COMPLEX NUMBERS

Some questions (Assertion–Reason type) are given below. Each question contains **Statement – 1 (Assertion)** and **Statement – 2 (Reason)**. Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct. So select the correct choice :

Choices are :

(A) **Statement – 1** is True, **Statement – 2** is True; **Statement – 2 is** a correct explanation for **Statement – 1**.
(B)**Statement – 1** is True, **Statement – 2** is True; **Statement – 2 is NOT** a correct explanation for **Statement – 1**.
(C) **Statement – 1** is True, **Statement – 2** is False.
(D) **Statement – 1** is False, **Statement – 2** is True.

344. Let $z = e^{i\theta} = \cos\theta + i\sin\theta$
Statement 1: Value of $e^{iA} . e^{iB} . e^{iC} = -1$ if $A + B + C = \pi$. **Statement 2:** $\arg(z) = \theta$ and $|z| = 1$.

345 Let $a_1, a_2, \ldots, a_n \in R^+$
Statement–1 : Minimum value of $\frac{a_1}{a_2} + \frac{a_2}{a_3} + \ldots + \frac{a_{n-1}}{a_n} + \frac{a_n}{a_1}$
Statement–2 : For positive real numbers, A.M ≥ G.M.

346. Let $\log\left(\frac{5c}{a}\right)$, $\log\left(\frac{3b}{5c}\right)$ and $\log\left(\frac{a}{3b}\right)$ then A.P., where a, b, c are in G.P. If a, b, c represents the sides of a triangle. Then : **Statement–1 :** Triangle represented by the sides a, b, c will be an isosceles triangle
Statement–2 : $b + c < a$

347. Let Z_1, Z_2 be two complex numbers represented by points on the curves $|z| = \sqrt{2}$ and $|z - 3 - 3i| = 2\sqrt{2}$. Then
Statement–1 : $\min |z_1 - z_2| = 0$ and $\max |z_1 - z_2| = 6\sqrt{2}$
Statement–2 : Two curves $|z| = \sqrt{2}$ and $|z - 3 - 3i| = 2\sqrt{2}$ touch each other externally

348. **Statement–1 :** If $|z - i| \le 2$ and $z_0 = 5 + 3i$, then the maximum value of $|iz + z_0|$ is 7
Statement–2 : For the complex numbers z_1 and z_2 $|z_1 + z_2| \le |z_1| + |z_2|$

349. Let z_1 and z_2 be complex number such that $|z_1 + z_2| = |z_1| + |z_2|$
Statement–1 : $\arg\left(\frac{z_1}{z_2}\right) = 0$
Statement–2 : z_1, z_2 and origin are collinear and z_1, z_2 are on the same side of origin.

350. Let fourth roots of unity be z_1, z_2, z_3 and z_4 respectively
Statement–1 : $z_1^2 + z_2^2 + z_3^2 + z_4^2 = 0$ **Statement–2** : $z_1 + z_2 + z_3 + z_4 = 0$.

351. Let $z_1, z_2, \ldots, z_n$ be the roots of $z^n = 1$, $n \in N$.
Statement–1 : $z_1 . z_2 \ldots z_n = (-1)^n$ **Statement–2** : Product of the roots of the equation $a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1 x + a_0 = 0$, $a_n \ne 0$, is $(-1)^n . \frac{a_0}{a_n}$.

352. Let z_1, z_2, z_3 and z_4 be the complex numbers satisfying $z_1 - z_2 = z_4 - z_3$.
Statement–1 : z_1, z_2, z_3, z_4 are the vertices of a parallelogram
Statement–2 : $\frac{z_1 + z_3}{2} = \frac{z_2 + z_4}{2}$.

353. **Statement–1** : The minimum value of $|\,|z| + |z - i|\,|$ is 0.
Statement–2 : For any two complex number z_1 and z_2, $|z_1 + z_2| \le |z_1| + |z_2|$.

354. **Statement–1** : Let z_1 and z_2 are two complex numbers such that $|z_1 - z_2| = |z_1 + z_2|$ then the orthocenter of ΔAOB is $\frac{z_1 + z_2}{2}$. (where O is the origin)
Statement–2 : In case of right angled triangle, orthocenter is that point at which triangle is right angled.

