cm.

$\therefore$ Volume of water that flows out of the pipe in 1 minute = $\pi\times\frac{1}{4}\times\frac{1}{4}\times 1000$ cm^3.

Also, volume of the cone = $\times \frac{1}{3}\times\pi \times 20\times 20\times 24$ cm^3.

Hence, the time needed to fill up this conical vessel $= \left(\frac{1}{3}\pi\times 20\times 20\times 24 \div \pi\times\frac{1}{4}\times\frac{1}{4}\times 1000\right)$ minutes

$= \left(\frac{20\times 20\times 24}{3}\times\frac{4\times 4}{100}\right) = \frac{4\times 24\times 16}{30}$ minutes $= \frac{256}{5}$ minutes = 51.2 minutes.

Hence, the required time is 51.2 minutes. **Ans.**

EXERCISE

OBJECTIVE DPP # 18.1

1. The height of a conical tent at the centre is 5m. The distance of any point on its circular base from the top of the tent is 13m. The area of the slant surface is :

(A) 144 π sq m (B) 130 π sq m (C) 156 π sq m (D) 169 π sq m

2. A rectangular sheet of paper 22 m long and 12 cm broad can be curved to form the lateral surface of a right circular cylinder in two ways. Taking $\pi = \frac{22}{7}$, the difference between the volumes of the two cylinders thus formed is :

(A) 200 c.c. (B) 210 c.c. (C) 250 c.c. (D) 252 c.c.

3. The percentage increase in the surface area of a cube when each side is increased to $\frac{3}{2}$ times the original length is

(A) 225 (B) 200 (C) 175 (D) 125

4. A cord in the form of a square enclose the area 'S' cm^2. if the same cord is bent into the form of a circle, then the area of the circle is

(A) $\frac{\pi S^2}{4}$ (B) $4\pi S^2$ (C) $\frac{S}{4\pi}$ (D) $\frac{4S}{\pi}$

5. If 'I', 'b' and 'h' if a cuboids are increased, decreased and increased by 1%, 3% and 2% respectively, then the volume of the cuboids

(A) increase

(B) decrease

(C) increase or decreases depending on original dimensions

(D) can't be calculated with given data

6. The radius and height of a cone are each increased by 20%, then the volume of the cone is increased by

(A) 20% (B) 40% (C) 60% (D) 72.8%

7. There is a cylinder circumscribing the hemisphere such that their bases are common. The ratio of their volume is

(A) 1 : 3 (B) 1 : 2 (C) 2 : 3 (D) 3 : 4

8. Consider a hollow cylinder of inner radius r and thickness of wall t and length ℓ. The volume of the above cylinder is given by

(A) $2\pi\ell(r^2 - \ell^2)$ (B) $2\pi r\ell t\left(\frac{t}{2r}+1\right)$ (C) $2\pi\ell(r^2 + t^2)$ (D) $2\pi r\ell(r + t)$

9. A cone and a cylinder have the same base area. They also have the same curved surface area. If the height of the cylinder is 3m, then the slant height of the cone (in m) is

(A) 3 (B) 4 (C) 6 (D) 7

10. A sphere of radius 3 cm is dropped into a cylindrical vessel of radius 4 cm. If the sphere is submerged completely, then the height (in cm) to which the water rises, is

(A) 2.35 (B) 2.30 (C) 2.25 (D) 2.15

SUBJECTIVE DPP # 18.2

1. The whole surface of a rectangular lock is 846 cm^2. Find the length, breadth and height, if these dimensions are in the ratio 5 : 4 : 3.
2. An open box is made of wood 3 cm thick. its external length, breadth and height are 1.48 m, 1.16 m and 8.3 dm. Find the cost of painting the inner surface at Rs 5 per m^2.
3. A room 8 m long 6 m board and 3 m high has two windows $1\frac{1}{2}$ m × 1 m and a door 2 m × $1\frac{1}{2}$ m. Find the cost of papering the walls will paper 50 cm wide at Rs. 40 per metre.
4. 50 circular plates, each of radius 7 cm and thickness $\frac{1}{2}$ cm, are placed one above the other to form a solid right circular cylinder. Find the total surface area.
5. A tent in the shape of a right circular cylinder surmounted by a right circular cone. The heights of the cylindrical and the conical parts are 40 m and 21 m respectively. If the base diameter of the tent is 56 m, find the area of the required canvas to make this tent if 20% of the area is consumed in folding and sewing.
6. A toy is in the form of a right circular cylinder closed at one end and with a hemisphere on the other end. The height and the radius of the base are 15 cm and 6 cm respectively. The radius of the hemisphere are cylinder are same. Calculate the total surface area and the volume of the toy. if the toy is painted at the rate of Rs. 2.50 per 10 cm^2, find the cost of painting the toy.
7. An iron pillar has some portion in the form of a right circular cylinder an remaining in the form of a right circular cone. The radius of the base of each of the cone and the cylinder is 8 cm. The cylindrical portion is 240 cm high and the conical part is 36 cm high. Find the weight of the pillar, if one cubic cm of iron weights 7.8 g.
8. A solid metallic sphere of diameter 28 is melted and recasted into a number of smaller cones, each of diameter $4\frac{2}{3}$ cm and height 3 cm. Find the number of cones so formed.

ANSWER KEY

(Objective DPP # 18.1)

Qus.	1	2	3	4	5	6	7	8	9	10
Ans.	C	B	D	D	B	D	C	B	C	C

(Subjective DPP # 18.2)

1. 15 cm, 12 cm, 9 cm
2. Rs. 27.97
3. Rs. 62.40
4. 1408 cm^2
5. Total surface area = 12144 cm^2
6. Surface area ≈ 678.86 cm^2, Volume ≈ 1470.86 cm^3, Cost ≈ Rs. 170
7. 395.37 kg.
8. 672 cones.

STATISTICS

ML - 19

INTRODUCTION

The branch of science known as Statistics has been used in India from ancient times. Statistics deals with collection of numerical facts i.e., data, their classification & tabulation and their interpretation. In statistics we shall try to study, in detail about collection, classification and tabulation of such data.

(a) Importance of Data :

Expressing facts with the helps of data is of great importance in our day-today life. For example, instead of saying that India has a large population it is more appropriate to say that the population of India, based on the census of 2000 is more than one billion.

(b) Collection of Data :

On the basis of methods of collection, data can be divided into two categories :

(i) Primary data : Data which are collected for the first time by the statistical investigator or with help of his workers is called primary data. As example if an investigator wants to study the condition of the workers working in a factory then fro this he collects some data like their monthly income, expenditure, number or brother, sisters, etc.

(ii) Secondary data : These are the data already collected by a person or a society and these may be in published or unpublished form. These data should be carefully used. These are generally obtained from the following two sources.

(A) Published sources

(B) unpublished sources

(c) Classification of Data :

When the data is complied in the same form and order in which it is collected, it is known as Raw Data, It is also **Crude Data.** For example, the marks obtained by 20 students of class X in English out of 10 marks are as follows :

7,	4,	9,	5,	8,	9,	6.	7,	9,	2,
0	3,	7,	6,	2,	1,	9,	8,	3,	8,

(i) Geographical basis : Here, the data is classified on the basis of place or region. For example the production of food grains of different state is shown in the following table :

S.No.	State	Production (in Tons)
1	Andhdra Pradesh	9690
2	Bihar	8074
3	Haryana	10065
4	Pubjab	17065
5	Uttar Pradesh	28095

(ii) Chronological classification : If data's classification is based on hour, day, week and month or year, then it is called chronological classification, For example, the population of India in different year is shown in following table :

S.No	Year	Production (in Crores)
1	1951	46.1
2	1961	53.9
3	1971	61.8
4	1981	68.5
5	1991	88.4
6	2001	100.01

(iii) Qualitative basis : When the data are classified into different groups on the basis of their descriptive qualities and properties, such a classification is known as descriptive or qualitative classification. Since the attributes can not be measured directly, they are counted on the basis of presence or absence of qualities. For example intelligence, literacy, unemployment, honesty etc. The following table shows classification on the basis of sex and employment.

Table Population (in lacs)

Gender →	**Male**	**Female**
Position of Employment ↓		
Employed	**16.2**	**13.7**
Unemployed	**26.4**	**24.8**
Total	**42.6**	**38.5**

(iv) Quantitative basis : if facts are such that they can be measured physically e.g. marks obtained height, weight, age, income, expenditure etc. Such facts are known as variable values. If such facts are kept into classes then it is called classification according to quantitative or class intervals.

Marks obtained	10-20	20-30	30-40	40-50
No. of students	7	9	15	6

DEFINITIONS

(i) Variate : The numerical quantify whose value varies in objective is called a variate, generally a variate is represented by x. There are two types of variate.

(A) Discrete variate : its magnitude is fixed. For example, the number of teacher in different branches of a institute are 30, 35, 40 etc.

(B) Continuous variate : is magnitude is not fixed. It is expressed in groups like 10 - 20, 20 - 30, ... etc.

(ii) Rage : The difference of the maximum and the minimum values of the variable x is called range.

(iii) Class frequency : In each class the number of times a data is repeated in known as its class frequency.

(iv) Class Interval = $\dfrac{\text{Range}}{\text{Number of classes}}$

It is generally denoted by h or i.

(v) Class limits : The lowest and the highest value of the class are known as lower and upper limited restively of that class.

(vi) Class mark : The average of the lower and the upper limits of a class is called the mid value or the class mark of that class. It is generally denoted by x.

If x be the mid value and h be the class interval, then the class limits are $\left(x-\dfrac{h}{2}, x+\dfrac{4}{2}\right)$.

Ex.1 The mid values of a distribution are 54, 64, 74, 84 and 94. Find the class interval and class limits.

Sol. The class interval is the difference of two consecutive class marks, therefore class interval (h) = 64- 54 = 10.

Here the mid values are given and the class interval is 10.

So class limits are

For 1st class $54-\dfrac{10}{2}$ to $54+\dfrac{10}{2}$ or 49 to 59

For 2nd class $64-\dfrac{10}{2}$ to $64+\dfrac{10}{2}$ or 59 to 69

For 3rd class $74-\dfrac{10}{2}$ to $74+\dfrac{10}{2}$ or 69 to 79

For 4th class $84-\dfrac{10}{2}$ to $84+\dfrac{10}{2}$ or 79 to 89

For 5th class $94-\dfrac{10}{2}$ to $94+\dfrac{10}{2}$ or 89 to 99

Therefore class limits are 49 - 59, 59 - 69, 79 - 89, and 89 - 99.

