EXERCISE-1

SECTION (A) : EQUATION OF SHM

A-1. A particle having mass 10 g oscillates according to the equation $x = (2.0 \text{ cm}) \sin [(100 \text{ s}^{-1}) t + \pi/6]$. Find (a) the amplitude, the time period and the force constant (b) the position, the velocity and the acceleration at t = 0.

A-2. The equation of motion of a particle started at t = 0 is given by $x = 5 \sin (20 t + \pi/3)$ where x is in centimetre and t in second. When does the particle
(a) first come to rest
(b) first have zero acceleration
(c) first have maximum speed ?

A-3. A simple harmonic motion has an amplitude A and time period T. Find the time required by it to travel directly from

(A) x = 0 to x = A/2 (B) $x = 0$ to $x = \frac{A}{\sqrt{2}}$ (C) x = A to x = A/2

(D) $x = -\frac{A}{\sqrt{2}}$ to $x = \frac{A}{\sqrt{2}}$ (E) $x = \frac{A}{\sqrt{2}}$ to x = A.

A-4. A particle is executing SHM with amplitude A and has maximum velocity v_0. Find its speed at displacement $\frac{A}{2}$.

A-5. A particle executes simple harmonic motion with an amplitude of 10 cm and time period 6 s. At t = 0 it is at position x = 5 cm going towards positive x-direction. Write the equation for the displacement x at time t. Find the magnitude of the acceleration of the particle at t = 4 s.

A-6. The position, velocity and acceleration of a particle executing simple hamonic motion are found to have magnitudes 2 cm, 1 m/s and 10 m/s^2 at a certain instant Find the amplitude and the time period of the motion.

A-7. The maximum speed and acceleration of a particle executing simple harmonic motion are 10 cm/s and 50 cm/s^2. Find the position(s) of the particle when the speed is 8 cm/s.

SECTION (B) : ENERGY

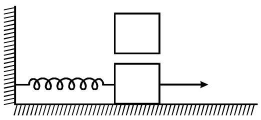
B-1. The particle executes simple harmonic motion with an amplitude of 10 cm. At what distance from the mean position are the kinetic and potential energies equal ?

B-2. A particle is oscillating in a straight line about a centre O, with a force directed towards O. When at a distance 'x' from O, the force is mn^2x where 'm' is the mass and 'n' is a constant. The amplitude is a = 15cm. When at a distance $\sqrt{3}\,\frac{a}{2}$ from O the particle receives a blow in the direction of motion which generates an extra velocity na. If the velocity is away from O, find the new amplitude. What is the answer, if the velocity of block was towards origin.

SECTION (C) : SPRING MASS SYSTEM

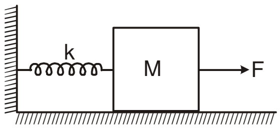
C-1. The pendulum of a clock is replaced by a spring - mass system with the spring having spring constant 0.1 N/m. What mass should be attached to the spring ? (time period of pendulum clock is 2 s)

C-2. A 1 kg block is executing simple harmonic motion of amplitude 0.1 m on a smooth horizontal surface under the restoring force of a spring of spring constant 100 N/m. A block of mass 3 kg is gently placed on it at the instant it passes through the mean position. Assuming that the two blocks move together, find the frequency and the amplitude of the motion.

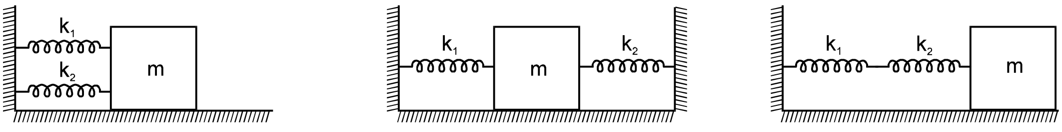


C-3. A spring stores 5 J of energy when stretched by 25 cm. It is kept vertical with the lower end fixed. A block fastened to its other end is made to undergo small oscillations. If the block makes 5 oscillations each second, what is the mass of the block ?

C-4. The spring shown in figure is unstretched when a man starts pulling on the cord. The mass of the block is M. If the man exerts a constant force F, find (a) the amplitude and the time period of the motion of the block (b) the energy stored in the spring when the block passes through the equilibrium position and (c) the kinetic energy of the block at this position.



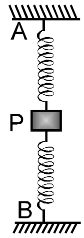
C-5. Find the time period of the oscillation of mass m in figures a, b, c. What is the equivalent spring constant of the pair of springs in each case ?



(a) (b) (c)

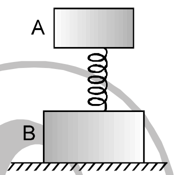
C-6. A spring of force constant 'k' is cut into two parts whose lengths are in the ratio 1 : 2. The two parts are now connected in parallel and a block of mass 'm' is suspended at the end of the combined spring. Find the period of oscillation performed by the block.

C-7. Two identical springs are attached to a small block P. The other ends of the springs are fixed at A and B. When P is in equilibrium the extension of top spring is 20 cm and extension of bottom spring is 10 cm. Find the period of small vertical oscillations of P about its equilibrium position. (use g = 9.8m/s^2).



C-8. A block of mass 1kg hangs without vibrating at the end of a spring with a force constant 1 N/cm attached to the ceiling of an elevator. The elevator initially is rising with an upward acceleration of $\frac{g}{4}$. The acceleration of the elevator suddenly ceases. What is the amplitude of the resulting oscillations?

C-9. A body A of mass m_1 = 1 kg and a body B of mass m_2 = 3 kg are interconnected by a spring as shown in (Fig). The body A performs free vertical harmonic oscillations with amplitude a = 1.6 cm and angular frequency ω = 25 s^{-1}. Neglecting the mass of the spring, find the maximum and minimum values of force that this system exerts on the bearing surface. (use g = 10 m/s^2).



SECTION (D) : SIMPLE PENDULUM

D-1. A pendulum having time period equal to two seconds is called a seconds pendulum. Those used in pendulum clocks are of this type. Find the length of a seconds pendulum at a place where g = π^2 m/s^2.

D-2. The angle made by the string of a simple pendulum with the vertical depends on time as $\theta = \frac{\pi}{90}\sin[\pi\, s^{-1})t]$. Find the length of the pendulum if g = π^2 m/s^2.

D-3. A pendulum clock giving correct time at a place where g = 9.8 m/s^2 is taken to another place where it loses 24 seconds during 24 hours. Find the value of g at this new place.

D-4. A pendulum is suspended in a lift and its period of oscillation is T_0 when the lift is stationary.
(i) What will the period T of oscillation of pendulum be, if the lift begins to accelerate downwards with an acceleration equal to $\frac{3g}{4}$?
(ii) What must be the acceleration of the lift for the period of oscillation of the pendulum to be $\frac{T_0}{2}$?

D-5. A simple pendulum of length ℓ is suspended through the ceiling of an elevator. Find the time period of small oscillations if the elevator (a) is going up with an acceleration a_0 (b) is going down with an acceleration a_0 and (c) is moving with a uniform velocity.

D-6. A simple pendulum fixed in a car has a time period of 4 seconds when the car is moving uniformly on a horizontal road. When the accelerator is pressed, the time period changes to 3.99 seconds. Making an approximate analysis, find the acceleration of the car.

D-7. A 0.1kg ball is attached to a string 1.2m long and suspended as a simple pendulum. At a point 0.2 m below the point of suspension a peg is placed, which the string hits when the pendulum comes down. If the mass is pulled a small distance to one side and released what will be the time period of the motion.

SECTION (E) : COMPOUND PENDULUM & TORSIONAL PENDULUM

E-1. Find the time period of small oscillations of the following systems. (a) A metre stick suspended through the 20 cm mark. (b) A ring of mass m and radius r suspended through a point on its perphery. (c) A uniform square plate of edge a suspended through a corner. (d) A uniform disc of mass m and radius r suspended through a point r/2 away from the centre.

E-2. A uniform disc of radius r is to be suspended through a small hole made in the disc. Find the minimum possible time period of the disc for small oscillations. What should be the distance of the hole from the centre for it to have minimum time period ?

E-3. A closed circular wire hung on a nail in a wall undergoes small oscillations of amplitude 2° and time period 2s. Find (a) the radius of the circular wire, (b) the speed of the particle farthest away from the point of suspnsion as it goes through its mean position, (c) the acceleration of this particle as it goes through its mean position and (d) the acceleration of this particle when it is at an exetreme position. Take $g = \pi^2$ m/s^2.

E-4. A physical pendulum is positioned so that its centre of gravity is above the point of suspension. From that position the pendulum started moving towards the stable equilibrium and passed it with an angular velocity ω. Neglecting the friction, find the period of small oscillations of the pendulum.

E-5. a uniform disc of mass m and radius r is suspended through a wire attached to its centre. If the time period of the torsional oscillations be T, what is the torsional constant of the wire.

SECTION (F) : SUPERPOSITION OF SHM

F-1. A particle is subjected to two simple harmonic motions of same time period in the same direction. The amplitude of the first motion is 3.0 cm and that of the second is 4.0 cm. Find the resultant amplitude if the phase difference between the motions is (a) 0°, (b) 60°, (c) 90°.

F-2. Three simple harmonic motions of equal amplitudes A and equal time periods in the same direction combine. The phase of the second motion is 60° ahead of the first and the phase of the third motion is 60° ahead of the second. Find the amplitude of the resultant motion.

F-3. A particle simultaneously participates in two mutually perpendicular oscillations $x = \sin \pi t$ & $y = 2\cos 2\pi t$. Write the equation of trajectory of the particle.