355. **Statement–1 :** If ω is complex cube root of unity then $(x - y)(x\omega - y)(x\omega^2 - y)$ is equal to $x^3 + y^2$
Statement–2 : If ω is complex cube root of unity then $1 + \omega + \omega^2 = 0$ and $\omega^3 = 1$

356. **Statement-1 :** If $|z| \le 4$, then greatest value of $|z + 3 - 4i|$ is 9.
Statement-2 : $\forall z_1, z_2 \in C$, $|z_1 + z_2| \le |z_1| + |z_2|$

357. **Statement-1:** The slope of line $(2 - 3i)\, z + (2 + 3i)\, \overline{z} - 1 = 0$ is $\dfrac{2}{3}$

Statement-2:: The slope of line $\overline{a}z + a\overline{z} + b = 0$ $b \in R$ & a be any non-zero complex. Constant is $-\dfrac{Re(a)}{Im(a)}$

358. **Statement-1:** The value of $\sum_{k=1}^{6}\left(\sin\dfrac{2\pi k}{7} - i\cos\dfrac{2\pi k}{7}\right)$ is i

Statement-2: The roots of the equation $z^n = 1$ are called the nth roots of unity where $z = \left(\dfrac{\cos 2\pi k}{n}\right) + i\sin\left(\dfrac{2\pi k}{n}\right)$ where k = 0, 1, 2, ... (n – 1)

359. **Statement-1:** $|z_1 - a| < a$, $|z_2 - b| < b$ $|z_3 - c| < c$, where a, b, c are +ve real nos, then $|z_1 + z_2 + z_3|$ is greater than $2|a + b + c|$ **Statement-2:** $|z_1 \pm z_2| \le |z_1| + |z_2|$

360. **Statement-1:** $(\cos 2 + i\sin 2)^{\pi} = 1$

Statement-2: $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ it is not true when n is irrational number.

361. **Statement-1 :** If $\alpha_1, \alpha_2, \alpha_3 \ldots. \alpha_8$ be the 8^{th} root of unity, then $\alpha_1^{16} + \alpha_2^{16} + \alpha_3^{16} + \ldots + \alpha_8^{16} = 8$

Statement-2 : In case of sum of pth power of nth roots of unity sum = 0 if $p \ne kn$ where p, k, n are integers sum = n if p = kn.

362. **Statement-1:** Locus of z, satisfying the equation $|z - 1| + |z - 8| = 16$ is an ellipse of eccentricity 7/16

Statement-2:: Sum of focal distances of any point is constant for an ellipse

363. **Statement-1:** $\arg\left(\dfrac{z_2}{z_1}\right) = \arg z_2 - \arg z_1$ & $\arg z^n = n(\arg z)$ **Statement-2:** If $|z| = 1$, then $\arg(z^2 + \overline{z}) = \dfrac{1}{2}\arg z$.

364. **Statement-1:** If $|z - z + i| \le 2$ then $\sqrt{5} - 2 \le |z| \le \sqrt{5} + 2$

Statement-2: If $|z - 2 + i| \le 2$ the z lies inside or on the circle having centre (2, –1) & radius 2.

365. **Statement-1:** The area of the triangle on argand plane formed by the complex numbers z, iz and z + iz is $\dfrac{1}{2}|z|^2$

Statement-2: The angle between the two complex numbers z and iz is $\dfrac{\pi}{2}$.

366. **Statement-1:** If $\left|\dfrac{zz_1 - z_2}{zz_1 + z_2}\right| = k$, $(z_1, z_2 \ne 0)$, then locus of z is circle.

Statement-2 : As, $\left|\dfrac{z - z_1}{z - z_2}\right| = \lambda$ represents a circle if, $\lambda \notin \{0, 1\}$

367. **Statement-1:** If z_1 and z_2 are two complex numbers such that $|z_1| = |z_2| + |z_1 - z_2|$, then $\text{Im}\left(\dfrac{z_1}{z_2}\right) = 0$.

Statement-2: arg (z) = 0 ⇒ z is purely real.

368. **Statement-1:** If $\alpha = \cos\left(\dfrac{2\pi}{7}\right) + i\sin\left(\dfrac{2\pi}{7}\right)$, $p = \alpha + \alpha^2 + \alpha^4$, $q = \alpha^3 + \alpha^5 + \alpha^6$, then the equation whose roots are p and q is $x^2 + x + 2 = 0$

Statement-2: If α is a root of $z^7 = 1$, then $1 + \alpha + \alpha^2 + \ldots. + \alpha^6 = 0$.

