THIS FILE CONTAINS

TWO DIMENSIONAL GEOMETRY

(COLLECTION # 2)

Very Important Guessing Questions For IIT JEE 2011 With Detail Solution

Junior Students Can Keep It Safe For Future IIT-JEEs

- ***Two Dimensional Geometry (2D)***
- ***The Point***
- ***Straight Lines***
- ***Circles***
- ***Parabola***
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For Collection # 2 (Page 39 to 54)

- ***Same As Above***

Single Correct Type

Q. 1 There are exactly two points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose distance from the centre of the ellipse are greatest and equal to $\sqrt{\frac{a^2+2b^2}{2}}$. Eccentricity of this ellipse is equal to

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{\frac{2}{3}}$

(codeV3T1PAQ6)

Q. 2 Length of the latus rectum of the parabola $25\left[(x-2)^2+(y-3)^2\right]=(3x-4y+7)^2$ is :

(A) 4 (B) 2 (C) $1/5$ (D) $2/5$

(codeV3T1PAQ7)

Q. 3 If the eccentricity of the hyperbola $x^2-y^2\sec^2\alpha=5$ is $\sqrt{3}$ times the eccentricity of the ellipse $x^2\sec^2\alpha+y^2=25,$ then a value of α is

(codeV3T2PAQ1)

(A) $\pi/6$ (B) $\pi/3$ (C) $\pi/2$ (D) $\pi/4$

Q. 4 Each member of the family of parabolas $y=ax^2+2x+3$ has a maximum or a minimum point depending upon the value of a. The equation to the locus of the maxima or minima for all possible values of 'a' is

(codeV3T2PAQ4)

(A) a straight line with slope 1 and y intercept 3.
(B) a straight line with slope 2 and y intercept 2.
(C) a straight line with slope 1 and x intercept 3. (D) a circle

Q. 5 The acute angle at which the line $y=3x-1$ intersects the circle $(x-2)^2+y^2=5$ is

(A) $30°$ (B) $45°$ (C) $60°$ (D) $75°$

(codeV3T2PAQ5)

Q. 6 Let ABC be a triangle with $\angle BAC=\frac{2\pi}{3}$ and $AB=x$ such that $(AB)(AC)=1.$ If x varies then the longest possible length of the angle bisector AD equals (codeV3T2PAQ7)

(A) $1/3$ (B) $1/2$ (C) $2/3$ (D) $3/2$

Q. 7 If $x^2+y^2=c^2\,(c\neq 0)$ then the least value of x^4+y^4 is equal to

(codeV3T2PAQ8)

(A) $\frac{c^4}{4}$ (B) $\frac{c^4}{2}$ (C) $\frac{3c^4}{4}$ (D) c^4

Q. 8 The area enclosed by the parabola $y^2=12x$ and its latus rectum is (codeV3T3PAQ1)

(A) 36 (B) 24 (C) 18 (D) 12

Q.9 Three distinct point $P(3u^2, 2u^3)$; $Q(3v^2, 2v^3)$ and $R(3w^2, 2w^3)$ are collinear then

(codeV3T3PAQ5)

(A) $uv+vw+wu=0$ (B) $uv+vw+wu=3$ (C) $uv+vw+wu=2$ (D) $uv+vw+wu=1$

Q. 10 At the end points A, B of the fixed segment of length L, line are drawn meeting in C and making angles θ and 2θ respectively with the given segment. Let D be the foot of the altitude CD and let x represents the length of AD. The value of x as θ tends to zero i.e. $\lim_{\theta\to 0} x$ equals (codeV3T4PAQ3)

(A) $\frac{L}{2}$ (B) $\frac{2L}{3}$ (C) $\frac{3L}{4}$ (D) $\frac{L}{4}$

Q. 11 Number of positive integral values of 'a' for which the curve $y=a^x$ intersects the line $y=x$ is

(A) 0 (B) 1 (C) 2 (D)More than 2

(codeV3T4PAQ7)

Q. 12 The area enclosed by the curve $y=\sqrt{x}$ & $x=-\sqrt{y}$, the circle $x^2+y^2=2$ above the x-axis, is

(A) $\frac{\pi}{4}$ (B) $\frac{3\pi}{2}$ (C) π (D) $\frac{\pi}{2}$

(codeV3T4PAQ9)

Q. 13 From the point $(-1, 2)$ tangent line are drawn to the parabola $y^2=4x$. The area of the triangle formed by the chord of contact and the tangents is (codeV3T5PAQ1)

(A) $4\sqrt{2}$ (B) 4 (C) 8 (D) $8\sqrt{2}$

Q. 14 The equation to the locus of the middle point of the portion of the tangent to the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ included between the co-ordinate axes is the curve : (codeV3T5PAQ2)

(A) $9x^2+16y^2=4x^2y^2$ (B) $16x^2+9y^2=4x^2y^2$ (C) $3x^2+4y^2=4x^2y^2$ (D) $9x^2+16y^2=x^2y^2$

Q. 15 Let P be a variable point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ (a>b)$ with foci F_1 & F_2. If A is the area of the triangle PF_1F_2, then the maximum value of A is (codeV3T5PAQ4)

(A) $a\sqrt{a^2-b^2}$ (B) $b\sqrt{a^2-b^2}$ (C) $2b\sqrt{a^2-b^2}$ (D) ab

Q. 16 All points on the curve $y^2=4a\left(x+a\sin\frac{x}{a}\right)$ at which the tangent t is parallel to x-axis lie on

(A) a circle (B) a parabola (C) an ellipse (D) a line

(codeV3T5PAQ5)

Q. 17 The number of possible tangents which can be drawn to the curve $4x^2-9y^2=36$, which are perpendicular to the straight line $5x+2y-10=0$ is : (codeV3T5PAQ6)

(A) zero (B) 1 (C) 2 (D) 4

Q. 18 Let $S\equiv(3, 4)$ and $S'\equiv(9, 12)$ be two foci of an ellipse. If the coordinates of the foot of the perpendicular form focus S to a tangent of the ellipse is $(1,-4)$ then the eccentricity of the ellipse is

(A) $\frac{5}{13}$ (B) $\frac{4}{5}$ (C) $\frac{5}{7}$ (D) $\frac{7}{13}$

(codeV3T5PAQ8)

Q. 19 The straight line joining any point P on the parabola $y^2=4ax$ to the vertex and perpendicular from the focus to the tangent at P, intersect at R, then the equation of the locus of R is (codeV3T5PAQ9)

(A) $x^2+2y^2-ax=0$ (B) $2x^2+y^2-2ax=0$ (C) $2x^2+y^2-ay=0$ (D) $2x^2+y^2-2ay=0$

Q. 20 The area of the quadrilateral with its vertices at the foci of the conics $9x^2-16y^2-18x+32y-23=0$ and $25x^2+9y^2-50x-18y+33=0$, is (codeV3T5PAQ10)

(A) 5/6 (B) 8/9 (C) 5/3 (D) 16/9

Q. 21 Locus of the feet of the perpendiculars drawn from either foci on a variable tangent to the hyperbola $16y^2 - 9x^2 = 1$ is
(codeV3T5PAQ12)
(A) $x^2 + y^2 = 9$ (B) $x^2 + y^2 = 1/9$ (C) $x^2 + y^2 = 7/144$ (D) $x^2 + y^2 = 1/16$

Q. 22 A line passing through the point $(21, 30)$ and normal to the curve $= 2\sqrt{x}$ can have the slope equal to
(A) 2 (B) 3 (C) –2 (D) –5
(codeV3T5PAQ13)

Q. 23 The magnitude of the gradient of the tangent at an extremity of latera recta of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is equal to (where e is the eccentricity of the hyperbola)
(codeV3T5PAQ14)
(A) be (B) e (C) ab (D) ae

Q. 24 If s, s' are the length of the perpendicular on a tangent from the foci a, a' are those from the vertices, c is that from the centre and e is the eccentricity of the ellipse, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then $\frac{ss' - c^2}{aa' - c^2} =$
(A) e (B) 1/e (C) 1/e^2 (D) e^2
(codeV3T5PAQ15)

Q. 25 TP & TQ are tangents to the parabola, $y^2 = 4ax$ at P & Q. If the chord PQ passes through the fixed point $(-a, b)$ then the locus of T is :
(codeV3T5PAQ16)
(A) $ay = 2b(x - b)$ (B) $bx = 2a(y - a)$ (C) $by = 2a(x - a)$ (D) $ax = 2b(y - b)$

Q. 26 Eccentricity of the hyperbola conjugate to the hyperbola $\frac{x^2}{4} - \frac{y^2}{12} = 1$ is
(codeV3T5PAQ17)
(A) $\frac{2}{\sqrt{3}}$ (B) 2 (C) $\sqrt{3}$ (D) $\frac{4}{3}$

Q. 27 Consider the system of equatioins $(4 - p^2)x + 2y = 0$ and $2x + (7 - p^2)y = 0.$ If the system has the solution other than $x = y = 0$ then the ratio $x : y$ can be
(codeV3T6PAQ8)
(for only this que. More than one Ans. May correct.)
(A) –1/2 (B) 1/2 (C) 2 (D) -2

Q. 28 An equilateral triangle has side length 8. The area of the region containing all points outside the triangle but not more than 3 units from a point on the triangle is : (codeV3T8PAQ1)
(A) $9(8 + \pi)$ (B) $8(9 + \pi)$ (C) $9\left(8 + \frac{\pi}{2}\right)$ (D) $8\left(9 + \frac{\pi}{2}\right)$

Q. 29 An object moves 8 cm in a straight line from A to B, turns at an angle α, measured in radians chosen at random from the interval $(0, \pi)$ and moves 5 cm in a straight line to C. The probability that $AC < 7$ is :
(codeV3T8PAQ3)
(A) $\frac{1}{6}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$

Comprehesion Type

1 Paragraph for Q. 1 to Q. 3

Consider a line $L: 2x+y=1$ and the points $A(1, 3/2)$ and $B(4, 5)$. P is a point on the line L.