EXERCISE-2

SECTION (A) : EQUATION OF SHM

A-1. Select the correct statements.
(A) a simple harmonic motion is necessarily periodic
(B) a simple harmonic motion is necessarily oscillatory
(C) an oscillatory motion is necessarily periodic
(D) a periodic motion is necessarily oscillatory

A-2. A particle moves in a circular path with a uniform speed. Its motion is
(A) periodic (B) oscillatory
(C) simple harmonic (D) angular simple harmonic

A-3. A particle is fastened at the end of a string and is whirled in a vertical circle with the other end of the string being fixed. The motion of the particle may be
(A) periodic (B) oscillatory
(C) simple harmonic (D) angular simple harmonic

A-4. A particle moves in a circular path with a continuously increasing speed. Its motion is
(A) periodic (B) oscillatory (C) simple harmonic (D) none of them

A-5. A function of time given by $(\cos \omega t + \sin \omega t)$ represents
(A) simple harmonic motion. (B) non-periodic motion
(C) periodic but not simple harmonic motion (D) oscillatory but not simple harmonic motion

A-6. A student says that he had applied a force $F = -k\sqrt{x}$ on a particle and the particle moved in simple harmonic motion. He refuses to tell whether k is a constant or not. Assume that he has worked only with positive x and no other force acted on the particle.
(A) As x increases k increases (B) As x increases k decreases

(C) As x increases k remains constant. (D) The motion cannot be simple harmonic.

A-7. For a particle executing simple harmonic motion, the acceleration is proportional to
(A) displacement from the mean position (B) distance from the mean position
(C) distance travelled since t = 0 (D) speed

A-8. The displacement of a particle in simple harmonic motion in one time period is
(A) A (B) 2A (C) 4A (D) zero

A-9. The distance moved by a particle in simple harmonic motion in one time period is
(A) A (B) 2A (C) 4A (D) zero

A-10. The time period of a particle in simple harmonic motion is equal to the time between consecutive appearance of the particle at a particular point in its motion. This point is
(A) the mean position
(B) an extreme position
(C) between the mean position and the positive extreme
(D) between the mean position and the negative extreme.

A-11. The time period of a particle in simple harmonic motion is equal to the smallest time between the particle acquiring a particular velocity $\vec{v}$. The value of v is
(A) v_{max} (B) 0 (C) between 0 and v_{max} (D) between 0 and $-v_{max}$

A-12. The displacement (in m) of a particle of mass 100 g from its equilibrium position is given by the equation:
$$y = 0.05 \sin 3\pi (5t + 0.4)$$
(A) the time period of motion is $\frac{1}{30}$ sec
(B) the time period of motion is $\frac{1}{7.5}$ sec
(C) the maximum acceleration of the particle is $11.25\pi^2$ m/s^2
(D) the force acting on the particle is zero when the displacement is 0.05 m.

A-13. If the maximum velocity and acceleration of a particle executing SHM are equal in magnitude, the time period will be
(A) π sec (B) $\frac{\pi}{2}$ sec (C) 2π sec (D) $\frac{2}{\pi}$ sec

A-14. Two SHM's are represented by $y = a \sin(\omega t - kx)$ and $y = b \cos(\omega t - kx)$. The phase difference between the two is:
(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{3\pi}{4}$

A-15. How long after the beginning of motion is the displacement of a harmonically oscillating particle equal to one half its amplitude if the period is 24s and particle starts from rest.
(A) 12s (B) 2s (C) 4s (D) 6s

A-16. The average acceleration in one time period in a simple harmonic motion is
(A) $A\omega^2$ (B) $A\omega^2/2$ (C) $A\omega^2/\sqrt{2}$ (D) zero.

A-17. The magnitude of average acceleration in half time period from equilibrium position in a simple harmonic motion is
(A) $\frac{2A\omega^2}{\pi}$ (B) $\frac{A\omega^2}{2\pi}$ (C) $\frac{A\omega^2}{\sqrt{2}\pi}$ (D) Zero

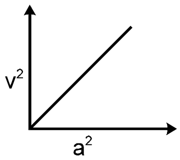
A-18. A particle moves on the X-axis according to the equation $x = A + B \sin \omega t$. The motion is simple harmonic with amplitude
(A) A (B) B (C) A + B (D) $\sqrt{A^2 + B^2}$

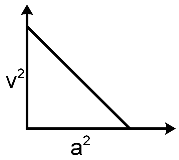
A-19. Equation of SHM is $x = 10 \sin 10\pi t$. Find the distance between the two points where speed is 50π cm/sec. x is in cm and t is in seconds.
(A) 10 cm (B) 20 cm (C) 17.32 cm (D) 8.66 cm.

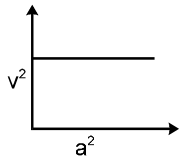
A-20. Two particles execute S.H.M. of same amplitude and frequency along the same straight line. They pass

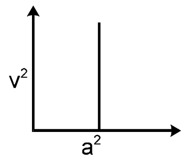
one another, when going in opposite directions, each time their displacement is half of their amplitude. The phase-difference between them is
(A) 0° (B) 120° (C) 180° (D) 135°

A-21. A mass M is performing linear simple harmonic motion, then correct graph for acceleration a and corresponding linear velocity ν is

(A) 

(B) 

(C) 

(D) 

SECTION (B) : ENERGY

B-1. A body executing SHM passes through its equilibrium. At this instant, it has
(A) maximum potential energy (B) maximum kinetic energy
(C) minimum kinetic energy (D) maximum acceleration

B-2. The K.E. and P.E of a particle executing SHM with amplitude A will be equal when its displacement is
(A) $\sqrt{2}A$ (B) $\frac{A}{2}$ (C) $\frac{A}{\sqrt{2}}$ (D) $\sqrt{\frac{2}{3}}A$

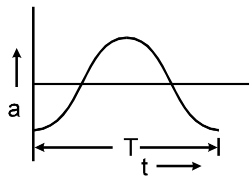
B-3. A point particle of mass 0.1 kg is executing S.H.M. of amplitude of 0.1 m. When the particle passes through the mean position, its kinetic energy is 8×10^{-3} J. The equation of motion of this particle when the initial phase of oscillation is 45° can be given by
(A) $0.1 \cos\left(4t + \frac{\pi}{4}\right)$ (B) $0.1 \sin\left(4t + \frac{\pi}{4}\right)$ (C) $0.4 \sin\left(t + \frac{\pi}{4}\right)$ (D) $0.2 \sin\left(\frac{\pi}{2} + 2t\right)$

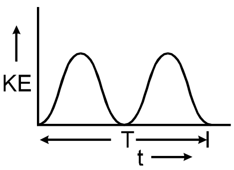
B-4. The total mechanical energy of a spring - mass system in simple harmonic motion is $E = \frac{1}{2} m\, \omega^2 A^2$. Suppose the oscillating particle is replaced by another particle of double the mass while the amplitude A remains the same. The new mechanical energy will
(A) become 2 E (B) become E/2 (C) become $\sqrt{2}$ E (D) remain E

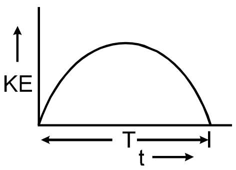
B-5. In a simple harmonic motion
(A) the potential energy is always equal to the kinetic energy
(B) the potential energy is never equal to the kinetic energy
(C) the average potential energy in any time interval is equal to the average kinetic energy in that time interval
(D) the average potential energy in one time period is equal to the average kinetic energy in this period.

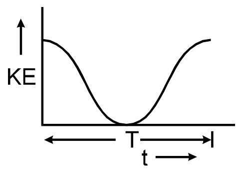
B-6. In a simple harmonic motion
(A) the maximum potential energy equals the maximum kinetic energy
(B) the minimum potential energy equals the minimum kinetic energy
(C) the minimum potential energy equals the maximum kinetic energy
(D) the maximum potential energy equals the minimum kinetic energy

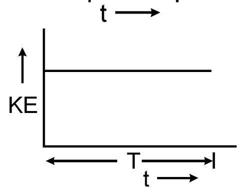
B-7. Acceleration a versus time t graph of a body in SHM is given by a curve shown below. T is the time peirod. Then corresponding graph between kinetic energy KE and time t is correctly represented by



(A) 

(B) 

(C) 

(D) 

B-8. The average energy in one time period in simple harmonic motion is
(A) $\frac{1}{2} m\, \omega^2 A^2$ (B) $\frac{1}{4} m\, \omega^2 A^2$ (C) $m\, \omega^2 A^2$ (D) zero

B-9. A particle executes simple harmonic motion with a frequency ν. The frequency with which the kinetic

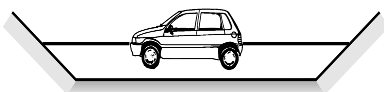
energy oscillates is
(A) $\nu/2$ (B) ν (C) 2ν (D) zero

B-10. A body is executing simple harmonic motion. At a displacement x, its potential energy is E_1 and at a displacement y, its potential energy is E_2. The potential energy E at a displacement (x + y) is
(A) $E_1 + E_2$ (B) $\sqrt{E_1^2 + E_2^2}$ (C) $E_1 + E_2 + 2\sqrt{E_1 E_2}$ (D) $\sqrt{E_1 E_2}$

SECTION (C) : SPRING MASS SYSTEM

C-1. Two bodies A and B of equal mass are suspended from two separate massless springs of spring cosntant k_1 and k_2 respectively. If the bodies socillate vertically such that their maximum velocities are equal, the ratio of the amplitude of A to that of B is
(A) k_1/k_2 (B) $\sqrt{k_1/k_2}$ (C) k_2/k_1 (D) $\sqrt{k_2/k_1}$