FREQUENCY DISTRIBUTION

The marks scored by 30 students of IX class, of a school in the first test of Mathematics our of 50 marks are as follows :

6	32	10	17	22	28	0	48	6	22
32	6	36	26	48	10	32	48	28	22
22	22	28	26	17	36	10	22	28	0

The number of times a mark is repeated is called its **frequency.** It is denoted by f.

Marks obtained	Taly mark	Frequency	Marks obtained	Tally mark	Frequency
0	II	2	26	II	2
6	III	3	28	IIII	4
10	III	3	32	III	3
17	II	2	36	II	2
22	IIII I	6	48	III	3

Above type of frequency distribution is called **ungrouped frequency distribution.** Although this representation of data is shorter than representation of raw data, but from the angle of comparison and analysis it is quite bit. So to reduce the frequency distribution, it can be classified into groups is following ways and it is called **grouped frequency distribution.**

Class	Frequency
0-10	8
11-20	2
21-30	12
31-40	5
41-50	3

(a) Kinds of Frequency Distribution :

Statistical methods like comparison, decision taken etc. depends of frequency distribution. Frequency distribution are of three types.

(i) Individual frequency distribution : Here each item or original price of unit is written separately. In n this category, frequency of each variable is one.

Ex.2 Total marks obtained by 10 students in a class.

S.No.	1	2	3	4	5	6	7	8	9	10
Marks obtained	46	18	79	12	97	80	5	27	67	54

(ii) Discrete frequency distribution : When number of terms is large and variable are discrete, i.e., variate can accept some particular values only under finite limits and is repeated then its called discrete frequency distribution. For example the wages of employees and their numbers is shown in following table.

Monthly wages	No. Of employees
4000	10
6000	8
8000	5
11000	7
20000	2
25000	1

The above table shows ungrouped frequency distribution the same facts can be written in grouped frequency as follows :

Monthly wages	No. of employees
0-10,000	23
11,000-20,000	9
21,000-30,000	1

NOTE :

If variable is repeated in individual distribution then it can be converted into discrete frequency distribution.

(iii) Continuous frequency distribution : When number of terms is large and variate is continuous. i.e., variate can accept all values under finite limits and they are repeated then it is called continuous frequency distribution. For example age of students in a school is shown in the following table :

Age (in year)	Class	No. of students
Less than 5 year	0-5	72
Between 5 and 10 y ear	5-10	103
Between 10 and 15 year	10-15	50
Between 15 and 20 year	15-20	25

NOTE :
Continuous frequency distribution is generally represented in form of grouped frequency distribution and variate is continuous in i, so 0 - 5, 6 - 10, 11 - 15, 16 - 20 types of classes can not be made here. If such classes are made in the table then students of age 5 to 6 year or 10 to 11 year or 15 to 16 years can not be classified. if such type of classes are given then they should be made continuous by following methods. Half of the difference between classes should be added to the upper limit of lower class and subtracted from lower limit o upper class. Thus the classes 0 - 5.5, 5.5 - 10.5, 10.5 - 15.5, 15.5 - 19.5 are obtained which are continuous.
Classes can be made mainly by two methods :
(i) Exclusive series : In this method upper limit of the previous class and lower limit of the next class is same. In this method the term of upper limit in a class is not considered in the same class, it is considered in the next class.
(ii) Inclusive series : In this method value of upper and lower limit are both contained in same class. In this method the upper limit of class and lower limit of other class are not same. Some time the value is not a whole number, it is a fraction or in decimals and lies in between the two intervals then in such situation the class interval can be constructed as follows

A		B	
Class	**Frequency**	**Or Class**	**Frequency**
0-9	4	0-9.99	4
10-19	7	10-19.99	7
20-29	6	20-29.99	6
30-39	3	30-39.99	3
40-49	3	40-49.99	3

CUMULIVE FREQUENCY

(i) Discrete frequency distribution : From the table of discrete frequency distribution, it can be identified that number of employees whose monthly income is 4000 or how many employees of monthly income 1100 are there. But if we want to know how many employees whose monthly income is upto 11000, then we should add 10 + 8 + 5 7 i.e., number of employees whose monthly income is upto 11000 is 30. Here we add all previous frequency and get cumulative frequency. If will be more clear from the following table

Class	Frequency (f)	Cumulative frequency (cf)	Explanation
4000	10	10	10 = 1 0
6000	8	18	10 + 8
8000	5	23	18 + 5
11000	7	30	23 + 7
20000	2	32	30 + 2
25000	1	33	32 + 1

(ii) Continuous frequency distribution : In the previous page we obtained cumulative frequency for discrete series. Similarly cumulative frequency table can be made from continuous frequency distribution also. For example, for table :

Monthly income	No. of employee	Cumulative	Explanation
Variate (x)	Frequency (f)	Frequency (cf)	
0 – 5	72	72	72 = 72
5 – 10	103	175	72 + 103 = 175
10 – 15	50	225	175 + 50 = 225
15 - 20	25	250	225 + 25 = 250

Above table can also be written as follows :

Clas	Cumulative Frequency
Less than 5	72
Less than 10	175
Less than 15	225
Less than 20	250

From this table the number of students of age less than the upper limit of a class, i.e., number of student whose age is less than 5, 10, 15, 20 year can determined by merely seeing the table but if we need the number students whose age is more than zero, more than 5, more than 10 or more than 15, then table should be constructed as follows :

Class	Frequency	Age Cumulative frequency	Explanation
0 – 5	72	0 and more 50	250 = 250
5 – 10	103	5 and more 78	250 – 72 = 178
10 - 15	50	10 and more 75	178 – 103 = 75
15 - 20	25	15 and more 25	75 – 50 = 25

GRAPHICAL REPRESANTATION OF DATA

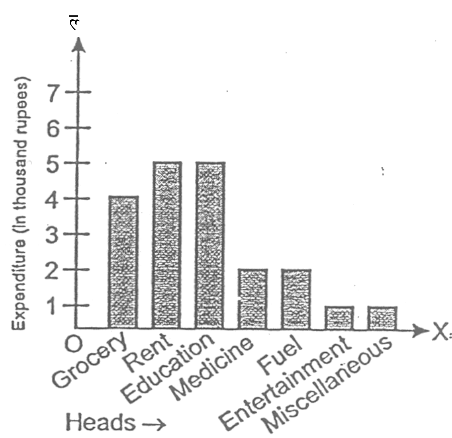
(i) Bar graphs
(ii) Histograms
(iii) Frequency polygons
(iv) Frequency curves
(v) Cumulative frequency curves or Ogives.
(vi) Pie Diagrams

(a) Bar Graphs :

Ex.3 A family with monthly income of Rs. 20,000 had planned the following expenditure per month under various heads: Draw bar graph for the data given below :

Heads	Expenditure (in Rs. 1000)
Grocery	4
Rent	5
Education of children	5
Medicine	2
Fuel	2
Entertainment	1
Miscellaneous	1

Sol.



Histogram : Histogram is rectangular representation of grouped and continuous frequency distribution in which class intervals are taken as base and height of rectangles are proportional to corresponding frequencies. To raw the histogram class intervals are marked along x-axis on a suitable scale. Frequencies are marked along y-axis on a suitable scale, such that the **areas of drawn rectangles are** proportional **to corresponding frequencies.**

Now we shall study construction of histograms related with four different kinds of frequency distributions.

(i) When frequency distribution is grouped and continuous and class intervals are also equal.

(ii) When frequency distribution is grouped and continuous but class interval are not equal.

(iii) When frequency distribution is grouped but not continuous.

(iv) When frequency distribution is ungrouped and middle points of the distribution are given.

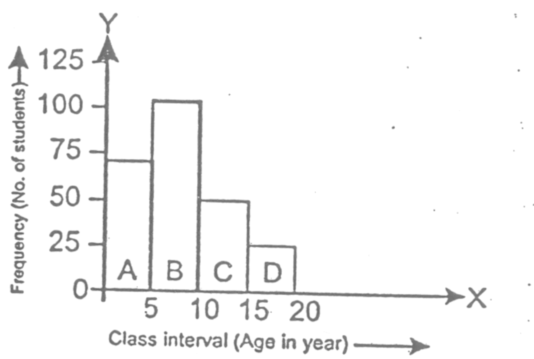
Now we try to make the above facts clear with some examples.

Ex.4 Draw a histogram of the following frequency distribution.

Clas (Age in year)	0 – 5	5 – 10	10 – 15	15 – 20
No. of students	72	103	50	25

Sol. Here frequency distribution is grouped and continuous and class intervals are also equal. So mark the class intervals on the x-axis i.e., age in year (scale 1 cm = 5 year). Mark frequency i.e., number of students (scale 1 cm = 25 students) on they y-axis.

Now, since the number of students in class interval 0 - 5 is 72, so draw a parallel line to x-axis in front of frequency to construct a rectangle on class interval 0 - 5. Repeating this procedure construct rectangle A, B, C and D.



Ex.5 The weekly wages of workers of a factory are given in the following table. Draw histogram for it.

Weekly wages	1000 - 2000	2000 – 2500	2500 – 3000	3000 – 5000	5000 – 5500
No. of workers	26	30	20	16	1

Sol. Here frequency distribution is grouped and continuous but class intervals are not same. Under such circumstances the following method is used to find heights of rectangle so that heights are proportional to frequencies.

(i) Write interval (h) of the least interval, here h = 500.

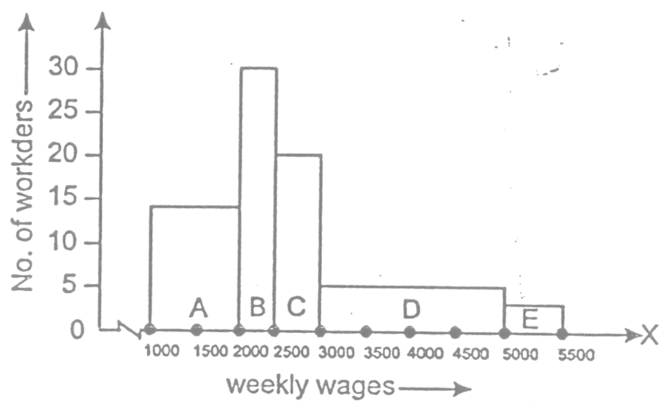
(ii) Redefine the frequencies of classes by the using the following formula.

