TOPIC = INTEGRAL CALCULUS

Single Correct Type

Que. 1. Let $f(x)=\sin^3 x+\sin^3\left(x+\frac{2\pi}{3}\right)+\sin^3\left(x+\frac{4\pi}{3}\right)$ then the primitive of f(x) w.r.t. x is

(a) $-\frac{3\sin 3x}{4}+C$ (b) $-\frac{3\cos 3x}{4}+C$ (c) $\frac{\sin 3x}{4}+C$ (d) $\frac{\cos 3x}{4}+C$

where C is an arbitrary constant. **(code-V2T3PAQ5)**

Que. 2. If the dependent variable y is changed to 'z' by the substitution y = tan z then the differential equation $\frac{d^2y}{dx^2}=1+\frac{2(1+y)}{1+y^2}\left(\frac{dy}{dx}\right)^2$ is changed to $\frac{d^2z}{dx^2}=\cos^2 z+k\left(\frac{dz}{dx}\right)^2$, then the value of k equals

(a) –1 (b) 0 (c) 1 (d) 2 **(code-V2T3PAQ10)**

Que. 3. $\int x^x \ell n(ex)\,dx$ is equal to **(code-V2T5PAQ1)**

(a) x^x+C (b) $x.\ell n\, x$ (c) $c(\ell n\, x)^x+C$ (d) None

Que. 4. The value of the definite integral $\int_{-2008}^{2008}\frac{f'(x)+f'(-x)}{(2008)^x+1}dx$ equals **(code-V2T5PAQ3)**

(a) $f(2008)+f(-2008)$ (b) $f(2008)-f(-2008)$

(c) 0 (d) $f(-2008)-f(2008)$

Que. 5. $\int_1^e\left(\frac{1}{\sqrt{x\,\ell n\, x}}+\sqrt{\frac{\ell n\, x}{x}}\right)dx$ equals **(code-V2T5PAQ4)**

(a) $\sqrt{e}$ (b) $2e$ (c) $2\sqrt{e}$ (d) $\sqrt{2e}$

Que. 6. If $g(x)=\int_0^x \cos^4 t\,dt$, then $g(x+\pi)$ equals **(code-V2T5PAQ5)**

(a) $g(x)+g(\pi)$ (b) $g(x)-g(\pi)$ (c) $g(x)g(\pi)$ (d) $[g(x)/g(\pi)]$

Que. 7. Let f be a positive function. Let $I_1=\int_{1-k}^{k} x\,f(x(1-x))\,dx;\ I_2=\int_{1-k}^{k} f(x(1-x))\,dx$, where $2k-1>0$.

Then $\frac{I_2}{I_1}$ is **(code-V2T5PAQ7)**

(a) k (b) 1/2 (c) 1 (d) 2

Que. 8. $\int\frac{dx}{x^2\sqrt{16-x^2}}$ has the value equal to **(code-V2T5PAQ8)**

(a) $C-\frac{1}{4}\text{arc sec}\left(\frac{x}{4}\right)$ (b) $\frac{1}{4}\text{arc sec}\left(\frac{x}{4}\right)+C$

(c) $C-\frac{\sqrt{16-x^2}}{16x}$ (d) $\frac{\sqrt{16-x^2}}{16x}+C$

Que. 9. If $\int x^2.e^{-2x}dx = e^{-2x}\left(ax^2+bx+c\right)+d$, then **(code-V2T5PAQ10)**

(a) $a=-\frac{1}{2}, b=-\frac{1}{2}, c=-\frac{1}{4}$ (b) $a=-\frac{1}{2}, b=-\frac{1}{2}, c=-\frac{1}{4}$

(c) $a=-\frac{1}{2}, b=-1, c=-\frac{1}{2}$ (d) $a=1, b=1, c=-\frac{1}{2}$

Que. 10. If $\int_0^1 \tan^{-1}x\,dx = \frac{\pi}{4}-\frac{1}{2}\ln 2$ then the value of the definite integral $\int_0^1 \tan^{-1}\left(1-x+x^2\right)dx$ equals

(a) $\ell n\,2$ (b) $\frac{\pi}{4}+\ell n\,2$ (c) $\frac{\pi}{4}-\ell n\,2$ (d) $2\ell n\,2$ **(code-V2T5PAQ11)**

Que. 11. $\lim_{n\to\infty}\frac{n}{2^n}\int_0^2 x^n dx$ equals **(code-V2T5PAQ12)**

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) 4

Que. 12. If f is continuous function and $F(x)=\int_0^x\left((2t+3).\int_t^2 f(u)du\right)dt$ then $F''(2)$ is equal to

(a) $-7f(2)$ (b) $7f'(2)$ (c) $3f'(2)$ (d) $7f(2)$ **(code-V2T5PAQ15)**

Que. 13. $\int_{\pi/2}^{\pi}\left(x^{\sin x}\right)\left(1+x\cos x\,\ell n\,x+\sin x\right)dx$ is equal to **(code-V2T5PAQ16)**

(a) $\frac{\pi^2}{2}$ (b) $\frac{\pi}{2}$ (c) $\frac{4\pi-\pi^2}{4}$ (d) $\frac{\pi}{2}-1$

Que. 14. Let $f:[0,\infty)\to R$ be a continuous strictly increasing function, such that **(code-V2T5PAQ17)**

$f^3(x)=\int_0^x t.f^2(t)dt$ for every $x \ge 0$. The value of $f(6)$ is

(a) 1 (b) 6 (c) 12 (d) 36

Que. 15. If the value of definite integral $\int_{\pi/6}^{\pi/4}\frac{1+\cot x}{e^x \sin x}dx$, is equal to $ae^{-\pi/6}+be^{-\pi/4}$ then (a + b) equals

(a) $2-\sqrt{2}$ (b) $2+\sqrt{2}$ (c) $2\sqrt{2}-2$ (d) $2\sqrt{3}-\sqrt{2}$ **(code-V2T5PAQ18)**

Que. 16. Let $J=\int_0^\infty\frac{\ell n\,x}{1+x^3}dx$ and $K=\int_0^\infty\frac{x\,\ell n\,x}{1+x^2}dx$ then **(code-V2T5PAQ20)**

(a) $J+K=0$ (b) $J-K=0$ (c) $J+K<0$ (d) none

Que. 17. The value of $x > 1$ satisfying the equation $\int_1^x t \ell n\, t\, dt = \frac{1}{4}$, is **(code-V2T5PAQ22)**

(a) $\sqrt{e}$ (b) e (c) e^2 (d) $e - 1$

Que. 18. If $F(x) = \int_1^x f(t)\,dt$ where $f(t) = \int_1^{t^2} \frac{\sqrt{1+u^4}}{u}\,du$ then the value of $F''(2)$ equals **(code-V2T5PAQ23)**

