EXERCISE–4

Part : (A) Only one correct option

1. There are 2 identical white balls, 3 identical red balls and 4 green balls of different shades. The number of ways in which they can be arranged in a row so that atleast one ball is separated from the balls of the same colour, is:
(A) 6 (7! – 4!) (B) 7 (6! – 4!) (C) 8! – 5! (D) none

2. The number of permutations that can be formed by arranging all the letters of the word 'NINETEEN' in which no two E's occur together is
(A) $\frac{8!}{3!\,3!}$ (B) $\frac{5!}{3! \times {}^6C_2}$ (C) $\frac{5!}{3!} \times {}^6C_3$ (D) $\frac{8!}{5!} \times {}^6C_3$.

3. The number of ways in which n different things can be given to r persons when there is no restriction as to the number of things each may receive is:
(A) ${}^nC_r$ (B) ${}^nP_r$ (C) n^r (D) r^n

4. The number of divisors of $a^p b^q c^r d^s$ where a, b, c, d are primes & p, q, r, s $\in$ N, excluding 1 and the number itself is:
(A) p q r s (B) (p + 1) (q + 1) (r + 1) (s + 1) – 4
(C) p q r s – 2 (D) (p + 1) (q + 1) (r + 1) (s + 1) – 2

5. The number of ordered triplets of positive integers which are solutions of the equation x + y + z = 100 is:
(A) 3125 (B) 5081 (C) 6005 (D) 4851

6. Number of ways in which 7 people can occupy six seats, 3 seats on each side in a first class railway compartment if two specified persons are to be always included and occupy adjacent seats on the same side, is (k). 5! then k has the value equal to:
(A) 2 (B) 4 (C) 8 (D) none

7. Number of different words that can be formed using all the letters of the word "DEEPMALA" if two vowels are together and the other two are also together but separated from the first two is:
(A) 960 (B) 1200 (C) 2160 (D) 1440

8. Six persons A, B, C, D, E and F are to be seated at a circular table. The number of ways this can be done if A must have either B or C on his right and B must have either C or D on his right is:
(A) 36 (B) 12 (C) 24 (D) 18

9. The number of ways in which 15 apples & 10 oranges can be distributed among three persons, each receiving none, one or more is:
(A) 5670 (B) 7200 (C) 8976 (D) none of these

10. The number of permutations which can be formed out of the letters of the word "SERIES" taking three letters together is:
(A) 120 (B) 60 (C) 42 (D) none

11. Seven different coins are to be divided amongst three persons. If no two of the persons receive the same number of coins but each receives atleast one coin & none is left over, then the number of ways in which the division may be made is:
(A) 420 (B) 630 (C) 710 (D) none

12. The streets of a city are arranged like the lines of a chess board. There are m streets running North to South & 'n' streets running East to West. The number of ways in which a man can travel from NW to SE corner going the shortest possible distance is:
(A) $\sqrt{m^2 + n^2}$ (B) $\sqrt{(m-1)^2 \,.\, (n-1)^2}$ (C) $\frac{(m+n)!}{m!\,.\,n!}$ (D) $\frac{(m+n-2)!}{(m-1)!\,.\,(n-1)!}$

13. In a conference 10 speakers are present. If S_1 wants to speak before S_2 & S_2 wants to speak after S_3, then the number of ways all the 10 speakers can give their speeches with the above restriction if the remaining seven speakers have no objection to speak at any number is:
(A) ${}^{10}C_3$ (B) ${}^{10}P_8$ (C) ${}^{10}P_3$ (D) $\frac{10!}{3}$

14. Two variants of a test paper are distributed among 12 students. Number of ways of seating of the students in two rows so that the students sitting side by side do not have identical papers & those sitting in the same column have the same paper is:

(A) $\frac{12!}{6!\,6!}$ (B) $\frac{(12)!}{2^5 . 6!}$ (C) $(6\,!)^2 . 2$ (D) $12\,! \times 2$

15. Sum of all the numbers that can be formed using all the digits 2, 3, 3, 4, 4, 4 is:
(A) 22222200 (B) 11111100 (C) 55555500 (D) 20333280

16. There are m apples and n oranges to be placed in a line such that the two extreme fruits being both oranges. Let P denotes the number of arrangements if the fruits of the same species are different and Q the corresponding figure when the fruits of the same species are alike, then the ratio P/Q has the value equal to:
(A) ${}^nP_2 . {}^mP_m . (n-2)!$ (B) ${}^mP_2 . {}^nP_n . (n-2)!$ (C) ${}^nP_2 . {}^nP_n . (m-2)!$ (D) none

17. The number of integers which lie between 1 and 10^6 and which have the sum of the digits equal to 12 is:
(A) 8550 (B) 5382 (C) 6062 (D) 8055

