THIS FILE CONTAINS (COLLECTION 1 & 2)

Very Important Guessing Questions
For IIT JEE 2010 With Detail Solution

Junior Students Can Keep It Safe For Future IIT JEEs

ALGEBRA

(I) Theory Equation
(II) Sequence and Series
(III) Binomial Theorem
(IV) Probability
(V) Determinate and Matrices
(VI) Permutation and Combination

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Single Correct Type

Que. 1. Let a > 1 be a real number. If S is the set of real number x that are solutions to the equation $a^{2\log_2 x} = 5 + 4x^{\log_2 a}$, then **(code-V1T2PAQ5)**

(a) S contains exactly one real number

(b) S contains more than two, but finitely many, real numbers

(c) S contains exactly two real numbers

(d) S contains infinitely many real numbers

Que. 2. If the quadratic equation $x^2 - 2(m-2)x + m^2 - 3 = 0$ has both roots negative then range of m is

(a) $(-\sqrt{3}, \sqrt{3})$ (b) $(-\infty, 2)$ (c) $(-\infty, -\sqrt{3})$ (d) none **(code-V1T4PAQ6)**

Que. 3. Maximum value of the sum of arithmetic progression $100 + 98 + 96 + 94 + ...$ is

(a) 5050 (b) 2550 (c) 2505 (d) 3505 **(code-V1T4PAQ5)**

Que. 4. Given the sequence $10^{\frac{1}{11}}, 10^{\frac{2}{11}}, 10^{\frac{3}{11}}, ..., 10^{\frac{n}{11}}$. The smallest value of $n \in N$ such that the product of the first n terms of the sequence exceeds one lac, is **(code-V1T5PAQ2)**

(a) 9 (b) 10 (c) 11 (d) 12

Que. 5. Let $a = \text{Min. value of } (x^2 + 2x + 3), x \in R$ **(code-V1T5PAQ6)**

and $b = \text{sum of the series}, \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} ....\infty$ then the value of $\sum_{r-0}^{n} a^r b^{n-r}$ equals

(a) $\frac{2^{n+1}+1}{3.2^n}$ (b) $\frac{4^{n+1}-1}{3.2^n}$ (c) $\frac{2^{n+1}-1}{3.2^n}$ (d) $\frac{4^n-1}{3.2^n}$ **(code-V1T5PAQ7)**

Que. 6. If exactly one root of the quadratic equation $x^2 - (a+1)x + 2a = 0$ lies in the interval (0,3) then the set of values 'a' is given by **(code-V1T5PAQ8)**

(a) $(-\infty, 0) \cup (6, \infty)$ (b) $(-\infty, 0] \cup (6, \infty)$

(c) $(\infty, 0] \cup [6, \infty)$ (d) $(0, 6)$

Que. 7. If $a, b \in R, a \neq 0$ and the quadratic equation $ax^2 - bx + 1 = 0$ has imaginary roots then $(a + b + 1)$ is:

(a) positive (b) negative **(code-V1T5PAQ11)**

(c) zero (d) dependent on the sign of b.

Que. 8. Cinsider the infinite series with sum $S = 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...$ and the following four statements

Statement - 1 : The sum increases without any limit. **(code-V1T5PAQ12)**

Statement - 2 : The sum decrease without any limit.

Statement - 3 : The difference between any tern of the sequence and zero is positive.

Statement - 4 : The difference (S – 4) is negative.

of these the correct ones are

(a) only 3 & 1 (b) only 4 & 2 (c) 1 and 2 only (d) only 3 and 4

Que. 9. If the quadratic equations, $3x^2+ax+1=0$ & $2x^2+bx+1=0$ have a common root, then the value of the expression $2a^2-5ab+3b^2$ is equal to $(2a\neq 3b)$ **(code-V1T5PAQ14)**

(a) 0 (b) 1 (c) – 1 (d) 2

Que. 10. If $a(b-c)x^2+b(c-a)xy+c(a-b)y^2$ is a perfect square, then a, b, c are in **(code-V1T5PAQ19)**

(a) A.P. (b) G.P. (c) H.P. (d) None

Que. 11. The first term of an infinite G.P. is the value of x for which the expression $\log_3(3^x-8)+x-2$ vanishes. If the common ratio of the G.P. $\cos\left(\frac{2005\pi}{3}\right)$ then sum of the G.P. is :**(code-V1T5PAQ21)**

(a) 1 (b) 3/2 (c) 4/3 (d) 4

Que. 12. The solution set of the inequality $\sqrt{\log_2 x-1}-\frac{1}{2}\log_2(x^3)+2>0$ is **(code-V1T5PAQ22)**

(a) [2,3] (b) [2,4) (c) (2,3] (d) (2,4]

Que. 13. If the sum of the roots of the quadratic equation, $ax^2+bx+c=0$ is equal to sum of the squares of their reciprocals, then $\frac{a}{c},\frac{b}{a},\frac{c}{b}$ are in : **(code-V1T5PAQ25)**

(a) A.P (b) G.P. (c) H.P (d) none of these

Que. 14. If the expression $y=8x-x^2-15$ is negative then x lies the interval **(code-V1T7PAQ1)**

(a) (3,5) (b) $(5,\infty)$ (c) $(3,\infty)$ (d) $(-\infty,3)\cup(5,\infty)$

Que. 15. If all possible solution to $\log_4(3-x)+\log_{0.25}(3+x)=\log_4(1-x)+\log_{0.25}(2x+1)$ are found, then there will be **(code-V1T10PAQ2)**

(a) only one prime solution (b) two real solutions
(c) no real solution (d) none of these

Que. 16. If the roots of the quadratic equation $(4p-p^2-5)x^2-(2p-1)x+3p=0$ lie on either side of unity then the number of integral values of p is **(code-V1T13PAQ3)**

(a) 1 (b) 2 (c) 3 (d) 4

Que. 17. The sum $\sum_{n=1}^{\infty}\left(\frac{n}{n^4+4}\right)$ is equal to **(code-V1T13PAQ6)**

(a) 1/4 (b) 1/3 (c) 3/8 (d) 1/2

Que. 18. Consider the sequence $S=7+13+21+31+.........+T_n$. The value of T_{70} is **(code-V1T13PAQ12)**

(a) 5013 (b) 5050 (c) 5113 (d) 5213

Que. 19. Number of integers satisfying the inquality $\log_2\sqrt{x}-2\log_{1/4}^2 x+1>0$ is **(code-V1T13PAQ13)**

(a) 1 (b) 2 (c) 3 (d) infinitely many

Que. 20. Suppose $f(n)=\log_2(3).\log_5(4).\log_4(5)..........\log_{n-1}(n)$ then the sum $\sum_{k=2}^{100}f(2^k)$ equals

(a) 5010 (b) 5050 (c) 5100 (d) 5049 **(code-V1T13PAQ15)**

Que. 21. In an A.P. the series containing 99 terms, the sum of all the odd numbered terms is 2550. The sum of all the 99 terms of the A.P. is **(code-V1T13PAQ17)**

(a) 5010 (b) 5050 (c) 5100 (d) 5049

Que. 22. Let r,s and t be the roots of the equation, $8x^3+1001x+2008=0.$ The value of $(r+s)^3+(s+t)^3+(t+r)^3$ is **(code-V1T13PAQ20)**

(a) 251 (b) 751 (c) 735 (d) 753

Que. 23. Let (a_1, b_1) and (a_2, b_2) are the pairs of real numbers such that 10, a, b, ab constitute an arithmetic progression. The value of the product $(a_1.b_1.a_2.b_2)$ is **(code-V1T13PAQ22)**

(a) 25 (b) – 50 (c) 75 (d) 100

Que. 24. If ℓ, m, n be the three positive roots of the equation $x^3-ax^2+bx-48=0$ then the minimum value of $\frac{1}{\ell}+\frac{2}{m}+\frac{3}{n}$ eqauls. **(code-V1T13PAQ23)**

(a) 1 (b) 2 (c) 3/2 (d) 5/2

Que. 25. The sequence $a_1, a_2, a_3, ...$ is a geometric sequence with common ratio r. **(code-V1T13PAQ25)**

The sequence $b_1, b_2, b_3, ...$ is also a geometric sequence.

If $b_1=1, b_2=\sqrt[4]{7}-\sqrt[4]{28}+1;\ a_1=\sqrt[4]{28}$ and $\sum_{n=1}^{\infty}\frac{1}{a_n}=\sum_{n=1}^{\infty}b_n,$ then the common ratio 'r' equals

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $2^{1/4}$ (d) $\sqrt{3}$

Que. 26. Let 'X' denotes the values of the product $(1+a+a^2+a^3+........\infty)(a+b+b^2+.........\infty)$ where 'a' and 'b' are the roots of the quadratic equation $11x^2-4x-2=0$ and 'Y' denotes the numerical vlaue of the infinite series $(\log_b 2)^0\left(\log_b 5^{4^0}\right)+(\log_b 2)^1\left(\log_b 5^{4^1}\right)+(\log_b 2)^2\left(\log_b 5^{4^3}\right)+.......\infty$ where $b=2000$ then the value of (XY) equals **(code-V1T15PAQ8)**

(a) $\frac{1}{5}$ (b) $\frac{11}{15}$ (c) $\frac{13}{6}$ (d) $\frac{22}{35}$

Que. 27. For which positive integers n is the ratio, $\frac{\sum_{k=1}^{n}k^2}{\sum_{k=1}^{n}k}$ an integer ? **(code-V1T18PAQ1)**

(a) odd n only (b) even n only

(c) n = 1 + 6k only, where $k \ge 0$ and $k \in I$ (d) n = 1 + 3k, integer $k \ge 0$

Que. 28. The value of the sum $\frac{1}{3^2+1}+\frac{1}{4^2+2}+\frac{1}{5^2+3}+\frac{1}{6^2+4}+..........\infty$ is equal to **(code-V1T18PAQ5)**

(a) $\frac{13}{36}$ (b) $\frac{12}{36}$ (c) $\frac{15}{36}$ (d) $\frac{18}{36}$

Que. 29. Let 'a' be a real number. Number of real roots of the equation $(x^2+ax+1)(3x^2+ax-3)=0,$ is

(a) at least two (b) atmost two (c) exactly two (d) all four. **(code-V2T1PAQ1)**

Que. 30. The value of the sum $\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\frac{k}{2^{n+k}}$ is equal to **(code-V2T1PAQ3)**

(a) 5 (b) 4 (c) 3 (d) 2

Que. 31. The sum $\sum_{k=1}^{10} k.k!$ equals **(code-V2T1PAQ5)**

(a) $(10)!$ (b) $(11)!$ (c) $(10)!+1$ (d) $(11)!-1$

Que. 32. If $F(x)=Ax^2+Bx+C$ and $f(x)=ax^2+bx+c$ are quadratic function with $F(x)\neq f(x)$. What is ture about the number of solution to $F(x)-f(x)=0$ **(code-V2T3PAQ1)**

I It is possible that there is no real solution

II It can not have more than 2 solution

III If is has one real solution then $A=a$

(a) I and II (b) II and III (c) III and I (d) I, II and III

Que. 33. Let α and β be the solution of the quadratic equation $x^2-1154x+1=0$ then the value of $\sqrt[4]{\alpha}+\sqrt[4]{\beta}$ is equal to **(code-V2T3PAQ6)**

(a) 4 (b) 5 (c) 6 (d) 8

Que. 34. Number of ways in which a person can walk up stairway which has 7 steps if he take 1 or 2 steps up the stairs at a time, is **(code-V2T3PAQ7)**

(a) 28 (b) 21 (c) 15 (d) 17

Que. 35. In the expansion of $(1+x+x^2+.......+x^{27})(1+x+x^2+........+x^{14})^2$, the coefficient of x^{28} is

(a) 195 (b) 224 (c) 378 (d) 405 **(code-V2T3PAQ8)**

Que. 36. If a,b,c are in A.P. then the quadratic equation $3ax^2-4bx+c=0$ has **(code-V2T8PAQ5)**

(a) both roots negative (b) both roots of opjposite sign

(c) both roots lying in (0,1) (d) atleast one root is (0,1)

Que. 37. The sum of all the roots of the equation $\sin\left(\pi\log_3\left(\frac{1}{x}\right)\right)=0$ in $(0,2\pi)$ is **(code-V2T8PAQ9)**

(a) 3/2 (b) 4 (c) 9/2 (d) 13/3

Que. 38. The sum $S={}^{20}C_2+2.{}^{20}C_3+3.{}^{20}C_4+............+19.{}^{20}C_{20}$ equal to **(code-V2T8PAQ11)**

(a) $1+5.2^{20}$ (b) $1+2^{21}$ (c) $1+9.2^{20}$ (d) 2^{20}

Que. 39. Number of permutions 1,2,3,4,5,6,7,8 and 9 taken all at a time are such that the digit **(code-V2T10PAQ3)**

1 appearing somewhere to the left of 2

3 apperaring to the left of 4 and

5 some where to the left of 6, is

(**e.g.** 815723946 would be one such permutation)

(a) $9.7!$ (b) $8!$ (c) $5!.4!$ (d) $8!.4!$

Que. 40. A box contains 10 tickets numbered from 1 to 10. Two tickets are drawn one by one without replacement. The probability that the "difference between the first drawn ticket number and the second is not less than 4" is **(code-V2T11PAQ1)**

(a) $\frac{7}{30}$ (b) $\frac{14}{30}$ (c) $\frac{11}{30}$ (d) $\frac{10}{30}$

Que. 41. A fair coin is flipped n times. Let E be the event "a head is obtained on the first filp", and let F_k be the event "exactly k heads are obtained." for which one of the following pairs (n,k) are E and F_k independent ? **(code-V2T11PAQ3)**

(a) (12, 4) (b) (20,10) (c) (40,10) (d) (100,51)

Que. 42. An urn contains 3 red balls and n white balls. **(code-V2T11PAQ5)**

Mr. Shuag Kariya draws two balls together from the urn. The probability that they have the same colour is 1/2. Mr. Vivek Jain draws one balls from the urn, notes its colour and replaces it. He then draws a second ball from the urn and finds that both have the same colour is, $5/8$. The possible value of n is **(code-V2T11PAQ6)**

(a) 9 (b) 6 (c) 5 (d) 1

Que. 43. Difine $a_k = (k^2+1)k!$ and $b_k = a_1 + a_2 + a_3 + ... + a_k$. Let $\frac{a_{100}}{b_{100}} = \frac{m}{n}$ where m and n are relatively prime natural numbers. The value of $(n-m)$ iw equal to **(code-V2T12PAQ3)**

(a) 99 (b) 100 (c) 101 (d) 102

Que. 44. Let m denote the number of four digit numbers such that the left most digit is odd, the second digit is even and all four digits are different and n denotes the number of four digit numbers such that the left most digit is even, an odd second digit and all four different digits. If m = nk then the value of k equals. **(code-V2T12PAQ4)**

(a) $\frac{6}{5}$ (b) $\frac{5}{4}$ (c) $\frac{4}{3}$ (d) $\frac{3}{2}$

Que. 45. The digit at a unit place of the sum **(code-V2T13PAQ10)**

$(1!)^2 + (2!)^2 + (3!)^2 + ... + (2008!)^2$, is

(a) 5 (b) 3 (c) 9 (d) 7

Que. 46. If the inequality $(k-1)x^2 - (k+1)x + (k+1)$ is positive $\forall\ x \in R$ then the sum of all the integral values of $k \in [1,100]$, is **(code-V2T13PAQ12)**

(a) 5050 (b) 5049 (c) 5051 (d) 5005

Que. 47. The value of n where n is a positive integer satisfying the equation **(code-V2T13PAQ13)**

$$2 + (6.2^2 - 4.2) + (6.3^2 - 4.3) + ... + (6.n^2 - 4.n) = 140$$

(a) 3 (b) 4 (c) 5 (d) 7

Que. 48. How many six-digit number can be formed using the digit 1,2,3,4,5 and 6 that have atleast two of the digits the same ? **(code-V2T13PAQ17)**

(a) $6(6^5 - 5!)$ (b) 6^6 (c) $6!$ (d) $6^6 - 5!$

Que. 49. A basket ball team consists of 12 pairs of twin brothers. On the first day of training, all 24 players stand in a circle in such a way that all pairs of twins brother are neighbours. Number of ways this can be done is **(code-V2T14PAQ5)**

(a) $(12)!2^{11}$ (b) $(11)!2^{12}$ (c) $(12)!2^{12}$ (d) $(11)!2^{11}$

Que. 50. Let a, b, c be the three sides of a triangle then the quadratic equation $b^2x^2 + \left(b^2 + c^2 - a^2\right)x + c^2 = 0$ has **(code-V2T14PAQ11)**

(a) both imaginary roots (b) both positive roots

(c) both negative roots (d) one positive and one negative roots.

Que. 51. The expression $\binom{n}{k} + \binom{n}{k+1}$ where $n \ge k \ge 1$ is the same as **(code-V2T14PAQ16)**

(a) $\binom{n+1}{k}$ (b) $\binom{n+1}{k-1}$ (d) $\binom{n}{k+1}$ (d) $\binom{n+1}{k+1}$

Que. 52. Let $S_n = 1 + 2 + 3 + \text{.........} + n$ and $P_n = \frac{S_2}{S_2 - 1} \cdot \frac{S_3}{S_3 - 1} \cdot \frac{S_4}{S_4 - 1} \cdot \text{.........} \frac{S_n}{S_n - 1}$ where $n \in N\left(n \ge 2\right). \lim\limits_{x \to \infty} P_n$ equals

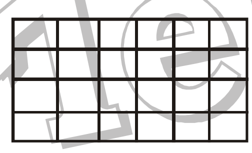
(a) 2 (b) 3 (c) 4 (d) 8 **(code-V2T14PAQ19)**

Que. 53. Number of ways in which n distinct objects can be kept in k different boxes (not more than one in each box) if there are more boxes than tnings, is **(code-V2T17PAQ1)**

(a) ${}^kP_n$ (b) k^n (c) n^k (d) ${}^kC_n$

Que. 54. If $e^{\left(\sin^2 x + \sin^4 x + \sin^6 x \text{........} \infty\right) \ell n\, 3}, x \in \left(0, \frac{\pi}{2}\right)$ satisfies the equation $t^2 - 28t + 27 = 0$ then the value of $\left(\cos x + \sin x\right)^{-1}$ equals **(code-V2T17PAQ9)**

Que. 55. Number of ractangles in the grid shown which are not squares is **(code-V2T19PAQ4)**



(a) 160 (b) 162 (c) 170 (d) 185

Que. 56. There are two urns marked A and B. Urn A contains 2 red and 1 blue. Urn B constains 1 red and 2 blue marbles. A fair coin is tossed. If it lands hends, a marble is drawn form A. If it lands tails a marble is drawn from B. Consider the events **(code-V2T20PAQ1)**

E_1: Hends and a red marble occur

E_2: Red marbles occurs

E_3: Blue marble occurs

E_4: Heads occurring if the marble drawn is red

Which one of the events described above is most porbable ?

(a) E_1 (b) E_2 (c) E_3 (d) E_4

Que. 57. Suppose A and B are two events with P(A) = 0.5 and $P\left(A \cup B\right) = 0.8$. Let $P(B) = p$ if A and B are mutually exclusive and $P(B) = q$ if A and B are independent then **(code-V2T20PAQ2)**

(a) p = q (b) p = 2q (c) 2p = q (d) p + q = 1.

Que. 58. A hat contains a number of cards with **(code-V2T20PAQ3)**

30% white on both sides

50% black on one side and white on the other side.

20% black on both sides.