C-2. A toy car of mass m is having two similar rubber ribbons attached to it as shown in the figure. The force constant of rubber ribbons is k. The car is displaced from mean position by x cm and released. At the mean position the ribbons are undeformed. Vibration period is



(A) $2\pi\sqrt{\frac{m(2k)}{k^2}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{m(2k)}{k^2}}$ (C) $2\pi\sqrt{\frac{m}{k}}$ (D) $2\pi\sqrt{\frac{m}{k+k}}$

C-3. A mass of 1 kg attached to the bottom of a spring has a certain frequency of vibration. The following mass has to be added to it in order to reduce the frequency by half :
(A) 1 kg (B) 2 kg (C) 3 kg (D) 4 kg

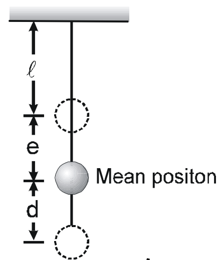
C-4. A force of 6.4 N stretches a vertical spring by 0.1 m. The mass that must be suspended from the spring so that it oscillates with a period of $(\pi/4)$ sec is :
(A) $(\pi/4)$kg (B) 1 kg (C) $(1/\pi)$ kg (D) 10 kg

C-5. A spring mass system oscillates with a frequency ν. If it is taken in an elevator slowly accelerating upward, the frequency will
(A) increase (B) decrease (C) remain same (D) become zero

C-6. A spring-mass system oscillates in a car. If the car accelerates on a horizontal road, the frequency of oscillation will
(A) increase (B) decrease (C) remain same (D) become zero.

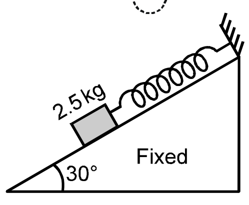
C-7. A ball of mass m kg hangs from a spring of spring constant k. The ball oscillates with a period of T seconds. If the ball is removed, the spring is shortened by
(A) $\frac{gT^2}{(2\pi)^2}$ metre (B) $\frac{3T^2g}{(2\pi)^2}$ metre (C) $\frac{Tm}{k}$ metre (D) $\frac{Tk}{m}$ metre

C-8. An elastic string of length ℓ supports a heavy particle of mass m and the system is in equilibrium with elongation produced being e. The particle is now pulled down below the equilibrium position through a distance d (d < e) and released. The angular frequency in radian per second is



(A) $\sqrt{\frac{g}{e}}$ (B) $\sqrt{\frac{g}{\ell}}$ (C) $\sqrt{\frac{g}{d+e}}$ (D) $\sqrt{\frac{g}{m\ell}}$

C-9. A smooth inclined plane having angle of inclination 30° with horizontal has a mass 2.5 kg held by a spring which is fixed at the upper end. If the mass is taken 2.5 cm up along the surface of the inclined plane, the tension in the spring reduces to zero. If the mass is then released, the angular frequency of oscillation in radian per second is
(A) 0.707 (B) 7.07 (C) 1.414 (D) 14.14

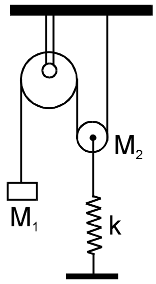


C-10. A particle executes simple harmonic motion under the restoring force provided by a spring. The time period is T. If the spring is divided in two equal parts and one part is used to continue the simple harmonic motion, the time period will
(A) remain T (B) becomes 2T (C) become T/2 (D) become $T/\sqrt{2}$

C-11. A body at the end of a spring executes S.H.M. with a period t_1, while the corresponding period for another spring is t_2. If the period of oscillation with the two spring in series is T, then

(A) $T = t_1 + t_2$ (B) $T^2 = t_1^2 + t_2^2$ (C) $\frac{1}{T} = \frac{1}{t_1} + \frac{1}{t_2}$ (D) $\frac{1}{T^2} = \frac{1}{t_1^2} + \frac{1}{t_2^2}$

C-12. The period of the free oscillations of the system shown here if mass M_1 is pulled down a little and force constant of the spring is k and masses of the fixed pulleys are negligible, is



(A) $T = 2\pi\sqrt{\frac{M_1 + M_2}{k}}$ (B) $T = 2\pi\sqrt{\frac{M_1 + 4M_2}{k}}$

(C) $T = 2\pi\sqrt{\frac{M_2 + 4M_1}{k}}$ (D) $T = 2\pi\sqrt{\frac{M_2 + 3M_1}{k}}$

SECTION (D) : SIMPLE PENDULUM

D-1. A pendulum clock that keeps correct time on the earth is taken to the moon. It will run

(A) at correct rate (B) 6 times faster (C) $\sqrt{6}$ times faster (D) $\sqrt{6}$ times slower

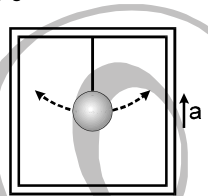
D-2. Two pendulums begin to swing simultaneously. The first pendulum maske 9 full oscillations when the other makes 7. The ratio of length of the two pendulums is

(A) $\frac{49}{81}$ (B) $\frac{7}{9}$ (C) $\frac{50}{81}$ (D) $\frac{1}{2}$

D-3. Energy of a simple pendulum is 1 J when its length is 3 m and amplitude of motion is 3 cm. If the length is unchanged but amplitude is made larger by 2 cm, the energy becomes

(A) 28 J (B) 0.28 J (C) 8.2 J (D) 25/9 J

D-4. A scientist measures the time period of a simple pendulum as T in a lift at rest. If the lift moves up with acceleration as one fourth of the acceleration of gravity, the new time period is



(A) $\frac{T}{4}$ (B) 4T (C) $\frac{2}{\sqrt{5}}T$ (D) $\frac{\sqrt{5}}{2}T$

D-5. A simple pendulum has some time period T. What will be the percentage change in its time period if its amplitudes is decreased by 5%?

(A) 6 % (B) 3 % (C) 1.5 % (D) 0 %

D-6. A simple pendulum with length ℓ and bob of mass m executes SHM of small amplitude A. The maximum tension in the string will be

(A) $mg(1 + A/\ell)$ (B) $mg(1 + A/\ell)^2$ (C) $mg[1 + (A/\ell)^2]$ (D) 2 mg

D-7. A wall clock uses a vertical spring-mass system to measure the time. Each time the mass reaches an extreme position, the clock advances by a second. The clock gives correct time at the equator. If the clock is taken to the poles it will

(A) run slow (B) run fast (C) stop working (D) give correct time.

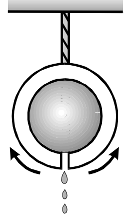
D-8. A pendulum clock keeping correct time is taken to high altitudes,

(A) it will keep correct time
(B) its length should be increased to keep correct time
(C) its length should be decreased to keep correct time
(D) it cannot keep correct time even if the length is changed.

D-9. The bob of a simple pendulum executes simple harmonic motion in water with a period t, while the period of oscillation of the bob in air is t_0. If the density of the material of the bob is (4/3) × 1000 kg m^{-3}, and the viscosity of water is neglected, the relationship between t and t_0 is

(A) $t = t_0$ (B) $t = \frac{t_0}{2}$ (C) $t = 2t_0$ (D) $t = 4t_0$

D-10. A simple pendulum has a hollow bob filled with a liquid of density ρ. If the liquid drains out of a small hole in the bottom of the bob, then frequency of oscillations



(A) goes on increasing (B) goes on decreasing
(C) remains same (D) first decreases and then increases

SECTION (E) : COMPOUND PENDULUM & TORSIONAL PENDULUM

E-1. The motion of a torsional pendulum is
(A) periodic (B) oscillatory (C) simple harmonic (D) angular simple harmonic

SECTION (F) : SUPERPOSITION OF SHM

F-1. When two mutually perpendicular simple harmonic motions of same frequency, amplitude and phase are superimposed
(A) the resulting motion is uniform circular motion.
(B) the resulting motion is a linear simple harmonic motion along a straight line inclined equally to the straight lines of motion of component ones.
(C) the resulting motion is an elliptical motion, symmetrical about the lines of motion of the components.
(D) the two S.H.M. will cancel each other.

