SHORT REVISION

DEFINITIONS :

1. **PERMUTATION :** Each of the arrangements in a definite order which can be made by taking some or all of a number of things is called a **PERMUTATION**.

2. **COMBINATION :** Each of the groups or selections which can be made by taking some or all of a number of things without reference to the order of the things in each group is called a **COMBINATION**.

FUNDAMENTAL PRINCIPLE OF COUNTING :
If an event can occur in 'm' different ways, following which another event can occur in '*n*' different ways, then the total number of different ways of simultaneous occurrence of both events in a definite order is $m \times n$. This can be extended to any number of events.

RESULTS : (i) A Useful Notation : $n! = n(n-1)(n-2)......... 3.\ 2.\ 1$; $n! = n.\ (n-1)!$
$0! = 1! = 1$; $(2n)! = 2^n.\ n!\ [1.\ 3.\ 5.\ 7...(2n-1)]$ Note that factorials of negative integers are not defined.

(ii) If ${}^nP_r$ denotes the number of permutations of n different things, taking r at a time, then
${}^nP_r = n(n-1)(n-2).....(n-r+1) = \dfrac{n!}{(n-r)!}$ Note that, ${}^nP_n = n!$.

(iii) If ${}^nC_r$ denotes the number of combinations of n different things taken r at a time, then
${}^nC_r = \dfrac{n!}{r!(n-r)!} = \dfrac{{}^nP_r}{r!}$ where $r \le n$; $n \in N$ and $r \in W$.

(iv) The number of ways in which (m + n) different things can be divided into two groups containing m & n things respectively is : $\dfrac{(m+n)!}{m!\,n!}$ If m = n, the groups are equal & in this case the number of subdivision is $\dfrac{(2n)!}{n!\,n!\,2!}$; for in any one way it is possible to interchange the two groups without obtaining a new distribution. However, if 2n things are to be divided equally between two persons then the number of ways $= \dfrac{(2n)!}{n!\,n!}$.

(v) Number of ways in which (m + n + p) different things can be divided into three groups containing m , n & p things respectively is $\dfrac{(m+n+p)!}{m!\,n!\,p!}$, $m \ne n \ne p$. If $m = n = p$ then the number of groups $= \dfrac{(3n)!}{n!\,n!\,n!\,3!}$.
However, if 3n things are to be divided equally among three people then the number of ways $= \dfrac{(3n)!}{(n!)^3}$.

(vi) The number of permutations of *n* things taken all at a time when *p* of them are similar & of one type, q of them are similar & of another type, *r* of them are similar & of a third type & the remaining $n-(p+q+r)$ are all different is : $\dfrac{n!}{p!\,q!\,r!}$.

(vii) The number of circular permutations of n different things taken all at a time is ; $(n-1)!$. If clockwise & anti–clockwise circular permutations are considered to be same, then it is $\dfrac{(n-1)!}{2}$.
Note : Number of circular permutations of n things when p alike and the rest different taken all at a time distinguishing clockwise and anticlockwise arrangement is $\dfrac{(n-1)!}{p!}$.

(viii) Given n different objects, the number of ways of selecting atleast one of them is , ${}^nC_1 + {}^nC_2 + {}^nC_3 + ... + {}^nC_n = 2^n - 1$. This can also be stated as the total number of combinations of n distinct things.

(ix) Total number of ways in which it is possible to make a selection by taking some or all out of p + q + r +...... things , where p are alike of one kind, q alike of a second kind , r alike of third kind & so on is given by : $(p+1)(q+1)(r+1)........ -1$.

(x) Number of ways in which it is possible to make a selection of m + n + p = N things , where p are alike of one kind, m alike of second kind & n alike of third kind taken r at a time is given by coefficient of x^r in the expansion of
$(1 + x + x^2 + ... + x^p)(1 + x + x^2 + ... + x^m)(1 + x + x^2 + ... + x^n)$.
Note : Remember that coefficient of x^r in $(1-x)^{-n} = {}^{n+r-1}C_r\ (n \in N)$. For example the number of ways in which a selection of four letters can be made from the letters of the word **PROPORTION** is given by coefficient of x^4 in $(1 + x + x^2 + x^3)(1 + x + x^2)(1 + x + x^2)(1 + x)(1 + x)(1 + x)$.

(xi) Number of ways in which n distinct things can be distributed to p persons if there is no restriction to the number of things received by men $= p^n$.

(xii) Number of ways in which n identical things may be distributed among p persons if each person may receive none, one or more things is ; ${}^{n+p-1}C_n$.

(xiii) **a.** ${}^nC_r = {}^nC_{n-r}$; ${}^nC_0 = {}^nC_n = 1$; **b.** ${}^nC_x = {}^nC_y \Rightarrow x = y$ or $x + y = n$
c. ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

(xiv) ${}^nC_r$ is maximum if : (a) $r = \dfrac{n}{2}$ if n is even. (b) $r = \dfrac{n-1}{2}$ or $\dfrac{n+1}{2}$ if n is odd.