The cards are mixed up, and a single cards is drawn at random and palced on the table, it's upper side shows up black. The probability that its other side is also black is

(a) 2/9 (b) 4/9 (c) 2/3 (d) 2/7

Que. 59. All the jacks, queen, kings and aces of a regular 52 cards deck are taken out. The 16 cards are throughly shuffled and my opponet, a preson who always tells the truth, simultaneously draws two cards at random and says, "I hold at least one ace." The probability that he holds two aces, is

(a) $\frac{1}{8}$ (b) $\frac{3}{16}$ (c) $\frac{1}{6}$ (d) $\frac{1}{9}$ **(code-V2T20PAQ4)**

Que. 60. Mr. Shuag Kariya lives at origin on the cartesian plain and has his office at (4,5). His friend Mr. Vivek Jain lives at (2,3) on the same plane. Mr. Shuag Kariya can go to his office travelling one block at a time either in the +y or +x direction. If all possible paths are equally likely then the probability that Mr. Shuag Kariya passed his friends house is **(code-V2T20PAQ5)**

(a) 1/2 (b) 10/21 (c) 1/4 (d) 11/21

Comprehesion Type

1 Paragraph for Q. 1 to Q. 3

The real roots of the equation $2x^3 - 19x^2 + 57x + k = 0$ are the first three terms of a geometric progression. **(code-V1T8PAQ1,2,3)**

1. The value of k equals

(a) 216 (b) 108 (c) – 54 (d) –108

2. If the geometric progression is Increasing then the sum of its first n terms equals

(a) $\left(\frac{3}{2}\right)^n - 1$ (b) $4\left(\left(\frac{3}{2}\right)^n - 1\right)$ (c) $6\left(\left(\frac{3}{2}\right)^n - 1\right)$ (d) $4\left(2^n - 1\right)$

3. If the geometric progresion is decreasing then the sum of its infinite number of term is

(a) 27/2 (b) 9 (c) 9/2 (d) 12

2 Paragraph for Q. 4 to Q. 6

Let $P(x)$ be quadratic polynomial with real coefficients such that for all real x the relation $2(1 + P(x)) = P(x-1) + P(x+1)$ holds. If P(0) = 8 and P(2) = 32 then

4. Sum of all the coeffcients of P(x) is **(code-V1T12PAQ1,2,3)**

(a) 20 (b) 19 (c) 17 (d) 15

5. If the range of P(x) is $[m, \infty)$ then the value of 'm' is

(a) – 12 (b) – 15 (c) – 17 (d) – 5

6. The value of P(40) is

(a) 2007 (b) 2008 (c) 2009 (d) 2010

 # 3 Paragraph for Q. 7 to Q. 9

Let equation $x^3+px^2+qx-q=0$ where $p,q\in R-\{0\}$ has 3 real roots α,β,γ in H.P., then

7. $(9p+2q)$ has the value equal to **(code-V1T16PAQ4,5,6)**

(a) 9 (b) – 18 (c)– 27 (d) 1

8. Minimum value of $\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}$ is (You may use the inequality $a^2+b^2+c^2\ge ab+bc+ca$ for any $a,b,c\in R$)

(a) $\frac{1}{3}$ (b) 1 (c) $\frac{4}{3}$ (d) 3

9. $\frac{p}{q}$ has the minimum value equal to

(a) $-\frac{1}{2}$ (b) $-\frac{1}{3}$ (c) $-\frac{1}{4}$ (d) – 1

 # 4 Paragraph for Q. 10 to Q. 12

Consider the cubic equation $x^3-(1+\cos\theta+\sin\theta)x^2+(\cos\theta\sin\theta+\cos\theta+\sin\theta)x-\sin\theta.\cos\theta=0$ whose roots are x_1,x_2 and x_3. **(code-V1T19PAQ17,18,19)**

10. The value of $x_1^2+x_2^2+x_3^2$ equals

(a) 1 (b) 2 (c) $2\cos\theta$ (d) $\sin\theta(\sin\theta+\cos\theta)$

11. Number of values of θ in $[0,2\pi]$ for which at least two roots are equal

(a) 3 (b) 4 (c) 5 (d)6

12. Greatest possible difference between two of the roots if $\theta\in[p,2\pi]$ is

(a) 2 (b) 1 (c) $\sqrt{2}$ (d) $2\sqrt{2}$

 # 5 Paragraph for Q. 13 to Q. 15

Consider a sequence whose sum to n terms is given by the quadratic function $S_n=4n^2+6n$.

13. The nature of the given series is **(code-V1T20PAQ14,15,16)**

(a) A.P. (b) G.P. (c) H.P. (d) A.G.P.

14. For the given sequence the number 5050 is the

(a) $(101)^{th}$ term (b) $(636)^{th}$ term (c) $(656)^{th}$ term (d) $(631)^{th}$ term

15. Sum of thesquares of the first 3 terms of the given series is

(a) 999 (b) 1100 (c) 799 (d) 1000

 # 6 Paragraph for Q. 16 to Q. 18 **(code-V2T2PAQ1,2,3)**

Consider a variable line L which passes through the point of intersection 'P' of the lines $3x+4y-12=0$ and $x+2y-5=0$, meeting the coordinate axes at the points A and B.

16. Locus of the middle point of the segment AB has the equation

(a) $3x+4y=4xy$ (b) $3x+4y=3xy$ (c) $4x+3y=4xy$ (d) $4x+3y=3xy$

17. Locus of the feet of the perpendicular from the origin on the variable line 'L' has the equation

(a) $2(x^2+y^2)-3x-4y=0$ (b) $2(x^2+y^2)-4x-3y=0$

(c) $x^2+y^2-2x-y=0$ (d) $x^2+y^2-x-2y=0$

18. Locus of the centroid of the varible triangle OAB has the equation (where 'O' is the origin)

(a) $3x+4y+6xy=0$ (b) $4x+3y-6xy=0$ (c) $3x+4y-6xy=0$ (d) $4x+3y+6xy=0$

 # 6 Paragraph for Q. 19 to Q. 21

Two fair dice are rolled. Let $P(A_i)>0$ denotes the event that the sum of the faces of the dice is divisible by i. **(code-V2T11PAQ7,8,9)**

19. Which one of the following events is most probable ?

(a) A_3 (b) A_4 (c) A_5 (d) A_6

20. For which one of the following pairs (i, j) are the events A_i and A_j are independent ?

(a) (3,4) (b) (4,6) (c) (2,3) (d) (4,2)

21. Number of all possible ordered pairs (i, j) for which the events A_i and A_j are independent.

(a) 6 (b) 12 (c) 13 (d) 25

 # 7 Paragraph for Q. 22 to Q. 24

Consider the cubic $f(x)=8x^3+4ax^2+2bx+a$ where $a, b \in R$. **(code-V2T16PAQ1,2,3)**

22. For $a=1$ if $y=f(x)$ is strictly increasing $\forall x \in R$ then maximum range of value of b is

(a) $\left(-\infty, \frac{1}{3}\right]$ (b) $\left(\frac{1}{3}, \infty\right)$ (c) $\left[\frac{1}{3}, \infty\right)$ (d) $(-\infty, \infty)$

23. For $b=1$, if $y=f(x)$ is non monotonic then the sum of all the integral values of $a \in [1,100]$, is

(a) 4950 (b) 5049 (c) 5050 (d) 5047

24. If sum of the base 2 logarithms of the roots of the cubic $f(x)=0$ is 5 then the value of 'a' is

(a) – 64 (b) – 8 (c) – 128 (d) – 256

 # 8 Paragraph for Q. 25 to Q. 27

A trial consists of rolling a red die and a blue die the dice being fair. The result R of the trial is defined as the sum of the two numbers showing when the numbers on the red and the blue dice are the same but as the product of these two numbers when they are different.

25. The probability that ruslt of a throw is 12, is **(code-V2T20PAQ6,7,8)**

(a) $\frac{1}{12}$ (b) $\frac{1}{9}$ (c) $\frac{5}{36}$ (d) $\frac{1}{6}$

26. If $R \geq 15$, then $P(R \geq 20)$ is

(a) $\frac{3}{5}$ (b) $\frac{2}{5}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

27. If the result of two such throws are added then $P(R \geq 45)$

(a) $\frac{5}{108}$ (b) $\frac{5}{648}$ (c) $\frac{5}{162}$ (d) None

Assertion & Reason Type

In this section each que. contains STATEMENT-1 (Assertion) & STATEMENT-2(Reason).Each question has 4 choices (A), (B), (C) and (D), out of which **only one is correct.**

Bubble (A) STATEMENT-1 is true, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1.

Bubble (B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1.

Bubble (C) STATEMENT-1 is True, STATEMENT-2 is False.

Bubble (D) STATEMENT-1 is False, STATEMENT-2 is True.

Que. 1. Let $ax^2+bx+c=0, a \neq 0 (a,b,c \in R)$ has no real and $a+b+2c=2$. **(code-V1T6PAQ2)**

Statement - 1 : $ax^2+bx+c>0 \ \forall \ x \in R$.

because

Statement - 2 : $a+b$ is be positive.

Que. 2. Consider the following statements **(code-V1T12PAQ9)**

Statement - 1 : The equation $x^2+(2m+1)x+(2n+1)=0$ where m and n are integers can not have any rational roots.

because

Statement - 2 : The quantity $(2m+1)^2-4(2n+1)$ where $m, n \in I$ can never be a perfect square.

Que. 3. Statement - 1 : If x, y, z are 3 positive numbers in G.P. then $\left(\frac{x+y+z}{3}\right)\left(\frac{3xyz}{xy+yz+zx}\right)=(xyz)^{\frac{2}{3}}$.

because **(code-V1T14PAQ1)**

Statement - 2 : (Arithmetic mean) (Harmonic mean) = (Geometric mean)^2.

Que. 4. Statement-1 2, 4 and 8 are in G.P. and 6, 8, 12 are in H.P. **(code-V1T14PAQ2)**

because

Statement-2 If t_1, t_2 and t_3 are 3 distinct number in G. P. then $t_1+t_2, 2t_2$ and t_2+t_3 are always in H.P.

Que. 5. Statement - 1 : If $xy+yz+zx=1$ where $x, y, z \in R^+$ then **(code-V1T14PAQ3)**

$$\frac{x}{1+x^2}+\frac{y}{1+y^2}+\frac{z}{1+z^2}=\frac{2}{\sqrt{(1+x^2)(1+y^2)(1+z^2)}}$$

because

Statement - 2 : In a triangle ABC $\sum \sin 2A = 4\prod \sin A$.

Que. 6. Statement - 1: If $27abc \geq (a+b+c)^3$ and $3a+4b+5c=12$ then $\frac{1}{a^2}+\frac{1}{b^3}+\frac{1}{c^5}$ where a, b, c are positive real numbers. **(code-V1T16PAQ10)**

because

Statement - 2 : For positive real numbers A.M. $\geq$G.M.

Que. 7. Statement - 1 : If $f(x)$ is a quadratic polynomial stsfying $f(2)+f(4)=0$. If unity is a root of f(x) = 0 then the other root is 3.5. **(code-V1T16PAQ12)**

Statement - 2 : If $g(x)=px^2+qx+r=0$ has roots α, β then $\alpha+\beta=-q/p$ and $\alpha\beta=\frac{r}{p}$.

Que. 8. Statement - 1 : The difference between the sum of the first 100 even natural numbers and the sum of the first 100 odd natural numbers is 100 **(code-V1T20PAQ12)**

because

Statement - 2 : The difference between the sum of the first n even natural numbers and the sum of the first n odd natural number is n.

Que. 9. Statement - 1 : Number of ways in which 7 identical coins can be distributed in 15 presons, if each preson receiving atmost one coin is the same as number of ways in which 8 identical coins can be distributed in 15 presons in a similiar manner. **(code-V1T20PAQ10)**

because

Statement - 2 : ${}^{n}C_{r} = {}^{n}C_{n-r}$

Que. 10. Consider the function $f(x) = \left({}^{x+1}C_{2x-8}\right)\left({}^{2x-8}C_{x+1}\right)$ **(code-V2T1PAQ9)**

Statement - 1 : Domain of f(x) is singleton.

because

Statment - 2 : Rangle of f(x) is singleton.

Que. 11. **Statement - 1 :** If a > b > c and $a^3 + b^3 + c^3 = 3abc$ then the quadratic equation $ax^2 + bx + c = 0$ has roots of oposite sign. **(code-V2T6PAQ5)**

Statement - 2 : If roots of a quadratic equation $ax^2 + bx + c = 0$ are of oposite sign then product of roots < 0 and | sum of roots | ≥ 0

Que. 12. Let a sample space S contains n elements. Two events A and B are difined on S, and $B \neq \phi.$.

Statement 1: The conditional probability of the event A given B, is the ratio of the number of elements in AB divided by the number of elements in B. **(code-V2T11PAQ10)**

because

Statement 2: The conditional probability modle given B, is equally likely model on B.

Que. 13. Consider an A.P. with 'a' as the first term and 'd' is the common difference such that S_n denotes the sum to n terms and a_n denotes the n^{th} term of the A.P. **(code-V2T15PAQ4)**

Given that for some $m, n \in N \dfrac{S_m}{S_n} = \dfrac{m^2}{n^2} (m \neq n)$

Statement 1: $d = 2a$

because

Statement 2: $\dfrac{a_m}{a_n} = \dfrac{2m+1}{2n+1}$

Que. 14. Consider tow quadratic function $f(x) = ax^2 + ax + (a+b)$ and $g(x) = ax^2 + 3ax + 3a + b$, where a and b are non-zero real numbers having same sign. **(code-V2T15PAQ7)**

Statement 1: Graphs of both $y = f(x)$ and $y = g(x)$ either completely lie above x-axis or lie completely below x-axis $\forall\, x \in R$.

because

statement 2: If discriminant of $f(x), D < 0,$ then $y = f(x)$ is of same sign $\forall\, x \in R$ and $f(x+1)$ will also be of same sign as that of $f(x) \forall x \in R$.

Que. 15. Let $y=f(x)$ is a polynomial of degree odd (≥ 3) with real coefficients and (a,b) is any point

Statement 1: There always exists a line passing through (a,b) and touching the curve $y=f(x)$ at some point **(code-V2T18PAQ7)**

because

Statement 2: A polynomial of degree odd with real coefficients have atleast one real root.

More than One May Correct Type

Que. 1. If $a\in R$ then numbers of distinct real solution of $x^2-|x|+a=0$ can be :

(a) 1 (b) 2 (c) 3 (d) 4 **(code-V1T4PAQ11)**

Que. 2. If the sum to n terms of the series $27+24+21+18+$......... is equal to 126 then the value of n can be **(code-V1T4PAQ12)**

(a) 7 (b) 9 (c) 11 (d) 12

Que. 3. If $\log 2, \log(2^x-1)$ and $\log(2^x+3)$ are in A.P. , then **(code-V1T6PAQ7)**

(a) 2^x is rational (b) x is irrational (c) $(\sqrt{2})^x$ is irrational (d) 2^{x^2} is rational

Que. 4. If α,β are the roots of the quadratic equation $ax^2+bx+c=0$ then which of the following expression will be the symmetric function of roots ? **(code-V1T6PAQ8)**

(a) $\left|\ln\frac{\alpha}{\beta}\right|$ (b) $\alpha^2\beta^5+\beta^2\alpha^2$ (c) $\tan(\alpha-\beta)$ (d) $\left(\ln\frac{1}{\alpha}\right)^2+(\ln\beta)^2$

Que. 5. If the quadratic equation $ax^2+bx+c=0 (a>0)$ has $\sec^2\theta$ and $\operatorname{cosec}^2\theta$ as its roots then which of the following must hold good ? **(code-V1T6PAQ10)**

(a) $b+c=0$ (b) $b^2-4ac\geq 0$ (c) $c\geq 4a$ (d) $4a+b\geq 0$

Que. 6. If one of the root of the equation $4x^2-15x+4p=0$ is the square of the other the the value of p is

(a) 125/64 (b) – 27/8 (c) $-125/8$ (d) 27/8 **(code-V1T6PAQ11)**

Que. 7. If x satisfies the inequality $\log_{(x+3)}(x^2-x)<1$ then **(code-V1T12PAQ14)**

(a) $x\in(-3,-2)$ (b) $x\in(-1,3]$ (c) $x\in(1,3)$ (d) $x\in(-1,0)$

Que. 8. If the roots of the equation, $x^3+px^2+qx-1=0$ form an increasing G.P. where p and q are real, then

(a) $p+q=0$ **(code-V1T14PAQ12)**

(b) $p\in(-3,\infty)$

(c) one of the roots is unity

(d) one root is smaller than 1 and one root is greater than 1

Que. 9. If the triplets $\log a, \log b, \log c$ and $(\log a-\log 2b), (\log 2b, \log 3c), (\log 3c-\log a)$ are in arithmetic progression then **(code-V1T15PAQ15)**

(a) $18(a+b+c)^2=18(a^2+b^2+c^2)+ab$ (b) a,b,c are in G.P.

(c) $a,2b,3c$ are in H.P. (d) a,b,c can be the lengths of the sides of a triangle

(Assume all logratithmic to be defined)

Que. 10. With usual notation in triangle ABC if $e^{\cos^2\frac{A}{2}}, e^{\cos^2\frac{B}{2}}, e^{\cos^2\frac{C}{2}}$ are in geometric progression. Which of the following statements are correct ? **(code-V1T15PAQ16)**

(a) $\cos A, \cos B, \cos C$ are in A.P. (b) $s-a, s-b, s-c$ are in H.P.

(c) r_1, r_2, r_3 are in A.P. (d) $\cos\frac{A}{2}, \cos\frac{B}{2}, \cos\frac{C}{2}$ are in H.P.

Que. 11. The graph of the quadratic trinomil $y = ax^2 + bx + c$ has its vertex at (4,–5) and two x-intercepts one positive and one negative. Whi;ch of the following holds good ? **(code-V1T19PAQ20)**

(a) $a > 0$ (b) $b < 0$ (c) $c < 0$ (d) $8a = b$

Que. 12. If S_n denotes the sum of first n terms of an Arithmetic progression and a_n denotes the n^{th} term of the same A.P. Given $S_n = n^2 p;\ S_k = k^2 p;$ where $k, p, n \in N$ and $k \neq n$ then **(code-V1T19PAQ23)**

(a) $a_1 = p$ (b)common difference = 2p

(c) $S_p = p^3$ (d) $a_p = 2p^2 - p$

Que. 13. Thirteen presons are sitting in a row. Number of ways in which four persons can beselected so that no two of them are consecutive is also equal to **(code-V1T19PAQ22)**

(a) Number of ways in which the letters of the word MRINAL can be arranged if vowels are never separated.

(b) Number of numbers lying between 100 and 100 using only the digits 1,2,3,4,5,6,7 without repetition.

(c) The number ofways in which 4 alike cadburies chocklate can be distributed in 10 children each child getting atmost one.

(d) Number of triangle that can be formed by joining 12 points in plane of which 5 are collinear.

Que. 14. The number a, b,c in that order form a three term A.P. and $a+b+c=60$. The number $(a-2), b(c+3)$ in the order form a three term G.P. All possible values of $(a^2+b^2+c^2)$ is/are

(a) 1218 (b) 1208 (c) 1288 (d) 1298 **(code-V2T4PAQ10)**

Que. 15. $\sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{k=0}^{n}\binom{n}{i}\binom{n}{j}\binom{n}{k}$, $\binom{n}{r} = {}^nC_r$ **(code-V2T6PAQ6)**

(a) is less than 500 if n = 3 (b) is greater than 600 if n = 3

(c) is less than 500 in n = 4 (d) is greater than 400 if n = 4

Que. 16. Number of ways in which n distinct things can be distributed to 3 children if each reciving none, one or more number of things, is NOT equal **(code-V2T6PAQ7)**

(a) The number of ways of all possible selections of one or more questions from n given questions, each question having an alternative.

(b) the sum of all the coefficients in the expansion of the binomial $(2p + q)^n$.

(c) Number of n digit number (containing at least one odd digit) that can be written, if each digit of the number selected from the set {1, 2, 3, 4, 5, 6}.

(d) Number of different signals that can be transmitted by making use of 3 different coloured flags keeping one above the other, if n different flages are available.

