Wishing You & Your Family A Very Happy & Prosperous Deepawali

Time Limit : 4 Sitting Each of 75 Minutes duration approx.

NOTE: This assignment will be discussed on the very first day after Deepawali Vacation, hence come prepared.
Q. B. on Compound angles, QE, Sol of triangle, Seq. & Prog. & trigo. equations and inequations

Select the correct alternative : (Only one is correct)

Q.1 A regular decagon $A_0, A_1, A_2, \ldots A_9$ is given in the xy plane. Measure of the $\angle A_0 A_3 A_7$ in degrees is
(A) 108°  (B) 96°  (C) 72°  (D) 54°

Q.2 If $a^2 + b^2 + c^2 = 1$ then $ab + bc + ca$ lies in the interval :
(A) $\left[ \frac{1}{2}, 2 \right]$  (B) $[-1, 2]$  (C) $\left[ -\frac{1}{2}, 1 \right]$  (D) $[-1, \frac{1}{2}]

Q.3 If the roots of the cubic $x^3 - px^2 + qx - r = 0$ are in G.P. then
(A) $q^3 = p^3r$  (B) $p^3 = q^3r$  (C) $pq = r$  (D) $pr = q$

Q.4 In a triangle ABC, $a : b : c = 4 : 5 : 6$. Then $3A + B =$
(A) $4C$  (B) $2\pi$  (C) $\pi - C$  (D) $\pi$

Q.5 An equilateral triangle has sides 1 cm long. An ant walks around the triangle, maintaining a distance of 1 cm from the triangle at all time. Distance travelled by the ant in one round is
(A) $3 + 3\sqrt{3}$  (B) $3 + 6\sqrt{3}$  (C) $3 + 2\pi$  (D) $3 + \frac{3\pi}{2}$

Q.6 If $P(x) = ax^2 + bx + c$ & $Q(x) = -ax^2 + dx + c$, where $ac \neq 0$, then $P(x) \cdot Q(x) = 0$ has
(A) exactly one real root  (B) atleast two real roots  (C) exactly three real roots  (D) all four are real roots.

Q.7 The set of all real numbers $x$ for which $\log_{2004}(\log_{2003}(\log_{2002}(\log_{2001} x)))$ is defined as
\{x \mid x > c\}. The value of $c$ is

Q.8 In a triangle ABC, CD is the bisector of the angle C. If $\cos \frac{C}{2}$ has the value $\frac{1}{3}$ and $l(CD) = 6$, then
\[ \left( \frac{1}{a} + \frac{1}{b} \right) \] has the value equal to
(A) $\frac{1}{9}$  (B) $\frac{1}{12}$  (C) $\frac{1}{6}$  (D) none

Q.9 The real values of 'a' for which the quadratic equation, $2x^2 - (a^3 + 8a - 1)x + a^2 - 4a = 0$ possesses roots of opposite signs is given by :
(A) $a > 5$  (B) $0 < a < 4$  (C) $a > 0$  (D) $a > 7$

Q.10 The arithmetic mean of the nine numbers in the given set \{9, 99, 999, \ldots, 999999999\} is a 9 digit number $N$, all whose digits are distinct. The number $N$ does not contain the digit
(A) 0  (B) 2  (C) 5  (D) 9
Q.11 If \( x = \frac{n\pi}{2} \), satisfies the equation \( \sin \frac{x}{2} - \cos \frac{x}{2} = 1 - \sin x \) & the inequality \( \left| \frac{x - \pi}{2} \right| \leq \frac{3\pi}{4} \), then:

(A) \( n = -1, 0, 3, 5 \)  
(B) \( n = 1, 2, 4, 5 \)  
(C) \( n = 0, 2, 4 \)  
(D) \( n = -1, 1, 3, 5 \)

Q.12 With usual notations, in a triangle ABC, \( a \cos(B - C) + b \cos(C - A) + c \cos(A - B) \) is equal to

(A) \( \frac{abc}{R^2} \)  
(B) \( \frac{abc}{4R^2} \)  
(C) \( \frac{4abc}{R^2} \)  
(D) \( \frac{abcR}{2R^2} \)

Q.13 If in a \( \Delta ABC \), \( \sin^3 A + \sin^3 B + \sin^3 C = 3 \sin A \cdot \sin B \cdot \sin C \) then

(A) \( \Delta ABC \) may be a scalene triangle  
(B) \( \Delta ABC \) is a right triangle  
(C) \( \Delta ABC \) is an obtuse angled triangle  
(D) \( \Delta ABC \) is an equilateral triangle

Q.14 If \( a, b, c \) are real numbers satisfying the condition \( a + b + c = 0 \) then the roots of the quadratic equation \( 3ax^2 + 5bx + 7c = 0 \) are:

(A) positive  
(B) negative  
(C) real & distinct  
(D) imaginary

Q.15 If \( \sin^3 x \cdot \cos 3x + \cos^3 x \cdot \sin 3x = \frac{3}{8} \), then the value of \( \sin 4x \) is

(A) \( \frac{1}{3} \)  
(B) \( \frac{1}{4} \)  
(C) \( \frac{1}{2} \)  
(D) \( \frac{3}{8} \)

Q.16 With usual notations in a triangle ABC, \( (I_1I) \cdot (I_2I) \cdot (I_3I) \) has the value equal to

(A) \( R^2r \)  
(B) \( 2R^2r \)  
(C) \( 4R^2r \)  
(D) \( 16R^2r \)

Q.17 Consider the pattern shown below

<table>
<thead>
<tr>
<th>Row 1</th>
<th>1</th>
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<tbody>
<tr>
<td>Row 2</td>
<td>3</td>
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<td>Row 3</td>
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<tr>
<td>Row 4</td>
<td>13</td>
<td>15</td>
<td>17</td>
<td>19</td>
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</tbody>
</table>

The number at the end of row 80, is

(A) 6479  
(B) 6319  
(C) 6481  
(D) 6531

Q.18 For all positive integers \( n \) let \( f(n) = \log_{2002}n^2 \). Let \( N = f(11) + f(13) + f(14) \) which of the following relations is true?

(A) \( 0 < N < 1 \)  
(B) \( N = 1 \)  
(C) \( 1 < N < 2 \)  
(D) \( N = 2 \)

Q.19 The roots of \( (x - 1)(x - 3) + K(x - 2)(x - 4) = 0 \), \( K > 0 \) are:

(A) real  
(B) real & equal  
(C) imaginary  
(D) one real & one imaginary

Q.20 An isosceles triangle has sides of length 2, 2, and \( x \). The value of \( x \) for which the area of the triangle is maximum, is

(A) 1  
(B) \( \sqrt{2} \)  
(C) 2  
(D) \( 2\sqrt{2} \)

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Q.21 If $x \sec \alpha + y \tan \alpha = x \sec \beta + y \tan \beta = a$, then $\sec \alpha \cdot \sec \beta =$

(A) $\frac{a^2 + y^2}{x^2 + y^2}$  (B) $\frac{a^2 + y^2}{x^2 - y^2}$  (C) $\frac{x^2 + y^2}{a^2 + y^2}$  (D) $\frac{x^2 - y^2}{a^2 - y^2}$

Q.22 Largest integral value of $m$ for which the quadratic expression

$y = x^2 + (2m + 6)x + 4m + 12$ is always positive, $\forall x \in \mathbb{R}$, is

(A) – 1  (B) – 2  (C) 0  (D) 2

Q.23 The general solution of the trigonometric equation

$tan x + tan 2x + tan 3x = tan x \cdot tan 2x \cdot tan 3x$ is

(A) $x = n\pi$  (B) $n\pi \pm \frac{\pi}{3}$  (C) $x = 2n\pi$  (D) $x = \frac{n\pi}{3}$

where $n \in \mathbb{I}$

Q.24 With usual notation in a $\triangle ABC$

\[ \left( \frac{1}{r_1} + \frac{1}{r_2} \right) \left( \frac{1}{r_2} + \frac{1}{r_3} \right) \left( \frac{1}{r_3} + \frac{1}{r_1} \right) = \frac{K}{a^2 + b^2 + c^2} \]

where $K$ has the value equal to :

(A) 1  (B) 16  (C) 64  (D) 128

Q.25 If the sum of the roots of the quadratic equation, $ax^2 + bx + c = 0$ is equal to sum of the squares of their reciprocals, then $\frac{a}{c}, \frac{b}{a}, \frac{c}{b}$ are in :

(A) A.P.  (B) G.P.  (C) H.P.  (D) none

Q.26 The set of values of $x$ satisfying the inequality

$\frac{1}{\log_4 \frac{x+1}{x+2}} \leq \frac{1}{\log_4 (x+3)}$ is :

(A) $(-3, -2)$  (B) $(-3, -2) \cup (-1, \infty)$  (C) $(-1, \infty)$  (D) none

Q.27 As shown in the figure AD is the altitude on BC and AD produced meets the circumcircle of $\triangle ABC$ at P where $DP = x$. Similarly $E Q = y$ and $F R = z$. If $a, b, c$ respectively denote the sides $BC, CA$ and $AB$ then

\[ \frac{a}{2x} + \frac{b}{2y} + \frac{c}{2z} \]

has the value equal to

(A) $\tan A + \tan B + \tan C$  (B) $\cot A + \cot B + \cot C$

(C) $\cos A + \cos B + \cos C$  (D) $\csc A + \csc B + \csc C$

Q.28 In a $\triangle ABC$, the value of

$\frac{a \cos A + b \cos B + c \cos C}{a + b + c}$ is equal to :