Redefined frequency of class = $\frac{h}{\text{clss int erval}}$ × frequency of class interval.

So here the redefined frequency table is obtained as follows :

Weekly wages (in Rs.)	No. of workers	Redefined of workers
1000 – 2000	26	$\frac{500}{1000}\times 26 = 13$
2000 – 2500	30	$\frac{500}{500}\times 30 = 30$
2500 – 3000	20	$\frac{500}{500}\times 20 = 20$
3000 – 5000	16	$\frac{500}{2000}\times 16 = 4$
5000 – 5500	1	$\frac{500}{500}\times 1 = 1$

Now mark class interval on x-axis (scale 1 cm = 500) and no. of workers on y-axis (scale 1 cm = 5). On the basis of redefined frequency distribution construct rectangle A, B, C D and E.



This is the required histogram of the given frequency distribution

(a) Difference Between Bar Graph and Histogram

(i) In histogram there is no gap in between consecutive rectangle as in bar graph.

(ii) The width of the bar is significant in histogram. In bar graph, width is not important at all.

(iii) In histogram the areas of rectangles are proportional to the frequency, however if the class size of the frequencies are equal then height of the rectangle are proportional to the frequencies.

Frequency polygon : A frequency polygon is also a form a graphical representation of frequency distribution. Frequency polygon can be constructed in two ways :

(i) With the help of histogram

(ii) Without the help of histogram

(A) Following procedure is useful to draw a frequency polygon with the help of histogram.

(a) Construct the histogram for the given frequency distribution.

(b) Find the middle point of each upper horizontal line of the rectangle.

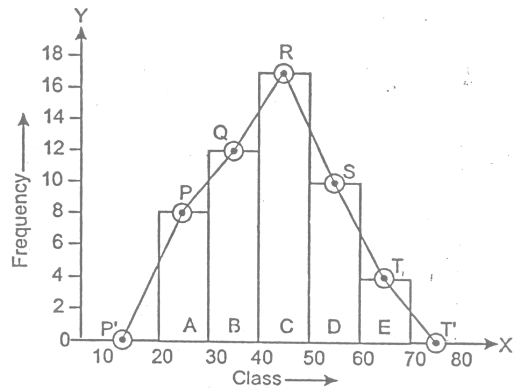
(c) Join these middle points of the successive rectangle by straight lines.

(d) Join the middle point of the initial rectangle with the middle point of the previous expected class interval on the x-axis.

Ex.6 For the following frequency distribution, draw a histogram and construct a frequency polygon with it.

Class	20 – 30	30 – 40	40 – 50	50 – 60	60 - 70
Frequency	8	12	17	9	4

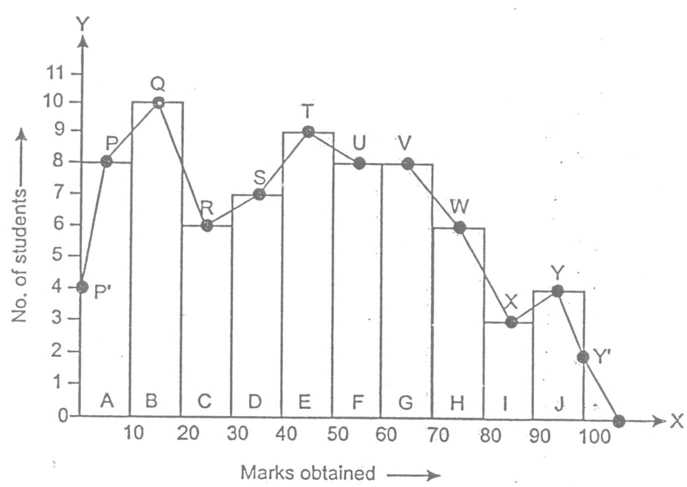
Sol. The given frequency distribution is grouped and continuous, so we construct a histogram by the method given earlier. Join the middle points P,Q,R,S,T of upper horizontal line of each rectangles A,B,C,D,E by straight lines.



Ex.7 Draw a frequency polygon of the following frequency distribution table.

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100
Frequency	8	10	6	7	9	8	8	6	3	4

Sol. Given frequency distribution is grouped and continuous. So we construct a histogram by using earlier method. Join the middle points of P,Q,R,S,T,U,V, W,X, Y of upper horizontal lines of each rectangle A,B,C,D,E,F,G,H,I,J by straight line in successions.



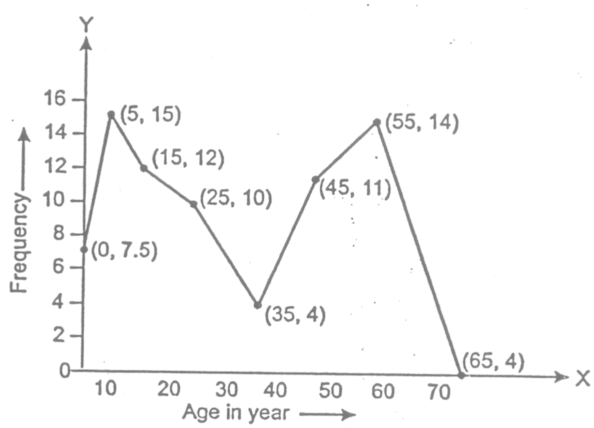
Ex.8 Draw a frequency polygon of the following frequency distribution.

Age (in years)	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Frequency	15	12	10	4	11	14

Sol. Here frequency distribution is grouped and continuous so here we obtain following table on the basis of class.

Age (in years)	0 – 10	10 – 20	20 – 20	30 – 40	40 – 50	50 – 60
Class mark	5	15	25	35	45	55
Frequency	15	12	10	4	11	14

Now taking suitable scale on graph mark the points (5, 15), (15,12), (25, 10) (35, 4), (45, 11), (55, 14). Since age can not be negative so instead of joining corner (5,15) with middle point of zero frequency of earlier assumed class, we draw vertical line from the lower limit of this class i.e., 0 and point of half frequency of this lie i.e., (0, 7.5) is joined by the end point. Joint the last point (55, 14) with the points of zero frequency of the next assumed class i.e, with (65, 0).



MEASURES OF CENTRAL TENDENCY

The commonly used measure of central tendency are -

(i) Mean

(ii) Median

(iii) Mode

(a) Mean :

The mean of a number of observation is the sum of the values of all the observations divided by the total number of observations. It is denoted by the symbol $\bar{x}$, read as x bar.

(i) **properties of mean :**

(a) If a constant real number 'a' is added to each of the observation than new mean will be $\bar{x}+a$.

(b) If a constant real number 'a' is subtracted from each of the observation then new mean will be $\bar{x}-a$

(c) If a constant real number 'a' is multiplied with each of the observation then new mean will be $\bar{x}$

(d) If each of the observation is dived by a constant no 'a' then new mean will be $\frac{\bar{x}}{a}$.

(ii) Mean of ungrouped data : If $x_1, x_2, x_3, \ldots, x_n$ are then n values (or observations) then A.M. (Arithmetic mean) is

$$\bar{x} = \frac{x_1 + x_1 + ... + x_n}{n} = \frac{\sum_{i-1}^{n} x_i}{n}$$

$$n\bar{x} = \text{Sum of observation} = = \sum_{i-1}^{n} x_i$$

i.e. product of means & no. of items given sum of observation.

Ex.9 Find the mean of the factors of 10

Sol. factors of 10 are 1,2,5 & 10.

$$\bar{x} = \frac{1+2+5+10}{4} = \frac{18}{4} = 4.5$$

Ex.10 If the mean of 6,4,7 P and 10 is 8 find P.

Sol. $8 = \frac{6+4+7+P+10}{5} \Rightarrow P = 13 \Rightarrow P = 13$

(iii) Method for Mean of ungrouped frequency distribution.

x_i	f_i	$f_i x_i$
x_1	f_1	$f_1 x_1$
x_2	f_2	$f_2 x_2$
x_3	f_3	$f_3 f_3$
.	.	.
.	.	.
.	.	.
x_n	f_n	$f_n x_n$
	$\sum f_i =$	$\sum f_i x_i =$

Then mean $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$

(iv) Method for Mean of grouped frequency distribution.

Ex.11 **(1) Direct Method : for finding mean**

Marks	No. of students fi	mid values xi	fixi
10 – 20	6	15	90
20 – 30	8	25	200
30 – 40	13	35	455
40 – 50	7	45	315
50 – 60	3	55	165
60 – 70	2	65	130
70 - 80	1	75	75
	$\sum f_1 = 40$		$\sum f_i x_i = 40$

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} \frac{1430}{40} = 35.75$$

(v) Combined Mean :

$$\bar{x} = \frac{n_1 \bar{x}_1 + n_2 + \bar{x}_2 + ...}{n_1 + n_2 + ...}$$

(vi) Uses of Arithmetic Mean

(A) It is used for calculating average marks obtained by a student.

(B) It is extensively used in practical statistics.

(C) It is used to obtain estimates.

(D) It is used by businessman to find out profit per unit article, output per machine, average monthly income and expenditure etc.

(b) Median :

Median of a distribution is the value of the variable which divides the distribution into two equal parts.

(i) Median or ungrouped data

(A) Arrange the data in ascending order.

(B) Count the no. of observations (Let there be 'n' observations)

(C) If n is odd then median = value of $\left(\frac{n+1}{2}\right)^{th}$ observation.

(D) If n is even the median = value of mean of $\left(\frac{n}{2}\right)^{th}$ observation and $\left(\frac{n}{2}+1\right)^{th}$ observation.

Ex.12 Find the median of the following values :

37, 31, 42, 43, 46, 25, 39, 45, 32

Sol. Arranging the data in ascending order, we have

25, 31, 32, 37, 39, 42, 43, 45, 46

Here the number of observations n = 9 (odd)

$\therefore$ Median $= \text{Value of } \left(\frac{9+1}{2}\right)^{th}$ observation

$= \text{Value of } 5^{th}$ observation

$= 39.$

Ex.13 The median of the observation 11, 12, 14, 18, x + 2, x + 4, 30, 32, 35, 41 arranged in ascending order is 24. Find the value of x.