369. **Statement-1:** If $|z| < \sqrt{2} - 1$ then $|z^2 + 2z\cos\alpha|$ is less than one.

Statement-2: $|z_1 + z_2| < |z_1| + |z_2|$. Also $|\cos\alpha| \le 1$.

370. **Statement-1:** The number of complex number satisfying the equation $|z|^2 + P|z| + q = 0$ $(p, q, \in R)$ is atmost 2.

Statement-2 : A quadratic equation in which all the co-efficients are non-zero real can have exactly two roots.

371. **Statement-1:** If $\left|\beta + \dfrac{1}{\beta}\right| = 1 (\beta \ne 0)$ is a complex number, then the maximum value of $|\beta|$ is $\dfrac{\sqrt{5}+1}{2}$.

Statement-2 :: On the locus $\left|\beta + \dfrac{1}{\beta}\right| = 1$ the farthest distance from origin is $\dfrac{\sqrt{5}+1}{2}$.

372. **Statement-1:** The locus of z moving in the Argand plane such that arg $\left(\frac{z-2}{z+2}\right)=\frac{\pi}{2}$ is a circle.

Statement-2: This is represent a circle, whose centre is origin and radius is 2.

ANSWER

344. B	**345. A**	**346. D**	**347. A**	**348. A**	**349. A**	**350. B**
351. D	**352. A**	**353. D**	**354. D**	**355. D**	**356. A**	**357.. A**
358. A	**359. D**	**360. D**	**361. A**	**362. A**	**363. B**	**364. A**
365. A	**366. D**	**367. A**	**368. A**	**369. A**	**370. D**	**371. A**
372. A						

SOLUTION

345. Using AM ≥ GM $\frac{a_1}{a_2}+\frac{a_2}{a_3}+...+\frac{a_{n-1}}{a_n}+\frac{a_n}{a_1} \ge n\left(\frac{a_1}{a_2}.\frac{a_2}{a_3}....\frac{a_n}{a_1}\right)^{1/n} \Rightarrow \frac{a_1}{a_2}+\frac{a_2}{a_3}+...+\frac{a_n}{a_1} \ge n$

Hence (A) is correct option.

346. $2\log\frac{3b}{5c}=\log\frac{5c}{a}+\log\frac{a}{3b} \Rightarrow \left(\frac{3b}{5c}\right)^2=\frac{5c}{a}.\frac{a}{3b} \Rightarrow 3b = 5c$

Also, $b^2 = ac \Rightarrow 9ac = 25c^2$ or $9a = 25c$ $\therefore \frac{9a}{5}=5c=3b \Rightarrow \frac{a}{5}=\frac{b}{3}=\frac{c}{9/5} \Rightarrow b + c < a$

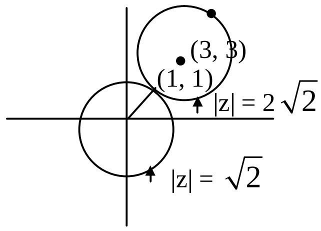
∴ (D) is the correct answer

347. From the diagram it is clear that both circles touch each other externally

∴ min $|z_1 - z_2| = 0$

max $|z_1 - z_2| = \sqrt{36+36}=6\sqrt{2}$]

Hence (A) is correct option.



348. $|iz + z_0| = |i(z - i) - 1 + 5 + 3i| = |i(z-i) + 4 + 3i|$

$\le |i|\,|z - i| + |4 + 3i| \le 7$ Hence (A) is the correct option.

349. (A) $\arg(z_1) = \arg(z_2)$

$\therefore \arg\left(\frac{z_1}{z_2}\right)=\arg(z_1)-\arg(z_2)=0$.

350. (B) Fourth roots of unity are – 1, 1, – i and i

$\therefore z_1^2+z_2^2+z_3^2+z_4^2=0$ and $z_1+z_2+z_3+z_4=0$.

351. Statement – II is true (a known fact).

Hence if $z_1, z_2, \ldots, z_n$ are roots of $z^n - 1 = 0$, then $z_1 . z_2 \ldots z_n = (-1)^n.\frac{(-1)}{1}=(-1)^{n+1}$,

which is never equal to $(-1)^n$ Hence (d) is the correct answer.