Q. 1 The abscissa of the point P for which the area of the ΔPAB is unity (Given P does not lie on the y-axis) is

(codeV3T2PAQ9)

(A) $\frac{4}{19}$ (B) $\frac{6}{19}$ (C) $\frac{8}{19}$ (D) None

Q. 2 A circle passes through A and B and has its centre on the x-axis. The x-coordinate of the centre is

(A) $\frac{151}{24}$ (B) $\frac{155}{7}$ (C) 7 (D) None (codeV3T2PAQ10)

Q. 3 If C is some point on L then the minimum distance $(AC+BC)$ is

(codeV3T2PAQ11)

(A) 7 (B) $\sqrt{\frac{181}{2}}$ (C) $\frac{\sqrt{181}}{4}$ (D) $\frac{\sqrt{181}}{2}$

2 Paragraph for Q. 4 to Q. 6

Consider the lines represented parametrically as

$L_1:\quad x=1-2t;\quad y=t;\quad z=-1+t$

$L_2:\quad x=4+x;\quad y=5+4s;\quad z=-2-s$ Find

Q. 4 acute angle between the line L_1 and L_2, is (codeV3T3PAQ14)

(A) $\cos^{-1}\left(\frac{1}{18}\right)$ (B) $\cos^{-1}\left(\frac{1}{3\sqrt{6}}\right)$ (C) $\cos^{-1}\left(\frac{1}{6\sqrt{3}}\right)$ (D) $\cos^{-1}\left(\frac{1}{3\sqrt{2}}\right)$

Q. 5 equation of a plane P containing the line L_2 and parallel to the line L_1, is (codeV3T3PAQ15)

(A) $5x+y+9z-7=0$ (B) $2x-3y+4z-15=0$ (C) $5x-y+9z+3=0$ (D) $9x-5y-z-13=0$

Q. 6 distance between the plane P and the line L_1 is

(codeV3T3PAQ16)

(A) $\frac{17}{\sqrt{29}}$ (B) $\frac{3}{\sqrt{87}}$ (C) $\frac{11}{\sqrt{107}}$ (D) $\frac{1}{\sqrt{107}}$

3 Paragraph for Q. 7 to Q. 9

Consider the circles $S_1: x^2+y^2-6y+5=0;\quad S_2: x^2+y^2-12y+35=0$

And a variable circle $S: x^2+y^2+2gx+2fy+c=0$

Q. 7 Number of common tangents to S_1 and S_2 is

(codeV3T5PAQ19)

(A) 1 (B) 2 (C) 3 (D) 4

Q. 8 Length of a transverse common tangent to S_1 and S_2 is

(codeV3T5PAQ20)

(A) 6 (B) $2\sqrt{11}$ (C) $\sqrt{35}$ (D) $11\sqrt{2}$

Q. 9 If the variable circle $S=0$ with centre C moves in such a way that it is always touching externally the circles $S_1=0$ and $S_2=0$ then the locus of the centre C of the variable circle is

(A) a circle (B) a parabola (C) an ellipse (D) a hyperbola

(codeV3T5PAQ21)

4 Paragraph for Q. 10 to Q. 12

From the point $P(h, k)$ three normals are drawn to the parabola $x^2=8y$ and m_1, m_2 and m_3 are the slopes of three normals

Q. 10 Algebraic sum of the slopes of these three normals is (codeV3T5PAQ22)

(A) zero (B) $\frac{k-4}{h}$ (C) $\frac{k-2}{h}$ (D) $\frac{2-k}{h}$

Q. 11 If two of the three normals are at right angles then the locus of point P is a conic whose latus rectum is

(A) 1 (B) 2 (C) 3 (D) 4

(codeV3T5PAQ23)

Q.12 If the two normals from P are such that they make complementary angles with the axis then the locus of point P is a conic then a directrix of conic is (codeV3T5PAQ24)

(A) $2y-3=0$ (B) $2y+3=0$ (C) $2y-5=0$ (D) $2y+5=0$

 # 5 Paragraph for Q. 13 to Q. 15

A conic C satisfies the differential equation, $(1+y^2)dx-xydy=0$ and passes through the point $(1, 0)$. An ellipse E which is confocal with C having its eccentricity equal to $\sqrt{2/3}$

Q. 13 Length of the latus rectumof the conic C, is (codeV3T5PAQ25)

(A) 1 (B) 2 (C) 3 (D) 4

Q. 14 Equation of the ellipse E is (codeV3T5PAQ26)

(A) $\frac{x^2}{3}+\frac{y^2}{1}=1$ (B) $\frac{x^2}{1}+\frac{y^2}{3}=1$ (C) $\frac{x^2}{4}+\frac{y^2}{9}=1$ (D) $\frac{x^2}{9}+\frac{y^2}{4}=1$

Q. 15 Locus of the point of intersection of the perpendicular tangents to the ellipse E, is (codeV3T5PAQ27)

(A) $x^2+y^2=4$ (B) $x^2+y^2=10$ (C) $x^2+y^2=8$ (D) $x^2+y^2=13$

 # 6 Paragraph for Q. 16 to Q. 18

Two equal parabola P_1 and P_2 have their vertices at $V_1(0, 4)$ and $V_2(6, 0)$ respectively. P_1 and P_2 are tangent to each other and have vertical axes of symmetry.

Q. 16 The sum of the abscissa and ordinate of their point of contact is (codeV3T5PAQ28)

(A) 4 (B) 5 (C) 6 (D) 7

Q. 17 Length of latus rectus is (codeV3T5PAQ29)

(A) 6 (B) 5 (C) 9/2 (D) 4

Q. 18 Area of the region enclosed by P_1, P_2 and the x-axis is (codeV3T5PAQ30)

(A) 1 (B) $4-2\sqrt{2}$ (C) $3-\sqrt{2}$ (D) $4(3-2\sqrt{2})$

 # 7 Paragraph for Q. 19 to Q. 21

Four A, B, C and D in order lie on the parabola $y=ax^2+bx+c$ and the coordinates of A, B and D are $(-2, 3)$; $(-1, 1)$ and $(2, 7)$ respectively.

Q. 19 $(a+b+c)$ has the value equal to (codeV3T8PAQ14)

(A) 1 (B) 2 (C) 3 (D) 4

Q. 20 If roots of the equation $ax^2+bx+c=0$ and α are β then the equation whose roots are α^{2009} and β^{2009} is
(codeV3T8PAQ15)
(A) $x^2-x+1=0$ (B) $x^2+x+1=0$ (C) $x^2+2x+3=0$ (D) $x^2-2x+3=0$

Q. 21 If the area of the quadrilateral ABCD is greatest, then the sum of the abscissa and ordinate of the point C is
(codeV3T8PAQ16)
(A) $\frac{9}{4}$ (B) $\frac{7}{4}$ (C) $\frac{5}{4}$ (D) $\frac{11}{4}$

 # 8 Paragraph for Q. 22 to Q. 24

Consider a line pair $ax^2+3xy-2y^2-5x+5y+c=0$ representing perpendicular lines intersecting each other at C and forming a triangle ABC with the x-axis.

Q. 22 If x_1 and x_2 are intercepts on the x-axis and y_1 and y_2 are the intercepts on the y-axis the sum $(x_1+x_2+y_1+y_2)$ is equal to
(codeV3T8PAQ17)
(A) 6 (B) 5 (C) 4 (D) 3

Q. 23 Distance between the orthocenter and circumcentre of the triangle ABC is (codeV3T8PAQ18)
(A) 2 (B) 3 (C) 7/4 (D) 9/4

Q. 24 If the circle $x^2+y^2-4y+k=0$ is orthogonal with the circumcircle of the triangle ABC then 'k' equals
(A) 1/2 (B) 1 (C) 2 (D) 3/2
(codeV3T8PAQ19)

 # 9 Paragraph for Q. 25 to Q. 27

Tangent is drawn at the point (x_i, y_i) on the curve $y=f(x)$, which intersects the x-axis at $(x_{i+1}, 0)$. Now again a tangent is drawn at (x_{i+1}, y_{i+1}) on the curve which intersects the x-axis at $(x_{i+2}, 0)$ and the prcess is reputed in times i.e., I = 1 ,2, 3,……………n.