Sol. Here, the number of observations n = 10. Since n is even, therefore

$$\text{Median } = \frac{\left(\frac{n}{2}\right)^{th} \text{conservation} + \left(\frac{n}{2}+1\right)^{th} \text{observation}}{2}$$

$$\Rightarrow \quad 24 = \frac{5^{th} \text{ observation} + 6^{th} \text{ observation}}{2}$$

$$\Rightarrow \quad 24 = \frac{(x+2)+(x+4)}{2}$$

$$\Rightarrow \quad 24 = \frac{2x+6}{2} \Rightarrow 24 = x+3 \Rightarrow x = 21.$$

Hence, x = 21

(ii) Uses of Median :

(A) Median is the only average to be used while dealing with qualitative data which cannot be measured quantitatively but can be arranged in ascending or descending order or magnitude.

(B) It is used for determining the typical value in problems concerning wages, distribution of wealth etc.

(c) Mode :

(i) Mode or ungrouped data (By inspection only) : Arrange the data in an array and then count the frequencies of each variate. The variate having maximum frequency is the mode.

Ex.13 Find the mode of the following array of an individual series of scores 7, , 10, 12, 12, 12, 11, 13, 13, 17.

Number	7	10	11	12	13	17
Frequency	2	1	1	3	2	1

$\therefore$ Mode is 12

(ii) Uses of Mode : Mode is the average to be used to find the ideal size, e.g., in business forecasting, in manufacture of ready-made garments, shoes etc.

(c) Empirical Relation between Mode, Median & Mean :

Mode = 3 Median - 2 Mean

RANGE

The range is the difference between the highest and lowest scores of a distribution. It is the simplest measure of dispersion. It gives a rough idea of dispersion. This measure is useful for ungrouped data.

(a) Coefficient of the Range :

If ℓ and h are the lowest and highest scores in a distribution then the coefficient of the Range $= \frac{h - \ell}{h + \ell}$

Ex.14 Find the range of the following distribution : 1, 3, 4, 7, 9, 10, 12, 13, 14, 16 and 19.

Sol. $\ell = 1$, h = 19

$\therefore$ Range = h - ℓ = 19 - 1 = 18 **Ans.**

Ex.15 Find the range of the following frequency distribution :

Class – Interval	Frequency
0 – 5	6
5 – 10	8
10 – 15	12
15 – 20	5
20 - 25	4

Sol. The range is the difference between the mid value of the least class-interval and the greatest class interval.

Mid value of least class interval = $\frac{0+5}{2}$ = 2.5

Mid value of greatest class interval = $\frac{20+25}{2}$ = 22.5

$\therefore$ Range = 22.5 - 2.5 = 20 **Ans.**

EXERCISE

OBJECTIVE DPP # 19.1

1. The median of following series is 520, 20, 340, 190, 35, 800, 1210, 50, 80

(A) 1210 (B) 520 (C) 190 (D) 35

2. If the arithmetic mean of 5, 7, 9, x is 9 then the value of x is

(A) 11 (B) 15 (C) 18 (D) 16

3. The mode of the distribution 3,5,7,4,2,1,4,3,4 is

(A) 7 (B) 4 (C) 3 (D) 1

4. If the mean and median of a set of numbers are 8.9 and 9 respectively, then the mode will be

(A) 7.2 (B) 8.2 (C) 9.2 (D) 10.2

5. A student got marks in 5 subjects in a monthly test is given below :

(A) 2,3,4,5,6, in these obtained marks, 4 is the

(A) Mean and median (B) Median but no mean (C) Mean but no median (D) Mode

6. What is the mode from the following table :

Marks obtained	3	1	23	33	43
Frequency (f)	7	11	15	8	3

(A) 13 (B) 43 (C) 33 (D) 23

7. If the class intervals in a frequency distribution are (72 - 73.9), (74 - 75.9), (76 - 77.9), (78 - 79.9) etc., the mid-point of the class (74 - 75.9) is

(A) 74.50 (B) 74.90 (C) 74.95 (D) 75.00

8. Which one of the following is not correct -

(A) Statistics is liable to be misused

(B) The data collected by the investigator to be used by himself are called primary data

(C) Statistical laws are exact

(D) Statistics do not take into account of individual cases

9. If the first five elements of a se replaced by $(x_1 + 5)$, where i = 1,2,3,...^{5} and the next five elements are replaced by $(x_i - 5)$, where = 6 ..... 10 then the mean will change by

(A) 25 (B) 10 (C) 5 (D) 0

10. The following numbers are given 61, 62, 63, 61, 63, 64, 64, 60, 65, 63, 64, 65, 66, 64. The difference between their mean and median is

(A) 0.4 (B) 0.3 (C) 0.2 (D) 0.1

11. The value of $\sum_{i=1}^{n}(x_i - \bar{x})$ where $\bar{x}$ is the arithmetic mean of x_1 is

(A) 1 (B) $n\bar{x}$ (C) 0 (D) None of these

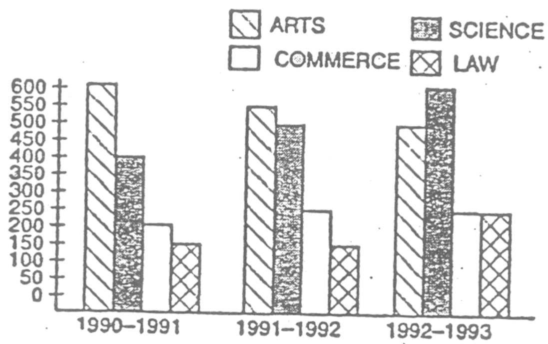
12. The average of 15 numbers is 18. The average of first 8 is 19 and that last 8 is 17, then the 8th number is

(A) 15 (B) 16 (C) 18 (D) 20

13. In an examination, 10 students scores the following marks in Mathematics 35, 19, 28, 32, 63, 02, 47, 31, 13, 98. It rage is

(A) 96 (B) 02 (C) 98 (D) 50

Direction : question 15 is based on the histogram given in the adjacent figure.



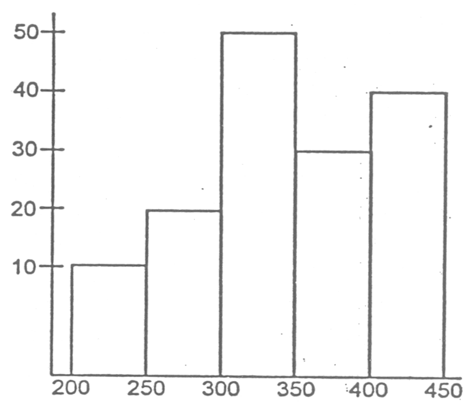
14. The percentage of students in science faculty in 1990-91 is :

(A) 26.9% (B) 27.8% (C) 29.6% (D) 30.2%

15. For the scores 8,6,10,12,1,5,6 and 6 the Arithmetic mean is

(A) 6.85 (B) 6.75 (C) 6.95 (D) 7

Direction : Each question from 16 to 18 is based on the histogram given in the adjacent figure.



16. What is the number of worker earning Rs. 300 to 350 ?

(A) 50 (B) 40 (C) 45 (D) 130

17. In which class interval of wages there is the least number of workers ?

(A) 400 - 450 (B) 350 - 400 (C) 250 - 300 (D) 200 - 250

18. What is the upper limit of the class-interval 200-250

(A) 200 (B) 250 (C) 225 (D) None of these

SUBJECTIVE DPP # 19.2

1. Find the mean of following data 13,17,16,14,11,13,10,16,11,18,.12,17.
2. Find the median of following data 38,70,48,34,42,55,63,46,54,44.
3. Find the mode of following data 2,2,6,5,4,3,4,5,7,9,4,5,3,1,10,4.
4. Find the median of :
(i) 5,30,15,6,18,22,26,32,6,9,18
(ii) 92,88,62,53,55,59,60,61,85,89
(iii) 66,69,108,72,78,82,98,99,102,101
5. Find the value of pm if the median of following observations is 48.
14, 17, 33, 35, p-5, p + 7, 57, 63, 69, 80. The above observation are in ascending order.]
6. Find the missing frequencies of the following distribution if it is known that mean of the distribution is 50.

x:	10	30	50	70	90	Total
f :	17	f_1	32	f_2	19	120

7. Find the mean for following data.

Age (Years)	25 – 30	30 – 35	35 – 40	40 – 45	45 – 50	50 – 55
No. of teachers	30	23	20	14	10	3

8. Calculate the mean of the following frequency distribution :

Marks	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No. of Students	3	6	13	15	14	5	4

9. The mean of a certain group of observations is 78. Find the resulting mean, if the value of each observation is :
(i) increased by 2 (ii) decreased by 3 (iii) multiplied by 1.5
(iv) divided by 2 (v) increased by 30% (iv) diminished by 25%
10. Draw a histogram to represent the following data :

Class-Interval	40-60	60-80	80-100	100-120	120-140	140-160	160-180	180-200
Frequency	20	40	30	50	30	20	10	40

11. Draw a bar-graph to represent the following

A	B	C	D	E	F
60	70	55	40	90	50

ANSWER KEY

(Objective DPP 19.1)

Qus.	1	2	3	4	5	6	7	8	9	10
Ans.	C	B	B	C	A	D	C	C	D	B
Qus.	11	12	13	14	15	16	17	18		
Ans.	C	C	A	C	B	A	D	B		

1. 14 **2.** 47 **3.** 4 **4.** (i) 18 (ii) 61.5 (iii) 90
5. P = 47 **6.** 28.24 **7.** 35.5 **8.** 55.33
9. (i) 80 (ii) 75 (iii) 117 (iv) 39 (v) 101.4 (iv) 58.5

➤➤➤ PROBABILITY ➤➤➤

ML - 20

PROBABILITY

Theory of probability deals with measurement of uncertainty of the occurrence of same event or incident in terms of percentage or ratio.

(i) Sample Space : Set of possible out comes.

(ii) Trial : Trial is an action which results in one of several outcomes.

(iii) An experiment : An experiment is any kind of activity such as throwing a die, tossing a coin, drawing a card. outcome of an experiment. The different possibilities which can occur during an experiment.

e.g. on throwing a dice, 1 dot, 2 dots, 3 dots, 4 dots, 5 dots, 6 dots can occur.

(iv) An event : getting a 'six', in a throw of dice, getting a head, in a toss of a coin.