352. Both statements – I and II are true and statement – II is the correct reasoning of statement – I, because $\frac{z_1+z_3}{2}=\frac{z_2+z_4}{2} \Rightarrow$ mid point of join of z_1, z_3 and z_2, z_4 are same, which is the necessary and sufficient condition for a quadrilateral ABCD, when $A \equiv A(z_1)$, $B \equiv B(z_2)$, $C \equiv C(z_3)$, $D \equiv D(z_4)$ to be a parallelogram Hence (A) is the correct answer.

353. $|z+i-z| \le |z|+|i-z|$ $\Rightarrow |z|+|z-i| \ge |i| = 1$

∴ Hence (d) is the correct answer.

354. $|z_1-z_2|^2=|z_1+z_2|^2$

$\Rightarrow z_1\bar{z}_2+\bar{z}_1z_2=0 \Rightarrow |z_1-z_2|^2=|z_1|^2+|z_2|^2$

⇒ ΔAOB is right angled at O.

∵ orthocenter is the origin. ∴ Hence (d) is the correct answer.

355. (D) $(x - y)(x\omega - y)(x\omega^2 - y)$

$= x^3\omega^2 - x^2y\omega - x^2y\omega^2 + xy^2 - x^2y\omega + xy^2\omega + xy^2\omega^2 - y^3 = x^3 - y^3$

356. Option (A) is correct
Since
$|z + 3 - 4i| \le |z| + |3-4i| = 9$ $(\therefore |z| \le 4)$.

357. Option (A) is correct.

358. $\sum_{k=1}^{6}(-i)\left(\cos\frac{2\pi k}{7} - i\sin\frac{2\pi k}{7}\right)$

$= (-i)\,)\sum_{k<1}^{6} z^k = (-i)\left(\frac{z - z^7}{1 - z}\right)$ $[\because z^7 = 1]$

$= (-i)(-1) = i$ Ans. (A)

359. $|z_1 + z_2 + z_3| = |z_1 - a + z_2 - b + z_3 - c + (a + b + c)$
$\le |z_1 - a| + |z_2 - b| + |z_3 - c| + |a + b + c| \le 2|a + b + c|$ Ans. (D)

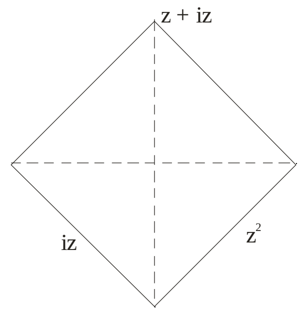
360. $(\cos 2 + i \sin 2)^{\pi}$ can not be evaluated because demoviers theorem does not hold for irrational index.
'd' is correct.

361. $1, \alpha, \alpha^2, \ldots \alpha^7$ are 8, 8th root of unity then after raising 16th power, we get $1, \alpha^{16}, \alpha^{32}, \alpha^{48} \ldots \alpha^{112}$
$1 + \alpha^{16} + \alpha^{32} + \alpha^{48} + \ldots + \alpha^{112}$
Now $\alpha^8 = 1$
So $\alpha^{16} = 1$
$1 + 1 + 1 + \ldots + 1 = 8$
'A' is correct.

365. (A)

$\frac{1}{2}|z||iz|$

$= \frac{|z|^2}{2}$



366. (D)

$$\left|\frac{zz_1 - z_2}{z_1 z + z_2}\right| = k \;\Rightarrow\; \left|\frac{z - \frac{z_2}{z_1}}{z + \frac{z_2}{z_1}}\right| = k$$

Clearly, if $k \ne 0, 1$; then z would lie on a circle. If k = 1, z would lie on the perpendicular bisector of line segment joining $\frac{z_2}{z_1}$ and $\frac{-z_2}{z_1}$ and represents a point, if k = 0.

367. We have, arg (z) = 0 $\Rightarrow$ z is purely real. R is true
Also, $|z_1| = |z_2| + |z_1 - z_2|$
$\Rightarrow (|z_1|^2 + |z_2|^2 - 2|z_1||z_2|\cos(\theta_1 - \theta_2)$
$= |z_1|^2 + |z_2|^2 - 2|z_1||z_2|$
$\Rightarrow \cos(\theta_1 - \theta_2) = 1 \Rightarrow \theta_1 - \theta_2 = 0$

$\Rightarrow \arg\left(\frac{z_1}{z_2}\right) = 0 \Rightarrow \frac{z_1}{z_2}$ is purely real.