(a) $\frac{7}{4\sqrt{17}}$ (b) $\frac{15}{\sqrt{17}}$ (c) $\sqrt{257}$ (d) $\frac{15\sqrt{17}}{68}$

Que. 19. Let f be a continuous function on $[a, b]$. If $F(x) = \left(\int_a^x f(t)dt - \int_x^b f(t)dt\right)\left(2x - (a+b)\right)$ then there exist some $c \in (a, b)$ such that **(code-V2T8PAQ6)**

(a) $\int_a^c f(t)dt = \int_c^b f(t)dt$

(b) $\int_a^c f(t)dt - \int_c^b f(t)dt = f(c)(a + b - 2c)$

(c) $\int_a^c f(t)dt - \int_c^b f(t)dt = f(c)\left(2c - (a+b)\right)$

(d) $\int_a^c f(t)dt + \int_c^b f(t)dt = f(c)\left(2c - (a+b)\right)$

Que. 20. The value of the definite integral $I = \int_0^{\pi/2} e^x \left\{\cos(\sin x)\cos^2\frac{x}{2} + \sin(\sin x)\sin^2\frac{x}{2}\right\} dx$, is

(a) $\frac{1}{2}\left[e^{\pi/2}(\cos 1 + \sin 1) - 1\right]$ (b) $\frac{e^{\pi/2}}{2}(\cos 1 + \sin 1)$ **(codeV2T10PAQ2)**

(c) $\frac{1}{2}\left(e^{\pi/2}\cos 1 - 1\right)$ (d) $\frac{e^{\pi/2}}{2}[\cos 1 + \sin 1 - 1]$

(a) $\frac{1}{2}\left[e^{\pi/2}(\cos 1 + \sin 1) - 1\right]$ (b) $\frac{e^{\pi/2}}{2}(\cos 1 + \sin 1)$

(c) $\frac{1}{2}\left(e^{\pi/2}\cos 1 - 1\right)$ (d) $\frac{e^{\pi/2}}{2}[\cos 1 + \sin 1 - 1]$

Que. 21. A tank with a capacity of 1000 liters originally contains 100 gms of salt dissolved in 400 liters of water. Beginning at time $t = 0$ and ending at time $t = 100$ minutes, water containing 1 gm of salt per liters enters the tank at the of 4 liters per minute, and the well mixed solution is drained from the tank at a rate of 2 liter/minute. The differential equation for the amount of salt y in the tank at time t is

(a) $\frac{dy}{dt} = 4 - \frac{y}{400 + 2t}$ (b) $\frac{dy}{dt} = 4 - \frac{y}{200 + t}$ **(code-V2T11PAQ2)**

(c) $\frac{dy}{dt} = 4 - \frac{y}{200(t+2)}$ (d) $\frac{dy}{dt} = 4 - \frac{y}{500 + t}$

Que. 22. Let $y = y(t)$ be solution to the differential equation $y' + 2ty = t^2$, then $\lim_{t\to\infty}\frac{y}{t}$ is **(code-V2T11PAQ4)**

(a) zero (b) $\frac{1}{2}$ (c) 1 (d) Non existent.

Que. 23. The area of the region bounded below by $y = \sin^{-1}x$, above by $y = \cos^{-1}x$ and on the left by y-axis, is

(a) $\sqrt{2}-1$ (b) $2-\sqrt{2}$ (c) $\sqrt{2}+1$ (d) $\sqrt{2}$ **(code-V2T11PAQ7)**

Que. 24. $\int_0^2 \sqrt{\frac{x}{4-x}}\,dx$ is equal to **(code-V2T13PAQ20)**

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{2}-1$ (c) $\pi-1$ (d) $\pi-2$

Que. 25. If $\int_{-2}^{2} x^4.\sqrt{4-x^2}\,dx$ has the value equal to $k\pi$ then the value of k equals **(code-V2T14PAQ20)**

(a) 0 (b) 2 (c) 8 (d) 4

Que. 26. Let $\int \frac{dx}{x^{2008}+x} = \frac{1}{p}\ell n\left(\frac{x^q}{1+x^r}\right)+C$ where $p, q.r \in N$ and need not be distinct, then the value of $(p+q+r)$ equals **(code-V2T14PAQ25)**

(a) 6024 (b) 6022 (c) 6021 (d) 6020

Que. 27. $\int_0^{\pi/2} (\sin x)^x \left(\ell n(\sin x) + x\cot x\right) dx$ is **(code-V2T17PAQ7)**

(a) – 1 (b) 1 (c) 0 (d) Indeterminant

Que. 28. Let $y = \ell n(1+\cos x)^2$ then the value of $\frac{d^2y}{dx^2}+\frac{2}{e^{y/2}}$ equals **(code-V2T17PAQ8)**

(a) 0 (b) $\frac{2}{1+\cos x}$ (c) $\frac{4}{(1+\cos x)}$ (d) $\frac{-4}{(1+\cos x)^2}$

Que. 29. The value of definite integral $\int_{-1}^{1}\frac{dx}{(1+e^x)(1+x^2)}$ is **(code-V2T18PAQ1)**

(a) $\pi/2$ (b) $\pi/4$ (c) $\pi/8$ (d) $\pi/16$

Que. 30. The expression $y^3\frac{d^2y}{dx^2}$ on the ellipse $3x^2+4y^2=12$ is equal to **(code-V2T19PAQ5)**

(a) $\frac{9}{4}$ (b) $-\frac{9}{4}$ (c) $\frac{4}{9}$ (d) $-\frac{4}{9}$

Que. 31. The value of the definite integral $\int_0^{n\pi}\frac{x|\sin x|}{1+|\cos x|}dx\,(n\in N)$ is equal **(code-V2T19PAQ6)**

(a) $n^2\pi\ell n 2$ (b) $n\pi^2\ell n 2$ (c) $n\pi\ell n 2$ (d) $\frac{n^2\pi\ell n 2}{2}$

Comprehesion Type

1 Paragraph for Q. 1 to Q. 3

A curve in represented parametrically by the equations $x = e^t \cos t$ and $y = e^t \sin t$ is a parameter. Then

1. The relation between the parameter 't' and the angle α between the tangent to the given curve and the x-axis is given by, 't' equals **(code-V2T4PAQ1,2,3)**

(a) $\frac{\pi}{2} - \alpha$ (b) $\frac{\pi}{4} + \alpha$ (c) $\alpha - \frac{\pi}{4}$ (d) $\frac{\pi}{4} - \alpha$

2. The value of $\frac{d^2y}{dx^2}$ at the point where $t = 0$ is

(a) 1 (b) 2 (c) – 2 (d) 3

3. If $F(t) = \int (x + y)\,dt$ the point the value of $F\left(\frac{\pi}{2}\right) - F(0)$ is

(a) 1 (b) – 1 (c) $e^{\pi/2}$ (d) 0

2 Paragraph for Q. 4 to Q. 6

Let f (x) is a derivable function satisfying $x f(x) - \int_0^x f(t)\,dt = x + \ell n\left(\sqrt{x^2+1} - x\right)$ with $f(0) = \ell n\ 2$. Let

$g(x) = xf'(x)$ then **(code-V2T4PAQ4,5,6,)**

4. Range of g (x) is

(a) $[0, \infty)$ (b) $[0, 1)$ (c) $[1, \infty)$ (d) $[-\infty, \infty)$

5. For the function f which one of the following is correct ?

(a) f is neither odd nor even (b) f is transcendental
(c) f is injective (d) f is symmetric w.r.t. origin.