F-2. The motion of a particle is given by $x = A \sin \omega t + B \cos \omega t$. The motion of the particle is
(A) not simple harmonic
(B) simple harmonic with amplitude $A + B$
(C) simple harmonic with amplitude $(A + B)/2$
(D) simple harmonic with amplitude $\sqrt{A^2 + B^2}$

F-3. A simple harmonic motion is given by $y = 5 (\sin 3\pi t + \sqrt{3} \cos 3\pi t)$. What is the amplitude of motion if y is in m?
(A) 100 cm (B) 5 m (C) 200 cm (D) 1000 cm

F-4. If a SHM is given by $y = (\sin \omega t + \cos \omega t)$ m, which of the following statements are true?
(A) The amplitude is 1m
(B) The amplitude is $(\sqrt{2})$ m
(C) Time is considered from y = 1 m
(D) Time is considered from y = 0 m

F-5. The displacement of a particle is given by $\vec{r} = A(\vec{i} \cos\omega t + \vec{j} \sin\omega t)$. The motion of the particle is
(A) simple harmonic
(B) on a straight line
(C) on a circle
(D) with constant acceleration

F-6. The displacement of a particle executing periodic motion is given by $y = 4 \cos^2 (0.5t) \sin (1000\, t)$. The given expression is composed by :
(A) nil SHM (B) four SHMs (C) three SHMs (D) one SHM

F-7. If a particle is in SHM of period T_1 under the action of one force F_1 and of period T_2 under force F_2 along the same direction, the period of motion under combined action of these forces is

(A) $\frac{T_1T_2}{T_1 + T_2}$ (B) $T_1 + T_2$ (C) $\frac{T_1T_2}{\sqrt{T_1^2 + T_2^2}}$ (D) $\frac{T_1^2\, T_2^2}{T_1 + T_2}$

EXERCISE-3

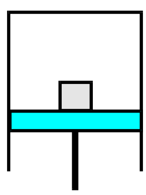
1. The potential energy of a particle of mass 'm' situated in a unidimensional potential field varies as $U(x) = U_0 [1 - \cos ax]$, where U_0 and a are constants. The time period of small oscillations of the particle about the mean position–

(A) $2\pi\sqrt{\frac{m}{aU_0}}$ (B) $2\pi\sqrt{\frac{am}{U_0}}$ (C) $2\pi\sqrt{\frac{m}{a^2U_0}}$ (D) $2\pi\sqrt{\frac{a^2m}{U_0}}$

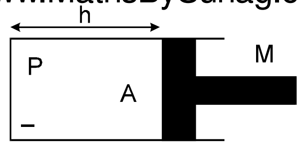
2. A person weighing M kg stands on a board which vibrates up and down simple harmonically at a frequency ν Hz. If the span is d m, the acceleration at top position is

(A) g (B) $-4\pi^2\nu^2 d$ (C) $-2\pi^2\nu^2 d$ (D) $\frac{2\pi^2\nu d}{M}$

3. A block of mass m is resting on a piston which is moving vertically with a SHM of period 1 s. The minimum amplitude of motion at which the block and piston separate is :
(A) 0.25 m (B) 0.52 m (C) 2.5 m (D) 0.15 m

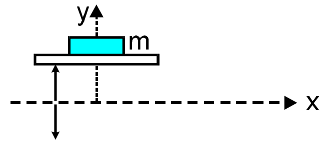


4. A cylindrical piston of mass M slides smoothly inside a long cylinder closed at one end, enclosing a certain mass of a gas. The cylinder is kept with its axis horizontal. If the piston is disturbed from its equilibrium position, it oscillates simple harmonically the period of oscillation will be



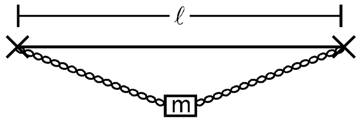
(A) $T = 2\pi\sqrt{\frac{Mh}{PA}}$ (B) $T = 2\pi\sqrt{\frac{MA}{Ph}}$ (C) $T = 2\pi\sqrt{\frac{M}{PAh}}$ (D) $T = 2\pi\sqrt{MPhA}$

5. A horizontal platform with a mass m placed on it is executing SHM along y-axis. If the amplitude of oscillation is 2.5 cm, the minimum period of the motion for the mass not to be detached from the platform is ($g = 10\ m/s^2$) :



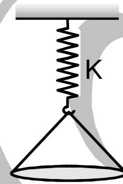
(A) $\frac{10}{\pi}$ s (B) $\frac{\pi}{10}$ s (C) $\frac{\pi}{\sqrt{10}}$ s (D) $\frac{1}{\sqrt{10}}$ s.

6. A massless rope is stretched between two fixed points a distance ℓ apart in such a way that the tension in it is T. A mass m is attached to the middle of the rope and given a slight displacement from its equilibrium position. If tension T remains unchanged during the motion, the period of oscillation of the rope is (assume gravity to be absent) :



(A) $2\pi\sqrt{\frac{m}{T}}$ (B) $2\pi\sqrt{\frac{m\ell}{2T}}$ (C) $2\pi\left(\frac{m\ell}{T}\right)^2$ (D) $\pi\left(\frac{m\ell}{T}\right)^{1/2}$

7. A solid ball of mass m is made to fall from a height H on a pan suspended through a spring of spring constant K. If the ball does not rebound and the pan is massless, then amplitude of oscillation is



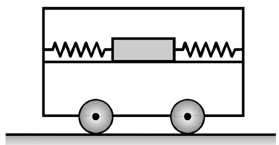
(A) $\frac{mg}{K}$ (B) $\frac{mg}{k}\left(1+\frac{2HK}{mg}\right)^{1/2}$

(C) $\frac{mg}{K}+\left(\frac{2HK}{mg}\right)^{1/2}$ (D) $\frac{mg}{K}\left[1+\left(1+\frac{2HK}{mg}\right)^{1/2}\right]$

8. In the previous question. If the pan also has mass 'm' then the amplitude is :

(A) $\frac{mg}{K}$ (B) $\frac{mg}{k}\left(2+\frac{HK}{mg}\right)^{1/2}$ (C) $\frac{mg}{K}+\left(\frac{2HK}{mg}\right)^{1/2}$ (D) $\frac{mg}{K}\left[1+\left(1+\frac{2HK}{mg}\right)^{1/2}\right]$

9. Two springs, each of spring constant k, are attached to a block of mass m as shown in the figure. The block can slide smoothly along a horizontal platform clamped to the opposite walls of the trolley of mass M. If the block is displaced by x cm and released, the period of oscillation is :

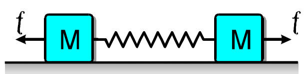


(A) $T = 2\pi\sqrt{\frac{Mm}{2k}}$ (B) $T = 2\pi\sqrt{\frac{(M+m)}{kmM}}$ (C) $T = 2\pi\sqrt{\frac{mM}{2k(M+m)}}$ (D) $T = 2\pi\frac{(M+m)^2}{k}$

10. A body executes simple harmonic motion under the action of a force F_1 with a time period $\frac{4}{5}$s. If the force is changed to F_2 it executes S.H.M. with time period $\frac{3}{5}$ s. If both the forces F_1 and F_2 act simultaneously in the same direction on the body, its time period in seconds is.

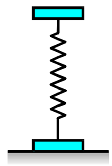
(A) $\frac{12}{25}$ (B) $\frac{24}{25}$ (C) $\frac{35}{24}$ (D) $\frac{15}{12}$

11. A spring of force constant α has two blocks of same mass M connected to each end of the spring. Same force f extends each end of the spring. If the masses are released, then period of vibration is :



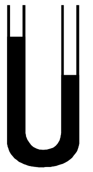
(A) $2\pi\sqrt{\frac{M}{2\alpha}}$ (B) $2\pi\sqrt{\frac{M}{\alpha}}$ (C) $2\pi\sqrt{\frac{2\alpha M}{\alpha^2}}$ (D) $2\pi\sqrt{\frac{M\alpha^2}{2\alpha}}$

12. Two plates of same mass are attached rigidly to the two ends of a spring. One of the plates rests on a horizontal surface and the other results a compression y of the spring when it is in steady state. The further minimum requirement of compression, so that when the force causing compression is removed the lower plate is lifted off the surface, will be :



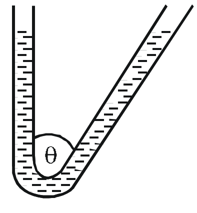
(A) 0.5 y (B) 3y (C) 2y (D) y

13. A U-tube contains 1 kg of mercury as shown and is disturbed so that it oscillates back and fourth from arm to arm. If we neglect friction and one centimetre of mercury column has a mass of 20 g, the period of oscillation is [Take $g = \pi^2$] :



(A) 0.5 s (B) 1.5 s (C) 2 s (D) 1 s

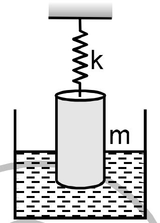
14. The period of oscillation of mercury of mass m and density ρ poured into a bent tube of cross sectional area S whose right arm forms an angle θ with the vertical is :



(A) $2\pi\sqrt{\frac{m}{\rho S(1+\sin\theta)g}}$ (B) $2\pi\sqrt{\frac{m}{\rho S\sin\theta g}}$

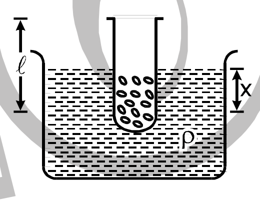
(C) $2\pi\sqrt{\frac{m}{\rho S(1+\cos\theta)g}}$ (D) $2\pi\sqrt{\frac{m}{\rho S\cos\theta g}}$

15. A uniform cylinder of mass m and length ℓ having area of cross-section a is suspended lengthwise with the help of a massless spring of constant k. The cylinder is half submerged in a liquid of density ρ. A small push and release makes it vibrate with small amplitude. The frequency of oscillation is :



(A) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{ka\rho g}{m}}$ (C) $\frac{1}{2\pi}\sqrt{\frac{m+a\rho g}{k}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{k+a\rho g}{m}}$

16. A test tube of length ℓ and area of cross-section A has some iron filings of mass M. The test tube floats normally in a liquid of density ρ with length x dipped in the liquid. A disturbing force makes the tube oscillate in the liquid. The time period of oscillation is given by T =



(A) $2\pi\sqrt{\frac{M\rho}{Ag}}$ (B) $2\pi\sqrt{\frac{x}{g}}$ (C) $2\pi\sqrt{\frac{\ell}{g}}$ (D) $2\pi\sqrt{\frac{M}{g}}$

17. The bob in a simple pendulum of length ℓ is released at t = 0 from the position of small angular displacement θ. Linear displacement of the bob at any time t from the mean position is given by