Que. 17. Consider the bonomial expansion of $\left(\sqrt{x}+\frac{1}{2.\sqrt[4]{x}}\right)^n, n \in N.$ where the terms of the expansion are written in decreasing powers of x. If the coefficients of the first three terms form an arithmetic progression then the statement(s) which hold good is/are **(code-V2T10PAQ9)**

(a) total number of terms in the expansion of the binomial is 8

(b) number of terms in the expansion with integral power of x is 3

(c) there is no term in the expansion which is independent of x

(d) fourth and fifth are the middle terms of the expansion.

Que. 18. For $P(A)=\frac{3}{8}; P(B)=\frac{1}{2}; P(A\cup B)=\frac{5}{8}$ which of the following do/does hold good ?

(a) $P(A^c/B)=2P(A/B^c)$ (b) $P(B)=P(A/B)$ **(code-V2T11PAQ12)**

(c) $15\,P(A^c/B^c)=8P(B/A^c)$ (d) $P(A/B^c)=P(A\cap B)$

Que. 19. Which of the following statement(s) is/are correct ? **(code-V2T11PAQ14)**

(a) 3 coins are tossed once. Two of them atleast must land the same way. No mater whether they land hends or tails, the third coin is equally likely to land either the same way or oppositely. So, the chance that all the three coins land the same way is 1/2.

(b) Let $0<P(B)<1$ and $P(A/B)=P(A/B^c)$ then A and B are independent.

(c) Suppose an urn contains 'w' white and 'b' black balls and a ball is drawn from it and is replaced along with 'd' additional balls of the same colour. Now a second ball is drawn from it. The probability that the second drawn ball is white is independent of the value of 'd'.

(d) A,B,C simultaneously satisfy $P(ABC)=P(A).P(B).P(C)$ and $P(AB\bar{C})=P(A).P(B).P(\bar{C})$ and $P(A\bar{B}C)=P(A).P(\bar{B}).P(C)$ and $P(\bar{A}BC)=P(\bar{A}).P(B).P(C)$ then A, B, C are independent.

Que. 20. Let $a>2$ be aconstant. If there are just 18 positive integers satisfying the inequality $(x-a)(x-2a)(x-a^2)<0$ then which of the option(s) is/are correct ? **(code-V2T12PAQ12)**

(a) 'a' is composite (b) 'a' is odd

(c) 'a' is greater than 8 (d) 'a' lies in the interval (3,11)

Que. 21. If a and b are distinct positive integers and the quadratic equation $(a-1)x^2-(a^2+2)x+(a^2+2a)=0$ and $(b-1)x^2-(b^2+2)x+(b^2+2b)=0$ have a common root. Then which of the following can be Ture ?

(a) $a^2+b^2=45$ (b) $a=2b$ (c) $b=2a$ (d) $ab=18$ **(code-V2T15PAQ13)**

Que. 22. Let $(\log_2 x)^2-4\log_2 x-m^2-2m-13=0$ be an equation in x and $m\in R$, the which of the following must be correct ? **(code-V2T18PAQ9)**

(a) For any $m\in R$, the equation has two distinct solution.

(b) The product of the solution of the equation does not depend on m.

(c) One of the solution of the equation is less than 1 while the other is greater than 1 for $\forall m\in R$.

(d) The minimum value of the larger solution is 2^6 and maximum value of the smaller solution is 2^{-2}.

Que. 23. Let $(a-1)\left(x^2+\sqrt{3}x+1\right)^2-(a+1)\left(x^4-x^2+1\right)\le \forall\, x\in R$, then which of the following is/are correct ?

(a) $a\in\left[-\frac{1}{\sqrt{3}},\frac{4}{\sqrt{3}}\right]$ (b) Largest possible value of a is $\sqrt{3}$ **(code-V2T19PAQ11)**

(c) Number of possible integral values of a is 3 (d) Sum of all possible integral values pf a is '0'

Que. 24. A and B are tow events. Suppose **(code-V2T20PAQ9)**

A: It rains today with $P(A)=40\%$ B : It rains tomorrow with $P(B)=50\%$

Also P(It rains today and tomorrow) = 30% Also $E_1 : P\left((A\cap B)/(A\cup B)\right)$ and $E_2 : P\left(\left\{\left(A\cap\overline{B}\right)\text{or}\left(B\cap\overline{B}\right)\right\}/(A\cap B)\right)$ then which of the following is/are true ?

(a) A and B are independent (b) $P(A/B)<P(B/A)$

(c) E_1 and E_2 are equiprobable (d) $P(A/A\cup B)=P(B/A\cup B)$

Que. 25. Two whole numbers are tandomly selected and multiplied. Consider two events E_1 and E_2 difined as

E_1 : Their product is divisible by 5 E_2 : Unit's place in their product is 5. **(code-V2T20PAQ10)**

Whose of the following statement(s) is/are correct ?

(a) E_1 is twice as likely to occur as E_2. (b) E_1 and E_2 are disjoint

(c) $P(E_2/E_1)=1/4$ (d) $P(E_1/E_2)=1$

Que. 26. Probability of n hends in 2n losses of a fair can be given by **(code-V2T20PAQ11)**

(a) $\prod_{r=1}^{n}\left(\frac{2r-1}{2r}\right)$ (b) $\prod_{r=1}^{n}\left(\frac{n+r}{2r}\right)$ (c) $\prod_{r=0}^{n}\left(\frac{{}^nC_r}{2^n}\right)^2$ (d) $\frac{\sum_{r=0}^{n}\left({}^nC_r\right)^2}{\left(\sum_{r=0}^{n}{}^nC_r\right)}$

Que. 27. Two fair dice are rolled and two evaents A and B are difined as follows **(code-V2T20PAQ12)**

A : Sum of hte points shown on the faces is odd

B : At least one of the dice shows up the face 3. Which of the following options are correct ?

(a) $P(A+B)=\frac{23}{36}$ (b) $P(A-B)=\frac{12}{36}$

(c) $P\left((A\cap B)\cup\overline{A}\right)=\frac{24}{36}$ (d) $P\left(A^o\cup B^o\right)-P\left(A^o\cup B^o\right)=\frac{17}{36}$

Que. 28. Which of the following statements is/are True ? **(code-V2T20PAQ13)**

(a) A fair coin is tossed n times where n is a positive integer. The probability that n^{th} toss rusults in head is 1/2.

(b) The conditional probability that the n^{th} toss result in head given that first (n–1) tosses rusult in head is $1/2^n$.

(c) Let E and F be the events such that F is neither impossible nor sure. If $P(E/F)>P(E)$ then $P\left(E/F^o\right)>P(E)$

(d) If A,B and C are independent then the events $(A\cup B)$ and C are independent.

Match Matrix Type

Que. 1. **Column - I** **(code-V1T8PBQ1)** **Column - II**

A. The harmonic mean of the roots of the equation $\left(5+\sqrt{2}\right)x^2-\left(4+\sqrt{5}\right)x+8+2\sqrt{5}=0$ is — **P.** 2.

B. Let $a_1, a_2, \ldots\ldots, a_{10}$, be in A.P. and $h_1, h_2, \ldots\ldots, h_2, \ldots\ldots h_{10}$ be in H.P. If $a_1 = h_1 = 2$ & $a_{10} = h_{10} = 3$ then a_4h_7 is — **Q.** 3.

C. The number of interger values of m, for which the x coordinate of the point of intersection of the lines $3x+4y=9$ and $y=mx+1$ is also an integer, is — **R.** 4.

D. Between 2 and 5 six geometric means are inserted. If their product can be expressed as $(10)^n$ then the value of n equals — **S.** 6.

Que. 2 **Column - I** **(code-V1T16PBQ2)** **Column - II**

A. If $\log_2 x + 4\log_4 y = 4 - 6\log_8 z$, then $(x+y+z)$ can not be equal to — **P.** 2

B. If $\left(3^{|\sin x|}\right)\left(2^{-|\sec y|}\right)+5\cos z = a$, where $x,, y, z \in R$ and $y \neq (2n+1)\frac{\pi}{2}, n \in I,$ then possible value(s) of 'a' can be — **Q.** 3.

C. In $\Delta ABC, \operatorname{cosec} A, \operatorname{cosec} B, \operatorname{cosec} C$ are in H.P., then possible integral values of $\frac{2b}{c}$ (where a,b,c denote the sides of ΔABC as in usual notation), can be — **R.** 4

D. Let $x^2-3x+p=0$ has two positi ew roots 'a' and 'b', then value of $\left(\frac{4}{b}+\frac{1}{b}\right)$ can not be equal to (You may use the fact that HM $\leq$ AM for 3 positive numbers) — **S.** 5

Que. 3. **Column - I** **(code-V2T11PBQ1)** **Column - II**

A. Two different numbers are taken from the set $\{1,2,3,4,5,6,7,8,9,10\}$. The probability that their sum and positive difference, are both multiple of 4, is x/55 then x equals — **P.** 4.

B. There are two red, two blue, two white and certain number (greater than 0) of green socks in a drawer. If two socks are taken are taken at random from the drawer without replacement, the probability that they are of the same colour is 1/5 then the number of green socks are — **Q.** 6.

C. A drawer contains a mixture of red socks and blue socks, at most 17 in all. It so happens that when two socks are selected randomly without replacement, there is a probability of exactly 1/2 that both are red or both are blue. The largest possible number of red socks in the drawer that is consistent with this data, is — **R.** 8. **S.** 10.

Que. 4. **Column - I** **(code-V2T20PBQ1)** **Column - II**

A. The probability of a bomb hitting a bridge is 1/2. Two direct hits are needed to destroy it. The least number of bombs required so that the probability of the bridge being destroyed is greater than 0.9, is — **P.** 4.

B. A bag contains 2 red, 3 white and 5 black balls, a ball is drawn its colour is noted and repaced. Minimum number of times, a ball must be drawn so that the probility of getting a red ball for thefirst time is a least even, is — **Q.** 5.

C. A hunter knows that a deer is hidden in one of the two near by bushes, the probability of its being hiddenin bush - I being 4/5. Thehunter having a rifle containing 10 bullets decides to fire them all at bush-I or II. It is known that each shot may hit one of hte two bushes, independently of the other with probability 1/2. Number of bullets must he fire on bush - I to hit the animal with maximum probability is (Assume that the bullet hitting the bush also hits the animal). — **R.** 6.

S. 7.

Que. 5. An uren contains four black and eitht white balls. Three balls are drawn from the urn without replacement. Three events are difined on this expreiement **(code-V2T20PBQ2)**

A : Exactly one black ball is drawn

B : All balls are drawn are of the same colour.

C : 3^{rd} drawn ball is black.

Match the entries of **column - I** with none, one or more entries of **column - II.**

Column - I		**Column - II**	
A.	The events A and B are	**P.**	Mutually exclusive
B.	The events B anc C are	**Q.**	Idependent
C.	The events C are A are	**R.**	Nither independent nor mutually exclusive
D.	The A, B and C	**S.**	Exhaustive

Subjective Type (Up to 4 digit)

Que. 1. Let r be a real number such that $\sqrt[3]{r}-\frac{1}{\sqrt[3]{r}}=2$, find the value of $r^3-\frac{1}{r^3}$. **(code-V1T2PDQ1)**

Que. 2. A quadratic equation is formed with rational coefficients whose one root α is given by $\frac{\sum_{r=1}^{8}\sin(5r)^o}{\sum_{r=1}^{8}\cos(5r)^o}$.

If the quadratic equation is expressed as $f(x)=x^2+bx+c=0,$ find f (50). **(code-V1T3PAQ1)**

Que. 3. The roots of the equation $x^3-12x^2+39x-28=0,$ are the first three consecutive terms of an arithmetic progression. Find the sum of n terms of the A.P. **(code-V1T3PAQ3)**

Que. 4. Let 'A' denotes the value of the expression $2x^4-x^3-19x^2-2x+35$ when $x=4\cos 36^o$. and 'B' denotes the value of $\alpha^{-2}+\beta^{-2}+\gamma^{-2}$ where α,β,γ are the roots of the cubic $x^3-x^2+8x-2=0.$ Find the value of (AB). **(code-V1T3PAQ5)**

Que. 5. If a_1, a_2, a_3............ is an arthmetic pogression with common difference 1 and $a_1+a_2+a_3$............$+a_{98}=137$ then find the value of $a_2+a_4+a_6+$..........$+a_{98}$. **(code-V1T4PDQ1)**

Que. 6. Find the value of x satisfying the equations. **(code-V1T9PAQ2)**

$$\log^2 x^3 - 20\log\sqrt{x} + 1 = 0 \text{ and } \log(x(x-9)) + \log\left(\frac{x-9}{x}\right) = 0 \text{ (Base to the logarithm is 10)}$$

Que. 7. Find all real solution(s) of the equation $2^{x+2}.5^{6-x} = 10^{x^2}$. **(code-V1T11PAQ2)**

Que. 8. Let $'\sigma'$ denotes the sum of the infinite series $\sum_{n=1}^{\infty}\left(\frac{n^2+2n+3}{2^n}\right)$. Compute the vlaue of $\left(1^3+2^3+3^3+........+\sigma^3\right)$. **(code-V1T15PDQ2)**

Que. 9. If the cubic equation $x^3+px^2+qx+r=0$ where $p,q,r \in R$ has root a^2, b^2, c^2 satisfying $a^2+b^2=c^2$, then the value of $\frac{p^3+8r}{pq}$ is equal to λ. Find the vlaue of λ^5. **(code-V1T15PDQ3)**

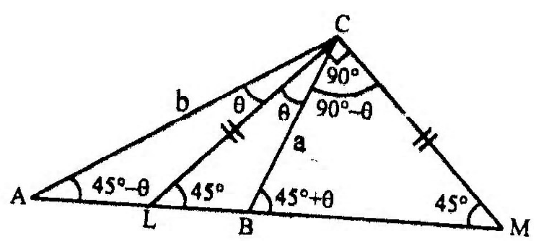
Que. 10. Let the equation $x^4-16x^3+px^2-256x+q=0$ has 4 positive real roots in G.P., then find (p+q). **(code-V1T18PDQ1)**

Que. 11. Compute the sum of the series $(20)^3-(19)^3+(18)^3-(17)^3+........+2^3-1^3$. **(code-V1T18PDQ2)**

Que. 12 . If the sum of all solution of the equation $\left(x^{\log_{10}3}\right)^2-\left(3^{\log_{10}x}\right)-2=0$ is $\left(a^{\log_b c}\right)$ where b and c are relatively prime and $a,b,c,\in N$. Find the value of $(a+b+c)$. **(code-V2T1PDQ1)**

Que. 13. If $x+y+z=12$ and $x^2+y^2+z^2=96$ and $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=36$. Find the value of $\left(x^3+y^3+z^3\right)$. **(code-V2T1PDQ3)**

Que. 14. How many ways in which 8 people can be arranged in a line If A and B must be next each other and C must b somewhere behind D. **(code-V2T1PDQ4)**



Que. 15. Let S denotes the sum of an infinite geometric progression whose first term is the value of the function $f(x)=\frac{\sin(x-(\pi/6))}{\sqrt{3}-2\cos x}$ at $x=\pi/6$, if f(x) is continuous at $x=\pi/6$ and whose common ratio is the limiting value of the function $g(x)=\frac{\sin(x)^{1/3}\ell n(1+3x)}{\left(\arctan\sqrt{x}\right)^2\left(e^{5x^{1/3}}-1\right)}$ as $x\to 0$. Find the value of (2008)S.

(code-V2T2PDQ2)

Que. 16. In the quadratic equation $A\left(\sqrt{3}-\sqrt{2}\right)x^2+\frac{B}{\left(\sqrt{3}+\sqrt{2}\right)}x+C=0$ with α,β as its roots.

If $A=\left(49+20\sqrt{6}\right)^{\frac{1}{4}}$; B = sum of the infinite G.P. as $8\sqrt{3}+\frac{8\sqrt{6}}{\sqrt{3}}+\frac{16}{\sqrt{3}}+..........\infty$ **(code-V2T10PDQ1)**

and $|\alpha-\beta|=\left(6\sqrt{6}\right)^{k}$ where $k=\log_6 10-2\log_6\sqrt{5}+\log_6\sqrt{(\log_6 18+\log_6 72)}$, then find the vlaue of C.

Que. 17. During a power blackout, 100 presons are arrested on suspect of looting. Each is given a polygraph test. From past experience it is know that the polygraph is 90% reliable when administered to a guilty person and 98% reliable when given to some one who is innocent. Suppose that of the100 presons taken into custody, only 12 were actually involved in any wrong doing. If the probadility that a given suspect in innocent given that the photograph says he is guilty is a/b where a and b are relatively prime, find the value of (a+b). **(code-V2T11PBQ2)**

Que. 18. A match between two players A and B is won by the player who first wins two games. A's chance of winning drawing or losing any particular games are 1/2, 1/6 or 1/3 respectively. If the probability of A's winning the match can be expressed in the form p/q, find $(p+q)$ **(code-V2T11PBQ3)**

Que. 19. Let $f(x)=x^3+x+1$. Suppose P(x) is a cubic polynomial such that $P(0)=-1$ and the roots of P(x) = 0 are the squares of the roots of f(x). Find the value of 50 P(4). **(code-V2T17PDQ2)**

Que. 20. If the integers a, b, c,d are in arithmetic progression and $a<b<c<d$ and $d=a^2+b^2+c^2$ then find the value of $(a+10b+100c+1000d)$. **(code-V2T18PDQ1)**

Que. 21. There are 4 urns. The first urn contains 1 white & 1 balck ball, the second urn contains 2 white & 3 black balls, the third urn contains 3 white & 5 black balls & the fourth urn contains 4 white % 7 black balls. The selection of each urn is not equally likely. The probability of selecting i^{th} urn is $\frac{i^2+1}{34}$ $(i=1,2,3,4)$. If we randomly select one of the urns & draw a ball, then the probability of ball being white is p/q where $q\in N$ are in their lowest form. Find $(p+q)$. **(code-V2T20PDQ1)**

Que. 22. A doctor is called to see a sick child. The doctor knows (Prior to the visit) that 90% of the sick children in that neighbourhood are sick with the flu, denoted by F, while 10% are sick with the measles, denoted by M. **(code-V2T20PDQ2)**

A well known symptom of measles is a rash, denoted by R. Theprobability of having a rash for a chlid sick with the measles is 0.95. However, occasionally children with the flu also develop a rash, with conditional probability 0.08. **(code-V2T20PDQ3)**

Upon examination the child, the dector finds a rash. What is theprobability that the child has the measles ? If the probability can be expressed in the form of p/q where $p,q\in N$ and are in their lowest form, find $(p+q)$. **(code-V2T20PDQ4)**

[SOLUTION]

Single Correct Type

Que. 1. (A)

$\left(a^{\log_2 x}\right)^2 = 5 + 4a^{\log_2 x}$

$t^2 - 4t - 5 = 0 \Rightarrow (t-5)(t+1) = 0 \Rightarrow t = 5$ or $t = -1$ (rejected)

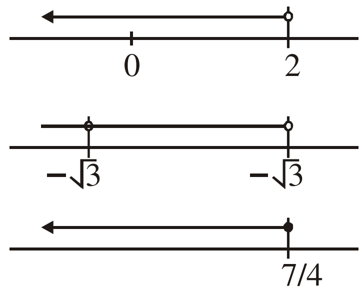
$\therefore \quad a^{\log_2 x} = 5 \Rightarrow x^{\log_2 a} = 5 \Rightarrow x = 5^{\log_a 2}$

Que. 2. (D) Sum < 0; product > 0 and $D \geq 0$

$m - 2 < 0 \Rightarrow m < 2 \Rightarrow m^2 - 3 > 0 \Rightarrow m > \sqrt{3}$ or $m < -\sqrt{3}$ and $4(m-2)^2 - 4(m^2-3) \geq 0$

$4 - 4m + 3 \geq 0;\ m \leq \frac{7}{4}$

$\Rightarrow \quad m \in \left(-\infty, -\sqrt{3}\right) \cup \left(\sqrt{3}, \frac{7}{4}\right]$



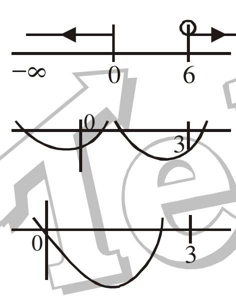
Que. 3. (B) $S = 2[1+2+3+.........+50] = 2.\frac{50.51}{2} = 2550$

Que. 4. (c) $10^{\frac{n(n+1)}{22}} > 10^5 \Rightarrow \frac{n(n+1)}{22} > 5 \Rightarrow n^2 + n > 110 \Rightarrow (n+11)(n-10) > 0 \Rightarrow n > 10 \Rightarrow n = 11$

Que. 5. **(B)** $a = (x+1)^2 + 2 = 2 \Rightarrow a_{min} = 2; \quad b = \frac{1/4}{1-(1/2)} = \frac{1}{2}$

$\therefore \quad \sum_{r=0}^{n} a^r b^{n-r} = \sum_{r=0}^{n} 2^r \left(\frac{1}{2}\right)^{n-r} = \frac{1}{2^n}\sum_{r=0}^{n} 4^r = \frac{1}{2^n}\left(1+4+4^2........+4^n\right) = \frac{1}{2^n}\left(\frac{4^{n+1}-1}{3}\right)$

Que. 6. **(B)** $f(0).f(3) < 0$ chek end points separately



Que. 7. **(A)** $D = b^2 - 4a < 0 \Rightarrow a > 0$ mouth opens upwards $\Rightarrow f(-1) > 0$

Que. 8. **(D)** Think. sum 4.