Q. 25 If $x_1, x_2, x_3, ...x_n$ form an arithmetic progression with common difference equal to $\log_2 e$ and curve passes through $(0, 2)$, the equation of the curve is (codeV3T9PAQ12)
(A) $y=2e^x$ (B) $y=2e^{-x}$ (C) $y=2^{1+x}$ (D) $y=2^{1-x}$

Q. 26 If $x_1, x_2, x_3, ..., x_n$ form a geometric progression with common ratio equal to 2 and the curve passes though $(1, 2)$, then the curve is
(codeV3T9PAQ13)
(A) a parabola (B) an ellipse
(C) a rectangular hyperbola (D) hyperbola which is not rectangular

Q. 27 The radius of the circle touching the curve obtained in **question no. -26** at $(1, 2)$ and passing through the point $(1, 0)$ is
(codeV3T9PAQ14)
(A) $\sqrt{5}$ (B) $\sqrt{4}$ (C) $\sqrt{3}$ (D) $\sqrt{13}$

 # 10 Paragraph for Q. 28 to Q. 30

Consider the two quadratic polynomials $C_a : y=\frac{x^2}{4}-ax+a^2+a-2$ and $C : y=2-\frac{x^2}{4}$

Q. 28 If the origin lies between the zeroes of the polynomial C_a then the number of integral value(s) of 'a' is

(A) 1 (B) 2 (C) 3 (D) more than 3

(codeV3T10PAQ9)

Q. 29 If 'a' varies then the equation of the locus of the vertex of C_a, is (codeV3T10PAQ10)

(A) $x-2y-4=0$ (B) $2x-y-4=0$ (C) $x-2y+4=0$ (D) $2x+y-4=0$

Q. 30 For $a=3$, if the lines $y=m_1x+c_1$ and $y=m_2x+c_2$ are common tangents to the graph of C_a and C then the value of (m_1+m_2) is equal to

(codeV3T10PAQ11)

(A) – 6 (B) – 3 (C) 1/2 (D) none

 # 11 Paragraph for Q. 31 to Q. 33

Two fixed points A and B are 4 units apart, and are on the same side of a moving line L. If perpendicular distances of A and B say P_1 and P_2 from the line L are such that $P_1+3P_2=k$, k being a constant, then the line L always touched a fixed circle C.

Q. 31 The centre of the circle C lies on

(codeV3T10PAQ12)

(A) line segment joining AB (B) perpendicular bisector of AB

(C) one of A or B (D) nothing definite can be said

Q. 32 If $k=4$ then the radius of the circle is

(codeV3T10PAQ13)

(A) 1 (B) 2 (C) 4 (D) 8

Q. 33 If A and B are $(-2, 0), (2, 0)$ respectively, then the centre of the circle C is

(A) $(0, 1)$ (B) $(1, 0)$ (C) $(3/2, 0)$ (D) can not be found

(codeV3T10PAQ14)

Assertion & Reason Type

In this section each que. contains STATEMENT-1 (Assertion) & STATEMENT-2(Reason). Each question has 4 choices (A), (B), (C) and (D), out of which **only one is correct.**

Bubble (A) STATEMENT-1 is true, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1.

Bubble (B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1.

Bubble (C) STATEMENT-1 is True, STATEMENT-2 is False.

Bubble (D) STATEMENT-1 is False, STATEMENT-2 is True.

Q. 1 Consider an expression $f(h,k)=h^2+3k^2-2hk$ where *h* and *k* are non zero real numbers.

Statement-1: $f(h,k)$ is always positive $\forall$ non zero and real *h* and *k*. (codeV3T1PAQ16)

Statement-2 : A quadratic expression ax^2+bx+c is always positive if $'a'>0$ and $b^2-4ac<0$.

Q. 2 Let C be a circle with centre 'O' and HK is the chord of contact of pair of the tangents from point A. OA intersects the circle C at P and Q and B is the midpoint of HK, then

Statement-1: AB is the harmonic mean of AP and AQ. (codeV3T3PAQ8)

Statement-2 : AK is the Geometric mean of AB & AO and OA is the arithmetic mean of AP and AQ.

Q. 3 **Statement-1:** The cuves $y^2 = x$ and $x^2 = y$ are not orthogonal.
Statement-2 : The angle of intersection between then at their point of intersection other than origin is $\tan^{-1}(1)$.
(codeV3T4PAQ13)

Q. 4 Consider the curve $C: y^2 = 3 + 2x - x^2$
Statement-1 : The tangent to C at its point $P(3, 0)$ and at its point $Q(-1, 0)$ is parallel to y-axis.
Statement -2 : At the point $P(3, 0)$ and $Q(-1, 0)$ $\frac{dy}{dx} \to \infty$
(codeV3T8PAQ10)

Q. 5 Consider a triangle whose vertices are $A(-2, 1)$, $B(1, 3)$ and $C(3x, 2x-3)$ where x is a real number.
Statement-1 : The area of the triangle ABC is independent of x (codeV3T10PAQ17)
Statement - 2 : The vertex C of the triangle ABC always moves on a line parallel to the base AB.

More than One Correct Type

Q. 1 Suppose $f(x, y) = 0$ is circle such that the equation $f(x, 0) = 0$ has coincident root equal to 1, and the equation $f(0, y) = 0$ also has coincident roots equal to 1. Also, $g(x, y) = 0$ is a circle centred at $(0, -1)$ and tangent to the circle $f(x, y) = 0$. The possible radii of the circle is (codeV3T1PA19)
(A) $4\sin 15°$ (B) $2\cos 15°$ (C) $4\sin 18°$ (D) $4\cos 36°$

Q. 2 Equation of a straight line on the complex plane passing through a point P denoting the complex number α and perpendicular to the vector $\overline{OP}$ where 'O' in the origin can be written as (codeV3T3PAQ22)
(A) $Im\left(\frac{z-\alpha}{\alpha}\right) = 0$ (B) $\text{Re}\left(\frac{z-\alpha}{\alpha}\right) = 0$ (C) $\text{Re}\left(\vec{\alpha} Z\right) = 0$ (D) $\vec{\alpha}z + \alpha\vec{z} - 2|\alpha|^2 = 0$

Match Matrix Type

Q. 1 Consider the conic $C_1: \quad x^2 - 3y + 2x + 3 = 0 \quad C_2: \quad 4x^2 + y^2 - 16x + 6y + 21 = 0$
$C_3: \quad x^2 - 4y^2 - 2x - 32y - 127 = 0$

Column-I	**Column-II** (codeV3T2PBQ1)
(A) Length of the latus rectum of C_1	(P) 4
(B) Length of the latus rectum of C_2	(Q) 3.
(C) Length of the latus rectum of C_3	(R) 2.
	(S) 1.

SOLUTION (COLLECTION # 2)

Single Correct Type

Q. 1 C [Sol. The given distance is clearly the length of semi major axis

Thus, $\sqrt{\frac{a^2+2b^2}{2}} = a \quad \Rightarrow \quad 2b^2 = a^2 \;\Rightarrow\; 2a^2\left(1-e^2\right) = a^2 \;\Rightarrow\; e^2 = \frac{1}{2} \Rightarrow \quad e = \frac{1}{\sqrt{2}}$

Q. 2 D [Hint: Note that focus is $(2, 3)$ and directrix is $3x - 4y + 7 = 0$ and distance from S to directrix is half the latus rectum]

Q. 3 D [Sol. $\frac{x^2}{5} - \frac{y^2}{5\cos^2\alpha} = 1$ $e_1^2 = 1 + \frac{b^2}{a^2} = 1 + \frac{5\cos^2\alpha}{5} = 1 + \cos^2\alpha$;
IIIly eccentricity of the ellipse

$\frac{x^2}{25\cos^2\alpha}+\frac{y^2}{25}=1$ is $\quad e_2^2=1-\frac{25\cos^2\alpha}{25}=\sin^2\alpha$; $\quad$ put $e_1=\sqrt{3}\,e_2 \Rightarrow e_1^2=3e_2^2$

$\Rightarrow \quad 1+\cos^2\alpha=3\sin^2\alpha \Rightarrow \quad 2=4\sin^2\alpha \quad \Rightarrow \quad \sin\alpha=\frac{1}{\sqrt{2}}$]