(A) $\ell\theta\cos\sqrt{\frac{g}{\ell}}t$ (B) $\ell\sqrt{\frac{g}{\ell}}t\ \cos\theta$ (C) $\ell g\sin\theta$ (D) $\ell\theta\sin\sqrt{\frac{g}{\ell}}t$

18. The period of small oscillations of a simple pendulum of length l if its point of suspension O moves a with a constant acceleration $\alpha = \alpha_1\hat{i} - \alpha_2\hat{j}$ with respect to earth is

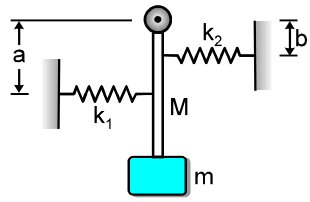
(A) $T = 2\pi\sqrt{\frac{\ell}{\{(g-\alpha_2)^2+\alpha_1^2\}^{1/2}}}$ (B) $T = 2\pi\sqrt{\frac{\ell}{\{(g-\alpha_1)^2+\alpha_2^2\}^{1/2}}}$

(C) $T = 2\pi\sqrt{\frac{\ell}{g}}$ (D) $T = 2\pi\sqrt{\frac{\ell}{(g^2+\alpha_1^2\}^{1/2}}}$

19. A rod of mass M and length L is hinged at its one end and carries a block of mass m at its lower end. A spring of force constant k_1 is installed at distance a from the hinge and another of force constant k_2 at a distance b as shown in the figure. If the whole arrangement rests on a smooth horizontal table top, the frequency of vibration is

(A) $\frac{1}{2\pi}\sqrt{\frac{k_1a^2+k_2b^2}{L^2(m+\frac{M}{3})}}$

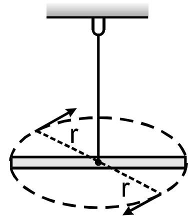
(B) $\frac{1}{2\pi}\sqrt{\frac{k_2+k_1}{M+m}}$



(C) $\frac{1}{2\pi}\sqrt{\frac{k_2+k_1\frac{a^2}{b^2}}{4\frac{M}{3}+m}}$

(D) $\frac{1}{2\pi}\sqrt{\frac{k_1+\frac{k_2b^2}{a^2}}{\frac{4}{3}m+M}}$

20. A metal rod is suspended in a horizontal position by a vertical wire attached to its centre. A horizontal couple of moment 10 N.m on the rod twists the wire and the rod is deflected through 18°. On releasing the rod, it oscillates as a torsion pendulum with a period of 1 seconds. The moment of inertia of the rod is



(A) 3.2 kg m^2 (B) 1.8 kg m^2 (C) 0.8 kg m^2 (D) 2.4 kg m^2

SECTION (B) : ONE OR MORE THAN ONE CORRECT CHOICE

21. A particle moves on the X-axis according to the equation $x = x_0 \sin^2 \omega t$. The motion is simple harnomic

(A) with amplitude x_0 (B) with amplitude $2x_0$

(C) with time period $\frac{2\pi}{\omega}$ (D)with time period $\frac{\pi}{\omega}$

22. An object is released from rest. The time it takes to fall through a distance h and the speed of the object as it falls through this distance are measured with a pendulum clock. The entire apparatus is taken on the moon and the experiment is repeated

(A) the measured times are same (B) the measured speeds are same

(C) the actual times in the fall are equal (D) the actual speeds are equal

23. The speed v of a particle moving along a straight line, when it is at a distance (x) from a fixed point of the line is given by $v^2 = 108 - 9x^2$ (all quantities are in cgs units) :

(A) the motion is uniformly accelerated along the straight line

(B) the magnitude of the acceleration at a distance 3cm from the point is 27 cm/sec^2

(C) the motion is simple harmonic about the given fixed point.

(D) the maximum displacement from the fixed point is 4 cm.

24. For a body executive SHM with amplitude A, time period T, maximum velocity v_{max} and phase constant zero, which of the following statements are correct (y is displacement from mean position) ?

(A) At y = (A/2), v > (v_{max}/2) (B) for v = (v_{max}/2) |y| > (A/2)

(C) For t = (T/8), y > (A/2) (D) For y = (A/2), t < (T/8)

25. If y, v, and a represent displacement, velocity and acceleration at any instant for a particle executing SHM, which of the following statements are true?

(A) v and y may have same direction (B) v and a may have same direction

(C) a and y may have same direction (D) a and v may have opposite direction

26. Which of the following functions represent SHM?

(A) $\sin 2\omega t$ (B) $\sin^2 \omega t$ (C) $\sin \omega t + 2 \cos \omega t$ (D) $\sin \omega t + \cos 2\omega t$

27. Which of the following examples represent periodic motion?

(A) A swimmer completing one return trip from one bank of a river flowing to the other and back

(B) A freely suspended bar magnet displaced from its N-S direction and released

(C) Halley's comet

(D) A hydrogen molecule rotating about its centre of mass

(E) An arrow released from a bow

28. The time period of a particle in simple harmonic motion is T. A time (T/6) sec after it passes its mean position. its :

(A) velocity will be one half its maximum velocity

(B) displacement will be one half its amplitude

(C) acceleration will be nearly 86% of its maximum acceleration
(D) KE = PE

29. The potential energy of a particle of mass 0.1 kg, moving along the x-axis, is given by U = 5x (x – 4) J, where x is in metres. It can be concluded that
(A) the particle is acted upon by a constant force (B) the speed of the particle is maximum at x = 2m
(C) the particle executes SHM (D) the period of oscillation of the particle is (π/5) sec

30. A horizontal plank has a rectangular block placed on it. The plank starts oscillating vertically and simple harmonically with an amplitude of 40 cm. The block just loses contact with the plank when the latter is at momentary rest. Then :
(A) the period of oscillation is $\left(\frac{2\pi}{5}\right)$
(B) the block weighs double its weight, when the plank is at one of the positions of momentary rest.
(C) the block weighs 0.5 times its weight on the plank halfway up
(D) the block weighs 1.5 times its weight on the plank halfway down
(E) the block weights its true weight on the plank when the latter moves fastest

31. A particle moves in the x-y plane according to the equation, $\vec{r} = (\hat{i} + 2\hat{j})$ A cos ωt. The motion of the particle is:
(A) on a straight line (B) on an ellipse (C) periodic (D) simple harmonic

32. A particle moves along the X-axis according to the equation x = 10 sin^3 (πt). The amplitudes and frequencies of component SHMs are
(A) amplitude 30/4, 10/4 ; frequencies 3/2, 1/2
(B) amplitude 30/4, 10/4 ; frequencies 1/2, 3/2
(C) amplitude 10, 10 ; frequencies 1/2, 1/2
(D) amplitude 30/4, 10 ; frequencies 3/2, 2

33. A certain mass m of mecury (density ρ) is poured into a glass U-tube (inner radius, r) and it oscillates freely up and down about its position of equilibrium. Which of the following statement(s) about it is/are correct?
(A) The effective spring constant for the oscillation is $2\pi r^2 \rho g$
(B) The period of oscillation is given as T = $\sqrt{m/2\pi r^2 \rho g}$
(C) The period of oscillation is given as T = $\sqrt{2\pi m / r^2 \rho g}$
(D) The amplitude of oscillation is independent of the period of oscillation

34. Which of the following will change the time period as they are taken to moon ?
(A) A simple pendulum (B) A physical pendulum
(C) a torsional pendulum (D) a spring mass system

35. A ball is hung vertically by a thread of length 'ℓ' from a point 'P' of an inclined wall that makes an angle 'α' with the vertical. The thread with the ball is then deviated through a small angle 'β' ($\beta > \alpha$) and set free. Assuming the wall to be perfectly elastic, the period of such pendulum is
(A) $2\sqrt{\frac{\ell}{g}}\left[\sin^{-1}\left(\frac{\alpha}{\beta}\right)\right]$ (B) $2\sqrt{\frac{\ell}{g}}\left[\frac{\pi}{2}+\sin^{-1}\left(\frac{\alpha}{\beta}\right)\right]$
(C) $2\sqrt{\frac{\ell}{g}}\left[\cos^{-1}\left(\frac{\alpha}{\beta}\right)\right]$ (D) $2\sqrt{\frac{\ell}{g}}\left[\cos^{-1}\left(-\frac{\alpha}{\beta}\right)\right]$

36. Suppose a tunnel is dug along a diameter of the earth. A particle is dropped from a point, a distance h directly above the tunnel. The motion of the particle as seen from the earth is
(A) simple harmonic (B) parabolic (C) on a straight line (D) periodic

37. Suppose a tunnel is dug along the diameter of the earth. A particle is dropped from a point a distant h directly above the tunnel. If earth's density is assumed uniform, and friction neglected, then :
(A) the particle will execute simple harmonic motion.
(B) the particle will have maximum speed when passing through the centre of the earth.
(C) the acceleration of the particle will be maximum just at the point of release.
(D) the particle will reach the same height h above the earth on the opposite end.

EXERCISE-4

SECTION (A) : EQUATION OF SHM AND ENERGY

A 1. Consider a particle moving in simple harmonic motion according to the equation

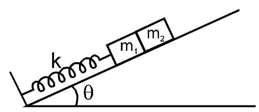
$$x = 2.0 \cos (50 \pi t + \tan^{-1} 0.75)$$

where x is in centimetre and t in second. The motion is started at t = 0. (a) When does the particle come to rest for the first time ? (b) When does the acceleration have its maximum magnitude for the first time ? (c) When does the particle come to rest for the second time ?