Que. 9. (C) $6x^2+2ax+2=0$ and $6x^2+3bx+3=0$

subtracting, $x(2a-3b)-1=0 \Rightarrow x=\dfrac{1}{2a-3b}$ (put in any equation)

$\therefore \quad \dfrac{2}{(2a-3b)^2}+\dfrac{b}{2a-3b}+1=0 \quad 2+b(2a-3b)+(2a-3b)^2=0 \Rightarrow 2+\left(2ab-3b^2\right)+4a^2+9b^2-12ab=0$

Or $\quad 4a^2+6b^2-10ab+2=0 \qquad 2a^2-5ab+3b^2=-1.$

Que. 10. (c)

Que. 11. (d) $\log_3\left(3^x-8\right)+x-2=0; \ \log_3\left(3^x-8\right)=2-x; \qquad 3^x-8=\dfrac{3^2}{3^x}$

let $3^x=t \quad t^2-8t-9=0 \quad t=9,-1 \Rightarrow 3^x \Rightarrow 9 \Rightarrow x=2$

$r=\cos\left(\dfrac{2005\pi}{3}\right)=\cos\left(668\pi+\dfrac{\pi}{3}\right)=\dfrac{1}{2}; \qquad S=\dfrac{x}{1-r}=\dfrac{2}{1-1/2}=\dfrac{2}{1/2}=4$

Que. 12. (b) $\sqrt{\log_2 x-1}-\dfrac{3}{2}\log_2 x+2>0 \quad (x>0) \Rightarrow \sqrt{\log_2 x-1}-\dfrac{3}{2}(\log_2 x-1)+\dfrac{1}{2}>0 \Rightarrow \left(2=\dfrac{3}{2}+\dfrac{1}{2}\right)$

let $\sqrt{\log_2 x-1}=t\geq 0$ ......(1) $\Rightarrow \log_2 x\geq 1 \Rightarrow x\geq 2 \quad \therefore \quad t-\dfrac{3}{2}t^2+\dfrac{1}{2}>0$

$\Rightarrow \quad 2t-3t^2+1>0 \Rightarrow \quad 3t^2-2t-1>0 \Rightarrow \quad -1/3<t<1$ ....(2) form (1) and (2)

$0\leq\sqrt{\log_2 x-1}<1 \Rightarrow 0\leq \log_2 x-1<1 \Rightarrow 1\leq \log_2 x<2 \Rightarrow 2\leq x<4$

Que. 13. (c) Let the roots of the equation are a $ax^2+bx+c=0$ are α and β

$\alpha+\beta=\dfrac{1}{\alpha^2}+\dfrac{1}{\beta^2}=\dfrac{(\alpha+\beta)^2-2\alpha\beta}{\alpha^2\beta^2} \qquad (\alpha+\beta)(\alpha\beta)^2=\left(\alpha+\beta^2\right)-2\alpha\beta \qquad \left(-\dfrac{b}{a}\right)\left(\dfrac{c^2}{a^2}\right)=\dfrac{b^2}{a^2}-\dfrac{2c}{a}$

$-bc^2=ab^2-2a^2c \qquad \Rightarrow \qquad ab^2+bc^2=2a^2c \qquad \Rightarrow \qquad \dfrac{b}{c}+\dfrac{c}{a}=2\dfrac{a}{b}$

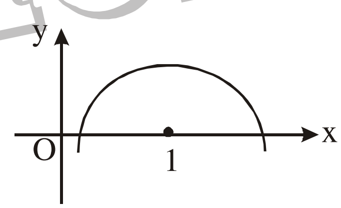
$\Rightarrow \qquad \dfrac{b}{c},\dfrac{a}{b},\dfrac{c}{a}$ are in A.P. $\Rightarrow \qquad \dfrac{c}{b},\dfrac{b}{a},\dfrac{a}{c}$ in H.P.

Que. 14. (d) $y=8x-x^2-15=(x-5)(3-x)\Rightarrow y<0\Rightarrow(x-5)(3-x)\Rightarrow(x-5)(x-3)>0 \quad \therefore \quad x>5$ or $x<3.$

Que. 15. (d) only $x=0$ is the solution. $x=7$ is to be rejected.

Que. 16. B. Note that $a<0$ hence $f(1)>0\Rightarrow 4p-p^2-5-2p+1+3p>0 \Rightarrow -p^2+5p-4>0$

$\Rightarrow p^2-5p+4<0 \Rightarrow (p-4)(p-1)<0 \Rightarrow 1<p<4 \Rightarrow p\in\{2,3\} \Rightarrow 2$ solutions $\Rightarrow 2$



Que. 17. C. $T_n = \dfrac{n}{(n^4+4n+4)-4n^2} = \dfrac{n}{(n^2+2)^2-(2n)^2} = \dfrac{n}{(n^2+2+2n)(n^2+2-2n)}$

$T_n = \dfrac{1}{4}\left[\dfrac{(n^2+2+2n)-(n^2-2n+2)}{(n^2+2+2n)(n^2-2n+2)}\right] = \dfrac{1}{4}\left[\dfrac{1}{(n-1)^2+1} - \dfrac{1}{(n+1)^2+1}\right]$ $\quad S_n = \sum_{n=1}^{\infty} T_n = \dfrac{3}{8}.$

Que. 18. C.

$$\begin{array}{l} S = 7+13+21+31+\ldots\ldots\ldots+T_n \\ S = \;\; +7 \;\; +13 \;\; +21+\ldots\ldots\ldots+T_{n-1}+T_n \\ \hline T_n = 7+6+8+10+\ldots\ldots\ldots\ldots+(T_n-T_{n-1}) \end{array}$$

$= 7+\dfrac{n-1}{2}\left[12+(n-2)2\right] = 7+\dfrac{n-1}{2}\left[6+n-2\right] = 7+(n-1)(n+4) = 7+n^2+3n-4 \Rightarrow T_n = n^2+3n+3$

$T_{70} = 4900+210+3 = 5113.$

Que. 19. C. $x>0, \dfrac{1}{2}\log_2 x - 2\left(\dfrac{\log_2 x}{2}\right)^2 + 1 > 0 \Rightarrow \log_2 x - (\log_2 x)^2 + 2 > 0 \Rightarrow (\log_2 x)^2 - \log_2 x - 2 < 0$

Let $\log_2 x = t \Rightarrow t^2 - t - 2 < 0 \Rightarrow (t-2)(t+1) < 0 \Rightarrow -1 < t < 2 \Rightarrow -1 < \log_2 x < 2 \Rightarrow \dfrac{1}{2} < x < 4$ hence number of integers $\{1,2,3\}$.

Que. 20. D. $f(n) = \dfrac{\log 3}{\log 2}.\dfrac{\log 4}{\log 3}\ldots\ldots\dfrac{\log n}{\log(n-1)} \Rightarrow f(n) = \dfrac{\log n}{\log 2} = \log_2(n) \quad \therefore \quad f(2^k) = \log_2(2^k) = k$

$\therefore \quad \sum_{k=2}^{100} f(2^k) = \sum_{k=2}^{100} k = 2+3+4+\ldots\ldots+100 = 5049$

Que. 21. D. A.P. is $a,(a+d),(a+2d)\ldots\ldots\ldots(a+98d)$ sum of odd terms $= 2550$

$\underbrace{a+(a+3d)2+(a+4d)\ldots\ldots\ldots(a+98d)}_{50 \text{ terms}} = 2550$

$\Rightarrow \dfrac{50}{2}\left[2a+98d\right] = 2550$ or $50\left[a+49d\right] = 2550$ or $a+49d = 51$ This is the 50th term of the A.P. Hence $S_{99} = 51\times 99 = 5049.$

Que. 22. D. $r+s+t=0, rst = -\dfrac{2008}{8}$ now let $r+s=A; s+t=B; t+r=C \therefore A+B+C = 2(r+s+t) = 0$

hence $A^3+B^3+C^3 = 3ABC \;\therefore\; (r+s)^3+(s+t)^3+(t+r)^3 = 3(r+s)(s+t)(t+r)$

$= 3(r+s+t-t)(s+t+r-r)(t+r+s-s) = 3\,rst$ (as $r+s+t=0$) $= 3\left(-\dfrac{2008}{8}\right) = 3(251) = 753.$

Alternatively : $(r+s)^3+(s+t)^3+(t+r)^3 = -(t^2+r^3+s^3)$ & now proceed

Que. 23. D. $10, a, b, ab$ are A.P. $\therefore$ $2a = 10 + b$ ......(1) also $2b = a(1+b)$ ....(2) substituting $a = \frac{10+b}{2}$

$$4b = (10+b)(1+b) \Rightarrow 4b = 10 + 11b + b^2 \Rightarrow b^2 + 7b + 10 = 0 \Rightarrow (b+5)(b+2) = 0 \Rightarrow b = -2 \text{ or } b = -5$$

$$\therefore\ a = 4 \quad \text{or} \quad 5/2 \quad \therefore \quad P = (-2)(-5)(4)\left(\frac{5}{2}\right) = 100.$$

Que. 24. C. Consider $\frac{1}{\ell}, \frac{2}{m}, \frac{3}{n}$ and use $AM \ge GM \Rightarrow \frac{1}{3}\left(\frac{1}{\ell} + \frac{2}{m} + \frac{3}{n}\right) \ge \left(\frac{1}{\ell}.\frac{2}{m}.\frac{3}{n}\right)^{1/3}$ or $\left(\frac{6}{\ell mn}\right)^{1/3}$

but $\ell mn = 48$ $\quad \therefore \frac{1}{3}\left(\frac{1}{\ell} + \frac{2}{m} + \frac{3}{n}\right) \ge \left(\frac{6}{48}\right)^{1/3} = \frac{1}{2}; \quad \therefore \left(\frac{1}{\ell} + \frac{2}{m} + \frac{3}{n}\right)_{min} = \frac{3}{2}.$

Que. 25. A. $a_1 (28)^{1/4}$; $a_2 = a_1 r$; $a_3 = a_1 r^2$ etc.; $b_1 = 1$; $b_2 = R$; $b_3 = R^2$ etc. where r and R thecommon ratio of the two G.P.'s and $R = \sqrt[4]{7} - \sqrt[4]{28} + 1$ now given $\sum_{n=1}^{\infty} \frac{1}{a_n} = \sum_{n=1}^{\infty} b_n$

$$\therefore \frac{1}{a_1}\left[1 + \frac{1}{r} + \frac{1}{r^2} + ...\infty\right] = \left(1 + R + R^2 + R^3 + ...\infty\right)$$

$$\frac{1}{a_1}\left[\frac{1}{1-(1/r)} = \frac{1}{1-R}\right]\left(R = b_2 = 7^{1/4}(28)^{1/4} + 1\right)$$

$$\frac{1}{(28)^{1/4}}\left(\frac{r}{r-1}\right) = \frac{1}{(28)^{1/4} - 7^{1/4}} \left(1 - R = (28)^{1/4} - (7)^{1/4}\right)$$

$$\therefore \frac{r}{r-1} = \frac{(28)^{1/4}}{(28)^{1/4} - 7^{1/4}} \Rightarrow \frac{r-1}{r} = \frac{(28)^{1/4} - 7^{1/4}}{(28)^{1/4}} \Rightarrow 1 - \frac{1}{r} = 1 - \left(\frac{1}{4}\right)^{1/4} \Rightarrow r = \sqrt{2}.$$

Que. 26. B. $X = \frac{1}{1-a}.\frac{1}{1-b} = \frac{1}{1-(a+b)+ab}$ where $a + b = \frac{4}{11}$ and $ab = -\frac{2}{11}$

$$\therefore \quad X = \left(1 - \frac{4}{11} - \frac{2}{11}\right)^{-1} = \left(1 - \frac{6}{11}\right)^{-1} \Rightarrow X = \frac{11}{5}$$

Roots are $\alpha, \beta = \frac{4 \pm \sqrt{16+88}}{22} = \frac{4 \pm \sqrt{104}}{22} = \frac{2 \pm \sqrt{26}}{11} |\alpha, \beta| < 1.$

for $Y : a = \log_b 5;\ r = \frac{4\log_b 5.\log_b 2}{\log_b 5} = 4\log_b 2$

$$\therefore \quad Y = \frac{\log_b 5}{1 - 4\log_b 2} = \frac{\log_b 5}{\log_b b - \log_b 16} = \frac{\log_b 5}{\log_b \frac{b}{16}} = \frac{\log_{2000} 5}{\log_{2000} 125} = \log_{125} 5 = \frac{1}{3} \Rightarrow Y = \frac{1}{3} \Rightarrow XY = \frac{11}{5}.\frac{1}{3} = \frac{11}{15}.$$

Que. 27. D. $\frac{n(n+1)(2n+1).2}{6.n(n+1)}$ must be an integer $\frac{2n+1}{3}$ must be an integer $\Rightarrow$ $(2n+1)$ is divisible by 3

$\Rightarrow$ $n \in 1, 4, 7, 10, ..., n$ is of the form of $(3k+1), k \ge 0, \in I.$

Que. 28.A. $T_n = \frac{1}{n^2+(n-2)} = \frac{1}{(n+2)(n-1)}, n = 3,4,5.. = \frac{1}{3}\left[\frac{1}{n-1} - \frac{1}{n+2}\right]$

$\therefore \quad S = \sum_{n=3}^{\infty} T_n = \frac{1}{3}\left(\frac{1}{2}-\frac{1}{5}\right) + \frac{1}{3}\left(\frac{1}{3}-\frac{1}{6}\right) + \frac{1}{3}\left(\frac{1}{4}-\frac{1}{7}\right) + \frac{1}{3}\left(\frac{1}{5}-\frac{1}{8}\right)$

$\Rightarrow S = \frac{1}{3}\left[\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right] = \frac{1}{3}\left[\frac{6+4+3}{12}\right] = \frac{13}{36}.$

Que. 29.A. $D_1 = a^2 - 4; \quad D_2 = a^2 + 36 > 0 \quad \Rightarrow \quad (A)$

Que. 30.D. $\sum_{k=1}^{\infty} \frac{k}{2^k} \sum_{n=1}^{\infty} \frac{1}{2^n} = \sum_{k=1}^{\infty} \frac{k}{2^k}\left[\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+......\infty\right] = \sum_{k=1}^{\infty} \frac{k}{2^k} = 2.$

Que. 31.D. General term $k!(k+1-1) \Rightarrow (k+1)! - k!$

$S = \sum_{k=1}^{10}\left((k+1)! - k!\right) = (2! - 1!) + (3! - 2!) + ... + (11! + 10!) = (11)! - 1.$

Que. 32. D. It is possible then $F(x) - f(x) = x^2 + x + 1 \quad \Rightarrow \quad$ **I**

qudratic equation can not have more then two solution $\Rightarrow$ II

If $F(x) - f(x)$ has one real solution $\Rightarrow$ $F(x) - f(x) = 0$ is a linear $\Rightarrow$ $A = a$ $\Rightarrow$ **III**.

Que. 33. C. $\alpha + \beta = 1154$ and $\alpha\beta = 1 \quad \Rightarrow \left(\sqrt{\alpha} + \sqrt{\beta}\right)^2 = \alpha + \beta + 2\sqrt{\alpha\beta} = 1154 + 2 = 1156 = (34)^2$

$\sqrt{\alpha} + \sqrt{\beta} = 34$ Again $\left(\alpha^{1/4} + \beta^{1/4}\right)^2 = \sqrt{\alpha} + \sqrt{\beta} + 2(\alpha\beta)^{1/4} = 34 + 2 = 36 \quad \Rightarrow \alpha^{1/4} + \beta^{1/4} = 6.$

Que. 34. B. x denotes the number of times he can take unit step and y denotes the number of times he can take 2 steps $\therefore \; x + y = 7$ Put $x = 1,3,5,7$ (why ? think !)

If $x = 1 \quad 1,222 \Rightarrow \frac{4!}{3!} = 4 \quad \Rightarrow x = 3 \quad 11122 \Rightarrow \frac{5!}{2!.3!} = 10$

$\Rightarrow 5 \quad 111112 \quad \Rightarrow {}^6C_1 = 6$

$x = 7 \quad 1111111 \quad \Rightarrow {}^7C_0 = \frac{1}{21}.$

Que. 35. B. Concept : Conefficient of x^r in $(1-x)^{-n}, n \in N$ is ${}^{n+r-1}C_r$.

Now given product is $\frac{1-x^{28}}{1-x} \cdot \left(\frac{1-x^{15}}{1-x}\right)^2 = \frac{\left(1-x^{28}\right)\left(1-x^{15}\right)^2}{(1-x)^3} = \frac{\left(1-x^{28}\right)\left(1-2x^{15}\right)}{(1-x)^3}$

$= \left(1-2x^{15}-x^{28}\right)(1-x)^{-3}$ Hence coefficient of $x^{28}\left(1-2x^5-x^{28}\right)(1-x)^{-3} - 2$. Coefficient of x^{13} in $(1-x)^{-3} - 1 = {}^{30}C_2 - 2.{}^{15}C_2 - 1 = 435 - 210 - 1 = 224.$

Que. 36. D. Consider $\int_0^1 (3ax^2 - 4bx + c)dx = a - 2b + c =$ zero as a, b, c are in A.P.

Hence f(x) = 0 must have atleast one root as f(x) is a quadratic equation.