(xv) Let $N = p^a \cdot q^b \cdot r^c \cdot .....$ where p , q , r...... are distinct primes & a , b , c..... are natural numbers then:
(a) The total numbers of divisors of N including 1 & N is $= (a+1)(b+1)(c+1).....$
(b) The sum of these divisors is
$= (p^0 + p^1 + p^2 + ... + p^a)(q^0 + q^1 + q^2 + ... + q^b)(r^0 + r^1 + r^2 + ... + r^c)....$
(c) Number of ways in which N can be resolved as a product of two

factors is $= \begin{cases} \frac{1}{2}(a+1)(b+1)(c+1).... & \text{if N is not a perfect square} \\ \frac{1}{2}\left[(a+1)(b+1)(c+1)....+1\right] & \text{if N is a perfect square} \end{cases}$

(d) Number of ways in which a composite number N can be resolved into two factors which are relatively prime (or coprime) to each other is equal to 2^{n-1} where n is the number of different prime factors in N. [Refer Q.No.28 of Ex–I]

(xvi) Grid Problems and tree diagrams.

DEARRANGEMENT : Number of ways in which n letters can be placed in n directed letters so that no letter goes into its own envelope is = $n!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}+..........+(-1)^n\frac{1}{n!}\right]$.

(xvii) Some times students find it difficult to decide whether a problem is on permutation or combination or both. Based on certain words / phrases occuring in the problem we can fairly decide its nature as per the following table :

PROBLEMS OF COMBINATIONS	PROBLEMS OF PERMUTATIONS
■ Selections , choose	■ Arrangements
■ Distributed group is formed	■ Standing in a line seated in a row
■ Committee	■ problems on digits
■ Geometrical problems	■ Problems on letters from a word

EXERCISE–1

Q.1 The straight lines l_1, l_2 & l_3 are parallel & lie in the same plane. A total of m points are taken on the line l_1 , n points on l_2 & k points on l_3. How many maximum number of triangles are there whose vertices are at these points ?

Q.2 How many five digits numbers divisible by 3 can be formed using the digits 0, 1, 2, 3, 4, 7 and 8 if each digit is to be used atmost once.

Q.3 There are 2 women participating in a chess tournament. Every participant played 2 games with the other participants. The number of games that the men played between themselves exceeded by 66 as compared to the number of games that the men played with the women. Find the number of participants & the total numbers of games played in the tournament.

Q.4 All the 7 digit numbers containing each of the digits 1, 2, 3, 4, 5, 6, 7 exactly once, and not divisible by 5 are arranged in the increasing order. Find the $(2004)^{th}$ number in this list.

Q.5 5 boys & 4 girls sit in a straight line. Find the number of ways in which they can be seated if 2 girls are together & the other 2 are also together but separate from the first 2.

Q.6 A crew of an eight oar boat has to be chosen out of 11 men five of whom can row on stroke side only, four on the bow side only, and the remaining two on either side. How many different selections can be made?

Q.7 An examination paper consists of 12 questions divided into parts A & B.
Part-A contains 7 questions & Part–B contains 5 questions. A candidate is required to attempt 8 questions selecting atleast 3 from each part. In how many maximum ways can the candidate select the questions ?

Q.8 In how many ways can a team of 6 horses be selected out of a stud of 16, so that there shall always be 3 out of A B C A′ B′ C′, but never A A′ , B B′ or C C′ together.

Q.9 During a draw of lottery, tickets bearing numbers 1, 2, 3,......, 40, 6 tickets are drawn out & then arranged in the descending order of their numbers. In how many ways, it is possible to have 4^{th} ticket bearing number 25.

Q.10 Find the number of distinct natural numbers upto a maximum of 4 digits and divisible by 5, which can be formed with the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 each digit not occuring more than once in each number.

Q.11 The Indian cricket team with eleven players, the team manager, the physiotherapist and two umpires are to travel from the hotel where they are staying to the stadium where the test match is to be played. Four of them residing in the same town own cars, each a four seater which they will drive themselves. The bus which was to pick them up failed to arrive in time after leaving the opposite team at the stadium. In how many ways can they be seated in the cars ? In how many ways can they travel by these cars so as to reach in time, if the seating arrangement in each car is immaterial and all the cars reach the stadium by the same route.

Q.12 There are n straight lines in a plane, no 2 of which parallel , & no 3 pass through the same point. Their point of intersection are joined. Show that the number of fresh lines thus introduced is $\frac{n(n-1)(n-2)(n-3)}{8}$.

Q.13 In how many ways can you divide a pack of 52 cards equally among 4 players. In how many ways the cards can be divided in 4 sets, 3 of them having 17 cards each & the 4^{th} with 1 card.

Q.14 A firm of Chartered Accountants in Bombay has to send 10 clerks to 5 different companies, two clerks in each. Two of the companies are in Bombay and the others are outside. Two of the clerks prefer to work in Bombay while three others prefer to work outside. In how many ways can the assignment be made if the preferences are to be satisfied.

Q.15 A train going from Cambridge to London stops at nine intermediate stations. 6 persons enter the train during the journey with 6 different tickets of the same class. How many different sets of ticket may they have had?