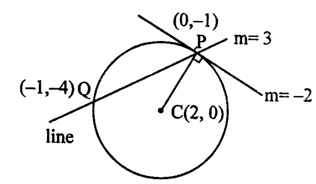
Q.4 A [Hint. Maxima/minima occurs at $=\left(-\frac{1}{a},\,3-\frac{1}{a}\right)$ $f(x)=ax^2+bx+c$ has a maxima or minima if $x=-\frac{b}{2a}$

$h=-\frac{1}{a}$ and $k=3-\frac{1}{a}$; eliminating 'a' we get $\Rightarrow k=3+x$ Locus is $y=x+3 \Rightarrow (A)$]

Q. 5 B [Sol. Solving $y=(3x-1)$ and $(x-2)^2+y^2=5$

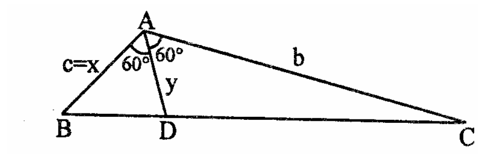
$(x-2)^2+(3x-1)^2=5 \Rightarrow \quad 10x^2-10x=0 \Rightarrow x=0$ or -1

i.e. $(0,-1)$ and $(-1,-4)$



Q. 6 B [Sol. $AD=y=\frac{2bc}{b+c}\cos\frac{A}{2}=\frac{bx}{b+x}(\text{as } c=x)$

but $\quad bx=1 \Rightarrow \quad b=\frac{1}{x} \quad \therefore \quad y=\frac{x}{1+x^2}=\frac{1}{x+\frac{1}{x}}$



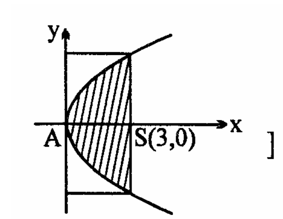
$y_{max}=\frac{1}{2}$ since minimum value of the denominator is 2 if $x>0 \Rightarrow \quad (B)$]

Q. 7 B [Sol. put $\quad x=c\cos\theta$; $\quad y=c\sin\theta$

$\therefore \quad E=x^4+y^4=c^4\left(\sin^4\theta+\cos^4\theta\right) \Rightarrow c^4\left(1-2\sin^2\theta\cos^2\theta\right)$

$=c^4\left[1-\frac{1}{2}\sin^2 2\theta\right]=E_{max}=\frac{c^4}{2}$ when $\sin^2 2\theta=1$]

Q. 8 B [Sol. $\frac{2}{3}(12.3)=24$ **Ans.**



]

Q. 9 A [Sol. $\begin{vmatrix} 3u^2 & 2u^3 & 1 \\ 3v^2 & 2v^2 & 1 \\ 3w^2 & 2w^2 & 1 \end{vmatrix}=0$

$R_1 \Rightarrow R_1-R_2$ and $R_2 \Rightarrow R_2-R_3$

$\Rightarrow \begin{vmatrix} u^2-v^2 & u^3-v^3 & 0 \\ v^2-w^2 & v^3-w^3 & 0 \\ w^2 & w^3 & 1 \end{vmatrix}=0 \Rightarrow \begin{vmatrix} u+v & u^2+v^2+vu & 0 \\ v+w & v^2+w^2+vw & 0 \\ w^2 & w^3 & 1 \end{vmatrix}=0$

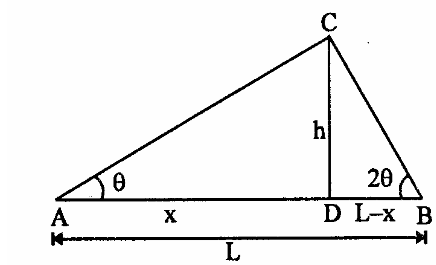
$R_1 \to R_1-R_2$

$\Rightarrow \begin{vmatrix} u-w & \left(u^2-w^2\right)+v(u-w) & 0 \\ v+w & v^2+w^2+vw & 0 \\ w^2 & w^3 & 1 \end{vmatrix}=0 \Rightarrow \begin{vmatrix} 1 & u+w+v & 0 \\ v+w & v^2+w^2+vw & 0 \\ w^2 & w^3 & 1 \end{vmatrix}=0$

$\Rightarrow \quad \left(v^2+w^2+vw\right)-(v+w)\left[(v+w)+u\right]=0$

$\Rightarrow \quad v^2+w^2+vw=(v+w)^2+u(v+w) \quad \Rightarrow \quad uv+vw+wu=0$ **Ans.]**

Q. 10 B **[Sol.** CD=h; $\tan\theta = \frac{h}{x}$; $\tan 2\theta = \frac{h}{L-x}$

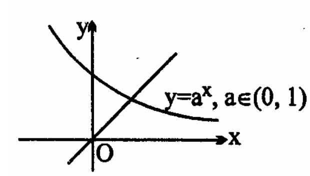


Now $x\tan\theta = (L-x)\tan 2\theta \Rightarrow x(\tan\theta + \tan 2\theta) = L\tan 2\theta$

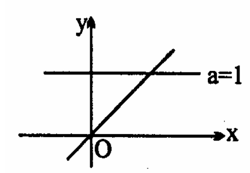
$$x = \left(\frac{\tan 2\theta}{\tan\theta + \tan 2\theta}\right)L \Rightarrow x = \left(\frac{2\theta\frac{\tan 2\theta}{2\theta}}{\theta\frac{\tan\theta}{\theta} + 2\theta\frac{\tan 2\theta}{2\theta}}\right)L$$

$\lim_{\theta\to 0} x = \frac{2L}{3}$ Ans.]

Q. 11 B **[Sol.** for $0 < a \le 1$ the line



Always cuts $y = a^x$

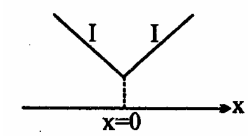


for $a > 1$ say $a = e$ consider $f(x) = e^x - x$

$f'(x) = e^x - 1 \Rightarrow x\ f'(x) > 0$ for $x > 0$ and $f'(x) < 0$ for $x < 0$

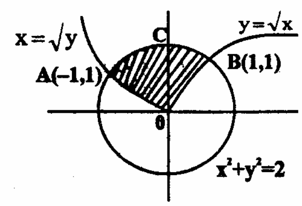
$\therefore$ $f(x)$ is increasing $(\uparrow)$ for $x > 0$

and decreasing $(\downarrow)$ for $x < 0 \Rightarrow$ $y = e^x$ always lies above $y = x$ i.e. $e^x - x \ge 1$ for $a > 1$ Hence never cuts $= a = (0, 1] \Rightarrow (B)$]



Q. 12 D **[Hint.** $A = \int_{-1}^{0}\left[\sqrt{2-x^2} - x^2\right]dx + \int_{0}^{1}\left[\sqrt{2-x^2} - \sqrt{x}\right]dx = \frac{\pi}{2}$

note that the area is equal to the sector AOB with central angle 90°



$\Rightarrow$ ¼ (the area of the circle)

required area $\pi - \frac{\pi}{2} = \frac{\pi}{2}$ Ans.

Q. 13 D **[Sol.** $A = \frac{\left(y_1^2 - 4ax_1\right)^{3/2}}{2a} = \frac{(4+4)^{3/2}}{2} = 8\sqrt{2}$ Ans.]

Q. 14 A Q. 15 B Q. 16 B

Q. 17 A **[Hint.** $y = -(5/2)x + 5 \Rightarrow m = 2/5 \Rightarrow a^2m^2 - b^2 = 9.\ 4/25 - 4 = (36-100)/25 < 0$

Note that the slope of the tangent $(2/5)$ is less than the slope of the asymptote which is 2/3 which is not possible]

Q. 18 A **[Sol.** $SS' = 2ae$, where a and e are length of semi-major axis and eccentricity respectively

$\therefore$ $\sqrt{(9-3)^2 + (12-4)^2} = 2ae \Rightarrow$ $\therefore$ $ae = 5 \Rightarrow \therefore$ centre is mid-point of SS'

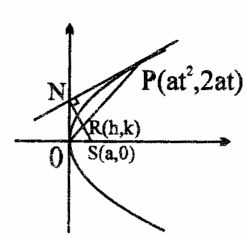
Centre =(6, 8), Let the equation of auxiliary circle be $(x-6)^2 = (y-8)^2 = a^2$

We know that the foot of the perpendicular from the focus on any tangent lies on the auxiliary circle

$\therefore$ $(1, -4)$ lies on auxiliary circle $\Rightarrow$ i.e. $(1-6)^2 + (-4-8)^2 = a^2$ $\Rightarrow$ $a = 13$

$\therefore$ $ae = 5 \Rightarrow$ $e = 5/13$ Ans.]