18. Number of ways in which a pack of 52 playing cards be distributed equally among four players so that each may have the Ace, King, Queen and Jack of the same suit is:
(A) $\frac{36\,!}{(9\,!)^4}$ (B) $\frac{36\,!\,.\,4\,!}{(9\,!)^4}$ (C) $\frac{36\,!}{(9\,!)^4\,.\,4\,!}$ (D) none

19. A five letter word is to be formed such that the letters appearing in the odd numbered positions are taken from the letters which appear without repetition in the word "MATHEMATICS". Further the letters appearing in the even numbered positions are taken from the letters which appear with repetition in the same word "MATHEMATICS". The number of ways in which the five letter word can be formed is:
(A) 720 (B) 540 (C) 360 (D) none

20. Number of ways of selecting 5 coins from coins three each of Rs. 1, Rs. 2 and Rs. 5 if coins of the same denomination are alike, is:
(A) 9 (B) 12 (C) 21 (D) none

21. Number of ways in which all the letters of the word " ALASKA " can be arranged in a circle distinguishing between the clockwise and anticlockwise arrangement is:
(A) 60 (B) 40 (C) 20 (D) none of these

22. If r, s, t are prime numbers and p, q are the positive integers such that the LCM of p, q is $r^2 t^4 s^2$, then the number of ordered pair (p, q) is **[IIT – 2006]**
(A) 252 (B) 254 (C) 225 (D) 224

Part : (B) May have more than one options correct

23. ${}^{n+1}C_6 + {}^nC_4 > {}^{n+2}C_5 - {}^nC_5$ for all ' n ' greater than:
(A) 8 (B) 9 (C) 10 (D) 11

24. In an examination, a candidate is required to pass in all the four subjects he is studying. The number of ways in which he can fail is
(A) ${}^4P_1 + {}^4P_2 + {}^4P_3 + {}^4P_4$ (B) $4^4 - 1$
(C) $2^4 - 1$ (D) ${}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4$

25. The kindergarten teacher has 25 kids in her class. She takes 5 of them at a time, to zoological garden as often as she can, without taking the same 5 kids more than once. Then the number of visits, the teacher makes to the garden exceeds that of a kid by:
(A) ${}^{25}C_5 - {}^{24}C_4$ (B) ${}^{24}C_5$ (C) ${}^{25}C_5 - {}^{24}C_5$ (D) ${}^{24}C_4$

26. The number of ways of arranging the letters AAAAA, BBB, CCC, D, EE & F in a row if the letter C are separated from one another is:
(A) ${}^{13}C_3 . \frac{12!}{5!\,3!\,2!}$ (B) $\frac{13!}{5!\,3!\,3!\,2!}$ (C) $\frac{14!}{3!\,3!\,2!}$ (D) $11 . \frac{13!}{6!}$

27. There are 10 points $P_1, P_2, ..., P_{10}$ in a plane, no three of which are collinear. Number of straight lines which can be determined by these points which do not pass through the points P_1 or P_2 is:
(A) ${}^{10}C_2 - 2 . {}^9C_1$ (B) 27 (C) ${}^8C_2$ (D) ${}^{10}C_2 - 2 . {}^9C_1 + 1$

28. Number of quadrilaterals which can be constructed by joining the vertices of a convex polygon of 20 sides if none of the side of the polygon is also the side of the quadrilateral is:
(A) ${}^{17}C_4 - {}^{15}C_2$ (B) $\frac{{}^{15}C_3 . 20}{4}$ (C) 2275 (D) 2125

29. You are given 8 balls of different colour (black, white,...). The number of ways in which these balls can

be arranged in a row so that the two balls of particular colour (say red & white) may never come together is:
(A) 8 ! – 2.7 ! (B) 6. 7 ! (C) 2. 6 !. $^7C_2$ (D) none

30. A man is dealt a poker hand (consisting of 5 cards) from an ordinary pack of 52 playing cards. The number of ways in which he can be dealt a "straight" (a straight is five consecutive values not of the same suit, eg. {Ace, 2, 3, 4, 5}, {2, 3, 4, 5, 6}......................... & {10, J, Q, K, Ace}) is
(A) 10 (4^5 – 4) (B) 4 !. 2^{10} (C) 10. 2^{10} (D) 10200

31. Number of ways in which 3 numbers in A.P. can be selected from 1, 2, 3,...... n is:
(A) $\left(\frac{n-1}{2}\right)^2$ if n is even (B) $\frac{n(n-2)}{4}$ if n is odd
(C) $\frac{(n-1)^2}{4}$ if n is odd (D) $\frac{n(n-2)}{4}$ if n is even

32. Consider the expansion, $(a_1 + a_2 + a_3 + ... + a_p)^n$ where $n \in N$ and $n \le p$. The correct statement(s) is/are:
(A) number of different terms in the expansion is $^{n+p-1}C_n$
(B) co-efficient of any term in which none of the variables $a_1, a_2 ..., a_p$ occur more than once is ' n '
(C) co-efficient of any term in which none of the variables $a_1, a_2, ..., a_p$ occur more than once is n ! if n = p
(D) Number of terms in which none of the variables $a_1, a_2, ..., a_p$ occur more than once is $\binom{p}{n}$.