(A) $\frac{r}{R}$  (B) $\frac{R}{2r}$  (C) $\frac{R}{r}$  (D) $\frac{2r}{R}$

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Q.29 If the sum of the first 11 terms of an arithmetical progression equals that of the first 19 terms, then the sum of its first 30 terms, is
(A) equal to 0    (B) equal to –1    (C) equal to 1    (D) non unique

Q.30 The sum of all the value of \( m \) for which the roots \( x_1 \) and \( x_2 \) of the quadratic equation \( x^2 - 2mx + m = 0 \) satisfy the condition \( x_1^3 + x_2^3 = x_1^2 + x_2^2 \), is
(A) \( \frac{3}{4} \)    (B) 1    (C) \( \frac{9}{4} \)    (D) \( \frac{5}{4} \)

Q.31 In an A.P. with first term 'a' and the common difference d (a, d ≠ 0), the ratio '\( \rho \) ' of the sum of the first \( n \) terms to sum of \( n \) terms succeeding them does not depend on \( n \). Then the ratio \( \frac{a}{d} \) and the ratio '\( \rho \) ', respectively are
(A) \( \frac{1}{2}, \frac{1}{4} \)    (B) 2, \( \frac{1}{3} \)    (C) \( \frac{1}{2}, \frac{1}{3} \)    (D) \( \frac{1}{2}, 2 \)

Q.32 AD, BE and CF are the perpendiculars from the angular points of a \( \Delta \) ABC upon the opposite sides. The perimeters of the \( \Delta \) DEF and \( \Delta \) ABC are in the ratio :
(A) \( \frac{2r}{R} \)    (B) \( \frac{r}{2R} \)    (C) \( \frac{r}{R} \)    (D) \( \frac{r}{3R} \)

Q.33 If \( \cos 25^\circ + \sin 25^\circ = p \), then \( \cos 50^\circ \) is
(A) \( \sqrt{2-p^2} \)    (B) \( -p \sqrt{2-p^2} \)    (C) \( p \sqrt{2-p^2} \)    (D) \( - \sqrt{2-p^2} \)

Q.34 Let \( r_1, r_2 \) and \( r_3 \) be the solutions of the equation \( x^3 - 2x^2 + 4x + 5074 = 0 \) then the value of \( (r_1 + 2)(r_2 + 2)(r_3 + 2) \) is
(A) 5050    (B) 5066    (C) – 5050    (D) – 5066

Q.35 If \( p, q, r \) in H.P. and \( p \) & \( r \) be different having same sign then the roots of the equation \( px^2 + qx + r = 0 \) are
(A) real & equal    (B) real & distinct    (C) irrational    (D) imaginary

Q.36 In a \( \Delta \) ABC if \( b + c = 3a \) then \( \cot \frac{B}{2} \cdot \cot \frac{C}{2} \) has the value equal to :
(A) 4    (B) 3    (C) 2    (D) 1

Q.37 Indicate the correct choice : If \( \log_{0.3} (x-1) < \log_{0.09} (x-1) \), then \( x \) lies in the interval ;
(A) (2, \( \infty \))    (B) (1, 2)    (C) (-2, -1)    (D) none of these

Q.38 Number of roots of the equation, \( \sin (\cos x) = \cos (\sin x) \) in \([0, 2\pi]\) is
(A) 0    (B) 1    (C) 2    (D) 4

Q.39 The sum of the roots of the equation \( (x + 1) = 2 \log_2(2^x + 3) - 2 \log_4(1980 - 2^{-x}) \) is
(A) 3954    (B) \( \log_2 11 \)    (C) \( \log_2 3954 \)    (D) indeterminate
Q.40 Let \( n \) be a positive integer such that \( \sin \frac{\pi}{2n} + \cos \frac{\pi}{2n} = \frac{\sqrt{n}}{2} \). Then

- (A) \( 6 \leq n \leq 8 \)
- (B) \( 4 \leq n \leq 8 \)
- (C) \( 4 \leq n < 8 \)
- (D) \( 4 < n < 8 \)

Q.41 Let \( f, g, h \) be the lengths of the perpendiculars from the circumcentre of the \( \Delta ABC \) on the sides \( a, b \) and \( c \) respectively. If \( \frac{a}{f} + \frac{b}{g} + \frac{c}{h} = \lambda \) then the value of \( \lambda \) is:

- (A) \( \frac{1}{4} \)
- (B) \( \frac{1}{2} \)
- (C) \( 1 \)
- (D) \( 2 \)

Q.42 The equation whose roots are \( \sec^2 \alpha \) and \( \csc^2 \alpha \) can be:

- (A) \( 2x^2 - x - 1 = 0 \)
- (B) \( x^2 - 3x + 3 = 0 \)
- (C) \( x^2 - 9x + 9 = 0 \)
- (D) none

Q.43 Minimum vertical distance between the graphs of \( y = 2 + \sin x \) and \( y = \cos x \) is

- (A) \( 2 \)
- (B) \( 1 \)
- (C) \( \sqrt{2} \)
- (D) \( 2 - \sqrt{2} \)

Q.44 Let \( C \) be a circle with centre \( P_0 \) and \( AB \) be a diameter of \( C \). Suppose \( P_1 \) is the mid point of the line segment \( P_0B \), \( P_2 \) is the mid point of the line segment \( P_1B \) and so on. Let \( C_1, C_2, C_3, \ldots \) be circles with diameters \( P_0 P_1, P_1 P_2, P_2 P_3 \ldots \) respectively. Suppose the circles \( C_1, C_2, C_3, \ldots \) are all shaded. The ratio of the area of the unshaded portion of \( C \) to that of the original circle \( C \) is

- (A) \( 8 : 9 \)
- (B) \( 9 : 10 \)
- (C) \( 10 : 11 \)
- (D) \( 11 : 12 \)

Q.45 If the orthocentre and circumcentre of a triangle \( ABC \) be at equal distances from the side \( BC \) and lie on the same side of \( BC \) then \( \tan B \tan C \) has the value equal to:

- (A) \( 3 \)
- (B) \( \frac{1}{3} \)
- (C) \(-3 \)
- (D) \(-\frac{1}{3} \)

Q.46 The graph of a quadratic polynomial \( y = ax^2 + bx + c \) \( (a, b, c \in \mathbb{R}) \) with vertex on the \( y \)-axis is as shown in the figure. Then which one of the following statement is INCORRECT?

- (A) Product of the roots of the corresponding quadratic equation is positive.
- (B) Discriminant of the quadratic equation is negative.
- (C) Nothing definite can be said about the sum of the roots, whether positive, negative or zero.
- (D) Both roots of the quadratic equation are purely imaginary.

Q.47 If in a triangle \( ABC \) \( \frac{2 \cos A}{a} + \frac{2 \cos C}{b} = \frac{a}{bc} + \frac{b}{ca} \) then the value of the angle \( A \) is:

- (A) \( \frac{\pi}{8} \)
- (B) \( \frac{\pi}{4} \)
- (C) \( \frac{\pi}{3} \)
- (D) \( \frac{\pi}{2} \)

Q.48 If \( \sin (\theta + \alpha) = a \) \& \( \sin (\theta + \beta) = b \) \( (0 < \alpha, \beta, \theta < \pi/2) \)

then \( \cos 2(\alpha - \beta) - 4ab \cos(\alpha - \beta) = \)

- (A) \( 1 - a^2 - b^2 \)
- (B) \( 1 - 2a^2 - 2b^2 \)
- (C) \( 2 + a^2 + b^2 \)
- (D) \( 2 - a^2 - b^2 \)

Q.49 Concentric circles of radii 1, 2, 3, \ldots 100 cms are drawn. The interior of the smallest circle is coloured red and the angular regions are coloured alternately green and red, so that no two adjacent regions are of the same colour. The total area of the green regions in sq. cm is equal to

- (A) \( 1000\pi \)
- (B) \( 5050\pi \)
- (C) \( 4950\pi \)
- (D) \( 5151\pi \)
Q.50 In a ΔABC if \( b = a \left( \sqrt{3} - 1 \right) \) and \( \angle C = 30^0 \) then the measure of the angle \( A \) is
(A) 15^0 (B) 45^0 (C) 75^0 (D) 105^0

Q.51 The number of solution of the equation \( e^{2x} + e^{x} + e^{-2x} + e^{-x} = 3(e^{-2x} + e^x) \) is
(A) 0 (B) 2 (C) 1 (D) more than 2

Q.52 If in a triangle \( \sin A : \sin C = \sin (A - B) : \sin (B - C) \) then \( a^2 : b^2 : c^2 \)
(A) are in A.P. (B) are in G.P.
(C) are in H.P. (D) none of these

Q.53 The number of natural numbers less than 400 that are not divisible by 17 or 23 is
(A) 382 (B) 359 (C) 360 (D) 376

Q.54 The minimum value of the expression \( \frac{9x^2 \sin^2 x + 4}{x \sin x} \) for \( x \in (0, \pi) \) is
(A) \( \frac{16}{3} \) (B) 6 (C) 12 (D) \( \frac{8}{3} \)