Q.16 Prove that if each of m points in one straight line be joined to each of n in another by straight lines terminated by the points, then excluding the given points, the lines will intersect $\frac{1}{4}mn(m-1)(n-1)$ times.

Q.17 How many arrangements each consisting of 2 vowels & 2 consonants can be made out of the letters of the word **'DEVASTATION'**?

Q.18 Find the number of words each consisting of 3 consonants & 3 vowels that can be formed from the letters of the word **"Circumference".** In how many of these c's will be together.

Q.19 There are 5 white , 4 yellow , 3 green , 2 blue & 1 red ball. The balls are all identical except for colour. These are to be arranged in a line in 5 places. Find the number of distinct arrangements.

Q.20 How many 4 digit numbers are there which contains not more than 2 different digits?

Q.21 In how many ways 8 persons can be seated on a round table
(a) If two of them (say A and B) must not sit in adjacent seats.
(b) If 4 of the persons are men and 4 ladies and if no two men are to be in adjacent seats.

(c) If 8 persons constitute 4 married couples and if no husband and wife, as well as no two men, are to be in adjacent seats?

Q.22 (i) If 'n' things are arranged in circular order , then show that the number of ways of selecting four of the things no two of which are consecutive is $\frac{n(n-5)(n-6)(n-7)}{4!}$

(ii) If the 'n' things are arranged in a row, then show that the number of such sets of four is $\frac{(n-3)(n-4)(n-5)(n-6)}{4!}$

Q.23(a)How many divisors are there of the number x = 21600. Find also the sum of these divisors.

(b)In how many ways the number 7056 can be resolved as a product of 2 factors.

(c)Find the number of ways in which the number 300300 can be split into 2 factors which are relatively prime.

Q.24 How many ten digits whole number satisfy the following property they have 2 and 5 as digits, and there are no consecutive 2's in the number (i.e. any two 2's are separated by at least one 5).

Q.25 How many different ways can 15 Candy bars be distributed between Ram, Shyam, Ghanshyam and Balram, if Ram can not have more than 5 candy bars and Shyam must have at least two. Assume all Candy bars to be alike.

Q.26 Find the number of distinct throws which can be thrown with 'n' six faced normal dice which are indistinguishable among themselves.

Q.27 How many integers between 1000 and 9999 have exactly one pair of equal digit such as 4049 or 9902 but not 4449 or 4040?

Q.28 In a certain town the streets are arranged like the lines of a chess board. There are 6 streets running north & south and 10 running east & west. Find the number of ways in which a man can go from the north-west corner to the south-east corner covering the shortest possible distance in each case.

Q.29 (i) Prove that : ${}^{n}P_r = {}^{n-1}P_r + r.\,{}^{n-1}P_{r-1}$ (ii) If ${}^{20}C_{r+2} = {}^{20}C_{2r-3}$ find ${}^{12}C_r$

(iii) Find the ratio ${}^{20}C_p$ to ${}^{25}C_r$ when each of them has the greatest value possible.

(iv) Prove that ${}^{n-1}C_3 + {}^{n-1}C_4 > {}^{n}C_3$ if $n > 7$. (v) Find r if ${}^{15}C_{3r} = {}^{15}C_{r+3}$

Q.30 There are 20 books on Algebra & Calculus in our library. Prove that the greatest number of selections each of which consists of 5 books on each topic is possible only when there are 10 books on each topic in the library.

EXERCISE–2

Q.1 Find the number of ways in which 3 distinct numbers can be selected from the set $\{3^1, 3^2, 3^3, ... 3^{100}, 3^{101}\}$ so that they form a G.P.

Q.2 Let n & k be positive integers such that $n \geq \frac{k(k+1)}{2}$. Find the number of solutions $(x_1, x_2, ..., x_k)$, $x_1 \geq 1$, $x_2 \geq 2, ...,$ $x_k \geq k$, all integers, satisfying $x_1 + x_2 + ... + x_k = n$.

Q.3 There are counters available in 7 different colours. Counters are all alike except for the colour and they are atleast ten of each colour. Find the number of ways in which an arrangement of 10 counters can be made. How many of these will have counters of each colour.

Q.4 For each positive integer k, let S_k denote the increasing arithmetic sequence of integers whose first term is 1 and whose common difference is k. For example, S_3 is the sequence 1, 4, 7, 10...... Find the number of values of k for which S_k contain the term 361.

Q.5 Find the number of 7 lettered words each consisting of 3 vowels and 4 consonants which can be formed using the letters of the word "DIFFERENTIATION".

Q.6 A shop sells 6 different flavours of ice-cream. In how many ways can a customer choose 4 ice-cream cones if
(i) they are all of different flavours (ii) they are non necessarily of different flavours
(iii) they contain only 3 different flavours (iv) they contain only 2 or 3 different flavours?

Q.7 6 white & 6 black balls of the same size are distributed among 10 different urns. Balls are alike except for the colour & each urn can hold any number of balls. Find the number of different distribution of the balls so that there is atleast 1 ball in each urn.

Q.8 There are 2n guests at a dinner party. Supposing that the master and mistress of the house have fixed seats opposite one another, and that there are two specified guests who must not be placed next to one another. Show that the number of ways in which the company can be placed is $(2n-2)!.(4n^2 - 6n + 4)$.