$I_m\left(\frac{z_1}{z_2}\right) = 0$ (A)

368. (A)
α is seventh root of unity $\Rightarrow 1 + \alpha + \alpha^2 + \ldots + \alpha^6 = 0$
$\Rightarrow p + q = -1$.
$pq = \alpha^4 + \alpha^6 + \alpha^7 + \alpha^5 + \alpha^7 + \alpha^8 + \alpha^7 + \alpha^9 + \alpha^{10} = 3 - 1 = 2$.

$\therefore$ $x^2 + x + 2 = 0$ is the req. equation.
Both A and R are true and R is correct explanation of A.

369. (A)
$|z^2 + 2z\cos\alpha| < |z^2| + |2z\cos\alpha| < |z^2| + 2|z|\,|\cos\alpha|$
$< (\sqrt{2}-1)^2 + 2(\sqrt{2}-1) < 1$.
($\because$ $|\cos\alpha| \le 1$).

372. $\frac{z-2}{z+2} = \left|\frac{z-2}{z+2}\right| e^{i\pi/2} = \left|\frac{z-2z+2}{}i\right|$ ... (i)

therefore $\frac{\bar{z}-2}{\bar{z}+2} = \left|\frac{\bar{z}-2}{\bar{z}+2}\right|(-1) = -\left|\frac{z-2}{z+2}\right| i$ ... (ii)

Then adding (i) & (ii)
$\frac{z-2}{z+2} + \frac{\bar{z}-2}{\bar{z}+2} = 0$
i.e., $(z-2)\ \bar{z}+2)$ + $(z+2)(\bar{z} - 2) = 0$, $2z\bar{z} - 8 = 0$
$|z|^2 = 4$ $\therefore$ $x^2 + y^2 = 4$.
Ans. (a)

Imp. Que. From Competitive Exams

1. The number of real values of a satisfying the equation $a^2 - 2a\sin x + 1 = 0$ is

(a) Zero (b) One
(c) Two (d) Infinite

2. For positive integers n_1, n_2 the value of the expression $(1+i)^{n_1} + (1+i^3)^{n_1} + (1+i^5)^{n_2} + (1+i^7)^{n_2}$ where $i = \sqrt{-1}$ is a real number if and only if **[IIT 1996]**

(a) $n_1 = n_2 + 1$ (b) $n_1 = n_2 - 1$
(c) $n_1 = n_2$ (d) $n_1 > 0, n_2 > 0$

3. Given that the equation $z^2 + (p+iq)z + r + is = 0$, where p, q, r, s are real and non-zero has a real root, then

(a) $pqr = r^2 + p^2 s$ (b) $prs = q^2 + r^2 p$
(c) $qrs = p^2 + s^2 q$ (d) $pqs = s^2 + q^2 r$

4. If $x = -5 + 2\sqrt{-4}$, then the value of the expression $x^4 + 9x^3 + 35x^2 - x + 4$ is **[IIT 1972]**

(a) 160 (b) –160
(c) 60 (d) –60

5. If $\sqrt{3} + i = (a+ib)(c+id)$, then $\tan^{-1}\left(\frac{b}{a}\right) + \tan^{-1}\left(\frac{d}{c}\right)$ has the value

(a) $\frac{\pi}{3} + 2n\pi, n \in I$ (b) $n\pi + \frac{\pi}{6}, n \in I$
(c) $n\pi - \frac{\pi}{3}, n \in I$ (d) $2n\pi - \frac{\pi}{3}, n \in I$

6. If $a = \cos\alpha + i\sin\alpha$, $b = \cos\beta + i\sin\beta$,

$c = \cos\gamma + i\sin\gamma$ and $\frac{b}{c} + \frac{c}{a} + \frac{a}{b} = 1$, then $\cos(\beta-\gamma) + \cos(\gamma-\alpha) + \cos(\alpha-\beta)$ is equal to **[RPET 2001]**