Q.55 In a ΔABC, \( a = a_1 = 2 \), \( b = a_2 \), \( c = a_3 \) such that \( a_{p+1} = \frac{5p}{3^{2-p}} a_p \left( \frac{2^2-p - 4p-2}{5^p} a_p \right) \)
where \( p \) = 1, 2 then
(A) \( r_1 = r_2 \) (B) \( r_3 = 2r_1 \) (C) \( r_2 = 2r_1 \) (D) \( r_2 = 3r_1 \)

Q.56 The sum \( \sum_{n=1}^{\infty} \frac{1}{n^2 + 2n} \) equals
(A) \( \frac{1}{2} \) (B) \( \frac{2}{3} \) (C) \( \frac{3}{4} \) (D) 1

Q.57 The product of the arithmetic mean of the lengths of the sides of a triangle and harmonic mean of the lengths of the altitudes of the triangle is equal to:
(A) \( \Delta \) (B) \( 2 \Delta \) (C) \( 3 \Delta \) (D) \( 4 \Delta \)
[where \( \Delta \) is the area of the triangle ABC]

Q.58 The set of real value(s) of \( p \) for which the equation, \( \left| 2x + 3 \right| + \left| 2x - 3 \right| = px + 6 \) has more than two solutions is:
(A) \( (0, 4) \) (B) \( (-4, 4) \) (C) \( R \setminus \{4, -4, 0\} \) (D) \{0\}

Q.59 If ‘O’ is the circumcentre of the \( \triangle \)ABC and \( R_1 \), \( R_2 \) and \( R_3 \) are the radii of the circumcircles of triangles OBC, OCA and OAB respectively then \( \frac{a}{R_1} + \frac{b}{R_2} + \frac{c}{R_3} \) has the value equal to:
(A) \( \frac{abc}{2R^2} \) (B) \( \frac{R^3}{a \cdot b \cdot c} \) (C) \( \frac{4\Delta}{R^2} \) (D) \( \frac{\Delta}{4R^2} \)
Q.60 If for an A.P. \(a_1, a_2, a_3, \ldots, a_n\),
\[a_1 + a_3 + a_5 = -12\] and \(a_1 a_2 a_3 = 8\),
then the value of \(a_2 + a_4 + a_6\) equals
(A) \(-12\) \hspace{1cm} (B) \(-16\) \hspace{1cm} (C) \(-18\) \hspace{1cm} (D) \(-21\)

Q.61 If in a \(\triangle ABC\), \(\cos A \cdot \cos B + \sin A \sin B \sin 2C = 1\) then, the statement which is incorrect, is
(A) \(\triangle ABC\) is isosceles but not right angled \hspace{1cm} (B) \(\triangle ABC\) is acute angled
(C) \(\triangle ABC\) is right angled \hspace{1cm} (D) least angle of the triangle is \(\frac{\pi}{4}\)

Q.62 The absolute term in the quadratic expression
\[
\sum_{k=1}^{n} \frac{1}{k+1} - \frac{1}{k} \text{ as } n \rightarrow \infty
\]
is
(A) 1 \hspace{1cm} (B) \(-1\) \hspace{1cm} (C) 0 \hspace{1cm} (D) 1/2

Q.63 The number of roots of the equation, \(\sin x + 2 \sin 2x = 3 + \sin 3x\) is:
(A) 0 \hspace{1cm} (B) 1 \hspace{1cm} (C) 2 \hspace{1cm} (D) infinite

Q.64 In a triangle the expression
\[
\frac{a^2 b^2 c^2 (\sin 2A + \sin 2B + \sin 2C)}{\Delta^3}
\]
simplifies to
(A) 8 \hspace{1cm} (B) 16 \hspace{1cm} (C) 32 \hspace{1cm} (D) 64

Q.65 Number of real values of \(x\) satisfying the equation
\[
\sqrt{x^2 - 6x + 9} + \sqrt{x^2 - 6x + 6} = 1
\]
is
(A) 0 \hspace{1cm} (B) 1 \hspace{1cm} (C) 2 \hspace{1cm} (D) more than 2

Q.66 If the roots of the equation \(x^3 - px^2 - r = 0\) are \(\tan \alpha, \tan \beta\) and \(\tan \gamma\) then the value of
\[
\sec^2 \alpha \cdot \sec^2 \beta \cdot \sec^2 \gamma
\]
is
(A) \(p^2 + r^2 + 2rp + 1\) \hspace{1cm} (B) \(p^2 + r^2 - 2rp + 1\) \hspace{1cm} (C) \(p^2 - r^2 - 2rp + 1\) \hspace{1cm} (D) None

Q.67 If \(r_1, r_2, r_3\) be the radii of excircles of the triangle \(ABC\), then
\[
\frac{\sum r_i}{\sqrt{\sum r_i r_2}}
\]
is equal to:
(A) \(\sum \cot \frac{A}{2}\) \hspace{1cm} (B) \(\sum \cot \frac{A}{2} \cot \frac{B}{2}\) \hspace{1cm} (C) \(\sum \tan \frac{A}{2}\) \hspace{1cm} (D) \(\prod \tan \frac{A}{2}\)