6. $\int_0^1 f(x)\,dx$ equals

(a) $\ell n\left(3 + 2\sqrt{2}\right) - 1$ (b) $2\,\ell n\left(1 + \sqrt{2}\right)$ (c) $\ell n\left(1 + \sqrt{2}\right) - 1$ (d) 1

3 Paragraph for Q. 7 to Q. 9

Suppose a and b are positive real numbers such that $ab = 1$. Let for any real parameter t, the distance from the origin the line $\left(ae^t\right)x + \left(be^{-t}\right)y = 1$ be denoted by D(t) then **(code-V2T16PAQ4.5.6)**

7. The value of the definite integral $I = \int_0^1 \frac{dt}{\left(D(t)\right)^2}$ is equal to

(a) $\frac{e^2-1}{2}\left(b^2 + \frac{a^2}{e^2}\right)$ (b) $\frac{e^2+1}{2}\left(a^2 + \frac{b^2}{e^2}\right)$ (c) $\frac{e^2-1}{2}\left(a^2 + \frac{b^2}{e^2}\right)$ (d) $\frac{e^2+1}{2}\left(b^2 + \frac{a^2}{e^2}\right)$

8. The value of 'b' at which I is minimum, is

(a) e (b) $\frac{1}{e}$ (c) $\frac{1}{\sqrt{e}}$ (d) $\sqrt{e}$

9. Minimum value of I is

(a) $e - 1$ (b) $e - \frac{1}{e}$ (c) e (d) $e + \frac{1}{r}$

Assertion & Reason Type

In this section each que. contains STATEMENT-1 (Assertion) & STATEMENT-2(Reason).Each question has 4 choices (A), (B), (C) and (D), out of which **only one is correct.**

Bubble (A) STATEMENT-1 is true, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1.

Bubble (B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1.

Bubble (C) STATEMENT-1 is True, STATEMENT-2 is False.

Bubble (D) STATEMENT-1 is False, STATEMENT-2 is True.

Que. 1. Statment 1: Let $f(x)=\int_0^x \sqrt{1+t^2}\,dt$ is odd function and $g(x)=f'(x)$ is an even function.

because **(code-V2T10PAQ8)**

Statement 2: For a differentiable function $f(x)$ if $f'(x)$ is an even function then f(x) is an odd function.

Que. 2. Statement 1: The solution of $(y\,dx - x\,dy)\cot\left(\frac{x}{y}\right) = ny^2\,dx$ is $\sin\left(\frac{x}{y}\right) = ce^{nx}$ **(code-V2T11PAQ11)**

because

Statement 2: Such type of differential equations can only be solved by the substitution $x = vy$.

Que. 3. Consider the following statements **(code-V2T16PAQ10)**

Statement 1: $\int_{-1}^{3}\frac{dx}{x^2} = \left.\frac{x^{-1}}{-1}\right]_{-1}^{3} = -\frac{1}{3}-1 = -\frac{4}{3}$

because

Statement 2: If f is continuous on [a,b] then $\int_a^b f(x)dx = F(b)-F(a)$ where F is any antiderivative of f, that is $F'=f$.

More than One May Correct Type

Que. 1. Which of the following definite intetgral(s) has/have their value equal to the value of atleast one of the remaining three ? **(code-V2T4PAQ13)**

(a) $\int_{-\pi/6}^{0}\sqrt{\frac{1+\sin t}{1-\sin t}}.\cos t\,dt$

(b) $\int_{-\pi/6}^{0}\left(\frac{\cos(t/2)+\sin(t/2)}{\cos(t/2)-\sin(t/2)}\right)\cos t\,dt$

(c) $\int_{-\pi/6}^{0}\left(\cos\frac{t}{2}+\sin\frac{t}{2}\right)^2 dt$

(d) $\int_{3/2}^{2}\sqrt{\frac{x-1}{3-x}}dx$

Que. 2. Which of the following definite integral vanishes ? **(code-V2T4PAQ16)**

(a) $\int_{1/2}^{2}\frac{x^n-1}{x^{n+2}+1}dx\,(n\in N)$

(b) $\int_{2}^{4}\left[\log_x 2-\frac{(\log_x 2)^2}{\ell n\,2}\right]dx$

(c) $\int_{1\backslash 2}^{2}\frac{1}{x}\sin\left(x-\frac{1}{x}\right)dx$

(d) $\int_0^{\pi}\cos mx.\sin nx\,dx$, where $(m, n\in I)$ and $(m-n)$ is even integer.

Que. 3. The function f is continuous and has the property $f(f(x))=1-x$ for all $x\in[0,1]$ and $J=\int_0^1 f(x)dx$ then

(a) $f\left(\frac{1}{4}\right)+f\left(\frac{3}{4}\right)=1$ (b) the value of J equal to $\frac{1}{2}$ **(code-V2T8PAQ14)**

(c) $f\left(\frac{1}{3}\right).f\left(\frac{2}{3}\right)=1$ (d) $\int_0^{\pi/2}\frac{\sin x\,dx}{(\sin x+\cos x)^3}$ has the same value as J.

Que. 4. The differential equation corresponding to the family of curves $y=A\cos(Bx+D)$, is

(a) of order 3 (b) of order 2 (c) degree 2 (d) degree 1 **(code-V2T11PAQ13)**

Que. 5. $\int\frac{x\,dx}{x^4+x^2+1}$ equals **(code-V2T17PAQ13)**

(a) $\frac{2}{3}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)+C$ (b) $\frac{1}{\sqrt{3}}\left[\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right]+C$

(c) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)+C$ (d) $\frac{1}{\sqrt{3}}\left[\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)+\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right]+C$

where C is an arbitrary constant.

Que. 6. If the independent variable x is changed to y then the differential equation $x\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2-\frac{dy}{dx}=0$ is changed to $x\frac{d^2x}{dy^2}\left(\frac{dx}{dy}\right)^2=k$ where k equals **(code-V2T17PAQ16)**

(a) 0 (b) 1 (c) – 1 (d) $\frac{dx}{dy}$

Que. 7. Let $L=\lim_{n\to\infty}\int_a^{\infty}\frac{n\,dx}{1+n^2x^2}$ where $a\in R$ then L can be **(code-V2T19PAQ9)**

(a) π (b) $\frac{\pi}{2}$ (c) 0 (d) 1

Subjective Type (Up to 4 digit)

Que. 1. If the value of the definite integral $\int_0^1 {}^{2007}C_7 x^{200}.(1-x)^7\,dx$ is equal to $\frac{1}{k}$ where $k\in N$. Find k.