(a) 3/2 (b) – 3/2
(c) 0 (d) 1

7. If $(1+i)(1+2i)(1+3i).....(1+ni) = a+ib$, then 2.5.10.... $(1+n^2)$ is equal to

[Karnataka CET 2002; Kerala (Engg.) 2002]

(a) $a^2 - b^2$ (b) $a^2 + b^2$

(c) $\sqrt{a^2 + b^2}$ (d) $\sqrt{a^2 - b^2}$

8. If z is a complex number, then the minimum value of $|z| + |z-1|$ is **[Roorkee 1992]**

(a) 1 (b) 0

(c) 1/2 (d) None of these

9. For any two complex numbers z_1 and z_2 and any real numbers a and b; $|(az_1 - bz_2)|^2 + |(bz_1 + az_2)|^2 =$ ***[IIT 1988]***

(a) $(a^2+b^2)(|z_1| + |z_2|)$ (b) $(a^2+b^2)(|z_1|^2 + |z_2|^2)$

(c) $(a^2+b^2)(|z_1|^2 - |z_2|^2)$ (d) None of these

10. The locus of z satisfying the inequality $\log_{1/3}|z+1| > \log_{1/3}|z-1|$ is

(a) $R(z) < 0$ (b) $R(z) > 0$

(c) $I(z) < 0$ (d) None of these

11. If $z_1 = a + ib$ and $z_2 = c + id$ are complex numbers such that $|z_1| = |z_2| = 1$ and $R(z_1\overline{z_2}) = 0$, then the pair of complex numbers $w_1 = a + ic$ and $w_2 = b + id$ satisfies

[IIT 1985]

(a) $|w_1| = 1$ (b) $|w_2| = 1$

(c) $R(w_1\overline{w_2}) = 0$, (d) All the above

12. Let z and w be two complex numbers such that $|z| \le 1$, $|w| \le 1$ and $|z + iw| = |z - i\overline{w}| = 2$. Then z is equal to ***[IIT 1995]***

(a) 1 or i (b) i or $-i$

(c) 1 or – 1 (d) i or –1

13. The maximum distance from the origin of coordinates to the point z satisfying the equation $\left|z + \frac{1}{z}\right| = a$ is

(a) $\frac{1}{2}(\sqrt{a^2+1} + a)$ (b) $\frac{1}{2}(\sqrt{a^2+2} + a)$

(c) $\frac{1}{2}(\sqrt{a^2+4} + a)$ (d) None of these

14. Find the complex number z satisfying the equations $\left|\frac{z-12}{z-8i}\right| = \frac{5}{3}, \left|\frac{z-4}{z-8}\right| = 1$ ***[Roorkee 1993]***

(a) 6 (b) $6 \pm 8i$

(c) $6 + 8i, 6 + 17i$ (d) None of these

15. If z_1, z_2, z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$, then $|z_1 + z_2 + z_3|$ is

[MP PET 2004; IIT Screening 2000]

(a) Equal to 1 (b) Less than 1

(c) Greater than 3 (d) Equal to 3

16. If $z_1 = 10 + 6i, z_2 = 4 + 6i$ and z is a complex number such that $amp\left(\frac{z-z_1}{z-z_2}\right) = \frac{\pi}{4}$, then the value of $|z - 7 - 9i|$ is equal to ***[IIT 1990]***

(a) $\sqrt{2}$ (b) $2\sqrt{2}$

(c) $3\sqrt{2}$ (d) $2\sqrt{3}$

17. If z_1, z_2, z_3 be three non-zero complex number, such that $z_2 \ne z_1, a = |z_1|, b = |z_2|$ and $c = |z_3|$ suppose that $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$, then $arg\left(\frac{z_3}{z_2}\right)$ is equal to

(a) $arg\left(\frac{z_2 - z_1}{z_3 - z_1}\right)^2$ (b) $arg\left(\frac{z_2 - z_1}{z_3 - z_1}\right)$

(c) $arg\left(\frac{z_3 - z_1}{z_2 - z_1}\right)^2$ (d) $arg\left(\frac{z_3 - z_1}{z_2 - z_1}\right)$

18. Let z and w be the two non-zero complex numbers such that $|z| = |w|$ and $arg\, z + arg\, w = \pi$. Then z is equal to

[IIT 1995; AIEEE 2002]