Q. 19 B **[Sol.** $T: ty = x + at^2$ ….(1)



Line perpendicular to (1) through $(a, 0)$

$tx + y = ta$ ….(2)

equation of $OP : y - \frac{2}{t}x = 0$ ....(3)

from (2) & (3) eliminating t we get locus]

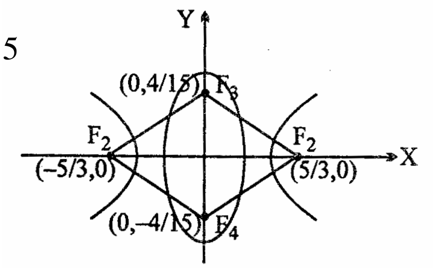
Q.20 B **[Sol.** 1^{st} is a hyperbola

$9(x-1)^2 - 16(y-1)^2 = 16$ with $e = 5/4$

and 2^{nd} is an ellipse $\Rightarrow 25(x-1)^2 + 9(y-1)^2 = 1$ with $e = 4/5$

With $x - 1 = X$ and $y - 1 = Y$ area $= \frac{1}{2}d_1d_2 = \frac{1}{2}.\frac{10}{3}.\frac{8}{15} = \frac{8}{9}$

Note that $e_E.e_H = 1$]



Q. 21 D **[Sol.** $\frac{y^2}{1/16} - \frac{x^2}{1/9} = 1$

Locus will be the auxiliary circle $x^2 + y^2 = 1/16$

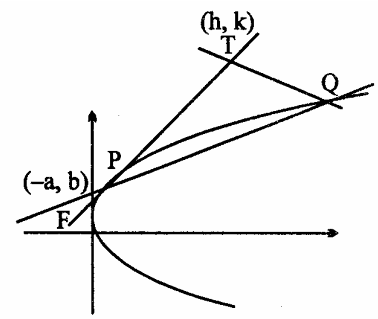
Q. 22 D **[Sol.** The point with slope 2 and 3 are normal at $(4, -4); (9, -6)$ where there is no curve, point of normal $(am^2, -2am)$]

Q. 23 B **[Sol.** $T : \frac{xx_1}{a^2} - \frac{yy_1}{b^2}; \frac{x.ae}{a^2} - \frac{y.b^2}{a.b^2} = 1$ or $\frac{ex}{a} - \frac{y}{a} = 1$ or

$ex - y = a \Rightarrow m = e$ Ans.]

Q. 24 D **[Sol.** Let the equation of tangent by $y = mx + \sqrt{a^2m^2 + b^2}$

Foci $\equiv (\pm ae, 0)$, vertices $\equiv (\pm a, 0)$, $C \equiv (0, 0)$



$\therefore \quad s = \left|\frac{mae + \sqrt{a^2m^2 + b^2}}{\sqrt{1+m^2}}\right|, \quad s' = \left|\frac{-mae + \sqrt{a^2m^2 + b^2}}{\sqrt{1+m^2}}\right|$

$a = \left|\frac{ma + \sqrt{a^2m^2 + b^2}}{\sqrt{1+m^2}}\right|, \quad a' = \left|\frac{-ma + \sqrt{a^2m^2 + b^2}}{\sqrt{1+m^2}}\right|$

$c = \left|\frac{\sqrt{a^2m^2 + b^2}}{\sqrt{1+m^2}}\right| \Rightarrow \therefore \quad \frac{ss' - c^2}{aa' - c^2} = \frac{\frac{m^2a^2e^2}{1+m^2}}{\frac{m^2a^2}{1+m^2}} = e^2$ **Ans.]**

Q. 25 C **[Hint.** Chord of contact of $(h, k) \Rightarrow ky = 2a(x+h)$. It passes through $(-a, b)$

$\Rightarrow bk = 2a(-a+h) \Rightarrow$ Locus is $by = 2a(x-a)$]

Q. 26 A **[Sol.** $e_1^2 = 1 + \frac{b^2}{a^2} = 1 + \frac{12}{4} = 4 \quad \Rightarrow \quad e_1 = 2 \quad ; \quad$ now $\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$

$\frac{1}{e_2^2} = 1 - \frac{1}{4} = \frac{3}{4} \quad \Rightarrow e_2^2 = \frac{4}{3} \quad \Rightarrow \quad e_2 = \frac{2}{\sqrt{3}}$]

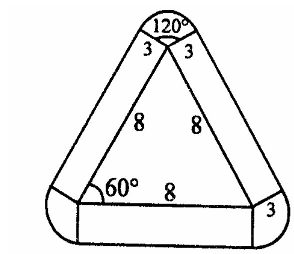
Q. 27 B, D **[Sol.** for non tribial solution

$(4-p^2)(7-p^2) - 4 = 0$

$p^4 - 11p^2 + 24 = 0 \quad \Rightarrow \quad p^2 = 3$ or $\quad p^2 = 8$

if $p^2 = 3, \quad x + 2y = 0 \Rightarrow \frac{x}{y} = -2 \quad \Rightarrow \quad (D)$

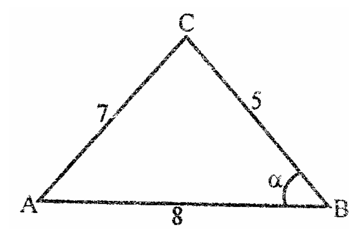
if $p^2 = 8, \quad -4x + 2y = 0 \Rightarrow \frac{x}{y} = \frac{1}{2} \quad \Rightarrow \quad (B)$]



Q. 28 [A] **[Sol. Area** $= 3.(8.3) + 3.\frac{1}{2}r^2\theta \Rightarrow 72 + \frac{3}{2}.9.\frac{2\pi}{3}$

$= 72 + 9\pi \quad \Rightarrow 9(8+\pi)$

Q. 29 [C] **[Sol.** If $AC = 7$, then



$\cos\alpha = \frac{8^2 + 5^2 - 7^2}{2.8.5} = \frac{64 + 25 - 49}{2.40}$

Hence $\cos\alpha = \frac{40}{2.40} = \frac{1}{2} \quad \Rightarrow \quad \alpha = 60°$

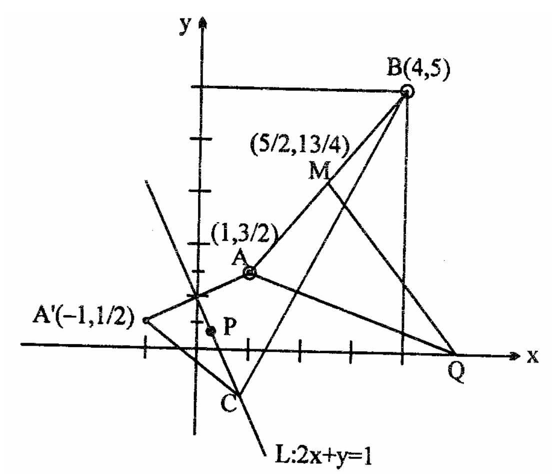
Now, $AC < 7 \quad \Rightarrow \quad \alpha \in (0, 60°)$

Hence $p = \frac{60}{180} = \frac{1}{3} \quad \Rightarrow \quad [C]$]

Comprehesion Type

Q. 1 C Q. 2 A

Q. 3 D [Sol. Let $P(x, 1-2x)$



Hence $\left\| \begin{matrix} x & 1-2x & 1 \\ 1 & 3/2 & 1 \\ 4 & 5 & 1 \end{matrix} \right\| = 2$

(1) $|4 - 19x| = 4 \quad \therefore \quad x = 0$ (rejected)

$19x - 4 = 4 \quad \Rightarrow \quad x = \frac{8}{19}$ Ans.

(2) Midpoint of AB is $M\left(\frac{5}{2}, \frac{13}{4}\right)$;

$m_{AB} = \frac{7}{6}$ Equation of perpendicular bisector

$y - \frac{13}{4} = -\frac{6}{7}\left(x - \frac{5}{2}\right)$ put $y = 0 \quad \Rightarrow \quad \therefore \quad x = \frac{5}{2} + \frac{91}{24} = \frac{151}{24}$ **Ans.**

(3) Image of A in the line L is $A'(-1, 1/2)$

now $AC + BC = A'C + BC \geq A'B = \sqrt{25 + \frac{81}{4}} = \frac{\sqrt{181}}{2}$ **Ans.**]

Q. 4 C Q. 5 A Q. 6 C [Sol. $L_1 : \frac{x-1}{-2} = \frac{y-0}{1} = \frac{z+1}{1}; L_2 : \frac{x-4}{1} = \frac{y-5}{4} = \frac{z+2}{-1}$

(i) $\vec{V_1} = -2\hat{i} + \hat{j} + \hat{k}; \quad \vec{V_2} = \hat{i} + 4\hat{j} - \hat{k}$

$\cos\theta = \left|\frac{-2+4-1}{\sqrt{6}.\sqrt{18}}\right| = \frac{1}{6\sqrt{3}} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{6\sqrt{3}}\right) \Rightarrow$ (C)