(code-V2T18PDQ2)

[SOLUTION]

Single Correct Type

Que. 1. D. Note that

$$\sin x+\sin\left(x+\frac{2\pi}{3}\right)+\sin\left(x+\frac{4\pi}{3}\right)=0 \Rightarrow \sin^3 x+\sin^3\left(x+\frac{2\pi}{2}\right)+\sin^3\left(x+\frac{4\pi}{3}\right)=-\frac{3}{4}\sin 3x$$

$$\left(a+b+c=0 \Rightarrow a^3+b^3+c^3=3abc\right) \qquad \therefore \qquad -\frac{3}{4}\int \sin 3x\,dx=\frac{\cos 3x}{4}+C.$$

Que. 2. D. Given $y=\tan z \qquad \Rightarrow \frac{dy}{dx}=\sec^2 z.\frac{dz}{dx}$ ....(1)

Now $\frac{d^2y}{dx^2}=\sec^2 z.\frac{d^2z}{dx^2}+\frac{dz}{dx}.\frac{d}{dx}(\sec^2 z)$ [using product rule] $=\sec^2 z.\frac{d^2z}{dx^2}+\frac{dz}{dx}.\frac{d}{dz}(\sec^2 z).\frac{dz}{dx}$

$$\frac{d^2y}{dx^2}=\sec^2 z.\frac{d^2z}{dx^2}+\left(\frac{dz}{dx}\right)^2.2\sec^2 z.\tan z \qquad ....(2)$$

Now $1+\frac{2(1-y)}{1+y^2}\left(\frac{dy}{dx}\right)^2=1+\frac{2(1+\tan z)}{\sec^2 z}.\sec^4 z.\left(\frac{dz}{dx}\right)^2=1+2(1+\tan z).\sec^2 z.\left(\frac{dz}{dx}\right)^2$

$$=1+2\sec^2 z\left(\frac{dz}{dx}\right)^2+2\tan z.\sec^2 z\left(\frac{dz}{dx}\right)^2 \qquad ....(3)$$

From (2) and (3) we have RHS of(2) = (3)

$$\sec^2 z.\frac{d^2z}{dx^2}=1+2\sec^2 z\left(\frac{dz}{dx}\right)^2 \quad \Rightarrow \quad \frac{d^2z}{dx^2}=\cos^2 z+2\left(\frac{dz}{dx}\right)^2 \quad \Rightarrow \quad k=2.$$

Que. 3. A. $I=\int x^x(\ell n\, ex)dx=\int x^x(1+\ell n\, x)dx$ Let $t=x^x=e^{x\ell n x} \Rightarrow \frac{dt}{dx}=x^x(1+\ell n\, x)dx$

$\Rightarrow \qquad I=\int dt=t+C=x^x+C$

Que. 4. B. $I=\int_{-a}^{a}\frac{f'(x)+f'(-x)}{a^x+1}dx$; use King and add $\Rightarrow$ Result

Que. 5. C. $I=\int_1^e \frac{1+\ell n\, x}{\sqrt{x\,\ell n\, x}}dx$ put $x\,\ell n\, x=t^2 \quad \Rightarrow (\ell n\, x+1)dx=2t\,dt \Rightarrow I=\int_0^{\sqrt{e}}\frac{2t\,dt}{t}=2\sqrt{e}$

Que. 6. A. $g(x+\pi)=\int_0^{x+\pi}\cos^4 t\,dt=\int_0^x \cos^4 t\,dt+\int_x^{x+\pi}\cos^4 t\,dt=g(x)+\int_0^{\pi}\cos^4 t\,dt=g(x)+g(\pi).$

Que. 7. D. $I_1=\int_{1-k}^{k}xf\left(x(1-x)\right)dx; I_2=\int_{1-k}^{k}f\left(x(1-x)\right)dx$ Using King

$$I_1=\int_{1-k}^{k}(1-k)f\left(x(1-x)\right)dx \Rightarrow 2I_1=\int_{1-k}^{k}f\left(x(1-x)\right)dx \Rightarrow 2I_1=\int_{1-k}^{k}f\left(x(1-x)\right)dx=I_2 \quad \therefore \frac{I_2}{I_1}=2.$$

Que. 8. C. $\int \frac{1}{x^2\sqrt{16-x^2}}dx$ put $x=\frac{1}{t^2}dt=\int \frac{-\frac{1}{t^2}dt}{\frac{1}{t}\times\frac{1}{t^2}\sqrt{16t^2-1}}=\int\frac{-t\,dt}{\sqrt{16t^2-1}}$

Let $16t^2-1=u^2; 32t\,dt 2u\,du; t\,dt=\frac{u}{16}du$ $\quad =-\frac{1}{16}\int\frac{u\,du}{u}=-\frac{u}{16}+C=-\sqrt{\frac{16-x^2}{16\,x}}+C.$

Que. 9. A. $\int x^2.e^{-2x}dx=e^{-2x}\left(ax^2+bx+c\right)+d$ differentiating both sides, we get

$x^2.e^{-2x}=e^{-2x}\left(2ax+b\right)+\left(ax^2+bx+c\right)\left(-2e^{-2x}\right)=e^{-2x}(-2ax^2+2(a-b)x+b-2c)$

$\Rightarrow\ a=-\frac{1}{2}, 2\left(a-b\right)=0, b-2c=0 \quad\Rightarrow\quad a=-\frac{1}{2}, b=-\frac{1}{2}, c=-\frac{1}{4}.$

Que. 10. A. $I=\int_0^1\tan^{-1}(1-x+x^2)dx=\int_0^1\cot^{-1}\left(\frac{1}{1-x+x^2}\right)dx=\int\left(\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{1-x+x^2}\right)\right)dx$

$=\frac{\pi}{2}-\int_0^1\tan^{-1}\left(\frac{1}{1-x+x^2}\right)dx=\frac{\pi}{2}-\left[\int_0^1\tan^{-1}\left(\frac{x+1-x}{1-x\left(1-x\right)}\right)dx\right]$

$=\frac{\pi}{2}-\left[\int_0^1\tan^{-1}x dx+\int_0^1\tan^{-1}\left(1-x\right)dx\right]=\frac{\pi}{2}-2\int_0^1\tan^{-1}x\,dx=\frac{\pi}{2}-2\left[\frac{\pi}{4}-\frac{1}{2}\ell n\,2\right]=\ell n\,2.$

Que. 11. B. $\lim_{n\to\infty}\frac{n}{2^n}.\frac{x^{n+1}}{n+1}\Big]_0^2=\lim_{n\to\infty}\frac{n}{2^n}.\frac{2^{n+1}}{n+1}\Rightarrow\lim_{n\to\infty}2.\frac{1}{1+(1/n)}=2.$