Q.9 Each of 3 committees has 1 vacancy which is to be filled from a group of 6 people. Find the number of ways the 3 vacancies can be filled if ;
(i) Each person can serve on atmost 1 committee.
(ii) There is no restriction on the number of committees on which a person can serve.
(iii) Each person can serve on atmost 2 committees.

Q.10 How many 15 letter arrangements of 5 A's, 5 B's and 5 C's have no A's in the first 5 letters, no B's in the next 5 letters, and no C's in the last 5 letters.

Q.11 5 balls are to be placed in 3 boxes. Each box can hold all 5 balls. In how many different ways can we place the balls so that no box remains empty if,
(i) balls & boxes are different (ii) balls are identical but boxes are different
(iii) balls are different but boxes are identical (iv) balls as well as boxes are identical
(v) balls as well as boxes are identical but boxes are kept in a row.

Q.12 In how many other ways can the letters of the word **MULTIPLE** be arranged;
(i) without changing the order of the vowels (ii) keeping the position of each vowel fixed &
(iii) without changing the relative order/position of vowels & consonants.

Q.13 Find the number of ways in which the number 30 can be partitioned into three unequal parts, each part being a natural number. What this number would be if equal parts are also included.

Q.14 In an election for the managing committee of a reputed club , the number of candidates contesting elections exceeds the number of members to be elected by r (r > 0). If a voter can vote in 967 different ways to elect the managing committee by voting atleast 1 of them & can vote in 55 different ways to elect (r – 1) candidates by voting in the same manner. Find the number of candidates contesting the elections & the number of candidates losing the elections.

Q.15 Find the number of three digits numbers from 100 to 999 inclusive which have any one digit that is the average of the other two.

Q.16 Prove by combinatorial argument that :
(a) ${}^{n+1}C_r = {}^nC_r + {}^nC_{r-1}$ (b) ${}^{n+m}C_r = {}^nC_0 \cdot {}^mC_r + {}^nC_1 \cdot {}^mC_{r-1} + {}^nC_2 \cdot {}^mC_{r-2} + \ldots\ldots + {}^nC_r \cdot {}^mC_0$.

Q.17 A man has 3 friends. In how many ways he can invite one friend everyday for dinner on 6 successive nights so that no friend is invited more than 3 times.

Q.18 12 persons are to be seated at a square table, three on each side. 2 persons wish to sit on the north side and two wish to sit on the east side. One other person insists on occupying the middle seat (which may be on any side). Find the number of ways they can be seated.

Q.19 There are 15 rowing clubs; two of the clubs have each 3 boats on the river; five others have each 2 and the remaining eight have each 1; find the number of ways in which a list can be formed of the order of the 24 boats, observing that the second boat of a club cannot be above the first and the third above the second. How many ways are there in which a boat of the club having single boat on the river is at the third place in the list formed above?

Q.20 25 passengers arrive at a railway station & proceed to the neighbouring village. At the station there are 2 coaches accommodating 4 each & 3 carts accommodating 3 each. Find the number of ways in which they can proceed to the village assuming that the conveyances are always fully occupied & that the conveyances are all distinguishable from each other.

Q.21 An 8 oared boat is to be manned by a crew chosen from 14 men of which 4 can only steer but can not row & the rest can row but cannot steer. Of those who can row, 2 can row on the bow side. In how many ways can the crew be arranged.

Q.22 How many 6 digits odd numbers greater than 60,0000 can be formed from the digits 5, 6, 7, 8, 9, 0 if (i) repetitions are not allowed (ii) repetitions are allowed.

Q.23 Find the sum of all numbers greater than 10000 formed by using the digits 0 , 1 , 2 , 4 , 5 no digit being repeated in any number.

Q.24 The members of a chess club took part in a round robin competition in which each plays every one else once. All members scored the same number of points, except four juniors whose total score were 17.5. How many members were there in the club? Assume that for each win a player scores 1 point , for draw 1/2 point and zero for losing.

Q.25 There are 3 cars of different make available to transport 3 girsls and 5 boys on a field trip. Each car can hold up to 3 children. Find (a) the number of ways in which they can be accomodated.
(b) the numbers of ways in which they can be accomodated if 2 or 3 girls are assigned to one of the cars.
In both the cars internal arrangement of childrent inside the car is to be considered as immaterial.

Q.26 Six faces of an ordinary cubical die marked with alphabets A, B, C, D, E and F is thrown n times and the list of n alphabets showing up are noted. Find the total number of ways in which among the alphabets A, B, C, D, E and F only three of them appear in the list.

Q.27 Find the number of integer betwen 1 and 10000 with at least one 8 and at least one 9 as digits.

Q.28 The number of combinations n together of 3n letters of which n are'a' and n are'b' and the rest unlike is $(n+2).2^{n-1}$.

Q.29 In Indo–Pak one day International cricket match at Sharjah , India needs 14 runs to win just before the start of the final over. Find the number of ways in which India just manages to win the match (i.e. scores exactly 14 runs) , assuming that all the runs are made off the bat & the batsman can not score more than 4 runs off any ball.