Que. 37. C. $r\log_3\left(\frac{1}{x}\right)=k\pi,\quad k\in I;\qquad \log_3\left(\frac{1}{x}\right)=k \Rightarrow x=3^{-k}$ possible values of k are are

$=1,0,1,2,3,.... \Rightarrow S=(3+1)+\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+............\infty\right)=4+\frac{(1/3)}{1-(1/3)}=4+\frac{1}{2}=\frac{9}{2}.$

Que. 38. C. Let $S={}^{20}C_2+2.{}^{20}C_3+3.{}^{20}C_4+..............+19.{}^{20}C_{20}$ .... (1)

and . ${}^{20}C_0+{}^{20}C_1+......................+{}^{20}C_{20}\left(=2^{20}\right)$ on both sides of equation (1)

$S+2^{20}=\ {}^{20}C_2+2.{}^{20}C_3+3.{}^{20}C_4+................19.{}^{20}C_{20}$

$+{}^{20}C_0+{}^{20}C_1+{}^{20}C_2+{}^{20}C_3+{}^{20}C_4+..........+20.{}^{20}C_{20}$ ....(2)

Now $(1+x)^n=C_0+C_1x+C_2x^2+.........................+C_nx^n$

$n(1+x)^{n-1}=C_1+2.C_2x+3.C_2x^2+............................+20{}^{20}C_{20}$

Hence $S+2^{20}=1+20.2^{19} \Rightarrow s=1+20.2^{19}-2^{20}=1+10.2^{20}-2^{20}=1+9.2^{20}.$

Que. 39. A. Number of digfits are 9 select 2 places for the digit 1 and 2 in ${}^9C_2$ ways from the remaining 7 places select any two places for 3 and 4 in ${}^7C_2$ ways and from the remaining 5 places select any two for 5 and 6 in ${}^5C_2$ ways now, theremaining 3 digits can be filled in 3! ways

$\therefore$ Total ways $={}^9C_2.{}^7C_2.{}^5C_2.3!=\frac{9!}{2!.7!}.\frac{7!}{2!.5!}.\frac{5!}{2!.3!}.3!=\frac{9!}{8}=\frac{9.8.7!}{8}=9.7!.$

Que. 40. A. 1 2 3 4 5 6 7 8 9 10 1st drawn is 5 then 2nd drawn can be 1 only. If 1st is 6 then 2nd is 1 or 2

$P(E)=\frac{1}{10}\left[\frac{1}{9}+\frac{2}{9}+\frac{3}{9}+\frac{4}{9}+\frac{5}{9}+\frac{6}{9}\right]=\frac{1}{90}\left[\frac{6.7}{2}\right]=\frac{7}{30}.$

Que. 41. B. $P(E)=\frac{1}{2}; P(F_k)={}^nC_k.\frac{1}{2^n} \Rightarrow P(E\cap F_k)=\frac{1}{2}.{}^{n-1}C_{k-1}\left(\frac{1}{2}\right)^{n-1} \therefore P(E\cap F_k)=P(E).P(F_k)$

${}^{n-1}C_{k-1}.\frac{1}{2^n}=\frac{1}{2}.{}^nC_k.\frac{1}{2^n} \Rightarrow 2.{}^{n-1}C_{k-1}={}^nC_k \Rightarrow n=2k.$

Que. 42. D. In the 1st case Urn < 3R, n white

P(they match) $\frac{{}^3C_2+{}^cC_2}{{}^{n+3}C_2}=\frac{1}{2};\ \frac{6+n(n-1)}{(n+3)(n+2)}=\frac{1}{2} \Rightarrow 2\left(n^2-n+6\right)=n^2+5n+6$

$\Rightarrow\ n^2-7n+6=0 \Rightarrow n+1$ or 6 ....(1) In the 2nd case

$\frac{3}{n+3}.\frac{3}{n+3}+\frac{n}{n+3}.\frac{n}{n+3}=\frac{5}{8}$ solving $n^2-10n+9=0 \Rightarrow n=9$ or 1 .... (2)

from (1) and (2) $\Rightarrow$ $n=1.$

Que. 43. A. $a_k = (k^2+1)k! = (k(k+1)-(k-1))K! = k(k+1)! - (k-1)k!$

so $k(k+1)! - (k-1)k! \Rightarrow a_1 = 1.2! - 0 \Rightarrow a_2 = 2.3! - 1.2! \Rightarrow a_3 = 3.4! - 2.3! \ldots a_k = k(k+1)! - (k-1)k!$

$a_1 + a_2 + ... + a_k = k(k+1)!$ hence $b_k = k(k+1)!$ $\quad \therefore \quad \frac{a_k}{b_k} = \frac{(k^2+1)k!}{k(k+1)!} = \frac{(k^2+1)}{k(k+1)} = \frac{k^2+1}{k^2+k}$

$\frac{a_{100}}{b_{100}} = \frac{10001}{10100} = \frac{m}{n}; \quad \therefore \quad (n-m) = 99.$

Que. 44. B. $m = 5.5.8.7 = 1400 \Rightarrow n = 1400 - (5.8.7) = 1400 - 280 = 1120 \Rightarrow k = \frac{1400}{1120} = \frac{5}{4}$

Que. 45. D. $S = 1+4+36+576+.........+(2008!)^2 = 617+$ all other terms and in zero hence digit at the unit place is 7.

Que. 46. B. $k \in \left(\frac{5}{3}, \infty\right) \Rightarrow \text{sum}\left[\{2,3,4,.........100\} = 5050 - 1 = 5049\right]$

Que. 47. B. $S = 1 + 6(2^2+3^2+4^2+...........+n^2) + 1 - 4(2+3+4+......+n) \Rightarrow 6\sum_{r=1}^{n} r^2 - 4\sum_{r=1}^{n} r = 140$

$n(n+1)(2n+1) - 2n(n+1) = 140 \Rightarrow n(n+1)(2n-1) = 4.5.7 \Rightarrow n = 4.$

Que. 48. A. $\underbrace{6^6}_{\text{total}} - 6!$ (all six different)

Que. 49. B. $\boxed{A_1A_2}\ \boxed{B_1B_2}$................$\boxed{L_1L_2}$ Number of ways in a circel $(11)!2^{12}$

Que. 50. A. $b^2x^2 + (b^2+c^2-a^2)x + c^2 = 0$ Note: $b^2+c^2-a^2 = 2bc\cos A$ (From cosine reul)

Let $f(x) = b^2x^2 + (2bc\cos A)x + c^2 = 0$ also $A \in (0,\pi)$ in a triangle $\cos A \in (-1,1)$

$\Rightarrow 2bc\cos A \in (-2bc, 2bc)$

$\Rightarrow D = (2bc\cos A)^2 - 4b^2c^2 = 4b^2c^2\underbrace{(\cos^2 A - 1)}_{-ve} \Rightarrow D < 0 \Rightarrow$ **A** is correct.

Que. 51. D. ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

Que. 52. B.

$S_n = \frac{n(n+1)}{2}$ and $S_n - 1 = \frac{(n+2)(n-1)}{2} \therefore \frac{S_n}{S_n-1} = \frac{n(n+1)}{2}.\frac{2}{(n+2)(n-1)} \Rightarrow \frac{S_n}{S_n-1} = \left(\frac{n}{n-1}\right)\left(\frac{n+1}{n+2}\right)$

$P_n = \left(\frac{2}{1}.\frac{3}{2}.\frac{4}{3}.\frac{5}{4}.........\frac{n}{n-1}\right)\left(\frac{3}{4}.\frac{4}{5}.\frac{5}{6}.........\frac{n+1}{n+2}\right) \Rightarrow P_n = \left(\frac{n}{1}\right)\left(\frac{3}{n+2}\right) \Rightarrow \lim_{x\to\infty} P_n = 3.$

Que. 53. A. Select n boxes out of k in ${}^{k}C_n$ ways and put n objects in n! ways

$\therefore$ Totao ways ${}^{k}C_n.n! = {}^{k}P_n$

Que. 54. A. $\sin^2 x + \sin^4 x + \sin^6 x + ... + \infty = \dfrac{\sin^2 x}{1-\sin^2 x} = \tan^2 x$ $\therefore$ we have $e^{\tan^2 x . \ell n 2} = 2^{\tan^2 x}$ satisfies the eqution $(t-27)(t-1)=0$

$\therefore$ $3^{\tan^2 x} = 27$ or $3^{\tan^2 x} = 1 \Rightarrow \tan^2 x3$ as $\tan^2 x = 0$ (rejcted think!) $\tan x = \sqrt{3}$ or $x \in \left(0, \dfrac{\pi}{2}\right) \Rightarrow x = \dfrac{\pi}{3}$

Now $\dfrac{1}{\sin x + \cos x} = \dfrac{\sec x}{1+\tan x} = \dfrac{\sqrt{1+\tan^2 x}}{1+\tan x} = \dfrac{2}{\sqrt{3}+1} = \dfrac{2\left(\sqrt{3}-1\right)}{2} = \left(\sqrt{3}-1\right).$

Que. 55. A. Total $= {}^{7}C_2 . {}^{5}C_2 = 210 -$ number of squares

number of squares $= \underbrace{24}_{14\text{ units}} + \underbrace{15}_{2\text{ units}} + \underbrace{8}_{3\text{ units}} + \underbrace{3}_{4\text{ units}} = 50$

$\therefore$ required number $= 210 - 50 = 160$ Ans.

Que. 56. D $P(E_1) = \dfrac{1}{3}$; $P(E_2) = P(E_3) = \dfrac{1}{2}$; $P(E_4) = \dfrac{2}{3}$ Urn - A$\langle^{2R}_{1B}$ Urn - B$\langle^{1R}_{2B}$]

Que. 57. C when A and B are mutually exclusive then $P(A \cap B) = 0$

$\therefore$ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ ....(1)

$0.8 = 0.5 + p - \underbrace{P(A \cap B)}_{\text{zero}} \Rightarrow p = 0.3$

when A and B are independent $P(A \cap B) = P(A).P(B)$

again $0.8 = 0.5 + q - (0.5)q$ from(1)

$0.3 = \dfrac{q}{2} \Rightarrow q = 0.6$ ....(3)

Hence $2p = q$ Ans.]

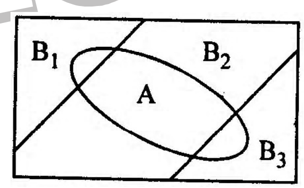
Que. 58. B A: card shows up black

B_1: It is the card with both side black

B_2 : card with both sides white

B_3 : card with one side white and one black

$P(B_1) = \dfrac{2}{10}$; $P(B_2) = \dfrac{3}{10}$; $P(B_3) = \dfrac{5}{10}$



$P(A/B_1)=1;\ \ P(A/B_2)=0;\ P(A/B_3)=\frac{1}{2}$ $\quad P(B_1/A)=\dfrac{\frac{2}{10}.1}{\frac{2}{10}.1+\frac{3}{10}.(0)+\frac{5}{10}.\frac{1}{2}}=\frac{4}{4+5}=\frac{4}{9}$

Que. 59. D A: exactly one ace ; B : both aces ; $E: A\cup B$

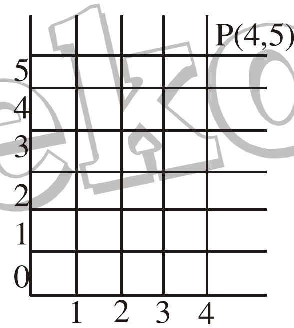
$P(B/A\cup B)=\dfrac{{}^4C_2}{{}^4C_1.{}^{12}C_1+{}^4C_2}=\frac{6}{54}=\frac{1}{9}$ **Ans.] Ans.]**

Que. 60. B. $n(S)=\dfrac{9!}{4!.5!}=126$

$n(A)=0$ to F and F to P

$=\dfrac{5!}{2!.3!}.\dfrac{4!}{2!.2!}=10.6=60$

$P(A)=\dfrac{60}{126}=\dfrac{10}{21}$ **Ans.]**



Comprehesion Type

1 Paragraph for Q. 1 to Q. 3

1. **C.** **2.** **B.** **3.** **A**

(i) Let $\frac{a}{r},a,ar$ the roots $\therefore\ a^3=-\frac{k}{2}$ ....(1) now $\frac{a}{r}+a+ar=\frac{19}{2}$ ....(2) and $\frac{a^2}{r}+a^2r+a^2=\frac{57}{2}$ or

$a\left(\frac{a}{r}+ar+a\right)=\frac{57}{2}\Rightarrow a.\frac{19}{2}=\frac{57}{2}\Rightarrow a=3$ form (1) $k=-2a^3=-54$.

(ii) $a=3$ now substituting in (2) $r=3/2$ or $2/3$ hence the GP's are 2, 3, 9/2, ...

or 9/2, 3, 2, ... $\Rightarrow$ hence $S_n=\dfrac{2\left(\left(\frac{3}{2}\right)^n-1\right)}{\frac{3}{2}-1}=4\left(\left(\frac{3}{2}\right)^n-1\right)$

(iii) $S_\infty=\dfrac{\frac{3}{2}}{1-\frac{2}{3}}=\frac{9}{2}.\frac{3}{1}=\frac{27}{2}$

2 Paragraph for Q. 4 to Q. 6

4. **B.** **5.** **C.** **6.** **B.**

(i) Put $x=1$ in $2(1+p(x))=P(x-1)+P(x+1)$ $\Rightarrow 2(1+P(1))=P(0)+P(2)$ $\Rightarrow 2+2P(1)=8+32$

$\Rightarrow 2P(1)=38\Rightarrow P(1)=19$ Hence sum of all the coefficient is 19.

(ii) Let $P(x)=ax^2+bx+c\Rightarrow P(0)=c\Rightarrow c=8$ also $P(2)=32\Rightarrow 4a+2b+8=32\Rightarrow 2a+b=12$

and $P(1)=19\Rightarrow a+b+c=19\Rightarrow a+b+8=19\Rightarrow a+b=11\Rightarrow a=1$ and $b=10$ $\Rightarrow P(x)=x^2+10x+8$

$=(x+5)^2-17\ \therefore P(x)|_{min}=-17\Rightarrow m=-17$

(ii) $P(40)=1600+400+8=2008$

 # 3 Paragraph for Q. 7 to Q. 9

7. **C.** **8.** **A.** **9.** **B.**

α,β,γ are in H.P., hence.

(i) $\frac{1}{\alpha},\frac{1}{\beta},\frac{1}{\gamma}$ are in A.P. $\Rightarrow$ $\frac{1}{\alpha}+\frac{1}{\gamma}=\frac{2}{\beta}$ $\Rightarrow$ $\beta=\frac{2\alpha\gamma}{\alpha+\gamma}$; $\frac{1}{\alpha}+\frac{1}{\gamma}=\frac{3}{\beta}$ $=\frac{\sum\alpha\beta}{\alpha\beta\gamma}$ $=\frac{q}{q}$ $=1$ $\beta=3$ which is a root $\Rightarrow$ $27+9p+3q-q=0$ $\Rightarrow$ $9p+2q+27=0$ $\Rightarrow$ $9p+2q=-27.$

(ii) $2\left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}\right)\ge 2\left(\frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\gamma\alpha}\right)$ using ($a^2+b^2+c^2\ge ab+bc+ca$ with $a=\frac{1}{\alpha},b=\frac{1}{\beta},c=\frac{1}{\gamma}$)

$\Rightarrow$ $3\left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}\right)\ge\left(\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}\right)^2=1$ $\left(\text{add }\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}\text{ both sides}\right)\Rightarrow\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}\ge\frac{1}{3}.$

(iii) For equality, $\alpha=\beta=\gamma=3\Rightarrow\sum\alpha\beta=27$ and $\alpha\beta\gamma=27$ $\frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\gamma\alpha}\le\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}$

$\left(ab+bc+ca\le a^2+b^2+c^2\right)\Rightarrow 3\left(\frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\gamma\alpha}\right)\le\left(\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}\right)^2=1\left(\text{adding }2\left(\frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\gamma\alpha}\right)\text{both sides}\right)$

$\frac{\alpha+\beta+\gamma}{\alpha\beta\gamma}\le\frac{1}{3}$ $\Rightarrow$ $-\frac{p}{q}\le\frac{1}{3}$ $\Rightarrow$ $\frac{p}{q}\ge-\frac{1}{3}.$

 # 4 Paragraph for Q. 10 to Q. 12

10. **B.** **11.** **C.** **12.** **A.**

(i) Given cubic $f(x)=(x-1)(x-\cos\theta)(x-\sin\theta)$ $\therefore$ roots are 1, $\sin\theta$ and $\cos\theta$

$\therefore$ $x_1^2+x_2^2+x_3^2+1=\sin^2\theta+\cos^2\theta=2.$

(ii) Now if $1=\sin\theta\Rightarrow\theta=\frac{\pi}{2}$ if $1=\cos\theta\Rightarrow\theta=0,2\pi$ and if $\sin\theta=\cos\theta\Rightarrow\tan\theta=1\therefore\theta=\frac{\pi}{4},\frac{5\pi}{4}$

$\therefore$ Nummber or values of θ in $[0,2\pi]$ is 5.

(iii) again maximum possible difference between the two roots is 2 $\underbrace{1-\sin\theta}_{\text{when }\theta=3\pi/2}$ or $\underbrace{1-\cos\theta}_{\text{when }\theta=\pi}$

 # 5 Paragraph for Q. 13 to Q. 15

13. **A.** **14.** **D.** **15.** **B.**

(i) $S_n=4n^2+6n$ $\Rightarrow$ $t_n=S_n-S_{n-1}=4n^2+6n-\left[4(n-1)^2+6(n-1)\right]=4\left(n^2-(n-1)^2\right)+6(n-n+1)$

$=4(2n-1)+6=8n+2$ $\Rightarrow$ A.P. with $d=8.$

(ii) If $t_n=5050$ $\therefore$ $5050=8n+2$ $\Rightarrow$ $n=\frac{5048}{8}=631.$

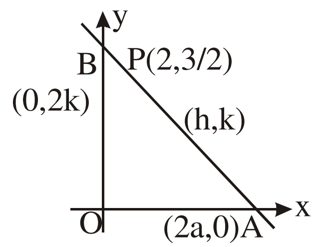
(iii) $t_1=10;\ t_n=18;\ t_3=26\Rightarrow t_1^2+t_2^2+t_3^2=100+324+676=1100.$

 # 6 Paragraph for Q. 16 to Q. 18

16. - A. **17. - B.** **18 - C.**

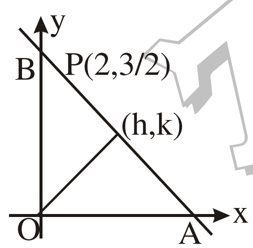
Point of intersection the line $3x+4y-12=0\Rightarrow x+2y-5=0$ is x = 2 and y = 3/2

(i). Equation of AB is



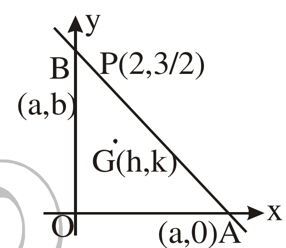
$$\frac{x}{2h}+\frac{y}{2k}=1 \Rightarrow \frac{2}{2h}+\frac{3}{4k}=1 \quad \therefore \quad 4k+3h=4kh \Rightarrow 3x+4y-4xy=0.$$

(ii).



$$\frac{k}{h}.\frac{k-(3/2)}{h-2}=-1 \Rightarrow \frac{k}{h}.\frac{2k-3}{h-2}=-2 \Rightarrow 2h(h-2)+k(2k-3)=0 \Rightarrow 2(x^2+y^2)-4x-3y=0.$$

(iii). Here, $a=3h$, and $b=3k$ $\therefore$ equation of AB is



$$\frac{x}{3h}+\frac{y}{3h}=1 \Rightarrow \frac{2}{3h}+\frac{1}{2k}=1 \Rightarrow 3x+4y-6xy=0.$$

 # 6 Paragraph for Q. 19 to Q. 21

19. **A.** **20.** **C.** **21.** **D.**

(i) $P(A_2)=\frac{18}{36}=\frac{12}{36};\ P(A_4)=\frac{1}{4}=\frac{9}{36};\ P(A_5)=\frac{7}{36}=\frac{7}{36};\ P(A_6)=\frac{6}{36}=\frac{6}{36} \Rightarrow A_3$ **is most probable.**

(ii) $P(A_2)=\frac{1}{2};\ P(A_3)=\frac{1}{3};\ P(A_6)=\frac{1}{6} \quad \therefore \quad P(A_2\cap A_3)=P(A_2).P(A_3) \Rightarrow P(A_2)=P(A_2).P(A_3)$

$\frac{6}{36}=\frac{1}{2}\times\frac{1}{3} \quad \Rightarrow \quad A_2$ and A_3 are independent.