Que. 12. A. $F'(x)=(2x+3)\int_x^2 f(u)\,du\ \ \therefore\ \ f''(x)=-(2x+3)f(x)+\left(\int_x^2 f(u)du\right).2$

$F''(2)=-7f(2)+0 \quad\Rightarrow\quad -7f(2).$

Que. 13. C. Integrand is $\left(x^{\sin x}.x\right)'$ $\quad\therefore\quad \int\left(x^{\sin x}.x\right)=x^{\sin x}.x\Big|_{\pi/2}^{\pi}=\pi^0.\pi-\frac{\pi}{2}.\frac{\pi}{2}=\pi-\frac{\pi^2}{4}=\frac{4\pi-\pi^2}{4}.$

Que. 14. B. Given $f^3(x)=\int_0^x t.f^2(t)dt$ differentiating, $3f^2(x)f'(x)=x\,f^2(x)\ \Rightarrow\ f(x)\neq 0\ \Rightarrow\ f'(x)=\frac{x}{3};$

$f(x)=\frac{x^2}{6}+C$ But $f(0)=0\ \Rightarrow\ C=0\ \Rightarrow\ f(6)=6.$

Que. 15. A. $\int_{\pi/6}^{\pi/4}e^{-x}\left(\operatorname{cosec}x+\cot x\operatorname{cosec}x\right)dx;$ put $-x=t;\ dx=-dt$

$\int_{-\pi/6}^{\pi/4}e^t\left(-\operatorname{cosec}(t)+\cot(t).\operatorname{cosec}(t)\right)dt=\int_{-\pi/6}^{-\pi/4}e^t\left(\operatorname{cosec}(t)-\cot(t).\operatorname{cosec}(t)\right)dt$

$=e^t\operatorname{cosec}(t)\Big]_{-\pi/6}^{-\pi/4}=-\sqrt{2}e^{-\frac{\pi}{4}}+2e^{-\frac{\pi}{6}}\quad\Rightarrow\quad 2e^{-\frac{\pi}{6}}-\sqrt{2}e^{-\frac{\pi}{4}}\quad\Rightarrow\quad a+b=2-\sqrt{2}.$

Que. 16. A. $J+K=\int_0^\infty \frac{(x+1)\ell n\, x}{1+x^3}dx=\int_0^\infty \frac{\ell n\, x\, dx}{x^2-x+1}$ start $x=\frac{1}{t}$ $\Rightarrow$ $J+K=-(J+K)\Rightarrow$ $J+K=0.$

Que. 17. A. $I=\int_1^x t\,\ell n\, t\, dt=\ell n\, t.\frac{t^2}{2}\Big]_1^x-\frac{1}{2}\int_1^x\frac{1}{t}.t^2dt=\frac{x^2}{2}\ell nx-\frac{1}{2}\left[\frac{t^2}{2}\right]_1^x=\frac{x^2\ell n\, x}{2}-\frac{1}{4}\left[x^2-1\right]=\frac{1}{4}$

$\therefore$ $\frac{x^2\ell n\, x}{2}-\frac{1}{4}x^2=0$ $\Rightarrow$ $[2\ell n\, x-1]=0(\text{as } x>1)\Rightarrow \ell n\, x=\frac{1}{2}$ $\Rightarrow$ $x=\sqrt{e}.$

Que. 18. C. $f'(t)=\frac{\sqrt{1+t^8}.2t}{t^2}=\frac{2.\sqrt{1+t^8}}{t}$ .... (1)

Now $F(x)=\int_1^x f(t)du$ $\Rightarrow$ $F'(x)=f(x)$ $\Rightarrow$ $F''(x)=f''(x)$ $\Rightarrow$ $F''(2)=f'(2)$

Form (1) $f'(2)=\sqrt{256+1}=\sqrt{257}$.

Que. 19. B. Given $F(x)=\left(\int_a^x f(t)dt-\int_x^b f(t)\,dt\right)(dx-(a+b))$ .... (1)

as f is continuous hence F(x) is also continuous. Also put x = a.

$F(a)=\left(-\int_a^b f(t)\,dt\right)(a-b)=(b-a)\int_a^b f(t)\,dt$ and put $x=b$ $F(b)=\left(\int_a^b f(t)\,dt\right)(b-a)$

hence F (a) = F(b) hence Roll's theorem is applicable to F (x)

$\therefore$ $\exists\text{some}\, c\in(a,b)$ such that $F'(c)=0$

Now $F'(x)=2\left(\int_a^x f(t)\,dt-\int_x^b f(t)\,dt\right)+(2x-(a+b))[f(x)+f(x)]=0$

$\therefore$ $F'(c)\left(\int_a^c f(t)\,dt-\int_c^b f(t)\,dt\right)=f(c)[(a+b)-2c]$

Que. 20. A. $I=\int_0^{\pi/2} e^x\left\{\cos(\sin x)\left(\frac{1+\cos x}{2}\right)+\sin(\sin x)\left(\frac{1-\cos x}{2}\right)\right\}dx$

$=\frac{1}{2}\int_0^{\pi/2} e^x\left[\underbrace{\{\cos(\sin x)+\sin(\sin x)\}}_{f(x)}+\underbrace{\cos x\{\cos(\sin x)-\sin(\sin x)\}}_{f'(x)}\right]dx=\frac{1}{2}\left[e^{\pi/2}(\cos 1+\sin 1)-1\right].$

Que. 21. B. $\frac{dy}{dt}=(1)(4)-\left(\frac{y}{400+2t}\right)2=4-\frac{y}{200+t}$

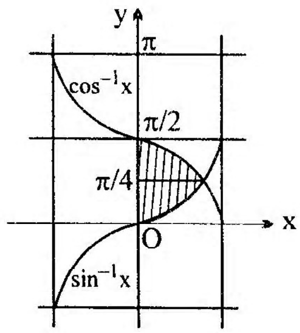
Que. 22. B. $\frac{dy}{dt}=(1)(4)-\left(\frac{y}{400+2t}\right)2=4-\frac{y}{200+t}$

Que. 23. (B) $\int_{\pi/4}^{\pi/2} \cos y\, dy + \int_{0}^{\pi/4} \sin y\, dy$

$= \sin y]_{\pi/4}^{\pi/2} - \cos y]_{0}^{\pi/4}$

$= \left(1 - \frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}} - 1\right)$

$= 2 - \sqrt{2}.$



Que. 24. D. $x = 4\sin^2\theta \Rightarrow dx = 8\sin\theta.\cos\theta\, d\theta \Rightarrow I = \int_0^{\pi/2} \frac{\sin\theta}{2\cos\theta} 8\sin\theta.\cos\theta d\theta = 8\int_0^{\pi/4} \sin^2\theta d\theta$

$\int_0^{\pi/4} 2\sin^2\theta d\theta = 4\left[\theta - \frac{1}{2}\sin 2\theta\right]_0^{\pi/4} = 4\left[\frac{\pi}{4} - \frac{1}{2}\right] = \pi - 2.$

Que. 25. D. $I = 2\int_0^2 x^4.\sqrt{4 - x^2}dx$ put $x = 2\sin\theta \Rightarrow 128\int_0^{\pi/2} \sin^4\theta\cos^2\theta d\theta.$ Use Walli's formula to get 4π.