(ii) Equation of the plane containing the line L_2 is

$A(x-4) + B(y-5) + C(z+2) = 0$ ...(1)

Where $A + 4B - C = 0$ since (1) is parallel to L_1

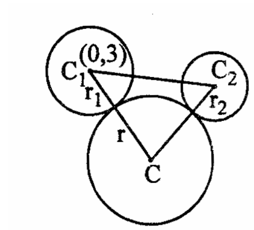
hence $-2A + B + C = 0 \Rightarrow \therefore \frac{A}{4+1} = \frac{B}{2-1} = \frac{C}{1+8} \Rightarrow A = 5k; B = k; C = 9k$

Hence equation of plane P $\Rightarrow 5(x-4) + y - 5 + 9(z+2) = 0 \Rightarrow 5x + y + 9z - 7 = 0$ (A)

(iii) distance between P and L_1 is $d = \left|\frac{5+0-9-7}{\sqrt{25+1+81}}\right| = \frac{11}{\sqrt{107}}$ **Ans.]**

Q. 7 D Q. 8 A

Q. 9 D **[Sol.** (iii) $r_1 = 2; r_2 = 1; C_1 = (0, 3); C_2 = (6, 0); C_1C_2 = 3\sqrt{5}$



clearly the circle with centre C_1 and C_2 are separated

$CC_1 = r + r_1 \Rightarrow CC_2 = r + r_2 \Rightarrow CC_1 - CC_2 = r_1 - r_2 =$ constant]

Q. 10 B Q. 11 B

Q. 12 A **[Sol.** Equation of normal in terms of slope is

$m^3x = (4-y)m^2 + 2 = 0$ point $P(h, k)$ satisfies this equation

$\therefore \quad m^2h + (4-k)m^2 + 2 = 0$ — m_1, m_2, m_3 .....(1)

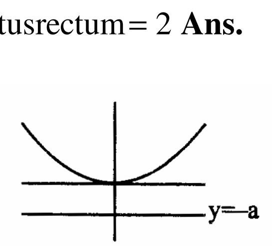
(1) $\therefore$ algebraic sum of slopes is $m_1 + m_2 + m_3 = \frac{k-4}{h}$ **Ans.**

(2) If two normals are perpendicular then $m_1m_2 = -1$ and $m_1m_2m_3 = -\frac{2}{h}$

substituting $m_3 = \frac{2}{h}$ in (1) we get

$\Rightarrow \frac{8}{h^2} + \frac{4(4-k)}{h^2} + 2 = 0 \Rightarrow 4 + 2(4-k) + h^2 = 0 \Rightarrow x^2 + 2(y-12)$ $\therefore$ latusrectum= 2 **Ans.**

(3) If the slopes are complementary then $m_1m_2 = 1 \Rightarrow m_3 = -\frac{2}{h}$



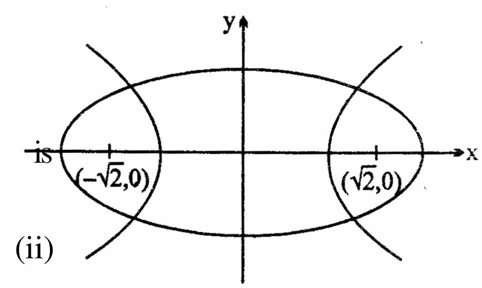
using $m_3 = -\frac{2}{h}$ in (1) we get

$-\frac{8}{h^2} + (4-k)\frac{4}{h^2} + 2 = 0 \Rightarrow -4 + 2(4-k) + h^2 = 0 \Rightarrow h^2 + 4 - 2k = 0$

$\therefore \quad x^2 = 2(y-2) \Rightarrow x^2 = 2Y$ let $Y = y - 2 \Rightarrow a = 1/2$

$\therefore$ directrix is $Y = -\frac{1}{2}$ or $y - 2 = -\frac{1}{2} \Rightarrow \therefore 2y - 3 = 0$ Ans.]

Q. 13 B Q. 14 A Q. 15 A



[Sol. $(1+y^2)dx = xydy \int \frac{dx}{x} = \int \frac{ydy}{1+y^2}$ $2\ln x = \ln(1+y^2)+C$

given $x=1, y=0 \Rightarrow C=0$ hence equation of C is

$x^2-y^2=1$ which is rectangular hyperbola with eccentricity $e=\sqrt{2}$.

(i) length of the latus rectum of rectangular hyperbola $=2a=2$ (ii)

Now for ellipse,

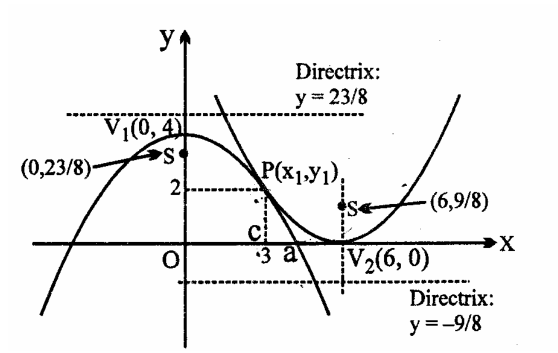
$$ae=\sqrt{2} \quad \Rightarrow \quad a^2e^2=2 \quad \Rightarrow \quad a^2.\frac{2}{3}=2 \quad \Rightarrow \quad a^2=3$$

and $b^2=a^2(1-e^2)=3\left(1-\frac{2}{3}\right)=1$. Hence equation of ellipse is $\frac{x^2}{3}+\frac{y^2}{1}=1$ **Ans.**

(ii) Locus of the point of intersection of the perpendicular tangents is the director circle of the ellipse equation is $x^2+y^2=4$.]

Q. 16 B Q. 17 C

Q 18 D **[Sol.** The parabolas will have their concavities in opposite direction otherwise they can not touch



Let $P_1: x^2=-\lambda(y-4)$ ....(1) $(\lambda>0)$

and $P_2: (x-6)^2=\lambda y$ ....(2)

Solving the two equation

$$x^2=-\lambda\left[\frac{(x-6)^2}{\lambda}-4\right] \Rightarrow x^2=-(x-6)^2+4\lambda$$

$$x^2+(x-6)^2-4\lambda=0 \Rightarrow 2x^2-12x+36-4\lambda=0$$

$$b^2-4ac=0;\ 144=4.2(36-4\lambda) \Rightarrow 18=(36-4\lambda)$$

$\Rightarrow 4\lambda=18 \Rightarrow \lambda=\frac{9}{2}$ Hence the parabola are

$x^2=-\frac{9}{2}(y-4)$; $(x-6)^2=\frac{9}{2}y \Rightarrow$ Latus rectum $=\frac{9}{2}$ **Ans (ii)**

again, $\left.\frac{dy}{dx}\right]_{P(x_1,\,y_1)}$ must be same for both

$$2x=-\lambda\frac{dy}{dx} \quad \Rightarrow \quad \left.\frac{dy}{dx}\right]_{x_1,\,y_1}=-\frac{2x_1}{\lambda}\text{(where } \lambda=\frac{9}{2}\text{)}$$

and $2(x-6)=\lambda\frac{dy}{dx} \quad \Rightarrow \quad \left.\frac{dy}{dx}\right]_{x_1,\,y_1}=\frac{2(x_1-6)}{\lambda}$ (where $\lambda=\frac{9}{2}$)

$\therefore \quad 2(x_1-6)=-2x_1 \Rightarrow 4x_1=12 \quad \Rightarrow \quad x_1=3$

when $x=3$ then $y_1=2$ $\therefore$ point of contact $=(3,2)$ $\Rightarrow$ sum $=5$ **Ans. (i)**

(iii) Mehtod – 1

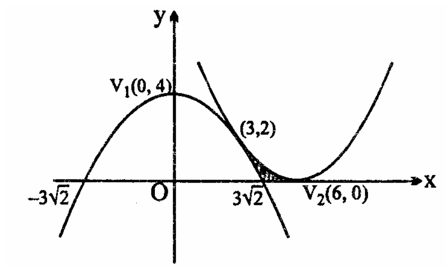
$$A_1=\int_3^6\frac{2}{9}(x-6)^2dx=\frac{2}{9}.\left.\frac{(x-6)^3}{3}\right]_3^6=\frac{2}{27}\left[0-(-3)^3\right]=2 \quad (A_1\equiv \text{Ar.PCV})$$

$$A_2=\int_3^{3\sqrt{2}}\left(4-\frac{2x^2}{9}\right)dx=4x.\left.\frac{2}{27}x^3\right]_3^{3\sqrt{2}}=\left(12\sqrt{2}-\frac{2.54\sqrt{2}}{27}\right)-\left(12-\frac{2}{27}.27\right)(A_2\equiv \text{Ar. PCQ})$$

$= \left(12\sqrt{2} - 4\sqrt{2}\right) - (12 - 2) = 12\sqrt{2} - 4\sqrt{2} - 10 = 8\sqrt{2} - 10$