(iii) Note A_1 is independent with all events $A_1, A_2, A_3, A_4, \ldots\ldots\ldots, A_{12}$ now total ordered pairs

$\underbrace{(1,1),(1,2),(1,3)\ldots\ldots\ldots(1,11)}_{22}+(1,12)=23$ pairs Also A_2, A_3 and A_3, A_3 are independent

$\Rightarrow$ 25 ordered pairs

 # 7 Paragraph for Q. 22 to Q. 24

22. **C.** **23.** **B.** **24.** **D.**

(i) $a=1 \Rightarrow f(x)=8x^3+4x^2+2bx+1 \Rightarrow f'(x)=24x^2+8x+2b=2\left(12x^2+4x+b\right)$ for increasing function, $f'(x)\ge 0\ \forall x\in R$ $\therefore$ $D\le 0 \Rightarrow 16-48b\le 0 \Rightarrow b\ge \frac{1}{3}$.

(ii) If $b=1 \Rightarrow f(x)=8x^3+4ax^2+2x+a \Rightarrow f'(x)=24x^2+8ax+2$ or $2\left(12x^2+4ax+1\right)$ for non monotonic $f'(x)=0$ must have distinct roots.

Hnece $D>0$ i.e. $16a^2-48>0 \Rightarrow a^2>3;\ \therefore\ a>\sqrt{3}$ or $a<-\sqrt{3}$ sum $=5050-1=5049$.

(iii) If x_1, x_2 and x_3 are the roots then $k\log_2 x_1+\log_2 x_2+\log_2 x_3=5 \Rightarrow \log_2\left(x_1x_2x_3\right)=5 \Rightarrow x_1x_2x_3=32$

$\Rightarrow -\frac{a}{8}=32 \quad \Rightarrow \quad a=-256.$

 # 8 Paragraph for Q. 25 to Q. 27

25. **C** **26.** **A** **27.** **C**

$$S=\begin{bmatrix}11&12&13&14&15&16\\21&22&23&24&25&26\\31&32&33&34&35&36\\41&42&43&44&45&46\\51&52&53&54&55&56\\61&62&63&64&65&66\end{bmatrix};\quad R=\begin{bmatrix}2&2&3&4&5&6\\2&4&6&8&10&12\\3&6&6&12&15&18\\4&8&12&8&20&24\\5&10&15&20&10&30\\6&12&18&24&30&12\end{bmatrix}$$

(iii) Possible ordered pairs each with probability $\frac{4}{1296}$

$(15, 30); (30, 15); (18, 30), (30, 18), (20, 30), (30, 20); (24, 24); (24, 30); (30, 24); (30, 30)]$

<u>Assertion & Reason Type</u>

Que. 1. (c) $f(x)=ax^2+bx+c$ given $f(0)+f(1)=2 \Rightarrow f(x)>0\forall x\in R \Rightarrow S-1$ is true.

Let $f(x)=x^2-x+1 \quad \Rightarrow \quad a+b=0 \quad \Rightarrow \quad S-2$ is False

Que. 2. A. $D=\underbrace{\underbrace{(2m+1)^2}_{\text{odd}}-\underbrace{4(2n+1)}_{\text{even}}}_{\text{odd}}$ for rational roots D must be a perfect square. As D is odd let D is a perfect square of $(2\ell+1)$ where $\ell\in I$ $\therefore$ $(2m+1)^2-4(2n+1)=(2\ell+1)^2$

Or $(2m+1)^2-(2\ell+1)^2=4(2n+1) \Rightarrow \left[(2m+1)+(2\ell+1)\right]\left[(2(m-\ell))\right]=4(2n+1)$

$4(m+\ell+1)(m-\ell)=4(2n+1)$ .... (1)

RHS of (1) is always odd but LHS is always even (think !) Hence D can not be a perfect square $\Rightarrow$ roots can not be rational hence Statement - 1 is true and Statement - 2 is true and is also the correct explanation for Statement - 1.

Que. 3. C. Reason is true only for 3 or more positive numbers in G.P.

Que. 4. A. a, ar, ar^2, in G.P

now $a+ar, 2ar, ar^2+ar$ will gbe in H.P.

only if $\frac{1}{a(1+r)}+\frac{1}{2ar}$ and $\frac{1}{ar(1+r)}$ in A.P. (only if $r \neq -1$)

now $\frac{1}{a(1+r)}+\frac{1}{ar(1+r)}=\frac{r+1}{ar(1+r)}=\frac{1}{ar}$

Que. 5. D. Let $x=\cot A; y=\cot B$ and $z=\cot C$ $\Rightarrow \sum \cot A \cot B = 1$ $\Rightarrow A+B+C=n\pi$

$\therefore$ LHS $=\frac{1}{2}[\sin 2A+\sin 2B+\sin 2C]=2\sin A\sin B\sin C$

RHS $=\frac{2}{\operatorname{cosec}A.\operatorname{cosec}B.\operatorname{cosec}C}=2\sin A\sin B\sin C=$ LHS $\Rightarrow S-2$ is obviously true.

Que. 6. D. Given $(a,b,c)^{1/3} \geq \frac{a+b+c}{3} \Rightarrow a=b=c$ (GM $\geq$ AM which is possible only if GM = AM)

$\therefore$ $3a+4b+5c=12 \Rightarrow a=b=c=1 \therefore \frac{1}{a^2}+\frac{1}{b^3}+\frac{1}{c^5}=3.$

Que. 7. A. $f(x)=(x-1)(ax+b) \Rightarrow f(2)=2a+b$ $\Rightarrow$ $f(4)=3(4a+b)=12a+3b$

$\Rightarrow$ $f(2)+f(4)=14a+4b=0$ $\Rightarrow$ $\frac{-b}{a}=3.5=\beta$

Que. 8. A.

Que. 9. C

Que. 10. B. Range : 1; Domain : $x = 9$]

Que. 11. A. $a>b>c$ $\Rightarrow$ a,b,c are distinct real also $a^2+b^2+c^2=0$ and $a>b>c \Rightarrow a$ and c are of oppsite sign otherwise $a+b+c \neq 0$ therefore $\frac{c}{a}$ negative.

Que. 12. A. $P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{n(AB)/N}{n(B)/N}=\frac{n(AB)}{n(B)}$ thus for $P(A/B)$ the sample space is the set B.

That is, the conditional probability model, gives B assign $\frac{1}{n(B)}$ to element of B and zero to each element of B^c.

Que. 13. **C.**

Que. 14.A. $f(x)=ax^2+ax+(a+b), D=a^2-4a(a+b)=-3a^2-4ab<0$ if $a>0, f(x)>0\ \forall x\in R.$

If $a<0, f(x)<0\ \forall x \in R.$ $\Rightarrow g(x)=a(x^2+2x+1)+a(x+1)+(a+b) \Rightarrow g(x)=f(x+1)$

Que. 15. A Equation of a tangent at (h,k) on $y=f(x)$ is

$y-k=f'(h)(x-h)$ ...(1)

suppose (1) passes through (a, b)

$b-k=f'(h)[a-h]$ must hold good for some (h,k)

now $hf'(h)-f(h)-af'(h)+b=0$ represents equation of degree odd in h

$\therefore$ $\exists$ some 'h' for which LHS vanishes.]

More than One May Correct Type

Que. 1. (B,C,D) Let $y=|x|$

$x^2|x|+a=0$ ....(1)

$y^2-y+a=0$ ....(2)

If both roots of (2) are positive then (1) have four solution. If one roots of (2) is positive then (1) have two solution and if $a=0$. $x^2-|x|=0$ has $x=-1,0,1$ as solutions.

Que. 2. (A,D) $126=\frac{n}{2}[54+(n-1)(-3)]$ $252=n(57-3n)$ $n^2-19n+84=0$ $\Rightarrow$ $(n-7)(n-12)=0$

$n=7$ or 12 $\Rightarrow$ A and D.

Que. 3. (A,B,C)

$\log 2, \log(2^x-1)$ and $\log(2^x+3)$ are in A.P. $2\log(2x^2-1)=\log(2(2^x+3))\Rightarrow(2^x-1)^2=2(2^x+3)$

$2^x+1-2^{x+1}=2^{x+1}+6\Rightarrow 2^{2x}-4.2^x-5=0\Rightarrow(2^x)^2-4.2^x-5=0$ or $t^2-4t-5=0$ where $(2^x=t)$

$\Rightarrow t=5$ ojr $-1\Rightarrow 2^x=5\Rightarrow x=\log_2 5\left(2^x=-1 \text{ is not possible}\right)\Rightarrow\left(\sqrt{2}\right)^{/n_2 5}=\sqrt{5}\Rightarrow$ (C)

Que. 4. (A,B,D)

Que. 5. (A,B,C) sum = product and roots are rals $-\frac{b}{a}=\frac{c}{a}\Rightarrow b+c=0\Rightarrow b^2-4ac\ge 0\Rightarrow a,b,c$

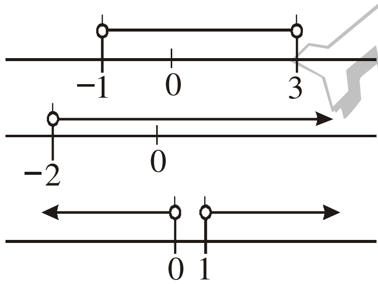
Que. 6. (C,D) If α is one root then $\alpha+\alpha^2=15/4$ and $\alpha^3=p\Rightarrow 4\alpha^2+4\alpha-15=0\Rightarrow 4\alpha^2+10\alpha-6\alpha-15=0$

$2\alpha(2\alpha+5)-3(2\alpha+5)=0\Rightarrow\alpha=-5/2$ or $\alpha=3/2\Rightarrow p=\alpha^2=-\frac{125}{8}$ or $\alpha=3/2$

$\Rightarrow$ $p=\alpha^2=-\frac{125}{8}$ or $\frac{27}{8}$.

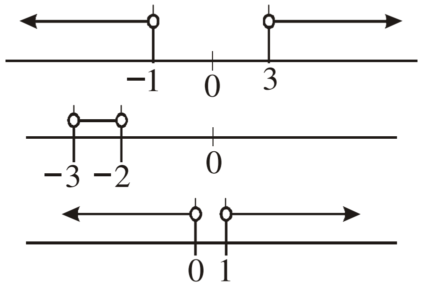
Que. 7. A,C,D. $\log_{x+3}(x^2-x)<1\Rightarrow x(x-1)>0\Rightarrow x>1$ or $x<0$ ....(1) let $x+3>1\Rightarrow x>-2$

here we have $x^2-x<x+3\Rightarrow x^2-2x-3<0\Rightarrow(x-3)(x+1)<0$ hence $x\in(-1,0)\cup(1,3)\Rightarrow$ (C), (D).



again, let $0<x+3<1\Rightarrow -3<x<-2$ .... (1)

then $x^2-x>x+3\Rightarrow x^2-2x-3>0\Rightarrow(x-3)(x+1)>0$ ....(2) hence $x\in(-3,-2)\Rightarrow$ (A)



Que. 8. A,C,D. roots are a/r, a, ar : where $a > 0, r > 1$ Now $a/r + a + ar = -p$ ....(1)

$a.a/r + a.ar + ar.a/r = q$ .... (2) $a/r.a.ar = 1$ ....(3)

$a^3 = 1 \Rightarrow a = 1 \Rightarrow$ (C) is correct

from (1) putting $a = 1$ we get $-p-3 > 0 \Rightarrow -p > 3 \Rightarrow p < -3 \Rightarrow 1/r + 1 + r = -p$ .... (4)

$\left(\sqrt{r} - \frac{1}{\sqrt{r}}\right)^2 + 3 = -p \Rightarrow -p-3 > 0 \Rightarrow -p > 3 \Rightarrow p < -3 \Rightarrow$ B is correct.

Form (2) putting $a = 1$ we get $1/r + r + 1 = q$ .... (5)

from (4) and (5) we have $-p = q \Rightarrow p + q = 0 \Rightarrow$ (A) si correct now $as, r > 1 \Rightarrow a/r = 1/r < 1$ and $ar = r > 1 \Rightarrow$ (D) is correct.

Que. 9. B,D. $\log a, \log b, \log c$ are in A.P. $\Rightarrow 2\log b = \log a + \log c \therefore b^2 = ac$ .... (1)

$\Rightarrow a, b, c$ are in G.P. $\Rightarrow$ (B). also given $(\log a - \log 2b), (\log 2b - \log 3c), (\log 3c - \log a)$ are in A.P. $\Rightarrow 2(\log 2b - \log 3c) = \log 3c0\log 2b \Rightarrow 3\log 2b = 3\log 3c \therefore 2b = 3c$ ....(2)

$\Rightarrow 4b^2 = 9c^2$ ....(3) from (1) and (3) $4ac = 9c^2 \Rightarrow a = \frac{9c}{4}$ and $b = \frac{3c}{2}$

$a = \frac{9c}{4}; b = \frac{3c}{2}$ and $c = c \therefore a, b, c$ forms the sides of triangle $\Rightarrow$ (D)

but 2, 2b and 3c are not in H.P. $|||\ell y$ Verify (A).

Que. 10. A,B,C,D. Given $2\cos^2\frac{B}{2} = \cos^2\frac{A}{2} + \cos^2\frac{C}{2} \Rightarrow \cos A, \cos B, \cos C$ are in A.P. $\Rightarrow$ (A)

Also $\sin^2\frac{A}{2}, \sin^2\frac{B}{2}, \sin^2\frac{C}{2}$ are in A.P. i.e., $\frac{(s-b)(s-c)}{bc}, \frac{(s-c)(s-a)}{ac}, \frac{(s-a)(s-b)}{ab}$ are in A.P.

$\Rightarrow \frac{a}{s-a}, \frac{b}{s-b}, \frac{c}{s-c}$ are in A.P. add one to all the terms. $\Rightarrow \frac{s}{s-a}, \frac{s}{s-b}, \frac{s}{s-c}$ are in A.P. ....(1)

now $s-a, s-b, s-c$ are in H.P. $\Rightarrow$ (B)

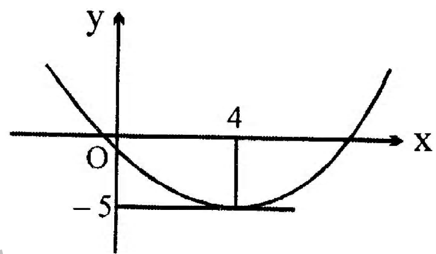
Multiply (1) by $\frac{\Delta}{s}$, we get r_1, r_2, r_3 are in A.P. $\Rightarrow$ (C).

agin multiply (1) by $\frac{\Delta}{S^2}$, we get $\tan\frac{A}{2}, \tan\frac{B}{2}, \tan\frac{C}{2}$ are in A.P.

$\Rightarrow \cot\frac{A}{2}, \cos\frac{B}{2}, \cot\frac{C}{2}$ are in H.P. $\Rightarrow$ (D)

Que. 11. A,B,C, From figure $a>0 \Rightarrow$ (A) and $-\frac{b}{2a}=4 \Rightarrow -\frac{b}{2a}>0;\ \therefore\ b<0 \Rightarrow$ (B) $f(0)=c<0 \Rightarrow$ (C)

also $-\frac{b}{2a}=4 \Rightarrow 8a+b=0 \Rightarrow$ **(D)** is incorrect



Que.12. A,B,C,D. $S_n=n^2p \Rightarrow S_p=p^3 \Rightarrow$ (C)

$t_n=S_n-S_{n-1}=p\left[n^2-(n-1)^2\right]=(2n-1)p \Rightarrow t_1=a_1=p \Rightarrow$ (A)

$t_p=a_p=2p^2-p \Rightarrow$ (D) . common difference $a_2-a_1=3p=2p \Rightarrow$ (B)

Que. 13. B,C,D. Acutual Answer is ${}^{10}C_4=210$

(A) MRINAL [IA] MRNL $\therefore$ number of words $=5!\times 2!=240 \Rightarrow$ (A) is correct.

(B) Now □□□ 7. 6. 5. $=210 \Rightarrow$ **(B)** is correct.

(C) ${}^{10}C_4\times 1=210-10=210 \Rightarrow$ **(D)** is correct.

Que. 14. B,D. Given $2b=a+c$ and $a+b+c=60 \Rightarrow 3b=60 \Rightarrow b=20 \therefore c=40-a$

Now $a-2, b, c+3$ in G.P. $\Rightarrow a-2, 20, 43-a$ in G.P. $\Rightarrow (a-2)(43-a)=400$

$45a-86-a^2=400 \Rightarrow a^2-45a+486=0 \Rightarrow a^2-27$ or $a=18$ If $a=27,\ c=13$ If $a=18,\ c=22$

$\therefore$ 27, 20, 13 or 18, 20, 22 $\Rightarrow a^2+b^2+c^2=729+400+160+1298 \Rightarrow$ **(D)**

$a^2+b^2+c^2=324+400+484=1208 \Rightarrow$ **(B)**

Que. 15. C,D. No of ways to distribute n different things among three boys.

$=\sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{k=0}^{n}\binom{n}{i}\binom{n}{j}\binom{n}{k}=2^{3n}$ for $n=3, E=2^9=512;$ for $n=4, E=2^{12}=4096 \Rightarrow$ **C,D.**

Que. 16. A,C,D. Answer is 3^n

(A) (3^n-1); (B). 3^n;

(C) Total - when all 3 digits are even $=6^n-3^n$; (D) ${}^nC_3.3!$

Que. 17. B,C. $\left(x^{1/2}+\frac{1}{2}x^{-1/4}\right)^n \Rightarrow T_{r+1}={}^nC_r.x^{\frac{n-r}{2}}.\frac{1}{2^r}x^{\frac{-r}{4}}$ coefficient of the 1^{st} 3 terms are

${}^nC_0, {}^CC_1.\frac{1}{4}=2.{}^nC_1.\frac{1}{2} \quad \therefore\ {}^nC_0+{}^nC_2.\frac{1}{4}=2.{}^nC_1.\frac{1}{2} \Rightarrow 1+\frac{n(n+1)}{8}=n$

$\therefore\ \frac{n(n-1)}{8}=(n-1) \Rightarrow n=8\ (\text{as } n\neq 1)\ \therefore\ T_{r+1}={}^8C_rx^{\frac{8-r}{2}}.\frac{1}{2^r}.x^{-\frac{r}{4}}={}^8C_r.\frac{1}{2^r}.x^{\left(4-\frac{3r}{4}\right)}$

terms of x with integer power occur when r = 0, 4, 8 $\Rightarrow$ 3 terms hence B/C are correct.

Que. 18. A,B,D. $P(A\cup B)=P(A)=P(B)-P(A\cap B) \Rightarrow \frac{5}{8}=\frac{3}{8}+\frac{4}{8}-P(A\cap B) \Rightarrow P(A\cap B)=\frac{2}{8}=\frac{1}{4}$

Now $P\left(A^c/B\right)=\frac{P\left(A^c\cap B\right)}{P(B)}=\frac{P(B)-P\left(A\cap B\right)}{P(B)}=1-\frac{2}{8}.\frac{8}{4}=\frac{1}{2}$

$2P\left(A/B^c\right)=\frac{2P\left(A\cap B^c\right)}{P(B^c)}=\frac{2\left(P(A)-P\left(A\cap B\right)\right)}{1-P(B)}=4\left(\frac{3}{8}-\frac{2}{8}\right)=\frac{1}{2} \Rightarrow$ **(A)** is correct

$P\left(A/B\right)=\frac{P\left(A\cap B\right)}{P(B)}=\frac{1}{4}.\frac{2}{1}=\frac{1}{2}=P(B) \Rightarrow$ **(B)** is correct.

again $P\left(A^c/B^c\right)=\frac{P\left(A^c\cap B^c\right)}{P\left(B^c\right)}=\frac{1-P\left(A\cup B\right)}{1-P(B)}=2\left(1-\frac{5}{8}\right)=\frac{3}{4}$

$P\left(B/A^c\right)=\frac{P\left(B\cap A^c\right)}{1-P(A)}=\frac{P(B)-P\left(A\cap B\right)}{5/8}=\frac{\frac{1}{2}-\frac{1}{4}}{\frac{5}{8}}=\frac{1}{4}.\frac{8}{5}=\frac{2}{5}$

Hence $8P\left(A^c/B^c\right)=15P\left(B/A^c\right) \Rightarrow$ **(C)** is correct.

again $2P\left(A/B^c\right)=\frac{1}{2}$ from (1) $\Rightarrow P\left(A/B^c\right)=\frac{1}{4}=P\left(A\cap B\right)$ Hcece **(D)** is correct.