Q.30 A man goes in for an examination in which there are 4 papers with a maximum of m marks for each paper; show that the number of ways of getting 2m marks on the whole is $\frac{1}{3}(m+1)(2m^2+4m+3)$.

EXERCISE–3

Q.1 Find the total number of ways of selecting five letters from the letters of the word INDEPENDENT.[REE '97, 6]

Q.2 Select the correct alternative(s). [JEE '98, 2 + 2]

(i) Number of divisors of the form $4n + 2$ ($n \geq 0$) of the integer 240 is
(A) 4 (B) 8 (C) 10 (D) 3

(ii) An n-digit number is a positive number with exactly 'n' digits. Nine hundred distinct n-digit numbers are to be formed using only the three digits 2, 5 & 7. The smallest value of n for which this is possible is :
(A) 6 (B) 7 (C) 8 (D) 9

Q.3 How many different nine digit numbers can be formed from the number 223355888 by rearranging its digits so that the odd digits occupy even positions ? [JEE '2000, (Scr)]
(A) 16 (B) 36 (C) 60 (D) 180

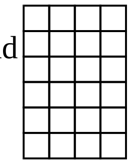
Q.4 Let T_n denote the number of triangles which can be formed using the vertices of a regular polygon of ' n ' sides. If $T_{n+1} - T_n = 21$, then ' n ' equals: [JEE '2001, (Scr)]
(A) 5 (B) 7 (C) 6 (D) 4

Q.5 The number of arrangements of the letters of the word BANANA in which the two N's do not appear adjacently is [JEE 2002 (Screening), 3]
(A) 40 (B) 60 (C) 80 (D) 100

Q.6 Number of points with integral co-ordinates that lie inside a triangle whose co-ordinates are (0, 0), (0, 21) and (21,0) [JEE 2003 (Screening), 3]
(A) 210 (B) 190 (C) 220 (D) None

Q.7 Using permutation or otherwise, prove that $\frac{(n^2)!}{(n!)^n}$ is an integer, where n is a positive integer.
[JEE 2004, 2 out of 60]

Q.8 A rectangle with sides 2m – 1 and 2n – 1 is divided into squares of unit length by drawing parallel lines as shown in the diagram, then the number of rectangles possible with odd side lengths is



(A) $(m + n + 1)^2$ (B) 4^{m+n-1}
(C) m^2n^2 (D) $mn(m + 1)(n + 1)$
[JEE 2005 (Screening), 3]

Q.9 If *r*, *s*, *t* are prime numbers and p, q are the positive integers such that their LCM of p, q is is $r^2t^4s^2$, then the numbers of ordered pair of (p, q) is
(A) 252 (B) 254 (C) 225 (D) 224 [JEE 2006, 3]

EXERCISE–4

Part : (A) Only one correct option

1. There are 2 identical white balls, 3 identical red balls and 4 green balls of different shades. The number of ways in which they can be arranged in a row so that atleast one ball is separated from the balls of the same colour, is:
(A) 6 (7 ! – 4 !) (B) 7 (6 ! – 4 !) (C) 8 ! – 5 ! (D) none

2. The number of permutations that can be formed by arranging all the letters of the word 'NINETEEN' in which no two E's occur together is
(A) $\frac{8!}{3!\,3!}$ (B) $\frac{5!}{3! \times {}^6C_2}$ (C) $\frac{5\,!}{3\,!} \times {}^6C_3$ (D) $\frac{8\,!}{5\,!} \times {}^6C_3$.

3. The number of ways in which n different things can be given to r persons when there is no restriction as to the number of things each may receive is:
(A) ${}^nC_r$ (B) ${}^nP_r$ (C) n^r (D) r^n

4. The number of divisors of $a^pb^qc^rd^s$ where a, b, c, d are primes & p, q, r, s ∈ N, excluding 1 and the number itself is:
(A) p q r s (B) (p + 1) (q + 1) (r + 1) (s + 1) – 4
(C) p q r s – 2 (D) (p + 1) (q + 1) (r + 1) (s + 1) – 2

5. The number of ordered triplets of positive integers which are solutions of the equation x + y + z = 100 is:
(A) 3125 (B) 5081 (C) 6005 (D) 4851

6. Number of ways in which 7 people can occupy six seats, 3 seats on each side in a first class railway compartment if two specified persons are to be always included and occupy adjacent seats on the same side, is (k). 5 ! then k has the value equal to:
(A) 2 (B) 4 (C) 8 (D) none

7. Number of different words that can be formed using all the letters of the word "DEEPMALA" if two vowels are together and the other two are also together but separated from the first two is:
(A) 960 (B) 1200 (C) 2160 (D) 1440

8. Six persons A, B, C, D, E and F are to be seated at a circular table. The number of ways this can be done if A must have either B or C on his right and B must have either C or D on his right is:
(A) 36 (B) 12 (C) 24 (D) 18

9. The number of ways in which 15 apples & 10 oranges can be distributed among three persons, each receiving none, one or more is:
(A) 5670 (B) 7200 (C) 8976 (D) none of these