Que. 26. C. $I = \int \frac{dx}{x\left(x^{2007} + 1\right)} = \int \frac{x^{2007} + 1 - x^{2007}}{x(x^{2007} + 1)}dx = \int\left(\frac{1}{x} - \frac{x^{2006}}{1 + x^{2007}}\right)dx$

$k = \ell n\, x - \frac{1}{2007}\ell\left(1 + x^{2007}\right) = \frac{\ell n\, x^{2007} - \ell n\left(1 + x^{2007}\right)}{2007} = \frac{1}{2007}\ell n\left(\frac{x^{2007}}{1 + x^{2007}}\right) + C \Rightarrow p + q + r = 6021.$

Que. 27. C. $I = \int_0^{\pi/2} \frac{d}{dx}\left((\sin x)^x\right)dx = (\sin x)^x\Big|_0^{\pi/2} = 1 - \lim_{x\to 0}(\sin x)^x = 1 - 1 = 0$

Que. 28. A. $y = 2\ell n(1 + \cos x) \Rightarrow y_1 \frac{-2\sin x}{1 + \cos x} \Rightarrow y_2 = -2\left[\frac{(1 + \cos x)\cos x - \sin x(-\sin x)}{(1 + \cos x)^2}\right]$

$= -2\left[\frac{\cos x + 1}{(1 + \cos x)^2}\right] = \frac{-2}{(1 + \cos x)} \quad \therefore\; 2e^{-y/2} = 2.e^{\frac{\ell n(1+\cos x)^2}{2}} = \frac{2}{(1 + \cos x)} \quad \therefore\; y_2 + \frac{2}{e^{y/2}} = 0.$

Que. 29. B. $I = \int_{-1}^{1} \frac{dx}{\left(1 + e^x\right)\left(1 + x^2\right)}$ ....(1) $\qquad = \int_{-1}^{1} \frac{dx}{1 + e^{-x}}.\frac{1}{1 + x^2}$ (using King)

$I = \int_{-1}^{1} \frac{dx}{\left(1 + e^x\right)\left(1 + x^2\right)}$ ....(2) $\qquad$ adding (1) and (2)

$2I = \int_{-1}^{1} \frac{\left(1 + e^x\right)dx}{\left(1 + e^x\right)\left(1 + x^2\right)} = \int_{-1}^{1} \frac{dx}{\left(1 + x^2\right)} = 2\int_0^1 \frac{dx}{\left(1 + x^2\right)}$

$I = \int_0^1 \frac{dx}{\left(1 + x^2\right)} = \tan^{-1}(1) = \pi/4$ $\qquad$ [convert it into value of definite integral T is same as]

Que. 30. B. Differentiating implicity we have

$$6x+8yy'=0 \quad \text{and hence } y'=-\frac{3x}{4y};\ 4\left[yy''+(y')^2\right]=-3$$

differentiating again and substitute for y' we have

$$3+4(y')^2+4yy''=0 \text{ and hence } 3+\frac{9x^2}{4y^2}+4yy''=0$$

multiplying by y^2, $3y^2+\frac{9x^2}{4}+4y^3\frac{d^2y}{dx^2}=0 \Rightarrow \frac{3y^2}{4}+\frac{9x^2}{16}+y^3y''=0 \Rightarrow \frac{3}{16}\left(3x^2+4y^2\right)+y^3y''=0$

but $3x^2+4y^2=12$ and hence $y^3y'''=-\frac{9}{4}$ at every point on the ellipse]

Que. 31. A. $I=\int_0^{n\pi}\frac{x|\sin x|}{1+|\cos x|}dx$ ....(1)

or $I=\int_0^{n\pi}\frac{(n\pi-x)|\sin x|}{1+|\cos x|}dx$ ....(2)

add (1) and (2)

$$2I=n\pi\int_0^{n\pi}\frac{|\sin x|}{1+|\cos x|}dx \Rightarrow 2I=n^2\pi\int_0^{\pi}\frac{\sin x}{1+|\cos x|}dx \quad \left(\text{Using } \int_0^{na}f(x)dx=n\int_0^{a}f(x)dx\right)$$

$$I=\frac{n^2\pi}{2}.2\int_0^{\pi/2}\frac{\sin x dx}{1+\cos x}=n^2\pi\int_0^{\pi/2}\frac{\sin x dx}{1+\cos x}=n^2\pi\int_0^{\pi/2}\frac{\cos x dx}{1+\sin x}$$

$=n^2\pi.\ln(1+\sin x)]_0^{\pi/2}=n^2\pi\ln 2$ **Ans.**

Comprehesion Type

1 Paragraph for Q. 1 to Q. 3

1. - C. **2. - B.** **3. - C.**

(i) $y=e^t\sin t \Rightarrow \frac{dy}{dt}=e^t[\cos t+\sin t] \Rightarrow x=e^t\cos t \Rightarrow \frac{dx}{dt}=e^t[\cos t\sin t]$

$\therefore \frac{dy}{dt}=\frac{\cos t+\sin t}{\cos t-\sin t}=\tan\alpha \quad \therefore \tan\left(\frac{\pi}{4}+t\right)=\tan\alpha \Rightarrow \left(\frac{\pi}{4}+t\right)\alpha \Rightarrow t=\alpha-\frac{\pi}{4}.$

(ii) $\frac{d^2y}{dx^2}=\frac{\sec^2\left(\frac{\pi}{4}+t\right)}{e^t(\cos t-\sin t)} \Rightarrow \left.\frac{d^2y}{dx^2}\right|_{t=0}=2.$

(iii) $F(t)=\int e^t(\cos t+\sin t)dt=e^t\sin t+C \Rightarrow F\left(\frac{\pi}{2}\right)-F(0)=\left(e^{\pi/2}+C\right)-0=e^{\pi/2}.$

 # 2 Paragraph for Q. 4 to Q. 6

4. - B. **5. - B.** **6. - A.**

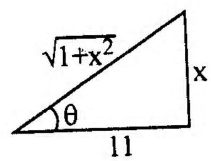
(I) $x\,f(x)-\int_0^x f(t)dt = x+\ell n\left(\sqrt{x^2+1}-x\right)$ differentiating $x\,f'(x)-f(x)=1+\dfrac{\dfrac{x}{\sqrt{x^2+1}}-1}{\left(\sqrt{x^2+1}-x\right)}$

$\Rightarrow \quad x\,f'(x)=1-\dfrac{1}{\sqrt{x^2+1}} \quad \therefore \quad$ range of $g(x)=x\,f'(x)$ is $[0,1)$.