Que. 19.B,C,D. **(A).** False; $P\left(TTT \text{ or } HHH\right)=\frac{1}{8}+\frac{1}{8}=\frac{1}{4}$

(B) $\frac{P\left(A\cap B\right)}{P(B)}=\frac{P\left(A\cap B^c\right)}{1-P(B)}=\frac{P(A)-P\left(A\cap B\right)}{1-P(B)} \Rightarrow P\left(A\cap B\right)\left[1-P(B)\right]=P(B).P(A)-P(B).P\left(A\cap B\right)$

$\Rightarrow P(A\cap B)=P(A).P(B) \Rightarrow$ **True.**

(D). To prove that A, B, C are pairwise independent only now $P\left(A\cap B\right)=P\left(A\cap B\cap \overline{C}\cup A\cap B\cap C\right)$ (from thevenn diagream)

$P\left(A\cap B\right)=P\left(A\cap B\cap \overline{C}\right)+P\left(A\cap B\cap C\right)=P(A).P(B).P(\overline{C})+P(A).P(B).P(C)$ (given)

$=P(A).P(B)\left[P(C)+P(\overline{C})\right]$ |||ℓy for other two $\Rightarrow$ **(D)** is correct.

Que. 20. B,D. as $a>2$ hence $a^2>2a>a>2$ now $\left(x-a\right)\left(x-2a\right)\left(x-a^2\right)<0 \Rightarrow$ the solution set is as shown

between (0,a) there are (a – 1) positive integers between $\left(2a,a^2\right)$ there are $\left(a^2-2a-1\right)$ positive integers $\therefore\ a^2-2a-1+a-1=18 \Rightarrow a^2-a-20=0 \Rightarrow (a-5)(a+4)=0 \therefore\ a=5 \Rightarrow$ **(B) & (D).**

Que. 21. B,C. $(a-1)x^2-(a^2+2)x+(a^2+2a)=0$ ....(1), $(b-1)x^2-(b^2+2)x+(b^2+2b)=0$ ..... (2)

Consider 1st equation $ax^2-x^2-a^2x-2x+a^2+2a=0 \Rightarrow ax(x-a)-(x-a)(x+a)-2(x-a)=0$

$(x-a)[ax-x-a-2]=0 \therefore\ x=a$ or $x=\dfrac{a+2}{a-1}$ |||ℓy from 2nd equation we get $x=b$ or $x=\dfrac{b+2}{b-1}$

Now (1) and (2) have a common root **Note :** a cannot be equal to b (as a and b are distinct)

also if $\dfrac{a+2}{a-1}=\dfrac{b+2}{b-1} \Rightarrow ab-a+2b-2=ab+2a-b=2 \Rightarrow 3a=3b \Rightarrow a=b$ (not possible)

$\therefore$ The only possiblity of common root is $a=\dfrac{b+2}{b-1}$ or $b=\dfrac{a+2}{a-1}$ $\therefore\ a=1+\dfrac{3}{b-1}$ since a is +ve integer $\therefore\ b-1=1$ or 3 $b=2$ or 4 $\left.\begin{array}{l}\text{if } b=2 \text{ then } a=4\\ \text{if } b=4 \text{ then } a=2\end{array}\right]$ if $a=2$ and $b=4$ or $a=4$ and $b=2$ then both the equations reduce to $x^2-6x+8=0 \Rightarrow$ they are identical and their both roots are common.

Que. 22. A, B, C, D $(\log_2 x)^2-4\log_2 x-12=(m+1)^2 \quad\Rightarrow\quad t^2-4t-\{12+(m+1)^2\}=0$

$t=\dfrac{4\pm\sqrt{16+4\left(12+(m+1)^2\right)}}{2} \quad \Rightarrow D>0 \quad\Rightarrow\quad$ (A) is correct

Now D_{min} when $m=-1$

$\log_2 x=\dfrac{4\pm 8}{2}=6$ or $-2 \Rightarrow \quad x=2^6$ or $2^{-2} \quad\Rightarrow$ (C) and (D) are correct

Also $\log_2 x_1+\log_2 x_2=4 \Rightarrow \log_2 x_1x_2=4 \Rightarrow \quad x_1x_2=2^4 \Rightarrow$ (B) is correct]

Que. 23. C, D $(a-1)\left(x^2+\sqrt{3}x+1\right)^2-(a+1)[(x^2+1)-\left(x\sqrt{3}\right)^2 \le 0$

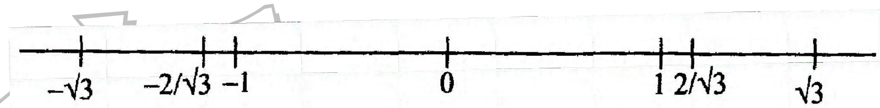
or $(a-1)\left(x^2+\sqrt{3}x+1\right)^2-(a+1)[x^2+x\sqrt{3}+1)\ \left(x^2-x\sqrt{3}-1\right)\le 0$

$\left(x^2+\sqrt{3}x+1\right)[(a-1)\left(x^2+\sqrt{3}x+1\right)-(a-1)\left(x^2-\sqrt{3}x+1\right)]\le 0\ \forall x\in R$

$\Rightarrow \quad -2\left(x^2+1\right)+2a\sqrt{3}x \le 0$

$\Rightarrow \quad x^2-a\sqrt{3}x+1\ge 0\ \forall\ x\in R \quad \Rightarrow 3a^2-4\le 0\ (D\le 0)$

$\Rightarrow \quad a\in\left[-\dfrac{2}{\sqrt{3}},\dfrac{2}{\sqrt{3}}\right]$



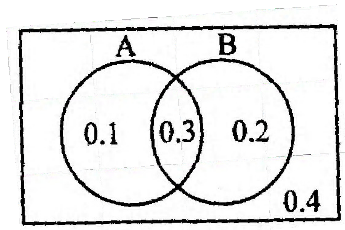
$\Rightarrow$ number of possible intergral value of 'a' is

$\{-1, 0, 1\} \quad\Rightarrow\quad 3 \quad$ Ans. $\Rightarrow$ (C)

and sum of all integral values of 'a' is $-1+0+1+0$ **Ans.** $\Rightarrow$ **(D)]**

Que. 24. B, C $P(E_1) = \frac{P(A \cap B)}{P(A \cup B)} = \frac{0.3}{0.6} = \frac{1}{2}$

$P(E_2) = \frac{0.3}{0.6} = \frac{1}{2}$



(B) $\frac{P(A \cap B)}{P(B)} = \frac{0.3}{0.5} = \frac{3}{5} = 0.60$; $P(B/A) = \frac{0.3}{0.4} = \frac{3}{4} = 0.75$]

Que. 25. C, D $P(E_1) = 1 - P(\text{unit's place in both is } 1,2,3,4,6,7,8,9)$

$P(E_1 = 0 \text{ or } 5) = 1 - \left(\frac{4}{5}\right)^2 = \frac{9}{25}$

$P(E_2 : 5) = P(1\ 3\ 5\ 7\ 9) - P(1\ 3\ 7\ 9)$ for 2 numbers

$= \frac{1}{4} - \frac{4}{25} = \frac{25-16}{100} = \frac{9}{100}$

$\frac{P(E_2)}{P(E_1)} = \frac{9}{100} \cdot \frac{9}{100} \cdot \frac{25}{9} = \frac{1}{4}$

$P(E_1) = 4P(E_2)$ $\Rightarrow$ A is not correct

$P(E_2 / E_1) = \frac{P(E_2 \cap E_1)}{P(E_1)} = \frac{P(E_2)}{P(E_1)} = \frac{9}{100} \cdot \frac{25}{9} = \frac{1}{4} \Rightarrow (C)$

$P(E_1 / E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{P(E_1)}{P(E_2)} = 1$ $\Rightarrow (D)$]

Que. 26. A, C, D $P(E) = {}^{2n}C_n \cdot \frac{1}{2^{2n}} = \frac{(2n)!}{n!.n!.2^n.2^n}$

verify all the alternatives, nothing the fact that $C_0 + C_1 + C_2 + \ldots\ldots\ldots\ldots\ldots + C_n = 2^n$ and $C_0^2 + C_1^2 + C_2^2 + \ldots\ldots\ldots\ldots\ldots + C_n^2 = {}^{2n}C_n$]

Que. 27. A, B, C, D $P(A) = \frac{18}{36}$; $P(B) = \frac{11}{36}$; $P(A \cap B) = \frac{6}{36}$]

Que. 28. A, D (D) $P(C \cap (A \cup B)) = P(C) \times P(A \cup B)$

$P((C \cap A) + (C \cap B))$

$= P(C \cap A) + P(C \cap B) - P(A \cap B \cap C)$

$= P(C).P(A) + P(C).P(B) - P(A).P(B).P(C)$

$= P(C)\left[P(A)+P(B)-P(A\cap B)\right]$

$= P(C).P(A\cup B) \quad \Rightarrow \quad C$ and $A\cup B$ are independent]

Match Matrix Type

Que. 1. A - R. B - S. C - P. D - Q.

A. $\left(5+\sqrt{2}\right)x^2-\left(4+\sqrt{5}\right)x+8+2\sqrt{5}\left\langle\begin{matrix}x_1\\ x_2\end{matrix}\right. \Rightarrow \text{H.M.}=\frac{2x_1x_2}{x_1+x_2}=\frac{2\left(8+2\sqrt{5}\right)}{4+\sqrt{5}}=4. \Rightarrow$ R.

B. $a_1+9d=3$ and $\frac{1}{h_{10}}=\frac{1}{h_1}=9d_1$

$2+9d=3\Rightarrow \quad d=\frac{1}{9}$ $\qquad \frac{1}{3}=\frac{1}{2}+9d_1$

$\therefore \; a_4=2+3d=2+\frac{1}{3}=\frac{7}{3}$ $\qquad 9d_1=-\frac{1}{6}\Rightarrow d_1=-\frac{1}{54}$

$\therefore \; a_4h_7=\frac{7}{3}\times\frac{18}{7}=6.\Rightarrow$ S. $\qquad \frac{1}{h_7}=\frac{1}{2}+6\left(-\frac{1}{54}\right)=\frac{1}{2}-\frac{1}{9}=\frac{7}{18}$

C. $3x+4(mx+1)=9\Rightarrow 3x+4mx=5\Rightarrow x=\frac{5}{3+4m}$ now intercept for x to be integer $m=-1$ or $m=-2$ $\Rightarrow$ 2 integral values $\Rightarrow$ P.

D. Product of n geometric means between two numbers is equal to n^{th} power of single geometric mean between them.

Que. 2. A - P,Q,R. B - P,Q,R,S. C - P,Q. D - P.

A. $\log_2 x+2\log_2 y+2\log_2 z=4=\log_2\left(xy^2z^2\right) \Rightarrow \quad xy^2z^2=16$

$$\frac{x+\frac{y}{2}+\frac{y}{2}+\frac{z}{2}+\frac{z}{2}}{5}\geq\left(x\frac{y^2}{4}\frac{z^2}{4}\right)^{1/5}=1 \quad (AM\geq GM) \quad \Rightarrow \quad \frac{x+y+z}{5}\geq 1 \quad \Rightarrow \quad x+y+z\geq 5 \;\Rightarrow\; P,Q,R.$$

B. $3^{|\sin x|}\in[1,3], 2^{-|\sec y|}\in\left(0,\frac{1}{2}\right], 5\cos z\in[-5,5] \quad \Rightarrow \quad P,Q,R,S.$

$a=5\cos z+32^{-|\sec y|}\in\left(-5+0,\frac{3}{2}+5\right]\in\left(-5,6\frac{1}{2}\right]$

C. $\operatorname{cosec}A, \operatorname{cosec}B, \operatorname{cosec}C$ are in H.P. $\Rightarrow$ $\sin A, \sin B, \sin C$ in are A.P.

$\therefore \quad 2b=a+c$

$a+c>b \quad \Rightarrow \quad 2b>b$ is true (No conclusion)

$a+b>c \quad \Rightarrow \quad \underbrace{2b-c}_{a}+b>c \quad \Rightarrow \quad b>\frac{2c}{3}\Rightarrow\frac{b}{c}>\frac{2}{3}\Rightarrow\frac{2b}{c}>\frac{4}{3}$

$b+c>a \quad \Rightarrow \quad b+c>2b-c \quad \Rightarrow \quad b<2c \quad \Rightarrow \quad \frac{b}{c}<2$

$\Rightarrow \quad \frac{2b}{c} \in \left(\frac{4}{3}, 4\right) \quad \Rightarrow \quad P, Q.$

D. $a + b = 3$ HM $\le$ AM of 3 numbers $\frac{a}{2}, \frac{a}{2}, b$ we have

$$\frac{3}{\frac{2}{a}+\frac{2}{a}+\frac{1}{b}} \le \frac{\frac{a}{2}+\frac{a}{2}+b}{3} = 1; \ \therefore \ 1 \ge \frac{3}{\frac{2}{a}+\frac{2}{a}+\frac{1}{b}} \Rightarrow \frac{2}{a}+\frac{2}{a}+\frac{1}{b} \ge 3 \Rightarrow \frac{4}{a}+\frac{1}{b} \ge 3 \quad \Rightarrow \quad P.$$

Que. 3. A - Q. B - P. C - S.

A. Let the two numbers are 'a' and 'b'

$\left.\begin{array}{l} a+b=4p \\ a-b=4q \end{array}\right] p, q \in I \ \ 2a = 4(p+q) \Rightarrow q = 2I_1 \Rightarrow 2b = 4(p-q) \Rightarrow b = 2I_2$ Hence both a and b even.

Also note that if (a–b) is a multiple of 4 then $(a+b)$ will automatically be a multiple of 4.

Hence $n(S) = {}^{11}C_2 \Rightarrow n(A) = (0,4), (0,8), (2,6), (2,10), (4,8), (6,10) = 6 \ \therefore \ P(A) = \frac{6}{{}^{11}C_2}$

(B). Let number of green socks are $x > 0$, E : two socks drawn are of the same colour

$$P(E) = P\left(RR \text{ or } BB \text{ or } WW \text{ or } GG\right) \begin{cases} 2R \\ 2B \\ 2W \\ xG \end{cases} = \frac{3}{{}^{6+x}C_2} + \frac{{}^{x}C_2}{{}^{6+x}C_2} = \frac{6}{(x+6)(x+5)} + \frac{x(x-1)}{(x+6)(x+5)} = \frac{1}{5}$$

$$5\left(x^2 - x + 6\right) = x^2 + 11x + 30 \Rightarrow 4x^2 - 16x = 0 \Rightarrow x = 4.$$

(C). Let there be x red socks and y blue socks. Then $\frac{{}^{x}C_2 + {}^{y}C_2}{{}^{x+y}C_2} = \frac{1}{2}$ let $x > y$ or

$$\frac{x(x-1) + y(y-1)}{(x+y)(x+y-1)} = \frac{1}{2}$$

Multiplying both sides by $2(x+y)(x+y-1)$ and expanding, e find that $2x^2 - 2x + 2y^2 - 2y$ $2x^2 - 2x + 2y^2 - 2y$. Rearranging, we have $x^2 - 2xy + y^2 = x + y \Rightarrow (x-y)^2 = x + y \Rightarrow |x - y| = x + y$

Since $x + y \le 17$, $x - y \le \sqrt{17}$. as $x - y$ must be an integer $\Rightarrow x - y = 4$. $\therefore$ $x + y = 16$. Adding both together and dividing by two yields $x \le 10$.

Que. 4. [(A) S; (B) P; (C) R]

(A) $P(S) = 1/2; \ P(F) = 1/2$

Let 'n' bombs are to be dropped

E: bridge is destroyed $\Rightarrow$ $P(E) = 1 - P(0 \text{ or } 1 \text{ success})$

$$= 1 - \left(\left(\frac{1}{2}\right)^n + {}^{n}C_1 . \frac{1}{2} . \left(\frac{1}{2}\right)^{n-1}\right) = 1 - \left(\frac{1}{2^n} + \frac{n}{2^n}\right) \ge 0.9$$

or $\frac{1}{10} \ge \frac{n+1}{2^n}$ or $\frac{2^n}{10(n+1)} \ge 1$

The value of n consitent with $n = 7$ or draw graph between $y = 2^x$ and $y = 10(x+1)$.

(B) Bag $\begin{cases} 2R \\ 3B \\ 5B \end{cases}$ 3B ; $P(S) = \frac{1}{5}$; $P(F) = \frac{4}{5}$; E: getting a red ball

$P(E) = P(S \text{ or } F\,S \text{ or } F\,F\,S \text{ or } ....] \ge \frac{1}{2}$; $\therefore P(F)^n \le \frac{1}{2}$; $\left(\frac{4}{5}\right)^n \le \frac{1}{2}$

The value of n consistent within 4 $\Rightarrow$ (P)]

Que. 5. $[(\mathbf{A})\mathbf{P}, (\mathbf{B})\mathbf{R}, (\mathbf{C})\mathbf{Q}; (\mathbf{D})\mathbf{P}]$

Urn $\begin{cases} BBBB \\ 8W \end{cases}$ $\xrightarrow{3 are drawn}$

$P(A) = \frac{{}^4C_1 . {}^8C_2}{{}^{12}C_3}$ $\left({}^{12}C_3 = 220;\ {}^8C_2 = 28\right)$

$P(BWW \text{ or } WBW \text{ or } BWW) = \frac{4.28}{220} = \frac{112}{220} = \frac{28}{55} = P(A)$

$P(B) = P(BBBB \text{ or } WWW) = \frac{{}^4C_3 + {}^8C_3}{{}^{12}C_3} = \frac{4+56}{220} = \frac{60}{220} = \frac{3}{11}$

$P(C) = P(WBB \text{ or } BWB \text{ or } WWB \text{ or } BBB)$

$= \frac{8}{12}.\frac{4}{11}.\frac{3}{10} + \frac{4}{12}.\frac{8}{11}.\frac{3}{10} + \frac{8}{12}.\frac{7}{11}.\frac{4}{10} + \frac{4}{12}.\frac{3}{11}.\frac{2}{10}$

$= \frac{96+96+224+24}{12.110} = \frac{440}{12.110} = \frac{1}{3}$

$A \cap B = \phi$ $\Rightarrow$ A and B are mutually exclusive

$P(B \cap C) = P(BBB) = \frac{4.3.2}{12.110} = \frac{1}{55}$

$P(C \cap A) = P(WWB) = \frac{8.7.4}{12.11.10} = \frac{28}{3.55}$

$P(C).P(A) = \frac{1}{3}.\frac{112}{220} = \frac{28}{3.55}$ $\Rightarrow$ C and A are independent

$A \cup B \cup C = \{BWW, WBW, WWB, BBB, WWW, WBB, BWB\}$]

Subjective Type (Up to 4 digit)

Que. 1. [2786]

$r^{\frac{1}{3}} - \frac{1}{r^{\frac{1}{3}}} = 2$ (given)

Now using $x^3 - y^3 = (x-y)^3 + 3xy(x-y$

$$r - \frac{1}{r} = \left(r^{\frac{1}{3}} - \frac{1}{r^{\frac{1}{3}}}\right)^3 + 3\left(r^{\frac{1}{3}} - \frac{1}{r^{\frac{1}{3}}}\right) = 8 + 6 = 14$$

again $r^3 - \frac{1}{r^3} = \left(r - \frac{1}{r}\right)^3 + 3\left[r - \frac{1}{r}\right] = (14)^3 + 42 = 2786$ Ans.