10. The number of permutations which can be formed out of the letters of the word "SERIES" taking three letters together is:
(A) 120 (B) 60 (C) 42 (D) none

11. Seven different coins are to be divided amongst three persons. If no two of the persons receive the same number of coins but each receives atleast one coin & none is left over, then the number of ways in which the division may be made is:
(A) 420 (B) 630 (C) 710 (D) none

12. The streets of a city are arranged like the lines of a chess board. There are m streets running North to South & 'n' streets running East to West. The number of ways in which a man can travel from NW to SE corner going the shortest possible distance is:
(A) $\sqrt{m^2 + n^2}$ (B) $\sqrt{(m-1)^2 \,.\, (n-1)^2}$ (C) $\frac{(m+n)\,!}{m!\,.\,n!}$ (D) $\frac{(m+n-2)\,!}{(m-1)\,!\,.\,(n-1)\,!}$

13. In a conference 10 speakers are present. If S_1 wants to speak before S_2 & S_2 wants to speak after S_3, then the number of ways all the 10 speakers can give their speeches with the above restriction if the remaining seven speakers have no objection to speak at any number is:
(A) ${}^{10}C_3$ (B) ${}^{10}P_8$ (C) ${}^{10}P_3$ (D) $\frac{10\,!}{3}$

14. Two variants of a test paper are distributed among 12 students. Number of ways of seating of the students in two rows so that the students sitting side by side do not have identical papers & those sitting in the same column have the same paper is:
(A) $\frac{12!}{6!\,6!}$ (B) $\frac{(12)!}{2^5\,.\,6!}$ (C) $(6\,!)^2 . 2$ (D) $12\,! \times 2$

15. Sum of all the numbers that can be formed using all the digits 2, 3, 3, 4, 4, 4 is:
(A) 22222200 (B) 11111100 (C) 55555500 (D) 20333280

16. There are m apples and n oranges to be placed in a line such that the two extreme fruits being both oranges. Let P denotes the number of arrangements if the fruits of the same species are different and Q the corresponding figure when the fruits of the same species are alike, then the ratio P/Q has the value equal to:
(A) ${}^nP_2 . {}^mP_m . (n-2)!$ (B) ${}^mP_2 . {}^nP_n . (n-2)!$ (C) ${}^nP_2 . {}^nP_n . (m-2)!$ (D) none

17. The number of integers which lie between 1 and 10^6 and which have the sum of the digits equal to 12 is:
(A) 8550 (B) 5382 (C) 6062 (D) 8055

18. Number of ways in which a pack of 52 playing cards be distributed equally among four players so that each may have the Ace, King, Queen and Jack of the same suit is:
(A) $\frac{36\,!}{(9\,!)^4}$ (B) $\frac{36\,!\,.\,4\,!}{(9\,!)^4}$ (C) $\frac{36\,!}{(9\,!)^4\,.\,4\,!}$ (D) none

19. A five letter word is to be formed such that the letters appearing in the odd numbered positions are taken from the

letters which appear without repetition in the word "MATHEMATICS". Further the letters appearing in the even numbered positions are taken from the letters which appear with repetition in the same word "MATHEMATICS". The number of ways in which the five letter word can be formed is:
(A) 720 (B) 540 (C) 360 (D) none

20. Number of ways of selecting 5 coins from coins three each of Rs. 1, Rs. 2 and Rs. 5 if coins of the same denomination are alike, is:
(A) 9 (B) 12 (C) 21 (D) none

21. Number of ways in which all the letters of the word " ALASKA " can be arranged in a circle distinguishing between the clockwise and anticlockwise arrangement is:
(A) 60 (B) 40 (C) 20 (D) none of these

22. If r, s, t are prime numbers and p, q are the positive integers such that the LCM of p, q is $r^2t^4s^2$, then the number of ordered pair (p, q) is **[IIT – 2006]**
(A) 252 (B) 254 (C) 225 (D) 224

Part : (B) May have more than one options correct

23. $^{n+1}C_6 + {}^{n}C_4 > {}^{n+2}C_5 - {}^{n}C_5$ for all 'n' greater than:
(A) 8 (B) 9 (C) 10 (D) 11

24. In an examination, a candidate is required to pass in all the four subjects he is studying. The number of ways in which he can fail is
(A) $^4P_1 + {}^4P_2 + {}^4P_3 + {}^4P_4$ (B) $4^4 - 1$ (C) $2^4 - 1$ (D) $^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4$

25. The kindergarten teacher has 25 kids in her class. She takes 5 of them at a time, to zoological garden as often as she can, without taking the same 5 kids more than once. Then the number of visits, the teacher makes to the garden exceeds that of a kid by:
(A) $^{25}C_5 - {}^{24}C_4$ (B) $^{24}C_5$ (C) $^{25}C_5 - {}^{24}C_5$ (D) $^{24}C_4$

26. The number of ways of arranging the letters AAAAA, BBB, CCC, D, EE & F in a row if the letter C are separated from one another is:
(A) $^{13}C_3 . \frac{12!}{5!\,3!\,2!}$ (B) $\frac{13!}{5!\,3!\,3!\,2!}$ (C) $\frac{14!}{3!\,3!\,2!}$ (D) $11 . \frac{13!}{6!}$