(a) w (b) $-w$

(c) $\overline{w}$ (d) $-\overline{w}$

19. If $|z - 25i| \le 15$, then $|\max.amp(z) - \min.amp(z)| =$

(a) $\cos^{-1}\left(\frac{3}{5}\right)$ (b) $\pi - 2\cos^{-1}\left(\frac{3}{5}\right)$

(c) $\frac{\pi}{2} + \cos^{-1}\left(\frac{3}{5}\right)$ (d) $\sin^{-1}\left(\frac{3}{5}\right) - \cos^{-1}\left(\frac{3}{5}\right)$

20. If z_1, z_2 and z_3, z_4 are two pairs of conjugate complex numbers, then $arg\left(\frac{z_1}{z_4}\right) + arg\left(\frac{z_2}{z_3}\right)$ equals

(a) 0 (b) $\frac{\pi}{2}$

(c) $\frac{3\pi}{2}$ (d) π

21. Let z, w be complex numbers such that $\overline{z} + i\overline{w} = 0$ and $arg\ zw = \pi$. Then $arg\ z$ equals **[AIEEE 2004]**

(a) $5\pi/4$ (b) $\pi/2$

(c) $3\pi/4$ (d) $\pi/4$

22. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + ... + C_n x^n$, then the value of $C_0 - C_2 + C_4 - C_6 + ...$ is

(a) 2^n (b) $2^n \cos\frac{n\pi}{2}$

(c) $2^n \sin\frac{n\pi}{2}$ (d) $2^{n/2}\cos\frac{n\pi}{4}$

23. If $x = \cos\theta + i\sin\theta$ and $y = \cos\phi + i\sin\phi$, then $x^m y^n + x^{-m} y^{-n}$ is equal to

(a) $\cos(m\theta + n\phi)$

(b) $\cos(m\theta - n\phi)$

(c) $2\cos(m\theta + n\phi)$

(d) $2\cos(m\theta - n\phi)$

24. The value of $\sum_{r=1}^{8}\left(\sin\frac{2r\pi}{9} + i\cos\frac{2r\pi}{9}\right)$ is

(a) -1 (b) 1

(c) i (d) $-i$

25. If a, b, c and u, v, w are complex numbers representing the vertices of two triangles such that $c = (1-r)a + rb$ and $w = (1-r)u + rv$, where r is a complex number, then the two triangles

(a) Have the same area (b) Are similar

(c) Are congruent (d) None of these

26. Suppose z_1, z_2, z_3 are the vertices of an equilateral triangle inscribed in the circle $|z| = 2$. If $z_1 = 1 + i\sqrt{3}$, then values of z_3 and z_2 are respectively **[IIT 1994]**

(a) $-2, 1 - i\sqrt{3}$ (b) $2, 1 + i\sqrt{3}$

(c) $1 + i\sqrt{3}, -2$ (d) None of these

27. If the complex number z_1, z_2 the origin form an equilateral triangle then $z_1^2 + z_2^2 =$ ***[IIT 1983]***

(a) $z_1 z_2$ (b) $z_1 \overline{z_2}$

(c) $\overline{z_2} z_1$ (d) $|z_1|^2 = |z_2|^2$

28. If at least one value of the complex number $z = x + iy$ satisfy the condition $|z + \sqrt{2}| = a^2 - 3a + 2$ and the inequality $|z + i\sqrt{2}| < a^2$, then

(a) $a > 2$ (b) $a = 2$

(c) $a < 2$ (d) None of these

29. If z, iz and $z + iz$ are the vertices of a triangle whose area is 2 units, then the value of $|z|$ is ***[RPET 2000]***

(a) – 2 (b) 2

(c) 4 (d) 8

30. If $z^2 + z|z| + |z|^2 = 0$, then the locus of z is

(a) A circle (b) A straight line

(c) A pair of straight lines (d) None of these

31. If $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$ then $\cos 3\alpha + \cos 3\beta + \cos 3\gamma$ equals to ***[Kar. CET 2000]***

(a) 0 (b) $\cos(\alpha + \beta + \gamma)$

(c) $3\cos(\alpha + \beta + \gamma)$ (d) $3\sin(\alpha + \beta + \gamma)$

32. If $z_r = \cos\frac{r\alpha}{n^2} + i\sin\frac{r\alpha}{n^2}$, where r = 1, 2, 3,….,n, then $\lim_{n\to\infty} z_1 z_2 z_3 ... z_n$ is equal to ***[UPSEAT 2001]***