Q.68 There is a certain sequence of positive real numbers. Beginning from the third term, each term of the sequence is the sum of all the previous terms. The seventh term is equal to 1000 and the first term is equal to 1. The second term of this sequence is equal to
(A) 246 \hspace{1cm} (B) \(\frac{123}{2}\) \hspace{1cm} (C) \(\frac{123}{4}\) \hspace{1cm} (D) 124

Q.69 If \(x, y\) and \(z\) are the distances of incentre from the vertices of the triangle \(ABC\) respectively then
\[
\frac{a \cdot b \cdot c}{\frac{x + y + z}{3}}
\]
is equal to
(A) \(\prod \tan \frac{A}{2}\) \hspace{1cm} (B) \(\sum \cot \frac{A}{2}\) \hspace{1cm} (C) \(\sum \tan \frac{A}{2}\) \hspace{1cm} (D) \(\sum \sin \frac{A}{2}\)
Q.70 If the equation \(a(x - 1)^2 + b(x^2 - 3x + 2) + x - a^2 = 0\) is satisfied for all \(x \in \mathbb{R}\) then the number of ordered pairs of \((a, b)\) can be
(A) 0    (B) 1    (C) 2    (D) infinite

Q.71 In a \(\triangle ABC\), a semicircle is inscribed, whose diameter lies on the side \(c\). Then the radius of the semicircle is
(A) \(\frac{2\Delta}{a + b}\)    (B) \(\frac{2\Delta}{a + b - c}\)    (C) \(\frac{2\Delta}{s}\)    (D) \(\frac{c}{2}\)
Where \(\Delta\) is the area of the triangle \(ABC\).

Q.72 Number of quadratic equations with real roots which remain unchanged even after squaring their roots, is
(A) 1    (B) 2    (C) 3    (D) 4

Q.73 Along a road lies an odd number of stones placed at intervals of 10 m. These stones have to be assembled around the middle stone. A person can carry only one stone at a time. A man carried out the job starting with the stone in the middle, carrying stones in succession, thereby covering a distance of 4.8 km. Then the number of stones is
(A) 15    (B) 29    (C) 31    (D) 35

Q.74 If in a \(\triangle ABC\), \(\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}\) then the triangle is
(A) right angled    (B) isosceles    (C) equilateral    (D) obtuse

Q.75 The quadratic equation \(ax^2 + bx + c = 0\) has imaginary roots if:
(A) \(a < -1\), \(0 < c < 1\), \(b > 0\)    (B) \(a < -1\), \(-1 < c < 0\), \(0 < b < 1\)
(C) \(a < -1\), \(c < 0\), \(b > 1\)    (D) none

Q.76 The equations \(x^3 + 5x^2 + px + q = 0\) and \(x^3 + 7x^2 + px + r = 0\) have two roots in common. If the third root of each equation is represented by \(x_1\) and \(x_2\) respectively, then the ordered pair \((x_1, x_2)\) is:
(A) \((-5, -7)\)    (B) \((1, -1)\)    (C) \((-1, 1)\)    (D) \((5, 7)\)

Q.77 If \(\cos A + \cos B + 2\cos C = 2\) then the sides of the \(\Delta ABC\) are in
(A) A.P.    (B) G.P    (C) H.P.    (D) none

Select the correct alternatives : (More than one are correct)

Q.78 If \(\sin \theta = \sin \alpha\) then \(\sin \frac{\theta}{3}\) equal to
(A) \(\sin \frac{\alpha}{3}\)    (B) \(\sin \left(\frac{\pi}{3} - \frac{\alpha}{3}\right)\)    (C) \(\sin \left(\frac{\pi}{3} + \frac{\alpha}{3}\right)\)    (D) \(-\sin \left(\frac{\pi}{3} - \frac{\alpha}{3}\right)\)

Q.79 \(\cos \alpha\) is a root of the equation \(25x^2 + 5x - 12 = 0\), \(-1 < x < 0\), then the value of \(\sin 2\alpha\) is :
(A) \(\frac{24}{25}\)    (B) \(-\frac{12}{25}\)    (C) \(-\frac{24}{25}\)    (D) \(\frac{20}{25}\)

Q.80 If \(\sin(x - y)\), \(\sin x\) and \(\sin(x + y)\) are in H.P., then \(\sin x . \sec \frac{y}{2}\) =
(A) \(2\)    (B) \(\sqrt{2}\)    (C) \(-\sqrt{2}\)    (D) \(-2\)