27. There are 10 points $P_1, P_2, ..., P_{10}$ in a plane, no three of which are collinear. Number of straight lines which can be determined by these points which do not pass through the points P_1 or P_2 is:
(A) $^{10}C_2 - 2.\,{}^9C_1$ (B) 27 (C) $^8C_2$ (D) $^{10}C_2 - 2.\,{}^9C_1 + 1$

28. Number of quadrilaterals which can be constructed by joining the vertices of a convex polygon of 20 sides if none of the side of the polygon is also the side of the quadrilateral is:
(A) $^{17}C_4 - {}^{15}C_2$ (B) $\frac{^{15}C_3 . 20}{4}$ (C) 2275 (D) 2125

29. You are given 8 balls of different colour (black, white,...). The number of ways in which these balls can be arranged in a row so that the two balls of particular colour (say red & white) may never come together is:
(A) 8 ! – 2.7 ! (B) 6. 7 ! (C) 2. 6 ! . $^7C_2$ (D) none

30. A man is dealt a poker hand (consisting of 5 cards) from an ordinary pack of 52 playing cards. The number of ways in which he can be dealt a "straight" (a straight is five consecutive values not of the same suit, eg. {Ace, 2, 3, 4, 5}, {2, 3, 4, 5, 6}.......................... & {10, J, Q, K, Ace}) is
(A) $10\,(4^5 - 4)$ (B) $4 !\ 2^{10}$ (C) $10 \cdot 2^{10}$ (D) 10200

31. Number of ways in which 3 numbers in A.P. can be selected from 1, 2, 3,...... n is:
(A) $\left(\frac{n-1}{2}\right)^2$ if n is even (B) $\frac{n\,(n-2)}{4}$ if n is odd (C) $\frac{(n-1)^2}{4}$ if n is odd (D) $\frac{n\,(n-2)}{4}$ if n is even

32. Consider the expansion $(a_1 + a_2 + a_3 + ... + a_p)^n$ where $n \in N$ and $n \le p$. The correct statement(s) is/are:
(A) number of different terms in the expansion is $^{n+p-1}C_n$
(B) co-efficient of any term in which none of the variables $a_1, a_2, ..., a_p$ occur more than once is 'n'
(C) co-efficient of any term in which none of the variables $a_1, a_2, ..., a_p$ occur more than once is n ! if n = p
(D) Number of terms in which none of the variables $a_1, a_2, ..., a_p$ occur more than once is $\binom{p}{n}$.

EXERCISE–5

1. In a telegraph communication how many words can be communicated by using atmost 5 symbols. (only dot and dash are used as symbols)

2. If all the letters of the word 'AGAIN' are arranged in all possible ways & put in dictionary order, what is the 50^{th} word.

3. A committee of 6 is to be chosen from 10 persons with the condition that if a particular person 'A' is chosen, then another particular person B must be chosen.

4. A family consists of a grandfather, m sons and daughters and 2n grand children. They are to be seated in a row for dinner. The grand children wish to occupy the n seats at each end and the grandfather refuses to have a grand children on either side of him. In how many ways can the family be made to sit?

5. The sides AB, BC & CA of a triangle ABC have 3, 4 & 5 interior points respectively on them. Find the number of triangles that can be constructed using these interior points as vertices.

6. How many five digits numbers divisible by 3 can be formed using the digits 0, 1, 2, 3, 4, 7 and 8 if, each digit is to be used atmost one.

7. In how many other ways can the letters of the word MULTIPLE be arranged ; (i) without changing the order of the vowels (ii) keeping the position of each vowel fixed (iii) without changing the relative order/position of vowels & consonants.

8. There are p intermediate stations on a railway line from one terminus to another. In how many ways can a train stop at 3 of these intermediate stations if no 2 of these stopping stations are to be consecutive?

9. Find the number of positive integral solutions of x + y + z + w = 20 under the following conditions:
(i) Zero values of x, y, z, w are include (ii) Zero values are excluded
(iii) No variable may exceed 10; Zero values excluded (iv) Each variable is an odd number
(v) x, y, z, w have different values (zero excluded).

10. Find the number of words each consisting of 3 consonants & 3 vowels that can be formed from the letters of the word **"CIRCUMFERENCE".** In how many of these C's will be together.

11. If 'n' distinct things are arranged in a circle, show that the number of ways of selecting three of these things so that no two of them are next to each other is, $\frac{1}{6}$ n (n – 4) (n – 5).

12. In maths paper there is a question on "Match the column" in which column A contains 6 entries & each entry of column A corresponds to exactly one of the 6 entries given in column B written randomly. 2 marks are awarded for each correct matching & 1 mark is deducted from each incorrect matching. A student having no subjective knowledge decides to match all the 6 entries randomly. Find the number of ways in which he can answer, to get atleast 25 % marks in this question.

13. Show that the number of combinations of n letters together out of 3n letters of which n are a and n are b and the rest unlike is, $(n + 2).\ 2^{n-1}$.