$f'(x)=\dfrac{\sqrt{x^2+1}-1}{x\sqrt{x^2+1}} \quad \Rightarrow \quad f'(x)$ is odd $\Rightarrow$ $f(x)$ is even.

Integrating (1), i.e. $f'(x)=\dfrac{1}{x}-\dfrac{1}{x\sqrt{x^2+1}} \Rightarrow f(x)=\int\dfrac{dx}{x}-\int\dfrac{dx}{x\sqrt{x^2+1}} \Rightarrow f(x)=\ell n\,(x)-I,$

where $I=\int\dfrac{dx}{x\sqrt{x^2+1}}$; put $x=\tan\theta \quad \Rightarrow \quad dx=\sec^2 d\theta$



$I=\int\dfrac{\sec^2\theta d\theta}{\tan\theta\sec\theta}=\int\mathrm{cosec}\,\theta d\theta=\ell n\left(\mathrm{cosec}\theta-\cot\theta\right)=\ell n\left(\dfrac{\sqrt{1+x^2}-1}{x}\right)+C$

$\therefore \quad f(x)\ell n\,x-\ell n\left(\dfrac{\sqrt{1+x^2}-1}{x}\right)+C \quad \therefore \quad f(x)=\ell n\,x-\ell n\left(\dfrac{x}{\sqrt{1+x^2}+1}\right)+C$

$f(x)=\ell m\left(\sqrt{1+x^2}+1\right)+C;$ put $x=0, f(0)\ell n\,2 \quad \Rightarrow \quad f(0)=\ell n\,2 \Rightarrow C=0$

$\Rightarrow f(x)=\ell n\left(\sqrt{1+x^2}+1\right).$

Now $\int_0^1 f(x)\,dx=\int_0^1 \underbrace{1}_{II}.\underbrace{\ell n\left(\dfrac{1}{\sqrt{x^2+1}}\right)}_{I}dx$

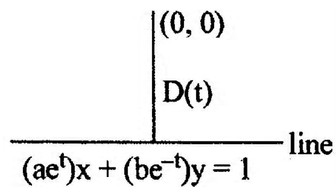
integrating by parts $f(x).x\Big]_0^1-\int_0^1 x\,f'(x)\,dx=f(1)-\int_0^1\left(1-\dfrac{1}{\sqrt{x^2+1}}\right)dx$

$=\ell n\left(1+\sqrt{2}\right)-\left[x-\ell n\left(x+\sqrt{x^2+1}\right)\right]_0^1=\ell n\left(1+\sqrt{2}\right)-\left[x-\ell n\left(x+\sqrt{x^2+1}\right)\right]_0^1$

$=\ell n\left(1+\sqrt{2}\right)-\left[\left\{1-\ell n\left(1+\sqrt{2}\right)-\{0\}\right\}\right] \quad =2\ell n\left(1+\sqrt{2}\right)-1=\ell n\left(2+2\sqrt{2}\right)-1.$

 # 3 Paragraph for Q. 7 to Q. 9

7. **C.** **8.** **D.** **9.** **B.**



(i) $D(t)=\left|\frac{1}{\sqrt{\left(ae^{t}\right)^{2}+\left(be^{-t}\right)^{2}}}\right|=\frac{1}{\sqrt{a^2e^{2t}+b^2e^{-2t}}} \Rightarrow \frac{1}{(D(t))^2}=\left(a^2e^{2t}+b^2e^{-2t}\right)$

$\therefore \quad I=\int_0^1\left(a^2e^{2t}+b^2e^{-2t}\right)dt=\left[\frac{a^2e^{2t}}{2}-\frac{b^2e^{-2t}}{2}\right]_0^1=\left(\frac{a^2e^2-b^2e^{-2}}{2}\right)-\left(\frac{a^2-b^2}{2}\right)$

$=\frac{a^2\left(e^2-1\right)-b^2\left(e^{-2}-1\right)}{2}=\frac{a^2\left(e^2-1\right)+\frac{b^2}{e^2}\left(e^2-1\right)}{2}=\frac{e^2-1}{2}\left(a^2+\frac{b^2}{e^2}\right).$

(ii) Now put $I=\frac{1}{a} \Rightarrow I=\frac{e^2-1}{2}\left(a^2+\frac{1}{a^2e^2}\right)=\frac{e^2-1}{2}\left(\left(a-\frac{1}{ae}\right)^2+\frac{2}{e}\right)$ I is minimum if $a=\frac{1}{ae}$

$\Rightarrow a^2=\frac{1}{e} \Rightarrow a=\frac{1}{\sqrt{e}} \Rightarrow b=\sqrt{e}.$

(iii) and $I_{min}=\frac{e^2-1}{2}\frac{2}{e}=e-\frac{1}{e}.$

Assertion & Reason Type

Que. 1. C. If $f(x)$ is odd $\Rightarrow$ $f'(x)$ is even but converse is not true

e.g. If $f'(x)=x\sin x$ then $f(x)=\sin x-x\cos x+C$; $\Rightarrow f(-x)=-\sin x+x\cos x+C$

$\Rightarrow$ $f(x)+f(-x)=$ constant which need not to be zero

For S-1: $f(x)=\int_0^x\sqrt{1+t^2}dt;\ g(x)=\sqrt{1+x^2}\ \ f(-x)=\int_0^{-x}\sqrt{1+t^2}dt;\ t=-y \Rightarrow f(-x)-\int_0^x\sqrt{1+y^2}dy$

$\therefore$ $f(x)+f(-x)=0$ $\Rightarrow$ f is odd and g is obviously even.

Que. 2. C.

Que. 3. D. $\int_{-1}^{3}\frac{dx}{x^2}$ does not exist.

More than One May Correct Type

Que. 1. A,B,C,D. Note that the integrand in A,B and C all reduces to (1+sin t)

$\therefore \quad I=\int_{-\pi/6}^{0}(1+\sin t)\,dt=t-\cos]_{-\frac{\pi}{6}}^{0}=(-1)-\left(-\frac{\pi}{6}-\cos\frac{\pi}{6}\right)=-1+\frac{\pi}{6}+\frac{\sqrt{3}}{2}$

Now, $D=\int_{3/2}^{2}\sqrt{\frac{x-1}{3-x}}\,dx$ Put $x=\cos^2\theta+3\sin^2\theta$ $\therefore$ $dx=4\sin\theta\cos\theta\,d\theta$

when $x=\frac{3}{2}$ then $\sin^2\theta=\frac{1}{2}$ $\Rightarrow$ $\theta=\frac{\pi}{6}$

when $x=2$ then $\sin^2\theta=\frac{1}{2}$ $\Rightarrow$ $\theta=\frac{\pi}{4}$

$$\therefore \quad I=\int_{\pi/6}^{\pi/4}\frac{\sin\theta}{\cos\theta}.4\sin\theta\cos\theta\,d\theta=2\int_{\pi/6}^{\pi/2}(1-\cos 2\theta)\,d\theta=2\left[\theta-\frac{1}{2}\sin 2\theta\right]_{\pi/6}^{\pi/4}=2\theta-\sin 2\theta\Big]_{\pi/6}^{\pi/4}$$

$$=\left(\frac{\pi}{2}-1\right)-\left(\frac{\pi}{3}-\frac{\sqrt{3}}{2}\right)=\frac{\pi}{6}-1+\frac{\sqrt{3}}{2} \quad \Rightarrow \mathbf{A,B,C,D.}$$

Que. 2. A,B,C,D.