$\therefore$ required area $= 2 - \left[8\sqrt{2} - 10\right] = 12 - 8\sqrt{2} = 4\left(3 - 2\sqrt{2}\right)$ **Ans. (iii)**

Method -2 $\int_0^2 \left[\left(6 - \frac{3\sqrt{y}}{\sqrt{2}}\right) - \left(\frac{3}{\sqrt{2}}\sqrt{4-y}\right)\right] dy$



$= 6y - \frac{3}{\sqrt{2}} \cdot \frac{2}{3} y^{3/2}\Big]_0^2 + \frac{3}{\sqrt{2}} \cdot \frac{2}{3}(4-y)^{3/2}\Big]_0^2$

$= \left(12 - \sqrt{2}.2\sqrt{2}\right) - (0) + \sqrt{2}\left(2\sqrt{2} - 8\right)$

$= (12 - 4) + \left(4 - 8\sqrt{2}\right) = 12 - 8\sqrt{2} = 4\left(3 - 2\sqrt{2}\right)$ **Ans. (iii)]**

Q. 19 [C] Q. 20 [B] Q. 21 [A] **[Sol.** (1)

$a = b = c = 1; \quad y = x^2 + x + 1 = 0 \lessdot^{w}_{w^2} \Rightarrow$ (C)

(2) $\alpha^{2009} = w^{2009} = w^2 \quad \Rightarrow \beta^{2009} = w^{4018} = w$

hence equation is $x^2 - \left(w + w^2\right)x + 1 = 0 \;\; x^2 + x + 1 \quad \Rightarrow$ (B)]

Q. 22 [B] Q. 23 [C] Q. 24 [D] **[Sol. (1)** As line are perpendicular

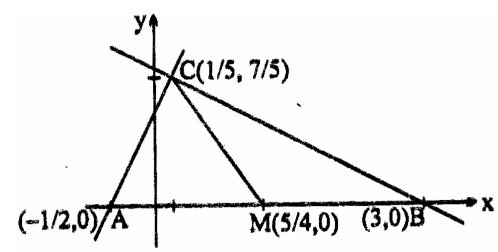
$\therefore$ $a - 2 = 0 \quad \Rightarrow \quad a = 2$ (coefficient of x^2+ coefficient of $y^2 = 0$)

using $\Delta = 0 \Rightarrow \quad c = -3 \left(D \equiv abc + 2fgh - af^2 - bg^2 - ch^2\right)$

hence the two lines are $x + 2y - 3 = 0$ and $2x - y + 1 = 0$

$\left.\begin{matrix} x\text{-intercepts} & y_1=3; & x_2=-1/2 \\ y\text{-intercepts} & y_1=3/2; & y_2=1 \end{matrix}\right] \Rightarrow \quad x_1 + x_2 + y_1 + y_2 = 5$ **Ans.**

(2) $(CM)^2 = \left(\frac{5}{4} - \frac{1}{5}\right)^2 + \frac{49}{25} = \left(\frac{25-4}{20}\right)^2 + \frac{49}{25}$



$= \frac{441}{400} + \frac{49}{25} = \frac{441 + 784}{400} = \frac{1225}{400} = \frac{49}{16} \quad \Rightarrow \quad CM = \frac{7}{4}$

(3) Circumcircle of ABC

$\left(x + \frac{1}{2}\right)(x - 3) + y^2 = 0 \quad \Rightarrow \quad (2x + 1)(x - 3) + 2y^2 = 0$

$\Rightarrow \quad 2\left(x^2 + y^2\right) - 5x - 3 = 0 \quad \Rightarrow \quad x^2 + y^2 - \frac{5}{2}x - \frac{3}{2} = 0$ ….(1)

Given $x^2 + y^2 - 4y + k = 0$ which is orthogonal to (1) using the condition of orthogonality

we get, $0 + 0 = k - \frac{3}{2} \Rightarrow \quad k = \frac{3}{2}$ Ans.]

Q. 25 D Q. 26 C Q. 27 A

[Sol. Equation of tangent to $y = f(x)$ at (x_i, y_i) $Y - y_i = m(X - x_i)$

(i) put $Y = 0,\; X = x_i - \frac{y_i}{m} = x_{i+1} \Rightarrow x_{i+1} - x_i = -\frac{Y_i}{m}$

(ii) $d = -\frac{y_i}{m} \quad \Rightarrow \quad m = -\frac{y_i}{d}$ ($x_i y_i$ lies on the curve, d is the common difference of A.P.) $\frac{dy}{dx} = -\frac{y}{\log_2 e} = -y \ln 2 \quad \Rightarrow \int \frac{dy}{y} = -\ln 2 \int dx \quad \Rightarrow \quad \ln y = -x \ln 2 + C$

Curve passing through $(0, 2) \Rightarrow \quad C = \ln 2$

$\therefore \quad \ln y = (1 - x).\ln 2 = \ln 2^{1-x} \quad \Rightarrow y = 2^{1-x}$ **Ans.**

(ii) again if $x_1, x_2, x_3, \dots\dots\dots, x_n$ in G.P.

Divide equation (1) by $x_i \Rightarrow \frac{x_{i+1}}{x_i} - 1 = -\frac{y_i}{mx_i}$

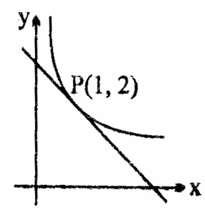
$r - 1 = -\frac{y}{mx}$ ($x_i y_i$ lies on curve, r is the common ratio of G.P.)

$2 - 1 = -\frac{y}{mx} \Rightarrow m = -\frac{y}{x} \Rightarrow \frac{dy}{dx} = -\frac{y}{x} \Rightarrow \frac{dy}{y} + \frac{dx}{x} = 0$

$\ln xy = \ln c \Rightarrow xy = c$ as curve passes through $(1, 2) \Rightarrow c = 2$

$xy = 2$ which is a rectangle hyperbola **Ans.**

(iii) Equation of tangent at $(1, 2)$ on $xy = 2$,



$\frac{x}{x_1} + \frac{y}{y_1} = 2;\ \frac{x}{1} + \frac{y}{2} = 2 \Rightarrow 2x + y = 4$ .....(1)

circle touching (1) at (1, 2) is

$(x-1)^2 + (y-2)^2 + \lambda(2x + y - 4) = 0$ .....(2)

it passé through $(1, 0) \Rightarrow 4 + \lambda(2 - 4) = 0 \Rightarrow \lambda = 2$

$x^2 + y^2 - 2x - 4y + 5 + 4x + 2y - 8 = 0 \Rightarrow x^2 + y^2 + 2x - 2y - 3 = 0$

$r^2 = 1 + 1 + 3 = 5 \Rightarrow r = \sqrt{5}$ **Ans.]**

Q. 28 B Q. 29 A Q. 30 B **[Sol.** $y = f(x) = \frac{x^2}{4} - ax + a^2 + a - 2$

(1) for zeroes to be on either side of origin

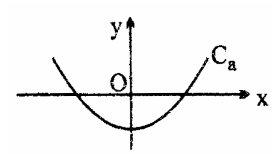
$f(0) < 0$

$a^2 + a - 2 < 0 \Rightarrow (a+2)(a-1) < 0 \Rightarrow -2 < a < 1 \Rightarrow 2$ integers i.e. $\{-1, 0\} \Rightarrow (B)$

(2) Vertex of C_a is $(2a, a-2)$

Hence $h = 2a$ and $k = a - 2$

$h = 2(k+2)$ Locus $x = 2y + 4 \Rightarrow x - 2y - 4 = 0$ **Ans.**



(3) Let $y = mx + c$ is a common tangent to $y = \frac{x^2}{4} - 3x + 10$

....(10) (for $a = 3$)

and $y = 2 - \frac{x^2}{4}$ ......(2) where $m = m_1$ or m_2 and $c = c_1$ or c_2

solving $y = mx = c$ with (1) $mx + c = \frac{x^2}{4} - 3x + 10$ or $\frac{x^2}{4} - (m+3)x + 10 - c = 0$

$D = 0$ gives $\Rightarrow (m+3)^2 = 10 - c \Rightarrow c = 10 - (m+3)^2$ ....(3)

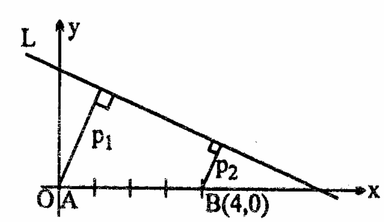
|||ly $mx + c = 2 - \frac{x^2}{4} \Rightarrow \frac{x^2}{4} + mx + c - 2 = 0 \Rightarrow D = 0$ gives

$m^2 = c - 2 \Rightarrow c = 2 + m^2$ .....(4)

From (3) and (4)