Que. 2. (2599)

$$\alpha = \frac{\sin 5^o + \sin 10^o + ... + \sin 40^o}{\cos 5^0 + \cos 10^0 + ... + \cos 40^0} = \tan 22.5 = -1 + \sqrt{2}$$

$\therefore \quad \beta = -1 - \sqrt{2}$

$\therefore \quad$ Sum $= -2;$ product $= 1 - 2 = -1$

$f(x) = x^2 + 2x - 1 \quad \Rightarrow \quad f(50) = 2500 + 99 = 2599$

Que. 3. $\frac{n(3n-1)}{2}; \frac{n}{2}[17 - 3n]$

$a - d, \quad a, \quad a + d$

$(a-d)a(a+d) = 28$

$a^2 - d^2 = 7 \quad \Rightarrow \quad d^2 = 9 \quad \Rightarrow \quad d = 3$ or -3

Hence AP' are

1, 4, 7, 10,.......................

7, 4, 1, –2,.......................

$$S_n = \frac{n}{2}\left[2 + (n-1)3\right] = \frac{n(3n-1)}{2}$$

Or $S_n = \frac{n}{2}\left[14 + (n-1)(-3)\right] = \frac{n}{2}(17 - 3n)$

Que. 4. (225)

$x = 4\cos 36^o = \sqrt{5} + 1$

$\therefore \quad (x-1)^2 = 5 \quad \Rightarrow \quad x^2 - 2x - 4 = 0$

Now $2x^4 - x^3 - 19x^2 - 2x + 35 = 0$

$$= 2x^2\underbrace{\left(x^2 - 2x - 4\right)}_{\text{zero}} + 3x^3 - 11x^2 - 2x + 35$$

$$= 3x\underbrace{\left(x^2 - 2x - 4\right)}_{\text{zero}} - 5x^2 + 10x + 35$$

$$= -5\underbrace{\left(x^2 - 2x - 4\right)}_{\text{zero}} + 15 \qquad \Rightarrow \qquad A = 15$$

Now $x^3 - x^2 + 8x - 2 = 0 \; \begin{matrix} \alpha \\ \beta \\ \gamma \end{matrix}$

$\alpha + \beta + \gamma = 1; \quad \alpha\beta + \beta\gamma + \gamma\alpha = 8; \qquad \alpha\beta\gamma = 2$

$$\therefore \qquad \alpha^{-2} + \beta^{-2} + \gamma^{-2} = \frac{\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2}{(\alpha\beta\gamma)^2} = \frac{(\alpha\beta + \beta\gamma + \gamma\alpha)^2 - 2\alpha\beta\gamma(\alpha + \beta + \gamma)}{(\alpha\beta\gamma)^2}$$

$$= \frac{(8)^2 - (2)(2)(1)}{(2)^2} = \frac{64 - 4}{4} = \frac{60}{4} = 15 \Rightarrow \qquad B = 15$$

$\therefore \qquad AB = 225$

Que. 5. (93) $d = 1 \quad$ let $a_1 = a$

$\therefore \qquad a_1 + a_2 + ... + a_{98} = 137 \;\; \frac{98}{2}[2a + 97] = 137$ .... (1)

$49(2a + 97) = 137 \;\; 2a + 97 = \frac{137}{49}$ .... (2)

To find $a_2 + a_4 + ... + a_{98}$ (49 terms) $\quad = \frac{49}{2}[a_2 + a_{98}] = \frac{49}{2}[(a+1) + a + 97]$

$$= \frac{49}{2}[2a + 97 + 1] = \frac{49}{2}\left[\frac{137}{49} + 1\right] = \frac{137}{2} + \frac{49}{2} = \frac{186}{2} = 93.$$

Que. 6. (eq. 1.) $\quad (\log x^3)^2 \, 10\log x + 1 = 0 \Rightarrow 9(\log x)^2 - 10\log x + 1 = 0 \Rightarrow 9(\log x)^2 - 9\log x - \log x + 1 = 0$

$9\log x(\log x - 1) - 1(\log x - 1) = 0 \Rightarrow (9\log x - 1)(\log x - 1) = 0 \Rightarrow \log x = 1/9; \;\; \log x = 1$

$\Rightarrow x = 10^{1/9}$ or $x = 10$

(eq. 2.) $\quad \log\left(x(x-9).\frac{x-9}{x}\right) = 0 \Rightarrow \log(x-9)^2 = 0 \Rightarrow (x-9)^2 = 1 \Rightarrow x - 9 = 1 \text{ or } -1 \Rightarrow x = 10 \text{ or } 8$

Hence $x = 10$ is the only value of x satistying both the equation.

Que. 7. $2^{x+2}.5^{6-x} = 2^{x^2}.5^{x^2} \Rightarrow 5^{6-x-x^2} = 2^{x^2-x-2} \; \therefore \; (6 - x - x^2)\log 5 = (x^2 - x - 2)\log 2$ (base 10)

$(6 - x - x^2)[1 - \log 2] = (x^2 - x - 2)\log 2 \Rightarrow 6 - x - x^2 = (\log 2)\left[(x^2 - x - 2) - x^2 - x + 6\right] \Rightarrow 6 - x - x^2$

$= (\log 2)[4 - 2x] \Rightarrow x^2 + x - 6 = 2(\log 2) \Rightarrow (x - 2) \Rightarrow (x + 3)(x - 2) = (\log 4)(x = 2) \;\; \therefore$ either $x = 2.$

Or $\quad x + 3 = \log 4 \Rightarrow x = \log 4 - 3 = \log\left(\frac{4}{1000}\right); \;\; x = -\log_{10}(250).$

Que. 8. (8281.00) $\sum_{n=1}^{\infty}\left(\frac{n^2+2n+3}{2^n}\right)=\underbrace{\sum_{n=1}^{\infty}\left(\frac{n^2}{2^n}\right)}_{\text{say x}}+\underbrace{\sum_{n=1}^{\infty}\left(\frac{2n}{2^n}\right)}_{\text{say y}}\underbrace{\sum_{n=1}^{\infty}\left(\frac{3}{2^n}\right)}_{\text{say z}}$

$$x=\frac{1}{2}+\frac{4}{2^2}+\frac{9}{2^3}+\frac{16}{2^4}+\frac{25}{2^5}+.........\infty \qquad ....(1)$$

$$\frac{x}{2}=+\frac{1}{2^2}+\frac{4}{2^3}+\frac{9}{2^4}+\frac{16}{2^5}+............\infty \qquad ....(2)$$

(1) - (2) gives

$$\frac{x}{2}=\frac{1}{2}+\frac{3}{2^2}+\frac{5}{2^3}+\frac{7}{2^4}+\frac{9}{2^5}+............ \qquad ....(3)$$

now $$\frac{x}{4}=+\frac{1}{2^2}+\frac{3}{2^3}+\frac{5}{2^4}+\frac{7}{2^5}+................ \qquad ....(4)$$

(3) - (4) gives

$$\frac{x}{4}=\frac{1}{2}+2\left[\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\frac{1}{2^4}+..........\right]\Rightarrow \frac{x}{4}=\frac{1}{2}+2\left(\frac{1/4}{1-(1/2)}\right)=\frac{1}{2}+2.\frac{1}{2}=\frac{3}{2} \qquad \Rightarrow \qquad x=6$$

agin $$y=\frac{2}{2}+\frac{4}{2^2}+\frac{6}{2^3}+\frac{8}{2^4}+\frac{10}{2^5}+........\infty$$

$$\frac{y}{2}=+\frac{2}{2^2}+\frac{4}{2^3}+\frac{6}{2^4}+\frac{8}{2^5}+..........\infty$$

$$\frac{y}{2}=\frac{2}{2}+\left[\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\frac{1}{2^5}+............\right]\Rightarrow \frac{y}{2}=1+2\left(\frac{1/4}{1-(1/2)}\right)=1+1\Rightarrow y=4$$

and $$z=\frac{3}{2}+\frac{3}{2^2}+\frac{3}{2^3}+\frac{3}{2^4}+.............\infty \qquad =3\left(\frac{1/2}{1-(1/2)}\right)=3\Rightarrow z=3.$$

Hence $x+y+z=13\Rightarrow \sigma=13 \therefore 1^3+2^3+3^3+........+(13)^3=\left(\frac{13.14}{2}\right)^2=(91)^2=8281.$

Que. 9. (1024.00) $x^3+px^2+qx+r=0$ (roots a^2, b^2, c^2)

$a^2+b^2+c^2=-p$ ....(1)

$a^2b^2+b^2c^2+c^2a^2=q$ ....(2)

$a^2b^2c^2=-r$ ....(3)

also given $a^2b^2=c^2$ ....(4)

(1) and (4) $\Rightarrow$ $c^2-\frac{p}{2}$ put $x=-\frac{p}{2}$ in the cubic to be t the answer.

Que. 10. (352) Let $x^4-16x^3+px^2-256x+q=0$ has roots x_1,x_2,x_3,x_4 which in order from G.P.

$\Rightarrow x_1x_4 \Rightarrow (x_1+x_4)+(x_2+x_3)=16$ (taken 1 at a time) ....(1)

$x_1x_4(x_2+x_3)+x_2x_3(x_1+x_4)=256$ (taken 3 at a time) ....(2)

$$x_1x_4\left[16-(x_1+x_4)\right]+x_2x_3(x_1+x_4)=256 \Rightarrow 16(x_1x_4)-\underbrace{x_1x_4(x_1+x_4)+x_2x_3(x_1+x_4)}_{\text{using (1)}}=256$$

$x_1x_4=16=x_2x_3 \Rightarrow x_1x_4=x_2x_3=16 \Rightarrow \dfrac{x_1+x_2+x_3+x_4}{4}=\dfrac{16}{4}=4$ form (1)

$(x_1x_2x_3x_4)^{1/4}=(16\times16)^{1/4}=4 \Rightarrow$ A.M. = G.M. $\Rightarrow x_1=x_2=x_3=x_4=4$

$\Rightarrow P=\sum x_1x_2=16\times6=36 \Rightarrow q=x_1x_2x_3x_4=256 \Rightarrow p+q=352.$

Que. 11. (4300) $S=\left(20^3+18^3+.........+2^2\right)-\left(1^3+3^3+5^3+.......+19^3\right)$

$$=2^3\left[1^3+2^3+.......+10^3\right)-\left[\sum_{n=1}^{10}(2n-1)^3\right]=8\left(\frac{10(10-1)}{2}\right)^2-\left[\sum_{n=1}^{10}\left(8n^3-12n^2+6n-1\right)\right]=4300.$$

Que. 12.(15) Let $\left(x^{\log_{10}3}\right)=t$ i.e., $\left(3^{\log_{10}x}\right)=t \;\therefore\; t^2-t-2=0 \Rightarrow (t-2)(t+1)=0 \Rightarrow t=2;\ t=-1$ (not possible) $\left(x^{\log_{10}3}\right)=2;\ x=\left(2^{\log_3 10}\right) \Rightarrow a+b+c=15.$

Que. 13. (866). $(x+y+z)^2=144$ (given)

$\sum x^2+2\sum xy=144 \Rightarrow 96+2\sum xy=144 \Rightarrow \sum xy=24$

again $\dfrac{xy+yz+zx}{xyz}=36 \Rightarrow xyz=\dfrac{24}{36}=\dfrac{2}{3}$ now $x^3+y^3+z^3-3xyz=(x+y+z)\left(\sum x^2-\sum xy\right)$

$\sum x^3-2=(12)(96-24)=(12)(72)=864 \Rightarrow \sum x^2=866.$

Que. 14. (5040). $\times\times\boxed{AB}\times\times\times\times$
C D

$\boxed{AB}$ and 6 other is $7!$ but A and B can be arranged in $2!$ ways $\therefore$ Total ways $=7!.2!$ when C is behind D. $\therefore$ required number of ways $\dfrac{7!.2!}{2!}=5040$ ways.

Que. 15. (5020) $a=f\left(\dfrac{\pi}{6}\right)=\lim\limits_{x\to\frac{\pi}{6}}\dfrac{\sin\left(x-(\pi/6)\right)}{\sqrt{3}-2\cos x}=\lim\limits_{x\to\pi/6}\dfrac{\sin\left(x-(\pi/6)\right)}{2\left(\cos(\pi/6)-\cos x\right)}$

$$=\lim_{x\to\pi/6}\frac{2\sin\left((x/2)-(\pi/12)\right)\cos\left((x/2)-(\pi/12)\right)}{4\sin\left((\pi/12)+(x/12)\right)\sin\left((\pi/12)+(x/2)\right)}=\frac{2}{2}=1 \qquad \text{hence } a=1$$

$$r=\lim_{x\to0}\frac{\sin(x)^{1/3}\ell n(1+3x)}{\left(\dfrac{\tan^{-1}\sqrt{x}}{\sqrt{x}}\right)^2\left(e^{5.x^{1/3}}-1\right)}\frac{\left(5x^{1/3}\right)}{\left(5x^{1/3}\right)}=\lim_{x\to0}\frac{3\ell n(1+3x)^{1/3x}}{5}=\frac{3}{5}$$

$\therefore\ \text{sum}(S)=\frac{a}{1-r}=\frac{1}{1-\frac{3}{5}}=\frac{5}{2}\ \Rightarrow\ 2008\times\frac{5}{2}=5020.$

Que. 16. (128) $A=\left(\left(5+2\sqrt{6}\right)^2\right)^{1/4}=\left(\left(5+2\sqrt{6}\right)\right)^{1/2}=\left[\left(\sqrt{3}+\sqrt{2}\right)^2\right]^{1/2}$ Hence $A=\sqrt{3}+\sqrt{2}$

$B=8\sqrt{3}+\frac{8\sqrt{6}}{\sqrt{3}}+\frac{16}{\sqrt{3}}+..........\infty\ \Rightarrow\ r=\frac{8\sqrt{6}}{\sqrt{3}}.\frac{1}{8\sqrt{3}}=\sqrt{\frac{2}{3}}\ \therefore\ B=\frac{8\sqrt{3}}{1-\left(\sqrt{2}/\sqrt{3}\right)}=\frac{\left(8\sqrt{3}\right)\sqrt{3}}{\sqrt{3}-\sqrt{2}}\Rightarrow B24\left(\sqrt{3}+\sqrt{2}\right)$

hence quadratic equation is

$$\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right)x^2+\frac{24\left(\sqrt{3}+\sqrt{2}\right)}{\left(\sqrt{3}+\sqrt{2}\right)}x+C=0\ \Rightarrow\ x^2+24x+C=0\ ....(1)$$

Now $|\alpha-\beta|=\left(6\sqrt{6}\right)^k\ \Rightarrow\ k=\log_6 10-\log_6 5+\frac{1}{2}\log_6\left(\log_6(18.73)\right)$

$=\log_6 2+\frac{1}{2}\log_6\left(\log_6 1296\right)=\log_6 2+\frac{1}{2}\log_6 4=2\log_6 2=\log_6 4$

$\therefore\ |\alpha-\beta|=\left(6\sqrt{6}\right)^{\log_6 4}=\left((6)^{3/2}\right)^{2\log_6 2}=6^{\log_6 8}=8$ Hence $(\alpha-\beta)^2=64\ \Rightarrow\ (\alpha+\beta)^2-4\alpha\beta=64$

$576-4C=64\ \Rightarrow\ 4C=512\ \Rightarrow\ C=128.$

Que. 17. (179). A : polygraph says person is guility; B : person in innocent $P(B_1)=0.88$

$\Rightarrow P(A/B_1)=0.02.\ P(A/B_2)=0.90\ \Rightarrow P(B_1/A)=\frac{P(B_1).P(A/B_1)}{P(B_1).P(A/B_1)+P(B_2).P(A/B_2)}$

$=\frac{0.88\times0.02}{0.88\times0.02+0.12\times0.90}=\frac{88\times2}{00\times2+12\times90}=\frac{176}{1256}=\frac{22}{157}\ \Rightarrow\ a+b=179.$

Que. 18. (206) $P(W)=\frac{1}{2};\ P(D)=\frac{1}{6};\ P(L)=\frac{1}{3}$ consider any 3 games in which the result of the game has come E : result of the game has come $P(E)=\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$ Now E_1: event when A wins the 1st two games given the result has come (irrespective of the draws between them)

and E_2: event when A wins the 3rd game and wins any one of the first two given that result has come

$\therefore\ P(A\text{ wins})=P(E_1\text{ or }E_2)$

$$=\frac{\frac{1}{2}.\frac{1}{2}}{\left(\frac{5}{6}\right)^2}+\frac{\frac{1}{2}}{\frac{5}{6}}\left[\frac{\frac{1}{2}.\frac{1}{2}}{\left(\frac{5}{6}\right)^2}+\frac{\frac{1}{3}.\frac{1}{2}}{\left(\frac{5}{6}\right)^2}\right]=\frac{1}{4}.\frac{36}{25}+\frac{1}{2}.\frac{6}{5}.\frac{36}{25}.\frac{2}{6}=\frac{9}{25}=\frac{36}{125}=\frac{45+36}{125}=\frac{81}{125}\ \Rightarrow\ 206.$$

Que. 19. (4950) Let $f(x)=(x-a)(x-b)(x-c)$ ....(1) where a, b, c are the root of f(x) = 0 hence the roots of P(x) are a^2,b^2,c^2 $\therefore\ P(x)=k(x-a^2)(x-b^2)(x-c^2)$ for some k ....(2)

put $x-0$ in (2) $P(0)-ka^2b^2c^2=-1$ (given) $ka^2b^2c^2=1$ but abc = 1 form (1) $\Rightarrow$ $k=1$

now, $P(x^2)=(x^2-a^2)(x^2-b^2)(x^2-c^2)=(x-a)(x-b)(x-c)(x+a)(x-b)(x+c)=-f(x).f(-x)$

put $x=2, P(4)=-f(2).f(-2)=-(11)(-9)=99.$

Que. 20. (2009) Let $a=x-t;$ $b=x,\ c=x+t$ and $d=x+2t$

where $x\in I$ and $t>0$ and t is an integer (as a<b<c<d)

now $d=a^2+b^2+c^2 \Rightarrow$ $x+2t=(x-t)^2+x^2+(x+t)^2 \Rightarrow x+2t=3x^2+2t^2$

$\Rightarrow x(1-3x)=2t(t-1)\ ...(1)$

as $t>0$ and $t\in I$ hence $t\geq 1$ $\Rightarrow$ $RHS\geq 0$

$\therefore$ LHS $x(1-3x)\geq 0$ $\Rightarrow$ $x\in\left[0,\frac{1}{3}\right]$ the only integer is zero $\Rightarrow$ $x=0$

$\therefore$ from (1) $t=0$ or $t=1$ $\Rightarrow$ but $t>0$

$\therefore$ $t=1$ $\Rightarrow$ $\therefore$ $a=-1;\ b=0;\ c=1,\ d=2$

$\Rightarrow$ $a+10b+100c+1000d=-1+0+100+2000+2099$ **Ans.**

Que. 21.[2065] $P(I)=\frac{2}{34};$ $P(II)=\frac{5}{34};\ P(III)=\frac{10}{34};\ P(IV)=\frac{17}{34}$

$P(\text{required even})=\frac{2}{34}.\frac{1}{2}+\frac{5}{34}.\frac{2}{5}+\frac{10}{34}.\frac{3}{8}+\frac{17}{34}.\frac{4}{11}]$ [Ans.569/1496]

Urn	W	B
I	1	1
II	2	3
III	3	5
IV	4	7

Que. 22. $\left[(p+q)=262 \text{ where } \frac{p}{q}=\frac{95}{167}\right]$

A: Dr. finds a rash

B_1 : Child has mesles $=\frac{1}{10}$ $P(S/F)=0.9$

B_2 : child has flu $=\frac{9}{10}$ $P(S/M)=0.10$

$P(A/B_1)=0.95$ $P(R/M)=0.95$

$P(A/B_2)=0.08$ $P(R/F)=0.08$

$P(B_1/A)=\frac{0.1\times 0.95}{0.1\times 0.95+0.9\times 0.08}=\frac{0.095}{0.095+0.072}=\frac{0.095}{0.167}=\frac{95}{167}]$