(v) A random experiment : Whenever we do some experiment at once.

(vi) Equally likely outcomes : there are equal uncertainty in getting 1 dot, 2 dots, 3 dots, 4 dots, 5 dots, 6 dots when we throw a single dice.

(vii) Probability of an event A: Written as P(A) in a random experiment and is defined as -

$$P(A) = \frac{\text{Number of outcomes in favour of A}}{\text{Total number of possible outcomes}}$$

(a) Important Properties :

(i) $0 \le P(A) \le 1$

(ii) P (not happening of (A) + P(happening of A) = 1

or $\quad P(\bar{A}) = P(A) = 1$

$\therefore \quad P(\bar{A}) = 1\text{-}P(A)$

Probability of the happening of A = $\dfrac{\text{Number of favourable outcomes}}{\text{Total number possible outcomes}}$

$$\frac{m}{m+n}$$

Probability of not happening of A (falling of A) = $\dfrac{n}{m+n}$

where is for an event A can happen in m ways and fail in n ways all these ways being equally likely to occur.

(b) Problems of Die :

(i) A die is thrown once. What is the probability of -

(A) Getting an even number in the throwing of a die, the total number of outcomes is 6.

Let be the event of getting an even number then there are three even numbers 2, 4, 6.

$\therefore$ number of favourable outcomes = 3.

$\therefore \quad P(A) = \dfrac{\text{no. of faourable outcomes}}{\text{total no. of outcomes}} = \dfrac{3}{6} = \dfrac{1}{2}.$

(B) Getting an odd number (A) total outcomes = 6, favourable outcomes = 3(1, 3, 5)

$\therefore \quad P(A) = \dfrac{3}{6} = \dfrac{1}{2}$

(C) Getting a natural number $P(A) = \dfrac{6}{6} = 1$

(D) Getting a number which is multiple of 2 and 3 $= \left(\dfrac{\text{Fabourable cases}}{6}\right)$

$\therefore$

(E) Getting a number $\geq 3(3,4,5,6)$

$P(A) = \dfrac{4}{6} = \dfrac{2}{3}$

(F) Getting a number 5 or 6 (5 or 6) $P(A) = \dfrac{2}{6} = \dfrac{1}{3}$

(G) Getting a number ≤ 5 $P(A) = \dfrac{5}{6}$ (1,2,3,4,5)

(c) Problems Concerning Drawing a Card :

(i) A pack of 52 cards

(ii) Face cards (King, Queen, Jack)

Ex.1 A card is drawn from a well shuffled deck of 52 cards. Find the probability of

(i) A king.

(ii) A heart.

(iii) A seven of heart.

(iv) A jack, queen or a king.

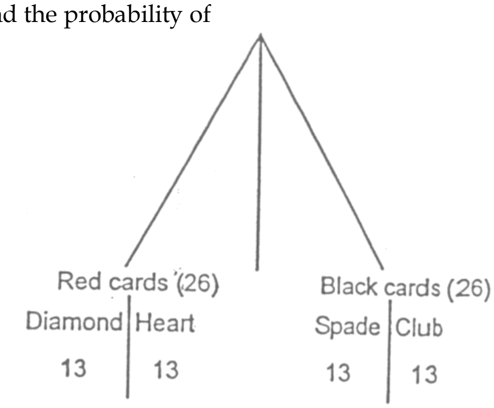
(v) A two of heart or a two of diamond.

(vi) A face card.

(vii) A black card.

(viii) Neither a heart nor a king.

(ix) Neither an ace nor a king.



Sol. Total no. of outcomes = 52

(i) A king.

No. of kings = 4 (favorable cases) $P(A) = \frac{4}{42} = \frac{1}{13}$.

(ii) A heart $P(A) = \frac{13}{52} = \frac{1}{4}$

(iii) A seven of heart $P(A) = \frac{1}{52}$

(iv) A jack, queen or a king $P(A) = \frac{12}{52} = \frac{3}{13}$

(v) A two of heat or a two of diamond. $P(A) = \frac{2}{52} = \frac{1}{26}$

(vi) A face card $P(A) = \frac{12}{52} = \frac{3}{13}$

(vii) A black card $P(A) = \frac{26}{52} = \frac{1}{2}$

(viii) Neither a heart nor a king (13 heart + 4 king, but 1 common)

$$P(A) = 1 - \frac{16}{52} = \frac{52-16}{52} = \frac{36}{52} = \frac{9}{13}$$

(ix) Neither an ace nor a king. $P(A) = \frac{44}{52} = \frac{11}{13}$

Ex.2 Two coins are tossed simultaneously. Find the probability of getting

(i) two heads (ii) at least one head (iii) no head

$\therefore$ On tossing two coins simultaneously, all the possible outcomes are

HH, HT, TH, TT.

(i) The probability of getting two heads = P (HH)

$$= \frac{\text{Even of occurence of two heads}}{\text{Total number of possible outcomes}} = \frac{1}{4}$$

(ii) The probability of getting at least on head

$$= \frac{\text{Favourable outcomes}}{\text{Total no. of outcomes}} = \frac{3}{4}$$

(iii) The probability of getting no head P(TT) = $\frac{1}{4}$

Ex.3 A bag contains 5 red balls, 8 white balls, 4 green balls and 7 black balls. If one ball is drawn at random, find the probability that it is

(i) Black

(ii) Not red

(iii) Green

Sol. Number of red balls in the bag = 5

Number of white balls in the bag = 8

Number of green balls in the bag = 4

Number of black balls in the bag = 7

$\therefore$ Total number of balls in the bag = 5 + 8 + 4 + 7 = 24.

Drawing balls randomly are equally likely outcomes.

$\therefore$ Total number of possible outcomes = 24

Now,

(i) There are 7 black balls, hence the number of such favourable outcomes = 7

$\therefore$ Probability of drawing a black ball = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}} = \frac{7}{24}$ **Ans.**

(ii) There are 5 red balls, hence the number of such favourable outcomes = 5.

$\therefore$ Probability of drawing a red ball = $\frac{\text{Numbe of favourable outcomes}}{\text{Total number of possible outcoes}} = \frac{5}{24}$ **Ans.**

$\therefore$ Probability of drawing not a red ball = P (Not Red ball) = $1 - \frac{5}{24} = \frac{19}{24}$ **Ans.**

(iii) There are 4 green balls.

$\therefore$ Number of such favourable outcomes = 4

Probability of drawing a green ball = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possibl outcomes}} = \frac{4}{24} = \frac{1}{6}$ **Ans.**

Ex.4 A card is drawn from a well - shuffled deck of playing cards. Find the probability of drawing

(i) a face card (ii) a red face card

Sol. Random drawing of cards ensures equally likely outcomes

(i) Number of face cards (King, Queen and jack of each suits) = 4 × 3 = 12

Total number of cards in deck = 52

$\therefore$ Total number of possible outcomes = 52

P (drawing a face card) = $\frac{12}{52} = \frac{3}{13}$

(ii) Number of red face cards = 2 × 3 = 6

Number of favourable outcomes of drawing red face card = 6

P (drawing of red face red) = $\frac{6}{52} = \frac{3}{26}$ **Ans.**

EXERCISE

OBJECTIVE DPP - 20.1

1. 3 Coins are tossed simultaneously. The probability of getting at least 2 heads is

(A) $\frac{3}{10}$ (B) $\frac{3}{4}$ (C) $\frac{3}{8}$ (D) $\frac{1}{2}$

2. Two cards are drawn successively with replacement from a pack of 52 cards. The probability of drowsing two aces is

(A) $\frac{1}{169}$ (B) $\frac{1}{221}$ (C) $\frac{1}{265}$ (D) $\frac{4}{663}$

3. In a single throw of two dice, the probability of getting more than 7 is

(A) $\frac{7}{36}$ (B) $\frac{7}{12}$ (C) $\frac{5}{12}$ (D) $\frac{5}{36}$

4. Two cards are drawn at random from a pack of 52 cards. The probability that both are the cards of space is

(A) $\frac{1}{26}$ (B) $\frac{1}{4}$ (C) $\frac{1}{17}$ (D) Nome of these

5. Two dice are thrown together. The probability that sum of the two numbers will be a multiple of 4 is

(A) $\frac{1}{9}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{5}{9}$

6. If the odds in favour of an event be 3 : 5 then the probability of non-happening of the event is

(A) $\frac{3}{5}$ (B) $\frac{5}{3}$ (C) $\frac{3}{8}$ (D) $\frac{5}{8}$

7. In a cricket match, a batswoman hits a boundary 6 times out of 30 balls she plays. Find the probability that she did not hit a boundary.

(A) 0.8 (B) 0.6 (C) 0.5 (D) 0.2

8. If the three coins are simultaneously tossed again compute the probability of 2 heads coming up.

(A) $\frac{3}{8}$ (B) $\frac{1}{4}$ (C) $\frac{5}{8}$ (D) $\frac{3}{4}$

9. A coin is tossed successively three times. The probability of getting one head or two heads is :

(A) 2/3 (B) 3/4 (C) 4/9 (D) 1/9

10. One card is drawn from a pack of 52 cards. What is the probability that the drawn card is either red or king:

(A) 15/26 (B) 1/2 (C) 7/13 (D) 17/32

SUBJECTIVE DPP - 20.2

1. Two dice are thrown together. Find the probability of getting a total of 9.

2. A coin and a dice are tossed simultaneously find the sample space.

3. A dice is thrown repeatedly until a six comes up. What is the sample space for this experiment.

4. On a simultaneous toss of three coins, find the probability of getting

(i) at least 2 heads

(ii) at most 2 heads

(iii) exactly 2 heads

5. Two dice are thrown simultaneously. Find the probability of getting
(i) an even number s the sum
(ii) the sum as a prime number
(iii) a doubled of even number
6. Three dice are thrown together. Find the probability of getting a total of a least 6.
7. Find the probability that a leap year selected at random will contain 53 Tuesday.
8. A coin is tossed 80 times with the following outcomes :
(i) head : 35
(ii) tail : 45
Find the probability of each event.
9. Two coins are tossed simultaneously 150 times and we get the following outcomes.
(a) No tail = 45
(b) One tail = 55
(c) Two tails = 50
Find the probability of each event.
10. In a cricket match a batsman hits a boundary 10 times out of 36 balls be play. Find the probability that he did not hit the boundary.
11. In a cricket match a batsman hits a boundary 3 times in 3 over he play. Find the probability that the did not hit the boundary.
12. A bag which contains 7 blue marbles, 4 black marbles and 9 white marbles. A marbles drawn at random from the bag then what is the probability that the drawn marble is
(i) blue
(ii) white or black
13. The odds in favour of an event are 3 : 5 find the probability of occurrence of this event.
14. Cards marked with the numbers 2 to 101 are placed in a box and mixed thoroughly. One card is drawn from this box, find the probability that the number on card is
(i) An even number
(ii) A number less than 14
(iii) A number which is a prefect square.
(iv) A prime number less than 20
15. An urn contains 6 oranges, 7 apples & 11 mango. A fruit is drawn at random, what is the probability of drawing.
(i) An orange
(ii) Not apple
(iii) An apple or a mango
16. A card is drawn at random from a well shuffled desk of playing cards. Find the probability that the card drawn is
(i) A card of spade or an ace
(ii) A red king
(iii) Neither a king nor a queen
(iv) Either a king or a queen

17. A box contains 19 balls bearing numbers 1,2,3..... 19. A ball is drawn at random from the box. Find the probability that the number on the balls is
(i) A prime number
(ii) Divisible by 3 or 5
(iii) Neither divisible by 5 nor by 10
(iv) An even number

18. There are 30 cards of same size in a bag containing numbers 1 to 30. One card is taken out from the bag at random. Find the probability that the number on the selected card is not divisible by 3.