$10 - (m+3)^2 = 2 + m^2 \Rightarrow 2m^2 + 6m + 1 = 0$

$\Rightarrow m_1 + m_2 = -\frac{6}{2} = -3$ **Ans]**



Q. 31 A Q. 32 A Q. 33 B **[Sol.**

(1) Let $A = (0, 0)$ and $B = (4, 0)$

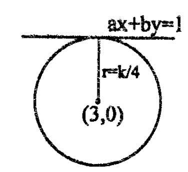
And the line be $ax + by = 1$

$$p_1 = \left|\frac{1}{\sqrt{a^2+b^2}}\right|; \quad p_2 = \left|\frac{4a-1}{\sqrt{a^2+b^2}}\right| \Rightarrow p_1 + 3p_2 = k$$

$$\left|\frac{1}{\sqrt{a^2+b^2}}\right| + 3\left|\frac{4a-1}{\sqrt{a^2+b^2}}\right| = k$$

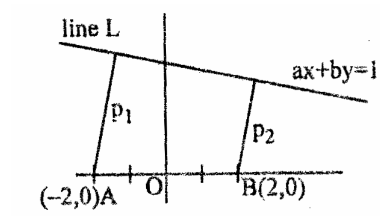
now $(0, 0)$ and $(4, 0)$ must give the same sign i.e. –ve with the line $L(4a-1<0)$

$$\therefore \quad \frac{1}{\sqrt{a^2+b^2}} + \frac{3(1-4a)}{\sqrt{a^2+b^2}} = k \Rightarrow \left|\frac{4(1-3a)}{\sqrt{a^2+b^2}}\right| = k; \quad \left|\frac{3a-1}{\sqrt{a^2+b^2}}\right| = \frac{k}{4}$$



hence centre of the fixed circle is $(3, 0)$ which lies on the line segment AB $\Rightarrow$ (A)

(2) If $k = 4 \Rightarrow r = 1$ **Ans.**



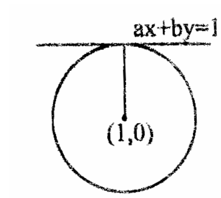
(3) $p_1 = \left|\frac{-2a-1}{\sqrt{a^2+b^2}}\right|; \quad p_2 = \left|\frac{2a-1}{\sqrt{a^2+b^2}}\right|$

hence with the same argument, $2a - 1 < 0$

and $-2a-1<0 \;\therefore\; p_1 + 3p_2 = k$

$$\frac{1+2a}{\sqrt{a^2+b^2}} + \frac{3(1-2a)}{\sqrt{a^2+b^2}} = k \Rightarrow \frac{4(1-a)}{\sqrt{a^2+b^2}} = k$$

$\frac{|a-1|}{\sqrt{a^2+b^2}} = \frac{k}{4}$ hence center is $(1, 0)$ Ans.]



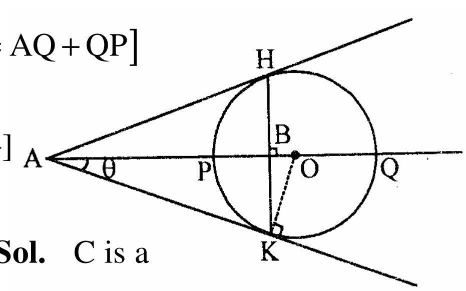
Assertion & Reason Type

Q. 1 A Q. 2 A [Sol. $\frac{(AK)}{(OA)} = \cos\theta = \frac{AB}{AK}$

$\Rightarrow \quad (AK)^2 = (AB)(OA) = (AP)(AQ)$ [$AK^2 = AP.AQ$ using power of point A]

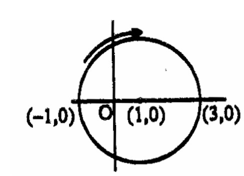
Also $OA = \frac{AP+AQ}{2}$ $[AQ - AO = r = AO - AP \Rightarrow 2AO = AQ + QP]$

$\Rightarrow \quad (AP)(AQ) = AB\left(\frac{AP+AQ}{2}\right) \Rightarrow \quad AB = \frac{2(AP)(AQ)}{(AP+AQ)}$]

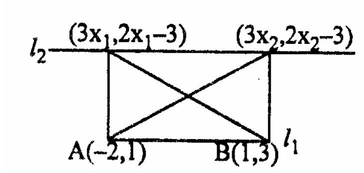


Q. 3 C **[Hint.** angle of intersection is $\tan^{-1}\left(\frac{3}{4}\right)$] Q. 4 [A] **[Sol.** C is a circle with centre $(+1, 0)$ and radius

$2(x-1)^2 + y^2 = 4$]



Q. 5 A **[Sol.** $m_{l_1} = \frac{2}{3}$

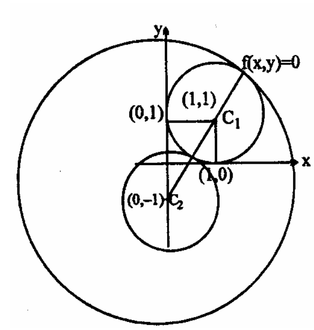


$m_{l_2} = \frac{2(x_2 - x_1)}{3(x_2 - x_1)} = \frac{2}{3}$

$A = \frac{1}{2}\begin{vmatrix} -2 & 1 & 1 \\ 1 & 3 & 1 \\ 3x & (2x-1) & 1 \end{vmatrix} = 8]$

More than One Correct Type

Q. 1 C, D [Sol. $f(x, y) = 0$ will have centre at $(1, 1)$ and radius unity

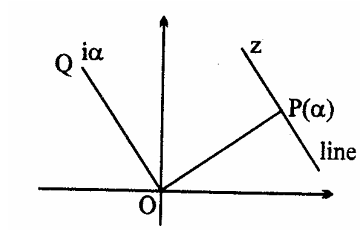


$= (x-1)^2 + (y-1)^2 = 1$ $C_1C_2 = \sqrt{5}$ $\left(\begin{matrix} \textit{seefigur, one circle external} \\ \textit{and other internlly tangent} \end{matrix}\right)$

Hence radius r of $g(x, y) = 0$ is

$\sqrt{5} + 1$ and $\sqrt{5} - 1 \Rightarrow$

Q. 2 B, D [Sol. Required line is passing through $P(\alpha)$ and parallel to the vector $\overrightarrow{OQ}$ Hence $z = \alpha + i\lambda a,\ \lambda \in R$



$\frac{z-\alpha}{\alpha}$ = purely imaginary $\Rightarrow$ $\text{Re}\left(\frac{z-\alpha}{\alpha}\right) = 0 \Rightarrow$ **(B)**

$\Rightarrow \quad \text{Re}\left((z-\alpha)\overline{\alpha}\right) = 0 \quad \Rightarrow \quad \text{Re}\left(z\overline{\alpha} - |\alpha|\right) = 0$

Also $\quad \frac{z-\alpha}{\alpha} + \frac{\overline{z}-\overline{\alpha}}{\overline{\alpha}} = 0$

$\Rightarrow \overline{\alpha}(z-\alpha) + \alpha(\overline{z} - \overline{\alpha}) = 0 \Rightarrow \overline{\alpha}z + \alpha\overline{z} - 2|\alpha|^2 = 0 \quad \Rightarrow$ **(D)**]

Match Matrix Type

Q. 1 (A) Q; (B) S; (C) P

[Sol. (A) $C_1:\quad x^2 - 3y + 2x + 3 = 0 \Rightarrow (x+1)^2 = 1 - 3 + 2y = 3x - 2 = 3\left(y - \frac{2}{3}\right)$ Hence $L_1L_2 = 3$ Ans.

(B) $C_2:\quad 4x^2 + y^2 - 16x + 6y + 21 = 0 \Rightarrow 4(x^2 - 4x) + (y+3)^2 + 21 - 9 = 0$

$4\left[(x-2)^2 - 4\right] + (y+3)^3 + 12 = 0 \Rightarrow 4(x-2)^2 + (y+3)^2 = 4$

$(x-2)^2 + \frac{(y+3)^2}{4} = 1$ Let $x - 2 = X;\quad y + 3 = Y$

$X^2 + \frac{Y^2}{4} = 1 \Rightarrow L_1L_2 = \frac{2b^2}{a} = \frac{2.1}{2}\quad (b = 1,\ a = 2) \Rightarrow \quad L_1L_2 = 1$

(C) $C_3:\quad x^2 - 4y^2 - 2x - 32y - 127 = 0 \Rightarrow \left[(x-1)^2 - 1\right] - 4\left[(y+4)^2 - 16\right] - 127 = 0$

$\frac{(x-1)^2}{64} - \frac{(y+4)^2}{16} = 1$ Let $x - 1 = X;\ y + 4 = Y \Rightarrow \frac{X^2}{64} - \frac{Y^2}{16} = 1 \Rightarrow L_1L_2 = \frac{2.16}{8} = 4$