14. Find the number of positive integral solutions of, (i) $x^2 - y^2 = 352706$ (ii) $xyz = 21600$

15. There are 'n' straight line in a plane, no two of which are parallel and no three pass through the same point. Their points of intersection are joined. Show that the number of fresh lines thus introduced is, $\frac{1}{8}\ n\,(n-1)\,(n-2)\,(n-3)$.

16. A forecast is to be made of the results of five cricket matches, each of which can be a win or a draw or a loss for Indian team. Find
(i) number of forecasts with exactly 1 error
(ii) number of forecasts with exactly 3 errors
(iii) number of forecasts with all five errors

17. Prove by permutation or otherwise $\frac{(n^2)!}{(n!)^n}$ is an integer $(n \in I^+)$. **[IIT – 2004]**

18. If total number of runs scored in n matches is $\left(\frac{n+1}{4}\right)(2^{n+1} - n - 2)$ where $n > 1$, and the rund scored in the k^{th} match are given by $k.\ 2^{n+1-k}$, where $1 \le k \le n$. Find n **[IIT – 2005]**

EXERCISE–1

Q.1 ${}^{m+n+k}C_3 - ({}^mC_3 + {}^nC_3 + {}^kC_3)$ **Q.2** 744 **Q.3** 13 , 156
Q.4 4316527 **Q.5** 43200 **Q.6** 145 **Q.7** 420
Q.8 960 **Q.9** ${}^{24}C_2\ .\ {}^{15}C_3$ **Q.10** 1106 **Q.11** $12!\ ;\ \frac{11!\ .\ 4!}{(3!)^4\ 2!}$
Q.13 $\frac{52!}{(13!)^4}\ ;\ \frac{52!}{3!(17!)^3}$ **Q.14** 5400 **Q.15** ${}^{45}C_6$ **Q.17** 1638
Q.18 22100 , 52 **Q.19** 2111 **Q.20** 576 **Q.21** (a) $5 \cdot (6!)$, (b) $3! \cdot 4!$, (c) 12
Q.23 **(a)** 72 ; 78120 ; **(b)** 23 ; **(c)** 32 **Q.24** 143 **Q.25** 440 **Q.26** ${}^{n+5}C_5$
Q.27 3888 **Q.28** $\frac{(14)!}{5!9!}$ **Q.29** **(ii)** 792 ; **(iii)** $\frac{143}{4025}$; **(v)** r = 3

EXERCISE–2

Q.1 2500 **Q.2** ${}^mC_{k-1}$ where $m = (1/2)\,(2n - k^2 + k - 2)$
Q.3 7^{10} ; $\binom{49}{6}$ 10 ! **Q.4** 24 **Q.5** 532770 **Q.6** (i) 15, (ii) 126, (iii) 60, (iv) 105
Q.7 26250 **Q.9** 120, 216, 210 **Q.10** 2252 **Q.11** **(i)** 150 ; **(ii)** 6 ; **(iii)** 25 ; **(iv)** 2 ; **(v)** 6
Q.12 **(i)** 3359 ; **(ii)** 59 ; **(iii)** 359 **Q.13** 61, 75 **Q.14** 10, 3
Q.15 121 **Q.17** 510 **Q.18** 2 ! 3 ! 8 !
Q.19 $\frac{24!}{(3!)^2(2!)^5}$; ${}^8C_1\ .\ \frac{23!}{(3!)^2(2!)^5}$ **Q.20** $\frac{(25)!}{(3!)^3\ (4!)^4\ .\ 4}$ **Q.21** $4\ .\ (4!)^2\ .\ {}^8C_4\ .\ {}^6C_2$
Q.22 240 , 15552 **Q.23** 3119976 **Q.24** 27 **Q.25** (a) 1680; (b) 1140
Q.26 ${}^6C_3[3^n - {}^3C_1(2^n - 2) - {}^3C_2]$ **Q.27** 974 **Q.29** 1506

EXERCISE–3

Q.1 72 **Q.2** **(i)** A ; **(ii)** B **Q.3** C **Q.4** B
Q.5 A **Q.6** B **Q.8** C **Q.9** C

EXERCISE–4

1. A **2.** C **3.** D **4.** D **5.** D **6.** C **7.** D **8.** D **9.** C **10.** C **11.** B
12. D **13.** D **14.** D **15.** A **16.** A **17.** C **18.** B **19.** B **20.** B **21.** C **22.** C
23. BCD **24.** CD **25.** AB **26.** AD **27.** CD **28.** AB **29.** ABC **30.** AD **31.** CD
32. ACD

EXERCISE–5

1. 62 **2.** NAAIG **3.** 154 **4.** (2n)! m! (m – 1) **5.** 205
6. 744 **7.** (i) 3359 (ii) 59 (iii) 359 **8.** ${}^{p-2}C_3$
9. (i) ${}^{23}C_3$ (ii) ${}^{19}C_3$ (iii) ${}^{19}C_3 - 4.{}^9C_3$ (iv) ${}^{11}C_8$ (v) 552
10. 22100, 52 **12.** 56 ways **14.** (i) Zero (ii) 1260 **16.** (i) 10 (ii) 80 (iii) 32 **18.** 7