EXERCISE

(Objective DPP # 20.1)

Qus.	1	2	3	4	5	6	7	8	9	10
Ans.	D	A	C	C	C	D	A	A	B	C

(Subjective DPP # 20.2)

1. $\frac{1}{9}$

2. (H, 1) (H, 2) (H, 3) (H, 4) (H, 5) (T, 1) (T, 2) (T, 3) (T, 4) (T, 5) (T, 6)

3. {6, (1, 6) (2, 6) (3, 6) (4, 6) (5, 6) (1, 1, 6) (1, 2, 6).... }

4. $\left(\frac{1}{2}, \frac{7}{8}, \frac{3}{8}\right)$

5. $\left(\frac{1}{2}, \frac{5}{12}, \frac{1}{12}\right)$

6. $\left(\frac{103}{108}\right)$

7. $\left(\frac{2}{7}\right)$

8. **(i)** $\frac{7}{16}$ **(ii)** $\frac{9}{16}$

9. **(a)** $\frac{3}{10}$ **(b)** $\frac{11}{30}$ **(c)** $\frac{1}{3}$

10. $\frac{13}{18}$

11. $\frac{3}{8}$

12. **(i)** $\frac{7}{20}$ **(ii)** $\frac{13}{20}$

13. $\frac{3}{8}$

14. **(i)** $\frac{1}{2}$ **(ii)** $\frac{3}{25}$ **(iii)** $\frac{2}{25}$ $\frac{9}{100}$ **(iv)** $\frac{2}{25}$

15. **(i)** $\frac{1}{4}$ **(ii)** $\frac{17}{24}$ **(iii)** $\frac{3}{4}$

16. **(i)** $\frac{4}{13}$ **(ii)** $\frac{1}{26}$ **(iii)** $\frac{11}{13}$ **(iv)** $\frac{2}{13}$

17. $\frac{8}{19}, \frac{8}{19}, \frac{16}{19}, \frac{9}{19}$

18. $\frac{2}{3}$

PROOF IN MATHEMATICS

ML - 21

STATEMENT

is a sentence which is neither an order nor a question nor an exclamatory sentence.
A sentence or statement can be

(a) a true statement (b) a false statement (c) an ambiguous statement

Examples for true statement

(i) 1 + 3 = 4, 7 + 3 = 10.
(ii) The number of days in a week is seven.
(iii) When 2x = 10, then x = 5.
(iv) There are three sides in a triangle.
(v) New Delhi the capital of India.

Examples for false statement

(i) 2 + 7 = 5 is a false statement
(ii) 9 × 2 = 15 is a false statement
(iii) 1 m = 1000 cm is a false statement
(iv) Patna is the capital of West Bengal is a false statement
(v) Sunday comes after Monday is a false statement

Examples for ambiguous statement

(i) The 7th of Mach falls on Monday.
(ii) The sum of any two angles of a triangle is 110^0.
(iii) Today is Friday.

Mathematically valid statement

Mathematically, a statement is valid or acceptable only if it is either always true or always false.

Deduction :

Deductive reasoning : To find the truth value of an unambiguous statement we use the deductive reasoning. This is the main logical tool.

AXIOM CONJECTURE AND THEOREM

(a) Axiom : Axiom is a statement which is accepted as a true statement. An axiom does not require a proof.

Example :

(i) $a = b, b = c \Rightarrow a = c$
(ii) $a > b, b > c \Rightarrow a > c$
(iii) $a = b \Rightarrow \frac{1}{2}a = \frac{1}{2}b$

(b) Conjecture : It is a statement whose truth ness or falseness has not been established mathematically.

(c) Theorem : A theorem is a mathematical statement whose truth has been established logically.

Proof of a theorem

The main parts of a proof are as under.

(i) The Hypothesis (i.e. what is given)

(ii) The conclusion (i.e. what is to be proved)

(iii) Consists of successive mathematical statements derived logically from the previous statement or axiom or hypothesis.

Ex.1 State whether the following statements are always true, always false or ambiguous, Justify your answer.

(i) There are 13 months in a year.

(ii) Diwali falls on Friday.

(iii) The temperature in Magadi is 26^0 C.

(iv) Dogs can fly.

(v) February has only 28 days.

Sol. (i) This statement is false because there are 12 months in a year.

(ii) This statement is always ambiguous because Diwali can fall on any day.

(iii) This statement is always ambiguous because it is not fixed.

(iv) This statement is always false.

(v) This is a false statement because February has 29 days in a leap year.

Ex.2 State whether the following statements are true or false. Give reasons for your answers.

(i) The sum of the interior angles of a quadrilateral is 350^0.

(ii) For any real number x, $x^2 \geq 0$.

(iii) A rhombus is a parallelogram.

(iv) The sum of two even numbers is even.

(v) The sum of two odd numbers is odd.

Sol. (i) This statement is false because the sum of the interior angles of a quadrilateral is 360^0.

(ii) This statement in always true. For example $(-2)^2 = 4$, then we can say $x^2 \geq 0$ for any real number x.

(iii) This statement is always true.

(iv) This statement is always true. For example, 2 + 2 = 4 and 6 + 4 = 10.

(v) This statement is always false. For example, 3 + 5 = 7 and 3 + 9 = 10.

Ex.3 State whether the following statements are true or false :

(i) Opposite angles of a cyclic quadrilateral are supplementary.

(ii) Every odd number greater than 1 is prime.

(iii) Exterior angle of a cyclic quadrilateral is equal to the opposite angle.

(iv) For any real number x, 5x + x = 6x.

(v) For every real number x, $x^3 \geq x$.

(vi) An exterior angle is greater than each interior opposite angle.

Sol. (i) This statement is true.

(ii) This statement is false ; for example, 9 is not a prime number

(iii) This statement is true.

(iv) This statement is true.

(v) This statement is false, for example $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$ and $\frac{1}{8}$ is not greater than $\frac{1}{2}$.

(vi) This statement is true.

Ex.4 Restate the following statements with appropriate condition so that they become true statements.

(i) Square of a real number is always greater than the number.

(ii) In a parallelogram the diagonals are equal.

(iii) There are four angles is a triangle.

Sol. (i) Square of a real number is always greater than the number when the magnitude of the number is greater than one.

(ii) In a rectangle, the diagonals are equal.

(iii) There are three and only three angles in a triangle.

Ex.5 Restate the following statements with appropriate conditions, so that they become true statements.

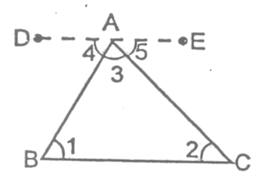
(i) All prime numbers are odd. (ii) Two times a real numbers is always even.

(iii) For any x, 3x + 1 > 4. (iv) For any x, $x^3 \geq 0$.

(v) In an equilateral triangle the medians are also an angle bisector.

Ex.6 The sum of the angles of a triangle is 180^0

Sol. **Statement :** The sum of the angles of a triangles is 180^0



Given : A Δ ABC

To Prove : $\angle 1 + \angle 2 + \angle 3 = 180^0$

Construction : Through A, draw a line DE parallel to BC.

Proof :

S.No.	Statement	Reason
1.	DE ‖ BC and AC is the transversal $\therefore \angle 1 = \angle 4$	Alternative interior $\angle$s
2.	Again DE ‖ BC and AC is the transversal $\therefore \angle 2 = \angle 2$ Adding. (1) and (2)	Alternate interior $\angle$s
3.	$\angle 1 + \angle 2 = \angle 4 + \angle 5$	Adding the corresponding side of (1) & (2)
4.	$\angle 1 + \angle 2 + \angle 3 = \angle 4 + \angle 5 + \angle 3$	Adding $\angle 3$ on the sides.
5.	But $\angle 4 + \angle 5 + \angle 3 = \angle DAE = 180^0$	$\angle DAE$ is a straight line angle.
6.	$\angle 1 + \angle 2 + \angle 3 = 180^0$	The sum of the angles of a triangle is 180^0.

Ex.7 For each natural number, n(n + 1) is multiple of 2.

Sol. We have to prove that the product (n + 1) is divisible by 2.

Now we have two cases. Either is even or odd. Let us examine each case. Suppose n is even. Then we can write n = 2 m, for same natural number m. And, then

n(n + 1) = 2m (2m + 1) which is clearly divisible by 2.

Next, suppose n is odd. Then n + 1 is even and we can write n + 1 = 2r, for some natural number 2.

We have n(n + 1) = (2r - 1 2r = 2r (2r - 1) which is clearly divisible by 2.

So, we can say that the natural number n(n + 1) is divisible by 2.

EXERCISE

SUBJECTIVE DPP # 22

1. Write down the truth value of each of the following statements.

(i) India is a democratic country.

(ii) Each prime number has exactly two factor.

(iii) $\sqrt{2}$ is an irrational number.

(iv) Jaipur is in U.P.

2. Write down the negation of the following

(i) Hindi is the mother tongue of India. (ii) India is progressing rapidly.