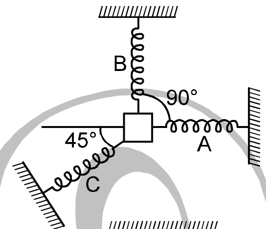
A 2. Two particles A and B are performing SHM along x and y-axis respectively with equal amplitude and frequency of 2 cm and 1 Hz respectively. Equilibrium positions of the particles A and B are at the co-ordinates (3, 0) and (0, 4) respectively. At t = 0, B is at its equilibrium position and moving towards the origin, while A is nearest to the origin and moving away from the origin. Find the maximum and minimum distances between A and B.

SECTION (B) : SPRING MASS SYSTEM

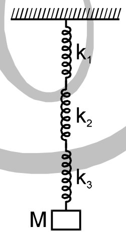
B 1. The block of mass m_1 shown in figure is fastened to the spring and the block of mass m_2 is placed against it. (a) Find the compression of the spring in the equilibrium position. (b) The blocks are pushed a further distance (2/k) $(m_1 + m_2)$ g sin θ against the spring and released. Find the position where the two blocks separate. (c) What is the common speed of blocks at the time of separation?



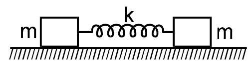
B 2. A particle of mass m is attatched to three springs A, B and C of equal force constant k as shown in figure. If the particle is pushed slightly against the spring C and released, find the time period of oscillations.



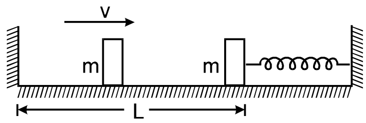
B 3. Find the elastic potential energy stored in each spring shown in figure, when the block is in equilibrium. Also find the time period of vertical oscillation of the block.



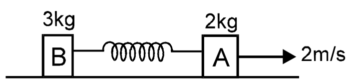
B 4. Consider the situation shown in figure. Show that if the blocks are displaced slightly in opposite directions and released, they will execute simple harmonic motion. Calculate the time period.



B 5. The left block in figure moves at a speed v towards the right block placed in equilibrium. All collisions to take place are elastic and the surface are frictionless. Show that the motions of the two blocks are periodic. Find the time period of these periodic motions. Neglect the widths of the blocks.



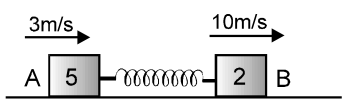
B 6. Two blocks A(2kg) and B(3kg) are resting upon a smooth horizontal surface are connected by a spring of stiffness 120 N/m. Initially the spring is underformed. A is imparted a velocity of 2m/s along the line of the spring away from B. Find the displacement of A, t seconds later.



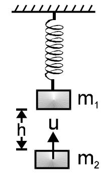
B 7. Two blocks A (5kg) and B (2kg) attached to the ends of a spring of spring constant 1120N/m are placed on a smooth horizontal plane with the spring undeformed. Simultaneously velocities of 3m/s and 10m/s along the line of the spring in the same direction are imparted to A and B then. Find out :

(a) The maximum extension in the spring.

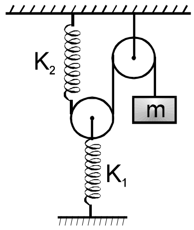
(b) Time after which maximum extension occurs after start.



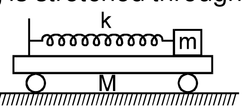
B 8. A body of mass $m_1 = 0.9$ kg is attached to the lower end of a spring (of negligible mass and spring constant K = 800N/m), the other end of which is fixed to a ceiling. Another body of mass $m_2 = 0.9$ kg is thrown vertically up with a velocity 4 m/s from distance 0.6 m below the mass m_1. The body m_2 sticks to m_1 on collision. Find the amplitude (in cm) of the resulting motion. (g = 10 m/s^2)



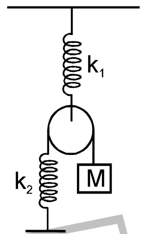
B 9. Find the time period of small oscillation of mass m about its mean position. Assume ideal conditions.



B 10. All the surfaces shown in figure are frictionless. The mass of the car is M, that of the block is m and the spring has spring constant k. Initially, the car and the block are at rest and the spring is stretched through a length x_0 when the system is released. (a) Find the amplitudes of the simple harmonic motion of the block and of the car as seen from the road. (b) Find the time period (s) ofthe two simple harmonic motions.



B 11. Find the time period of small oscillations of mass 'M' about its equilibrium position. Also find the extension in each spring when 'M' is in equilibrium. Springs, pulley & strings are of negligible mass.



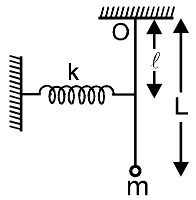
SECTION (C) : SIMPLE PENDULUM

C 1. A simple pendulum of length 40 cm is taken inside a deep mine. Assume for the time being that the mine is 1600 km deep. Calculate the time period of the pendulum there. Radius of the earth = 6400 km.

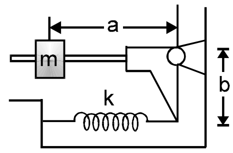
C 2. A simple pendulum of length ℓ is suspended from the ceiling of a car moving with a speed v on a circular horizontal road of radius r. (a) Find the tension in the string when it is at rest with respect to the car. (b) Find the time period of small oscillation.

SECTION (D) : ANGULAR SHM

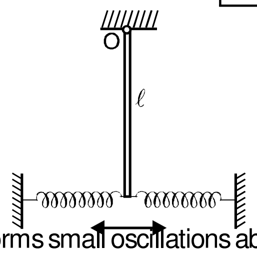
D 1. A particle of mass m is suspended at the lower end of a thin rod of negligible mass. The upper end of the rod is free to rotate in the plane of the page about a horizontal axis passing through the point O. The spring is undeformed when the rod is vertical as shown in fig. Show that the motion of the particle is SHM, if it is displaced from its mean position and hence find the period of oscillation.



D 2. Determine the expression of the natural frequency f for small oscillations of the weighted rod about O. The stiffness of the spring is k & its length is adjusted so that the rod is in equilibrium in the horizontal position as shown in figure. Neglect the masses of the spring & rod as compared to 'm'.



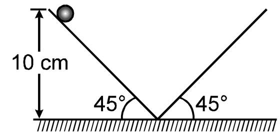
D 3. Find the frequency of small oscillations of a thin uniform vertical rod of mass m and length ℓ hinged at the point O (Fig.). The combined stiffness of the springs is equal to K. The spring are of negligible mass.



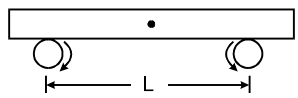
D 4. A thin uniform plate shaped as an equilateral triangle with a height h performs small oscillations about the horizontal axis coinciding with one of its sides. Find the oscillation period and the reduced length of the given pendulum.

SECTION (E) : MISCELLANEOUS

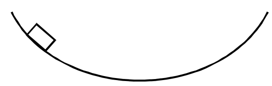
E 1. Find the time period of the motion of the particle shown in figure. Neglect the small effect of the bend near the bottom. (g = 10 m/s^2)



E 2. A uniform plate of mass M stays horizontally and symmetricaly on two wheels rotating in opposite directions (figure). The separation between the wheels is L. The friction coefficient between each wheel and the plate is μ. Find the time period of oscillation of the plate if it is slightly displaced along its length and released.



E 3. A small block oscillates back and forth on a smooth concave surface of radius R (figure). Find the time period of small oscillation and forth on a smooth concave surface of radius R (figure). Find the time period of small oscillation.



E 4. A spherical ball of mass m and radius r rolls without slipping on a rough concave surface of large radius R. It makes small oscillations about the lowest point. Find the time period.

E 5. Assume that a tunnel is dug across the earth (radius = R) passing through its centre. Find the time a particle takes to cover the length of the tunnel if (a) it is projected into the tunnel with a speed of $\sqrt{gR}$ (b) it is released from a height R above the tunnel (c) it is thrown vertically upward along the length of tunnel with a speed of $\sqrt{gR}$.

E 6. Assume that a tunnel is dug along a chord of the earth, at a perpendicular distance R/2 from the earth's centre where R is the radius of the earth. The wall of the tunnel is frictionless. (a) Find the gravitational force exerted by the earth on a particle of mass m placed in the tunnel at a distance x from the centre of the tunnel. (b) Find the component of this force along the tunnel and perpendicular to the tunnel. (c) Find the normal force exerted by the wall on the particle. (d) find the resultant force on the particle. (e) Show that the motion of the particle in the tunnel is simple harmonic and find the time period.

QUESTIONS FOR SHORT ANSWERS :

1. Determine weather the following motions are periodic, oscillatory or SHM.
(a) Earth spinning about its own axis (b) Earth revolving around the sun.
(c) Motion of a pendulum in pendulum wall clock.
(d) Motion of a particle attached to a spring if it is displaced along the length of spring.

2. A vibrating simple pendulum of time period T is placed in a lift which is accelerating downwards. What will be the effect on the time period of simple pendulum ?

3. Given below are five examples of accelerated motion : (a) a particle executing S.H.M. (B) a body falling under gravity near the surface of the earth (C) a body falling under gravity from a height comparable to the earth's radius (D) a stone revolving in a circle with constant speed and (e) a stone revolving in a circle with variable speed.
Match each example with one of the following categories :
(i) acceleration of constant magnitude and direction,
(ii) acceleration of constant magnitude but changing direction,
(iii) acceleration of changing magnitude but constant direction,
(iv) acceleration of changing magnitude and direction.