(a) $\cos\alpha + i\sin\alpha$ (b) $\cos(\alpha/2) - i\sin(\alpha/2)$

(c) $e^{i\alpha/2}$ (d) $\sqrt[3]{e^{i\alpha}}$

33. If the cube roots of unity be $1, \omega, \omega^2$, then the roots of the equation $(x-1)^3 + 8 = 0$ are ***[IIT 1979; MNR 1986; DCE 2000; AIEEE 2005]***

(a) $-1, 1+2\omega, 1+2\omega^2$

(b) $-1, 1-2\omega, 1-2\omega^2$

(c) $-1, -1, -1$

(d) None of these

34. If $1, \omega, \omega^2, \omega^3 ...., \omega^{n-1}$ are the n, n^{th} roots of unity, then $(1-\omega)(1-\omega^2).....(1-\omega^{n-1})$ equals ***[MNR 1992; IIT 1984; DCE 2001; MP PET 2004]***

(a) 0 (b) 1

(c) n (d) n^2

35. The value of the expression $1.(2-\omega)(2-\omega^2) + 2.(3-\omega)(3-\omega^2) + ...$

$.... + (n-1).(n-\omega)(n-\omega^2)$,

where ω is an imaginary cube root of unity, is***[IIT 1996]***

(a) $\frac{1}{2}(n-1)n(n^2 + 3n + 4)$

(b) $\frac{1}{4}(n-1)n(n^2 + 3n + 4)$

(c) $\frac{1}{2}(n+1)n(n^2 + 3n + 4)$

(d) $\frac{1}{4}(n+1)n(n^2 + 3n + 4)$

36. If $i=\sqrt{-1}$, then $4+5\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{334}+3\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{365}$ is equal to ***[IIT 1999]***

(a) $1-i\sqrt{3}$ (b) $-1+i\sqrt{3}$

(c) $i\sqrt{3}$ (d) $-i\sqrt{3}$

37. If $a=\cos(2\pi/7)+i\sin(2\pi/7)$, then the quadratic equation whose roots are $\alpha=a+a^2+a^4$ and $\beta=a^3+a^5+a^6$ is ***[RPET 2000]***

(a) $x^2-x+2=0$ (b) $x^2+x-2=0$

(c) $x^2-x-2=0$ (d) $x^2+x+2=0$

38. Let z_1 and z_2 be n^{th} roots of unity which are ends of a line segment that subtend a right angle at the origin. Then n must be of the form ***[IIT Screening 2001; Karnataka 2002]***

(a) $4k+1$ (b) $4k+2$

(c) $4k+3$ (d) $4k$

39. Let ω is an imaginary cube roots of unity then the value of

$2(\omega+1)(\omega^2+1)+3(2\omega+1)(2\omega^2+1)+.....+(n+1)(n\omega+1)(n\omega^2+1)$ is ***[Orissa JEE 2002]***

(a) $\left[\frac{n(n+1)}{2}\right]^2+n$ (b) $\left[\frac{n(n+1)}{2}\right]^2$

(c) $\left[\frac{n(n+1)}{2}\right]^2-n$ (d) None of these

40. ω is an imaginary cube root of unity. If $(1+\omega^2)^m=(1+\omega^4)^m$, then least positive integral value of m is ***[IIT Screening 2004]***

(a) 6 (b) 5

(c) 4 (d) 3

ANSWER

1	**c**	**2**	**d**	**3**	**d**	**4**	**b**	**5**	**b**
6	**d**	**7**	**b**	**8**	**a**	**9**	**b**	**10**	**a**
11	**d**	**12**	**c**	**13**	**c**	**14**	**c**	**15**	**a**
16	**c**	**17**	**c**	**18**	**d**	**19**	**b**	**20**	**a**
21	**c**	**22**	**d**	**23**	**c**	**24**	**d**	**25**	**b**
26	**a**	**27**	**a**	**28**	**a**	**29**	**b**	**30**	**c**
31	**c**	**32**	**c**	**33**	**b**	**34**	**c**	**35**	**b**
36	**c**	**37**	**D**	**38**	**d**	**39**	**a**	**40**	**d**