(iii) $(a + b)^3 = a^3 + 3a^2b + 3\ ab^2 + b^3$ (iv) 2 is the real part of 2 + 4i.

(v) 4 is multiple of 20. (vi) 2nd October is the birthday of Mahatma Gandhi.

(vii) Republic day of India held o 26th January.

(viii) The roots of the equation $x^4 + 4x^3 + 6x^2 + 4x + 1= 0$ are equal.

(ix) New-York is in England.

3. State whether the following statements are true or false. Give reason for your answers

(i) 1 m = 100 cm (ii) The isosceles triangles have to sides equal.

(iii) The sum of two odd number is even. (iv) Three and three makes six.

(v) February has 30 days.

Prove that all $n \in N$ (Q. No. 4 to 6)

4. $1^2 + 2^2 + 3^2 + 4^2 + ... + n^2 = \frac{n(n+1(2n+1)}{6}$

5. $2^{2n} - 1$ is divisible by 8.

6. $1 + 3 + 5 + ... + (2n - 1) = n^2$.

7. Whish of the following sentences are statements :

(i) It is hot day. (ii) Qutubminar is in Lucknow.

(iii) Don't talk, please (iv) Hurrah ! India has won the match.

(v) Rasika is a sincere girl. (vi) Will it rain today ?

(vii) 2 + 3 = 5. (viii) 5 + 7 = 10.

(ix) x + 2 = 11. (x) Every prime number has only one factor.

8. A number can be divided into three equal parts if the sum of its digits is divisible by 3. Based on the above statement can 9875340 be divided into three equal groups.

9. Look at the following pattern :

$11^1 = 11 =$

$11^2 = 121 =$

$11^3 = 1331 =$

$11^4 = 14641 =$

$11^5 = 161051 =$

$11^6 = 1771561 =$

Is 19487171 power of 11.[Hint : Sum of digits at odd places - Sum of digits at even places = 0]

10. $a^2 + b^2$ is a prime for all whole numbers a, b.

ANSWER KEY

(Subjective DPP # 22)

1. (i) T (ii) T (iii) T (iv) F

2. (i) Hindi is not the mother tongue of India

(ii) India is not progressing rapidly.

(iii) $(a + b)^3 \neq a^3 + 3a^2b + 3\,ab^2 + b^3$.

(iv) 2 is not the real part of 2 + 4i.

(v) 4 is not multiple of 20.

(vi) 2nd October is not the birthday of Mahatma Gandhi.

(vii) Republic day of India held not on 26th January.

(viii) The roots of the equation $x^4 + 4x^3 + 6x^2 + 4x + 1 = 0$ are not equal.

(ix) New-York is not in England.

3. (i) T

(ii) T

(iii) T

(iv) F

(v) F

7. (i), (ii). (v), (vii), (ix)

8. Number 9 8 7 5 3 4 0 can be divided into three equal groups.

9. Yes $11^7 = 19487171 = [1 + 4 + 7 + 7] - [9 + 8 + 1 + 1] = 0$

10. For a = 3, b = 4, $a^2 + b^2$, is not a prime.

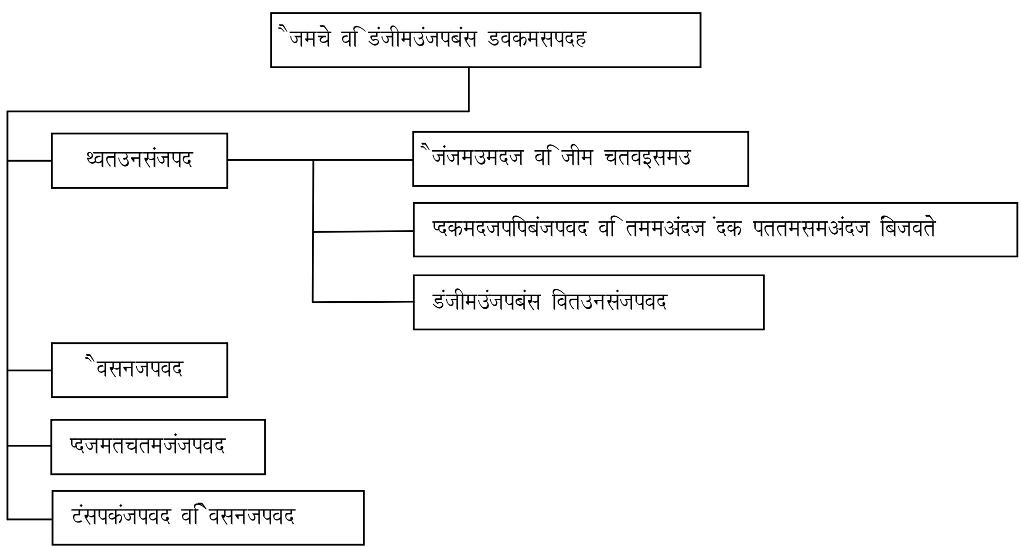
MATHMETICAL MODELING

ML - 22

Definition : mathematical model is a mathematical relation that describes some real life situation. e.g. To find the area of an equilateral triangle we can use

Area = $\frac{\sqrt{3}}{4}$ (side)^2

This formula is an example of mathematical model.



Ex.1 A car travelled 416 kilometres on 52 litres of petrol. I have to go by same car to a place which is 96 km away. How much petrol do I need?

Following steps include to solve the problem.

Sol. **Formulation :** Farther we travel, the more petrol we require, that is, the amount of petrol we need varies directly with the distance we travel.

Petrol needed for travelling 416 km = 52 litres

Petrol needed for travelling 96 km = ?

Mathematical Description :

Let x = distance traveled

y = need of Petrol and y varies directly with x

So, y = Kx, where K is a constant.

I can travel 416 kilometres with 52 litres of petrol.

So, $y = 52, x = 416$

$\therefore \quad K = \frac{y}{x} = \frac{52}{416} = \frac{1}{8}$

$\therefore \quad y = \frac{1}{8}x$ ....(1)

Equation (1) describes the relationship between the petrol needed and distance travelled.

Step 2 : We want to find the petrol we need to travel 96 kilometres. So we have to find the value of y when $x = 96$. Putting $x = 96$ in (1), we have $= \frac{96}{8} = 12$

Step 3 . Interpretation:

Since $y = 12$, we need 12 litres of petrol to travel 96 kilometres.

Step 4. Validation of the result :

This result is valid only if all the conditions remain same i.e. mileage of car track on which car is running, gradient of the track (road), etc.

Ex.2 Suppose Rakesh has invested Rs. 20,000 at 12% simple interest per year. With the return from the investment, he wants to buy a colour. T.V. that cost Rs. 25,000. For what period should he invest Rs. 20,000 so that he has enough money to buy a colour T.V. ?

Sol. **Step 1. Formulation of the problem :**

Here, we know the principal and the rte of interest in the amount Rakesh needs in addition to Rs. 20,000 to buy the colour T.V. We have to find the number of years.

Mathematical Description :

The formula for simple interest is

$$S.I = \frac{Pnr.}{100}$$

Where P = Principal

n = Number of years

r% = Rate of interest

S.I. = Interest earned

Here, the principal = Rs. 20,000

The money required by Rakesh for buying a colour T.V. = Rs. 25000

So, the interest to be earned = Rs. (25,000 - 20,000) = Rs. 5,000

The number of years for which Rs. 20,000 is deposited = n

The Interest of Rs. 20,000 for n years at the rate of 12% = S.I.

Then, $S.I. = \frac{20{,}000 \times n \times 12}{100}$

So, $S.I. = 2400\, n$ ....(2)

Give the relationship between the number of years and interest, if Rs. 20,000 is invested an annual interest rate of 12%. We have to find the period in which the interest earned is Rs. 5,000. Putting S.I. = 5,000 in (1), we have

$5{,}000 = 2400\, n$

Step 2. Solution of the problem :

Solving equations (2), we get

$$n = \frac{5000}{2400} = \frac{50}{24} = 2\frac{1}{12}$$

Step 3 : Interpretation : Since n = $2\frac{1}{12}$ and one twelfth of a year is one month Rakesh can buy a colour T.V. after 2 years and one month.

Step 4. : Validation of result

We have to assume that the interest rate remains the same for the period for which we calculate the interest. Otherwise, the formula S.I. = $\frac{pnr}{100}$ will into be valid. We have also assumed that the price of the colour T.V. machine does not increase by the time $2\frac{1}{12}$ year.

Ex.3 A motor boat goes upstream on a river and covers the distance between two town on the river bank in & hours. It covers this distance downstream in five hours. If the speed of the stream is 4 km/h, find the speed of the boat in still water.

Sol. **Step 1 : Formulation :** We know the speed of the river and the time taken to cover the distance between two places. We have to find the speed of the boat in still water.

Mathematical Description : Let us write x for the speed of the boat, t for the time taken and y for the distance travelled. Then y = tx ....(1)

Let d be the distance between the two places. While going upstream.

The actual speed of boat = speed of the boat - speed of the river

∴ The boat is travelling against the flow of the river.

So, the speed of the boat in upstream = (x - 4) km/h.

It takes 8 hours to cover the distance between the towns upstream. So from (1), we have

$$d = 8\,(x + 4) \qquad ....(2)$$

When going downstream,

The speed of the boat in downstream = (x + 4) km/h

The boat takes five hours to cover the same distance downstream, so

$$d = 5\,(x + 4) \qquad .....(3)$$

From (2) and (3), we have

$$5(x + 4) = 8(x - 4)$$

Step 2. Finding the solution.

Solving for a in equation (4), we get $x = \frac{52}{3}$

Step 3. Interpretation.

Since $x = \frac{52}{3}$, therefore the speed of the motorboat in still water is $\frac{52}{3}$ km/h.

We have assumed that

1. The speed of the river and the boat remains constant all the time.
2. The effect of the friction between the boat and water and the friction due to air is negligible.

Step 4. Validation of result :

The speed of the motor boat is $\frac{52}{3}$ km/h and the distance between two towns,

$y = 8(x - 4)$

$\Rightarrow \quad y = 8\left(\frac{52}{3} - 4\right)$

$\Rightarrow \quad y = 8\left(\frac{52 - 12}{3}\right)$

$\Rightarrow \quad y = \frac{8 \times 40}{3} = \frac{320}{3}$

$\Rightarrow$ y = 106.66 km.