4. A girl is sitting on a swing. Another girl sits by her side. What will be the effect on the periodic-time of the swing?

5. The girl sitting on a swing stand up. What will be the effect on the periodic-time of the swing?

6. The bob of a simple pendulum is a ball full of water. If a fine hole is made in the bottom of the ball, what will be its effect on the time-period of the pendulum ?

7. A simple pendulum executing SHM is falling freely along with the support. Will its time-period change ?

8. Can a pendulum-clock be used in an artificial satellite ?

REASONING AND ASSERTION : –

In each of the following questions, a statement of Assertion (A) is given followed by a corresponding statement of Reason (R) just below it. Of the statements, mark the correct answer.
(a) If both assertion and reason are true and reason is the correct explanation of assertion
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false

(d) If both assertion and reason are false

9. **Assertion :** In SHM, the velocity of the body is maximum at the mean position.
Reason : SHM is a periodic motion.

10. **Assertion :** Earth is in periodic motion around the sun.
Reason : The motion of earth around the sun is not simple harmonic motion (SHM).

11. **Assertion :** All oscillatory motions are necessarily periodic motion but all periodic motion are not oscillatory.
Reason : Simple pendulum is an example of oscillatory motion.

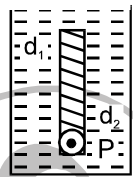
12. **Assertion :** The percentage change in time period is 2%, if the length of simple pendulum increases by 3%.
Reason : Time period is directly proportional to length of pendulum.

13. **Assertion :** In a SHM, kinetic and potential energies become equal when the displacement is $1/\sqrt{2}$ times the amplitude.
Reason : In SHM, kinetic energy is zero when potential energy is maximum.

14. **Assertion :** If the amplitude of a simple harmonic oscillator is doubled, its total energy also becomes doubled.
Reason : The total energy is directly proportional to the amplitude of vibration of the harmonic oscillator.

EXERCISE-5

1. A thin rod of length L and area of cross section S is pivoted at its lowest point P inside a stationary, homogeneous and non viscous liquid. The rod is free to rotate in a vertical plane about a horizontal axis passing through P. The density d_1 of the material of the rod is smaller than the density d_2 of the liquid. The rod is displaced by a small angle θ from its equilibrium position and then released. Show that motion of the rod is simple harmonic and determine its angular frequency in terms of the given parameters. **[JEE - 96]**
[Note : This can be done after studying fluid mechanics and Rotational motion]



2. A block is kept on a horizontal table. The table is undergoing simple harmonic motion of frequency 3 Hz in a horizontal plane. The coefficient of static friction between block and the table surface is 0.72. Find the maximum amplitude of the table at which the block does not slip on the surface. **[REE - 96]**

3. A block is kept on a horizontal table. The table is undergoing simple harmonic motion of frequency 3 Hz in a horizontal plane. The coefficient of static friction between block and the table surface is 0.72. The maximum amplitude of the table at which the block does not slip on the surface is **[REE - 96]**
(A) 2 cm (B) 2.5 cm (C) 3.0 cm (D) 4.0 cm

4. A particle is subjected to two SHMs. $X_1 = A_1 \sin\omega t$ and $X_2 = A_2 \sin\left(\omega t + \frac{\pi}{4}\right)$. The resultant SHM will have an amplitude of : **[JEE 1996]**
(A) $(A_1 - A_1)/2$ (B) $\sqrt{A_1^2 + A_2^2}$ (C) $\sqrt{A_1^2 + A_2^2 + \sqrt{2}A_1A_2}$ (D) $(A_1 + A_1)/2$

5. A particle of mass M is executing oscillations about the origin on the x-axis. Its potential energy is $|U| = k|x|^3$ where k is a positive constant. If the amplitude of oscillation is a, then its period T is : **[JEE 1998]**
(A) proportional to $1/\sqrt{a}$ (B) independent of a (C) proportional to $\sqrt{a}$ (D) proportional to $a^{3/2}$

6. A particle free to move along the x-axis has potential energy given by $U(x) = k[1 - e^{-x^2}]$
for $-\infty < x < +\infty$, where k is a positive constant of appropriate dimensions. Then
(A) at point away from the origin, the particle is in unstable equilibrium.
(B) for any finite non-zero value of x, there is a force directed away from the origin.
(C) if its total mechanical energy is k/2, it has its minimum kinetic energy at the origin.
(D) for small displacements from x = 0, the motion is simple harmonic. **[JEE - 99]**

7. A small bar magnet having a magnetic moment of 9×10^{-9} Wb-m is suspended at its center of gravity by a light torrionless string at a distance of 10^{-2} m vertically above a long straight horizontal wire carrying a current of 1.0 A. Find the frequency of oscillation of the magnet about its equilibrium position assuming that the motion is undamped. The moment of inertia of the magnet is 6×10^{-9} kg-m^2.[Note : This is for class XII. Chapter Magnetic Effects of Current] **[REE - 99]**

8. Three simple harmonic motions in the same direction having the same amplitude a and same period are superposed. If each differs in phase from the next by 45^0, then, **[I.I.T. 1999]**

(A) the resultant amplitude is $(1+\sqrt{2})a$
(B) the phase of the resultant motion relative to the first is 90^0.
(C) the energy associated with the resulting motion is $(3+2\sqrt{2})$ times the energy associated with any single motion.
(D) the resulting motion is not simple harmonic.

9. A bob of mass M is attached to the lower end of a vertical string of length L and cross-sectional area A. The Young's modulus of the material of the string is Y. If the bob executes SHM in the vertical direction, find the frequency of these oscillations. [Note : This can be done after studying Elasticity] **[REE-2000]**

10. The period of oscillation of simple pendulum of length L suspended from the roof of a vehicle which moves without friction down on inclined plane of inclination α is given by **[I.I.T. (Scr.) 2000]**

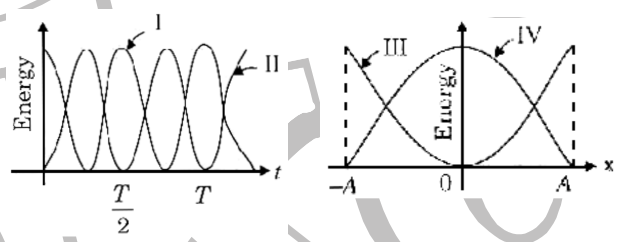
(A) $2\pi\sqrt{\frac{L}{g\cos\alpha}}$ (B) $2\pi\sqrt{\frac{L}{g\sin\alpha}}$ (C) $2\pi\sqrt{\frac{L}{g}}$ (D) $2\pi\sqrt{\frac{L}{g\tan\alpha}}$

11. A particle executes simple harmonic motion between X = – A and x = +A. The time taken for it to go from 0 to A/2 is T_1 and to go from A/2 to A is T_2, then **[I.I.T. Scr 2001]**
(A) $T_1 < T_2$ (B) $T_1 > T_2$ (C) $T_1 = T_2$ (D) $T_1 = 2T_2$

12. A simple pendulum has a time period T_1 when on the earth's surface, and T_2 when taken to height R above the earth's surface, where R is the radius of the earth. The value of $\frac{T_2}{T_1}$ is: **[JEE - MAINS -2001]**

(A) 2 (B) 1 (C) $\sqrt{2}$ (D) 4

13. The displacement of a linear harmonic oscillator is given by $x = A\cos\omega t$. The curves showing the variation of the potential energy with t and x (see figure) are displayed respectively by : **[JEE Sc. 2003]**



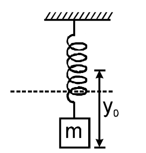
(A) I and III (B) I and IV
(C) II and III (D) II and IV

14. A solid sphere of radius R is half immersed in a liquid of density ρ. Find out the frequency of oscillation of the sphere for small displacement. **[JEE - 2004]**

15. A simple pendulum has time period T_1. When the point of suspension moves vertically up according to the equation $y = kt^2$ where $k = 1\ m/s^2$ and 't' is time then the time period of the pendulum is T_2 then $\left(\frac{T_1}{T_2}\right)^2$ is

(A) $\frac{5}{6}$ (B) $\frac{11}{10}$ (C) $\frac{6}{5}$ (D) $\frac{5}{4}$ **[JEE Scr. 2005]**

16. A block is performing SHM of amplitude 'A' in vertical direction. When block is at 'y' (measured from mean position), it detaches from spring, so that spring contracts and does not affect the motion of the block. Find 'y*' such that block attains maximum height from the mean position. (Given $A\omega^2 > g$) **[JEE 2005' 4]**



17. Function $x = A\sin^2\omega t + B\cos^2\omega t + C\sin\omega t\cos\omega t$ represents SHM **[JEE 2006' 5]**
(A) for any value of A,B and C (except C = 0) (B) If A = – B, C = 2B, amplitude = $|B\sqrt{2}|$
(C) If A = B ; C = 0 (D) If A = B ; C = 2B, amplitude = |B|

ANSWER

EXERCISE - 1

SECTION : (A)

A-1. (a) 2.0 cm, $\frac{\pi}{50}$ s = 0.063 s, 100 N/m
(b) 1.0 cm, $\sqrt{3}$ m/s, 100 m/s^2

A-2. (a) $\frac{\pi}{120}$ s (b) $\frac{\pi}{30}$ s (c) $\frac{\pi}{30}$ s

A-3. (a) T/12 (b) T/8 (c) T/6
(d) T/4 , (e) T/8

A-4. $\frac{\sqrt{3}v_0}{2}$

A-5. x = (10 cm) sin $\left[\left(\frac{\pi}{3}s^{-1}\right)t+\frac{\pi}{6}\right]$, $\frac{10}{9}\pi^2 \approx 11$ cm/s^2

A-6. 4.9 cm, $\frac{2\pi}{10\sqrt{5}}$ s = 0.28 s

A-7. ± 1.2 cm from the mean position

SECTION : (B)

B-1. $5\sqrt{2}$ cm **B-2.** $15\sqrt{3}$ cm , $15\sqrt{3}$ cm

SECTION : (C)

C-1. ≈ 10 g **C-2.** $\frac{5}{2\pi}$ Hz, 5 cm

C-3. 0.16 kg

C-4. (a) $\frac{F}{k}$, $2\pi\sqrt{\frac{M}{k}}$ (b) $\frac{F^2}{2k}$ (c) $\frac{F^2}{2k}$

C-5. (a) $2\pi\sqrt{\frac{m}{k_1+k_2}}$, $k_{eq.} = k_1 + k_2$;
(b) $2\pi\sqrt{\frac{m}{k_1+k_2}}$, $k_{eq.} = k_1 + k_2$;
(c) $2\pi\sqrt{\frac{m\,(k_1+k_2)}{k_1k_2}}$, $k_{eq.} = \frac{k_1k_2}{k_1+k_2}$

C-6. $T = 2\pi\sqrt{\left(\frac{2m}{9k}\right)}$ **C-7.** $\frac{\pi}{7}$ sec.