Q.81 Which of the following functions have the maximum value unity ?
(A) \(\sin^2 x - \cos^2 x\)    (B) \(\frac{\sin 2x - \cos 2x}{\sqrt{2}}\)
(C) \(-\frac{\sin 2x - \cos 2x}{\sqrt{2}}\)    (D) \(\sqrt{\frac{6}{5}} \left(\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{3}} \cos x\right)\)
Q.82 In a triangle, the lengths of the two larger sides are 10 and 9 respectively. If the angles are in A.P., then the length of the third side can be:
(A) $5 - \sqrt{6}$  
(B) $3\sqrt{3}$  
(C) 5  
(D) $6 \pm \sqrt{5}$

Q.83 $\left(\frac{\log_b x}{x}\right)^2 - 5x^{\log_b a} + 6 = 0$, where $a > 0$, $b > 0$ & $ab \neq 1$, then the value of $x$ can be equal to
(A) $2^{\log_b a}$  
(B) $3^{\log_b b}$  
(C) $b^{\log_b 2}$  
(D) $a^{\log_b 3}$

Q.84 If the roots of the equation, $x^3 + px^2 + qx - 1 = 0$ form an increasing G.P. where $p$ and $q$ are real, then
(A) $p+q = 0$  
(B) $p \in (-\infty, -3)$  
(C) one of the roots is one  
(D) one root is smaller than 1 & one root is greater than 1.

Q.85 The graph of the quadratic polynomial:
$y = ax^2 + bx + c$ is as shown in the figure. Then:
(A) $b^2 - 4ac > 0$  
(B) $b < 0$  
(C) $a > 0$  
(D) $c < 0$

Q.86 If $\left(\frac{\log_b x}{x}\right)^2 - 5x^{\log_b a} + 6 = 0$, where $a > 0$, $b > 0$ & $ab \neq 1$, then the value of $x$ can be equal to
(A) $2^{\log_b a}$  
(B) $3^{\log_b b}$  
(C) $b^{\log_b 2}$  
(D) $a^{\log_b 3}$

Q.87 Let $\Delta^2$ be the discriminant and $\alpha, \beta$ be the roots of the equation $ax^2 + bx + c = 0$. Then $2a\alpha + \Delta$ and $2a\beta - \Delta$ can be the roots of the equation:
(A) $x^2 + 2bx + b^2 = 0$  
(B) $x^2 - 2bx + b^2 = 0$  
(C) $x^2 + 2bx - 3b^2 + 16ac = 0$  
(D) $x^2 - 2bx - 3b^2 + 16ac = 0$

Q.88 Which of the following statement(s) is/are true?
(A) $\log_{10} 2$ lies between $\frac{1}{3}$ & $\frac{1}{4}$  
(B) $\log_{\csc (\pi/6)} \cos \frac{5\pi}{3} = -1$  
(C) $e^{3 \ln 3} \text{ is smaller than } 1$  
(D) $\log_{10} 1 + \frac{1}{2} \log_{10} 3 + \log_{10} \left(2 + \sqrt{3}\right) = \log \left(1 + \sqrt{3} + (2 + \sqrt{3})\right)$

Q.89 Select the statement(s) which are true in respect of a triangle ABC, all symbols have their usual meaning.
(A) The inradius, circumradius and one of the exradii of an equilateral triangle are in the ratio of $1 : 2 : 3$.  
(B) $abc = \frac{1}{4}Rrs$  
(C) If $r = 3$ then the value of $\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{3}$  
(D) If the diameter of any escribed circle is equal to the perimeter then the triangle must be a right triangle.

Q.90 The graph of a quadratic polynomial $y = ax^2 + bx + c$ ($a, b, c \in \mathbb{R}, a \neq 0$) is as shown. Then the incorrect statement(s) is/are
(A) $c > 0$  
(B) $b < 0$  
(C) product of the roots is negative  
(D) sum of the roots is positive
Q.91 The points A \((x_1, y_1)\); B\((x_2, y_2)\) and C\((x_3, y_3)\) lie on the parabola \(y = 3x^2\). If \(x_1, x_2, x_3\) are in A.P. and \(y_1, y_2, y_3\) are in G.P. then the common ratio of the G.P. is
(A) \(3 + 2\sqrt{2}\)  
(B) \(3 + \sqrt{2}\)  
(C) \(3 - \sqrt{2}\)  
(D) \(3 - 2\sqrt{2}\)

Q.92 Which of the following when simplified, vanishes ?
(A) \(\frac{1}{\log_3 2} + \frac{2}{\log_5 4} - \frac{3}{\log_{27} 8}\)
(B) \(\log_2 \left(\frac{2}{3}\right) + \log_4 \left(\frac{9}{4}\right)\)
(C) \(-\log_2 \log_4 \log_2 16\)
(D) \(\log_{10} \cot 1^\circ + \log_{10} \cot 2^\circ + \log_{10} \cot 3^\circ + \ldots + \log_{10} \cot 89^\circ\)