Hence the distance between two towns = 106.66 km.

Ex.4 Four hundred entrance tickets were sold for a school fair. The cost of the ticket for adults was Rs. 20 and that for students was Rs. 10. The total collection from of the sale of entrance tickets was Rs. 6000. How many adults visited the fair ?

Sol. **Step 1. Formulation of the problem**

We know that

The total number of tickets sold = 400

The cost of a ticket for adults = Rs. 20

The cost of the ticket for student = Rs. 10

and the total proceedings were = Rs. 6000

Mathematical formulation :

Let the number of adults who visited the fair be x

$\therefore$ number of students visited = (400 - x)

Total amount received from adults = Rs. 20 x

Total amount received from students = Rs. 10(400 - x)

Total amount collected = Rs. 6000

$\therefore$ the model (relation) is

$20x + 10(400 - x) = 6000$

Step 2. Finding the solutions

$20x + 4000 - 10x = 6000$

$\Rightarrow$ $10x + 4000 = 6000$

$\Rightarrow$ $10x = 2000$

$\Rightarrow$ $x = 200$

Step 3. Interpretation of the solution

We assumed that the number of adults who visited the fair was x $\therefore$ 200 adults visited the fair.

Step 4. Validation of the result

No. of adults who visited the fair = 200

No. of students who visited the fair = (400 - 200) = 200

$\therefore$ Total receipts = Rs (20 × 200) + Rs. (10 × 200)

= Rs. 4000 + Rs. 2000

= Rs. 6000

Thus, the total collection from 200 adults and 200 students is Rs. 6000.

Ex.5 The price of sugar has gone up by 40%. By what percent should a family reduce the consumption of sugar so that the expenditure on sugar may remain the same ?

Sol. Let us consider various steps of mathematical modeling and solve this problem.

Step 1. Formulation of the problem : Price of sugar goes up by 40% i.e. if the family spends Rs. 100 on sugar, then with the increase in price, the family will have to spend Rs. 140. But the family decides not to increase the expenditure, instead it prefers to reduce the consumption of sugar.

Mathematical formulation :

Suppose the family consumed × kg of sugar for Rs. 100 before the price hike.

$\therefore$ the increased prince of x k of sugar is Rs. 140.

Quantity of sugar that can be bought for Rs. 100 at the increased price $= \frac{x}{140} \times 100 = \frac{10x}{14}\text{kg}$

$\therefore$ reduction in quantity of sugar

$= \quad x - \frac{10x}{14} = \frac{4x}{14}\text{kg}$

Step 2. Finding the solution : Percent reduction in consumption of sugar $= \frac{\frac{4x}{14}}{x} \times 100 = \frac{400}{14} = \frac{200}{7}$

Step 3. Interpretation of the solution :

The family should reduce the consumption of sugar by $28\frac{4}{7}\%$

Step 4. Validation of result :

After increase in price of sugar, the amount of sugar bought by the family.

$= \left(100 - 28\frac{4}{7}\right)\text{kg} = \left(100 - \frac{200}{7}\right)\text{kg}$

$= \frac{700-200}{7}\text{kg} = \frac{500 \times 2}{7 \times 2} = \frac{1000}{14}\text{kg}$

The cost of $\frac{1000}{14}$ kg sugar at Rs. 140 for 100 kg

$=$ Rs. $\frac{1000}{14} \times \frac{140}{100}\text{Rs.}100$

Hence the result

Ex.6 Suppose company need a computer for some period of time. The company can either hire a computer for Rs. 2,000 per month or buy one for Rs. 25,000. If the company has to use the computer for a long period, the company will pay such a high rent, that buying a computer will be cheaper. On the other hand, if the company has to use the computer for say, just one month, then hiring a computer will be cheaper. For the number of months beyond which it will be cheaper to buy a computer.

Sol. **Step 1. Formulation :**

We know that the company can hire a computer for Rs. 2,000 per month or the company can buy the compute for Rs. 25,000. The company has to use the computer for 0 just one month then the hiring the computer will be cheaper. Here we have find out number of months beyond which it will be cheaper to buy a computer.

Mathematical formulation :

Let the number of months beyond which it will be cheaper to buy a compute = x months.

Rate of hiring computer = Rs. 2,000 per month

The amount of hiring a computer for x months = Rs. 2,000 x

The cost of the compute = Rs. 25,000

The company will not have to pay more if cost of computer is less than the hiring charges for computer.

⇒ 25,000 < 2,000 x

Step 2. Solution :

⇒ $\frac{25,000}{2,000} < x$

⇒ $\frac{25}{2} < x$

Step 3. Interpretation :

If $\frac{25}{2} < x$. The least value of x is 13 month (more than $12\frac{1}{2}$) It will be cheaper for the company to buy a computer if it has to hire a computer 13 months or more than 13 months.

we have assumed that :

(i) The rate of hiring a computer remains same throughout the period.

(ii) After 13 months the cost of computer may not increase

Ex.7 We have given the timings of the gold medalists in the 400-metre race from the time the event was included in the Olympics, in the table below. Construct a mathematical model relating the years and timings. Use it to estimate the timing in the next Olympics.

Year	Timing (in seconds)
1964	52.01
1968	52.03
1972	51.08
1976	49.28
1978	48.88
1984	48.83
1988	48.65
1992	48.83
1996	48.25
2000	49.11
2004	49.41

Sol. **Formulation :** In the figure, the times of the gold medalist of 400 metres race are given of the Olympics (1964 - 2004). We take 1964 as zeroth years and write 1 for 1968, 2 for 1972 and 3 for 1976. We prepare a new table.

Year	**Timing (in seconds)**
0	52.01
1	52.03
2	51.08
3	49.28
4	48.88
5	48.83
6	48.65
7	48.83
8	48.25
9	49.11
10	49.41

The reduction in timings of gold medalist in 400 metres rave in Olympics given in the following table.

Year	Timings	Change in timings
0	52.01	0
1	52.03	+0.02
2	51.08	-0.95
3	49.28	+1.80
4	48.88	-0.40
5	48.3	-0.05
6	48.65	-0.18
7	48.83	+0.18
8	48.25	-0.58
9	49.11	+.086
10	49.41	+0.30

At the end of 4 years period from 1964 - 1968 the timing has increased by 0.02 second from 52.01 to 52.03 second.

At the end of second Olympic the reduction in timing is 0.95 second from 52.03 to 51.08. From the table above we cannot find a definite relationship between the number of years and change in timing. But the reduction is fairly steady except in the first 7th year, 9th year and 10th year.

The mean of the value is

$$= \frac{0.02 - 0.95 - 1.80 - 0.40 - 0.05 - 0.18 + 0.18 - 0.58 + 0.86 + 0.30}{10}$$

$$= \frac{-2.6}{10} = -0.26$$

Le us assume that the timings in 400 m race of Olympic reduced at the rate of 0.26 per Olympic.

Mathematical description :

We have assumed that the timings reduces at the rate of 0.26 second per Olympic.

So, the reduction in timings in the first Olympic = 52.01 - 0.26

Reduction in the second Olympic = 52.01 - 0.26 - 0.26 = 52.01 - 2 × 0.26

Reduction in the third Olympic = 52.01 - 0.26 - 0.26 - 0.26 = 52.01 - 3 × 0.26

So, the reduction in the 11th Olympic = 52.01 - 11 × 0.26

Now, we have to estimate the timings in the next (11th) Olympic i.e., 2008

But the timings in the nth Olympic will be = 52.01 - 0.26 n ....(1)

Step 2. Solution : Substituting n = 11, in (1), we get

52.01 - 0.26 × 11 = 52.01 - 2.86 = 49.15

Step 3. Interpretation : The timings for 400 m race in the next Olympic i.e., (2008) is estimated as 49.15 sec.

Step 4. Validation : Let us check if formula (1) is in agreement with the reality. Let us find the values for the years we already know using formula (1) and compare it with known values by finding the difference.

EXERCISE

SUBJECTIVE DPP # 23

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1. A sailor goes 8 kg downstream in 40 minutes and returns in 1 hour. Determine the speed of the sailor in still water and the speed of the current.

2. While covering a distance of 30 km, Ajeet takes 2 hours more than Amit. If Ajeet doubles his speed,. he would take 1 hour less than Amit. Find their speed of walking.

3. Places A and B are 80 k m apart from each other on a highway. A car starts from A and other from B at the same speed. If they move in the same direction, they meet in 8 hours and if they more in opposite directions, they meet in 1 hour and 20 minutes. Find the speed of the cars.

4. A bag contains one rupee, 50 paise and 25 paise coin in the ratio 5 : 6 : 7. if total amount is Rs. 390, find the number of coins of each kind.

5. The ages of two person are in the ratio of 5 : 7. Sixteen years ago, the ratio was 3 : 5. Find their present ages.

6. Suppose Sudhir has invested Rs. 15,000 at 8% simple interest per year with the return from the investment. He wants to buy a washing machine that costs Rs. 19,000. For what period should he invest Rs. 15,000 so that he has enough money to buy a washing machine ?

7. A motorboat goes upstream on a river and covers the distance between two points on the riverbank in six hours. It covers this distance downstream in five hours. If the speed of the steam is 2km/hr. find the speed of the boat in still water.

8. Suppose you have a room of length 6m and breadth 5m. You want to cover the floor of the room with square mosaic tiles of side 30 cm. How many tiles will you need ? Solve this by constructing a mathematical model.

9. A travelled 432 kilometers on 48 litres of petrol in my car. I have to go by my cat to a place which is 180 km away. how much petrol do I need ?

10. Suppose a car starts from Delhi at a speed of 70 km/h towards Chandigarh. At that instance, a motorcycle starts from Chandigarh towards Delhi at a speed of 55 km/h. If the distance between Delhi and Chandigarh is 250 km, after how much time will the car and motorcycle meet ?

ANSWER KEY

(Subjective DPP # 23)

1. Speed of sailor = 10 km/h., Speed of current = 2km/h.

2. Ajeet's speed = 5km/h., Amit's speed = 7.5km/h.

3. 35km/h., 25km/h.

4. No. of one rupee coins = 200 ; No. of 50 paise coin = 240 ; No. of 25 paise coin = 280

5. 40 and 56 years.

6. 3 years, 4 months

7. 22km/hr.

8. 340 tiles

9. 20 litres

10. 2 hours