C-8. 2.5 cm

C-9. F = $(m_1 + m_2)$ g ± $m_1 a\omega^2$ = 50 N and 30 N.

SECTION : (D)

D-1. 1 m **D-2.** 1m

D-3. 9.795 m/s^2

D-4. (i) $2T_0$ (ii) 3g upwards

D-5. (a) $2\pi\sqrt{\frac{\ell}{g+a_0}}$ (b) $2\pi\sqrt{\frac{\ell}{g-a_0}}$ (c) $2\pi\sqrt{\frac{\ell}{g}}$

D-6. g/10 **D-7.** 2.1 sec.

SECTION : (E)

E-1. (a) 1.51 s (b) $2\pi\sqrt{\frac{2r}{g}}$
(c) $2\pi\sqrt{\frac{\sqrt{8}\,a}{3\,g}}$ (d) $2\pi\sqrt{\frac{3\,r}{2\,g}}$

E-2. $2\pi\sqrt{\frac{r\sqrt{2}}{g}}$, r/$\sqrt{2}$

E-3. (a) 50 cm (b) 11 cm/s
(c) 1.2 cm/s^2 towards the point of suspension
(d) 34 cm/s^2 towards the mean position

E-5. $\frac{2\pi^2 mr^2}{T^2}$

F-1. (a) 7.0 cm (b) $\sqrt{37}$ cm = 6.1 cm (c) 5.0 cm

F-2. 2 A **F-3.** $2x^2 + \frac{y}{2} = 1$

EXERCISE - 2

SECTION : (A)

A-1. AB	**A-2.** A	**A-3.** A
A-4. D	**A-5.** A	**A-6.** A
A-7. A	**A-8.** D	**A-9.** C
A-10. B	**A-11.** A	**A-12.** BC
A-13. C	**A-14.** A	**A-15.** C
A-16. D	**A-17.** A	**A-18.** B
A-19. C	**A-20.** B	**A-21.** B

SECTION : (B)

B-1. B	**B-2.** C	**B-3.** B	**B-4.** D
B-5. D	**B-6.** AB	**B-7.** A	**B-8.** A
B-9. C	**B-10.** C		

SECTION : (C)

C-1. D	**C-2.** C	**C-3.** C
C-4. B	**C-5.** C	**C-6.** C
C-7. A	**C-8.** A	**C-9.** D
C-10. D	**C-11.** B	**C-12.** C

SECTION : (D)

D-1. D	**D-2.** A.	**D-3.** D
D-4. C	**D-5.** D	**D-6.** C
D-7. D	**D-8.** C	**D-9.** C
D-10. D		

SECTION : (E)

E-1. ABD

SECTION : (F)

F-1. B	**F-2.** D	**F-3.** D
F-4. BC	**F-5.** C	**F-6.** C
F-7. C		

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EXERCISE - 3

1.	C	2.	C	3.	A	4.	A
5.	B	6.	D	7.	B	8.	B
9.	C	10.	A	11.	A	12.	C
13.	D	14.	C	15.	D	16.	B
17.	A	18.	A	19.	A	20.	C
21.	D	22.	AB	23.	BC	24.	ABCD
25.	ABD	26.	ABC	27.	BCD	28.	AC
29.	BCD	30.	ABCDE			31.	ACD
32.	B	33.	ACD	34.	AB	35.	BD
36.	CD	37.	BD				

EXERCISE - 4

SECTION (A) :

A 1. (a) 1.6×10^{-2} s (b) 1.6×10^{-2} s
(c) 3.6×10^{-2} s

A 2. $x = 3 - A\cos\omega t, Y = 4 - A\sin\omega t$, Min = 3, Max = 7

SECTION (B) :

B 1. (a) $\dfrac{(m_1+m_2)\,g\sin\theta}{k}$
(b) when the spring acquires its natural length
(c) $\sqrt{\dfrac{3}{k}(m_1+m_2)g\sin\theta}$

B 2. $2\pi\sqrt{\dfrac{m}{2k}}$

B 3. $\dfrac{M^2g^2}{2k_1}$, $\dfrac{M^2g^2}{2\,k_2}$ and $\dfrac{M^2g^2}{2\,k_3}$

from above, time period $= 2\pi\sqrt{M\left(\dfrac{1}{k_1}+\dfrac{1}{k_2}+\dfrac{1}{k_3}\right)}$

B 4. $2\pi\sqrt{\dfrac{m}{2k}}$ **B 5.** $\left(\pi\sqrt{\dfrac{m}{k}}+\dfrac{2L}{v}\right)$

B 6. $0.8t + 0.12 \sin 10t$ **B 7.** (a) 25 cm (b) $\dfrac{\pi}{56}$ sec.

B 8. $\dfrac{38}{8} = 4.575$ cm **B 9.** $T = 2\pi\sqrt{\dfrac{m(K_1+4K_2)}{K_1K_2}}$

B 10. (a) $\dfrac{Mx_0}{M+m}$, $\dfrac{mx_0}{M+m}$ (b) $2\pi\sqrt{\dfrac{mM}{k\,(M+m)}}$

B 11. $T = 2\pi\sqrt{\dfrac{m(K_1+4K_2)}{K_1K_2}}$

SECTION (C) :

C 1. 1.47 s

C 2. (a) ma (b) $2\pi\sqrt{\ell/a}$ where $a = \left[g^2+\dfrac{v}{r^2}\right]^{1/2}$

SECTION (D) :

D 1. $T = 2\pi\sqrt{\left\{\dfrac{mL^2}{(k\ell^2+mgL)}\right\}}$ **D 2.** $f = \dfrac{1}{2\pi}\dfrac{b}{a}\sqrt{\dfrac{K}{m}}$

D 3. $\omega = \sqrt{\dfrac{3g}{2\ell}\left(1+\dfrac{2k\ell}{mg}\right)}$ **D 4.** $T = \pi\sqrt{\dfrac{2h}{g}}$, $\ell_{red} = \dfrac{h}{2}$

SECTION (E) :

E 1. ≈ 0.8 s **E 2.** $2\pi\sqrt{\dfrac{\ell}{2\mu g}}$

E 3. $2\pi\sqrt{R/g}$ **E 4.** $2\pi\sqrt{\dfrac{7(R-r)}{5g}}$

E 5. $\dfrac{\pi}{2}\sqrt{\dfrac{R}{g}}$ in each case

E 6. (a) $\dfrac{GMm}{R^3}\sqrt{x^2+R^4/4}$ (b) $\dfrac{GMm}{R^3}x$, $\dfrac{GMm}{2R^2}$
(c) $\dfrac{GMm}{2R^2}$ (d) $\dfrac{GMm}{R^3}x$
(e) $2\pi\sqrt{R^3/(GM)}$

SHORT ANSWERS :

1. (a) Periodic (b) periodic
(c) periodic and oscillatory
(d) Periodic, oscillatory and SHM.

2. T' > T

3. (A) In S.H.M, acceleration is always proportional to displacement but directed opposite to thedisplacement. So in this case, magnitude as well as direction of acceleration changes. Hence it corresponds to (iv)
(B) In this case acceleration due to gravity is constant in magnitude and is always directed towards the centre of the earth. So it corresponds to (i).
(C) In this case, acceleration due to gravity increases continuously. So it corresponds to (iii).
(D) in this case, acceleration is due to change in direction. So it corresponds to (ii).
(e) In this case, acceleration is due to change in magnitude as well as direction. So it corresponds to (iv).

9.	(b)	10.	(b)	11.	(b)
12.	(d)	13.	(b)	14.	(d)

EXERCISE - 5

1. $\sqrt{\dfrac{3g}{2L}\,\dfrac{d_2-d_1}{d_1}}$ **2.** 2cm **3.** A

4. C **5.** A **6.** D

7. 8.7×10^{-4} Hz **8.** A, C

9. $f = \dfrac{1}{2\pi}\sqrt{\dfrac{YA}{mL}}$ **10.** A **11.** A

12. A **13.** A

14. $f = \dfrac{1}{2\pi}\sqrt{\dfrac{3g}{2R}}$ **15.** (C)

16. $y^* = \dfrac{g}{\omega^2}$ **17.** (B, D)