Q.93 If one root of the quadratic equation, \(px^2 + qx + r = 0\) \((p \neq 0)\) is a surd \(\frac{\sqrt{a}}{\sqrt{a} + \sqrt{a - b}}\)
where \(p, q, r\); \(a, b\) are all rationals then the other root is
(A) \(\frac{\sqrt{a}}{\sqrt{a} - \sqrt{a - b}}\)  
(B) \(a + \frac{\sqrt{a(a-b)}}{b}\)  
(C) \(\frac{a + \sqrt{a(a-b)}}{b}\)  
(D) \(\frac{\sqrt{a} - \sqrt{a-b}}{\sqrt{b}}\)

SUBJECTIVE:

Q.94 Find whether the diagonals of the parallelogram formed by the lines \(\frac{x}{a} + \frac{y}{b} = 1\), \(\frac{x}{a} + \frac{y}{b} = 1\), \(\frac{x}{a} + \frac{y}{b} = 2\)
and \(\frac{x}{a} + \frac{y}{b} = 2\) are perpendicular to each other or not.

Q.95 Find the area of the parallelogram formed by the lines
\(4y - 3x - 1 = 0\)  
\(3y - 4x + 1 = 0\)  
\(4y - 3x - 3 = 0\)  
\(3y - 4x + 2 = 0\)

Q.96 Find the value of \(a\) for which the lines \(2x + y - 1 = 0\), \(ax + 3y - 3 = 0\), \(3x + 2y - 2 = 0\) are concurrent.

Q.97 Find the coordinates of the points on the line \(y = x + 1\), which are at a distance of 5 from the line \(4x - 3y + 20 = 0\).

Q.98 An equilateral triangle has its centroid at the origin and one side is \(x + y = 1\). Find the other two sides.

Q.99 The three lines \(x + 2y + 3 = 0\), \(x + 2y - 7 = 0\), \(2x - y - 4 = 0\) form the three sides of two squares. Find the equation to the fourth side of each square.

Q.100 Each side of a square is of length 4. The centre of the square is \((3, 7)\) and one of its diagonals is parallel to \(y = x\). Find the coordinates of its vertices.
ANSWER KEY

Select the correct alternative: (Only one is correct)

Q.1 D  Q.2 C  Q.3 A  Q.4 D  Q.5 C
Q.6 B  Q.7 B  Q.8 A  Q.9 B  Q.10 A
Q.11 B  Q.12 A  Q.13 D  Q.14 C  Q.15 C
Q.16 D  Q.17 A  Q.18 D  Q.19 A  Q.20 D
Q.21 B  Q.22 C  Q.23 D  Q.24 C  Q.25 C
Q.26 C  Q.27 A  Q.28 A  Q.29 A  Q.30 D
Q.31 C  Q.32 C  Q.33 C  Q.34 C  Q.35 D
Q.36 C  Q.37 A  Q.38 A  Q.39 B  Q.40 D
Q.41 A  Q.42 C  Q.43 D  Q.44 D  Q.45 A
Q.46 C  Q.47 D  Q.48 B  Q.49 B  Q.50 D
Q.51 C  Q.52 A  Q.53 C  Q.54 C  Q.55 D
Q.56 C  Q.57 B  Q.58 D  Q.59 C  Q.60 D
Q.61 C  Q.62 A  Q.63 A  Q.64 C  Q.65 A
Q.66 B  Q.67 C  Q.68 B  Q.69 B  Q.70 B
Q.71 A  Q.72 C  Q.73 C  Q.74 C  Q.75 D
Q.76 A  Q.77 A  Q.78 A, B, D  Q.79 A, C  Q.80 B, C
Q.86 B, C  Q.87 A, C  Q.88 A, B, D  Q.89 A, C, D  Q.90 A, C, D
Q.91 A, D  Q.92 A, B, C, D  Q.93 A, C

SUBJECTIVE:

Select the correct alternatives: (More than one are correct)

Q.78 A, B, D  Q.79 A, C  Q.80 B, C  Q.81 A, B, C, D
Q.82 A, D  Q.83 B, C  Q.84 A, B, C, D  Q.85 A, B, C, D
Q.86 B, C  Q.87 A, C  Q.88 A, B, D  Q.89 A, C, D  Q.90 A, C, D
Q.91 A, D  Q.92 A, B, C, D  Q.93 A, C

77° 77° 90° 17° 96° 67° 98° 8° 47° 50° 59° 47° 72° 92° 82°
9° 1° 37° 8° 23° 4° 8° 1° 7° 3° 2° 8° 1° 1°