EXERCISE–5

1. In a telegraph communication how many words can be communicated by using atmost 5 symbols. (only dot and dash are used as symbols)

2. If all the letters of the word 'AGAIN' are arranged in all possible ways & put in dictionary order, what is the 50th word.

3. A committee of 6 is to be chosen from 10 persons with the condition that if a particular person 'A' is chosen, then another particular person B must be chosen.

4. A family consists of a grandfather, m sons and daughters and 2n grand children. They are to be seated in a row for dinner. The grand children wish to occupy the n seats at each end and the grandfather refuses to have a grand children on either side of him. In how many ways can the family be made to sit?

5. The sides AB, BC & CA of a triangle ABC have 3, 4 & 5 interior points respectively on them. Find the number of triangles that can be constructed using these interior points as vertices.

6. How many five digits numbers divisible by 3 can be formed using the digits 0, 1, 2, 3, 4, 7 and 8 if, each digit is to be used atmost one.

7. In how many other ways can the letters of the word MULTIPLE be arranged ; (i) without changing the order of the vowels (ii) keeping the position of each vowel fixed (iii) without changing the relative order/ position of vowels & consonants.

8. There are p intermediate stations on a railway line from one terminus to another. In how many ways can a train stop at 3 of these intermediate stations if no 2 of these stopping stations are to be consecutive?

9. Find the number of positive integral solutions of x + y + z + w = 20 under the following conditions:
(i) Zero values of x, y, z, w are include
(ii) Zero values are excluded
(iii) No variable may exceed 10; Zero values excluded
(iv) Each variable is an odd number
(v) x, y, z, w have different values (zero excluded).

10. Find the number of words each consisting of 3 consonants & 3 vowels that can be formed from the letters of the word **"CIRCUMFERENCE".** In how many of these C's will be together.

11. If ' n ' distinct things are arranged in a circle, show that the number of ways of selecting three of these things so that no two of them are next to each other is, $\frac{1}{6}$ n (n – 4) (n – 5).

12. In maths paper there is a question on "Match the column" in which column A contains 6 entries & each entry of column A corresponds to exactly one of the 6 entries given in column B written randomly. 2 marks are awarded for each correct matching & 1 mark is deducted from each incorrect matching. A student having no subjective knowledge decides to match all the 6 entries randomly. Find the number of ways in which he can answer, to get atleast 25 % marks in this question.

13. Show that the number of combinations of n letters together out of 3n letters of which n are a and n are b and the rest unlike is, $(n + 2).\ 2^{n-1}$.

14. Find the number of positive integral solutions of, (i) $x^2 - y^2 = 352706$ (ii) $xyz = 21600$

15. There are 'n' straight line in a plane, no two of which are parallel and no three pass through the same point. Their points of intersection are joined. Show that the number of fresh lines thus introduced is, $\frac{1}{8}$ n (n – 1) (n – 2) (n – 3).

16. A forecast is to be made of the results of five cricket matches, each of which can be a win or a draw or a loss for Indian team. Find
(i) number of forecasts with exactly 1 error
(ii) number of forecasts with exactly 3 errors
(iii) number of forecasts with all five errors

17. Prove by permutation or otherwise $\frac{(n^2)!}{(n!)^n}$ is an integer ($n \in I^+$). **[IIT – 2004]**

18. If total number of runs scored in n matches is $\left(\frac{n+1}{4}\right)$ $(2^{n+1} - n - 2)$ where n > 1, and the rund scored in the k^{th} match are given by k. 2^{n+1-k}, where $1 \le k \le n$. Find n **[IIT – 2005]**

ANSWER KEY

EXERCISE–4

1. A **2.** C **3.** D **4.** D **5.** D **6.** C **7.** D
8. D **9.** C **10.** C **11.** B **12.** D **13.** D **14.** D
15. A **16.** A **17.** C **18.** B **19.** B **20.** B **21.** C
22. C **23.** BCD **24.** CD **25.** AB **26.** AD
27. CD **28.** AB **29.** ABC **30.** AD **31.** CD **32.** ACD

EXERCISE–5

1. 62 **2.** NAAIG **3.** 154
4. (2n)! m! (m – 1) **5.** 205 **6.** 744
7. (i) 3359 (ii) 59 (iii) 359
8. ${}^{p-2}C_3$
9. (i) ${}^{23}C_3$ (ii) ${}^{19}C_3$ (iii) ${}^{19}C_3 - 4.{}^{9}C_3$ (iv) ${}^{11}C_8$ (v) 552
10. 22100, 52 **12.** 56 ways
14. (i) Zero (ii) 1260 **16.** (i) 10 (ii) 80 (iii) 32
18. 7