(A). Put $x=1/t$ and add to get result

(B). $\int_2^4\left(\frac{\ell n\,2}{\ell n\,x}-\frac{\ell n\,2}{\ell n^2\,x}\right)dx$ if $f(x)=\frac{1}{\ell n\,x}\Rightarrow x f'(x)=-\frac{1}{\ell n^2\,x}$

$$\Rightarrow I=\ell n2\left(\frac{x}{\ell n\,x}\right)_2^4=\ell n\,2\left[\frac{4}{\ell n\,4}-\frac{2}{\ell n\,2}\right]=0$$

(C). $x=\frac{1}{t}\Rightarrow dx=-\frac{1}{t^2}dt$

$$I=\int_2^{1/2}t\sin\left(\frac{1}{t}-t\right)\left(-\frac{1}{t^2}\right)dt=\int_2^{1/2}\frac{1}{t}\sin\left(t-\frac{1}{t}\right)dt=-\int_{1/2}^{2}\frac{1}{t}\sin\left(t-\frac{1}{t}\right)dt=-I \qquad \Rightarrow \qquad 2I=0 \;\Rightarrow\; I=0.$$

Alternatively for (C); put $x=e^t\Rightarrow I\int_{-\ell n2}^{\ell n2}\sin\left(e^t-e^{-t}\right)dt=0$ (odd function)

(D). $\frac{1}{2}\int_0^{\pi}2\sin nx.\cos mx\,dx=\frac{1}{2}\left(\int_0^{\pi}\sin(n+m)x+\sin(n-m)x\,dx\right)$

$$=-\frac{1}{2}\left(\frac{\cos(n+m)x}{n+m}+\frac{\cos(n-m)x}{n-m}\right)_0^{\pi}=-\frac{1}{2}\left[\frac{1}{n+m}+\frac{1}{n-m}-\frac{1}{n+m}-\frac{1}{n-m}\right]=0$$

Que. 3. A,B,D. Given $f(f(x))=-x+1$ replacing $x\to f(x)$ $\Rightarrow f(f(f(x)))=-f(x)+1$

$\Rightarrow f(1-x)=-f(x)+1 \Rightarrow f(x)+f(1-x)=1$ .... (1) $\Rightarrow$ **(A)**

Now $J=\int_0^1 f(x)dx=\int_0^1 f(1-x)\,dx$ (Using King)

$2J=\int_0^1\left(f(x)+f(1-x)\right)dx;$ $2J=\int_0^1 dx=1 \Rightarrow J=\frac{1}{2}.$

Que. 4. A,D. $y=A\left[\cos Bx\cos D-\sin Bx\sin D\right]\Rightarrow y=C_1\cos Bx+C_2\sin Bx$ .... (1)

$(A\cos D=C_1;-A\sin D=C_2)\Rightarrow$ $y=BC_1\sin Bx+BC_2\cos Bx \Rightarrow y_1=-BC_1\sin Bx+BC_2\cos Bx$

$\Rightarrow =-B^2(C_1\cos Bx+C_2\sin Bx)\Rightarrow y_2=-B^2y\Rightarrow \frac{y_2}{y}=-B^2\Rightarrow yy_3-yy_2=0\Rightarrow y\frac{d^2y}{dx^3}=\frac{dy}{dx}.\frac{d^2y}{dx^2}$

Que. 5. B,C. $I=\frac{1}{2}\int\frac{dt}{t^2+t+1}=\frac{1}{2}\int\frac{dt}{\left(t+(1/2)\right)^2+(3/4)}=\frac{1}{2}.\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{t+(1/2)}{\sqrt{3}/2}\right)=\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2t+1}{\sqrt{3}}\right)$

$$=\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)+C.$$

Alternatively : $I=\int\frac{xdx}{(x^4+2x^2+1)-x^2}=\int\frac{xdx}{(x^2+x+1)(x^2-x+1)}\frac{1}{2}\int\frac{(x^2+x+1)-(x^2-x+1)}{(x^2+x+1)(x^2-x+1)}dx$

$$=\frac{1}{2}\int\frac{dx}{x^2-x+1}-\frac{1}{2}\int\frac{dx}{x^2+x+1}=\frac{1}{2}\int\frac{dx}{\left(x-(x/2)\right)^2-\left(\sqrt{3}/2\right)^2}-\frac{1}{2}\int\frac{dx}{\left(x+(1/2)\right)^2-\left(\sqrt{3}/2\right)^2}$$

$$=\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)=\frac{1}{\sqrt{3}}\left[\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right]+C.$$

Que. 6. A,C,D.

Que. 7. A, B, C Consider $I=\int_a^\infty\frac{ndx}{n^2\left(x^2+\frac{1}{n^2}\right)}=\frac{1}{n}.n\left(\tan^{-1}nx\right)_a^\infty=\left(\frac{\pi}{2}-\tan^{-1}an\right)$

$$\therefore L=\lim_{x\to\infty}\left(\frac{\pi}{2}-\tan^{-1}an\right)=\begin{bmatrix}\pi \text{ if } a<0\\ \pi/2 \text{ if } a=0\\ 0 \text{ if } a>0\end{bmatrix}\Rightarrow\quad (A),(B),(C)$$

Subjective Type (Up to 4 digit)

Que. 1. 208 Let $I=\int {}^{207}C_7\underbrace{x^{200}}_{II}\underbrace{(1-x)^7}_{I}dx$

$$I={}^{207}C_7\left[\underbrace{\left.(1-x)^7.\frac{x^{201}}{201}\right|_0^1}_{zero}+\frac{7}{201}\int_0^1(1-x)^6.x^{201}dx\right]={}^{207}C_7.\frac{7}{201}\int_0^1(1-x)^6.x^{201}dx$$

I.B.P. again 6 more times

$$={}^{207}C_7.\frac{7!}{201.202.203.204.205.206.207}\int_0^1x^{207}dx=\frac{(207)!}{7!(200)!}.\frac{7!}{201.202.......207}.\frac{1}{208}$$

$$=\frac{(207)!}{(207)!7!}.\frac{7!}{208}=\frac{1}{208}=\frac{1}{k}\quad\Rightarrow\quad k=208$$ **Ans.]**