ANSWERSHEET (TOPIC = ALGEBRA) COLLECTION #2

Question Type = A.Single Correct Type

Q. 1 (B) Sol

$$\frac{1}{14!}\left(2^{14}C_1+2^{14}C_3+2^{14}C_5+2^{14}C_7\right)=\frac{1}{14!}\left({}^{14}C_1+{}^{14}C_3+{}^{14}C_5+{}^{14}C_7+{}^{14}C_9+{}^{14}C_{11}+{}^{14}C_{13}\right)=\frac{1}{14!}.2^{14-1}=\frac{2^{13}}{14!}\ ]$$

Q. 2 (A) Sol $\frac{{}^{n}C_{k}}{{}^{n}C_{k+1}}=\frac{1}{2}$ $\Rightarrow$ $\frac{n!}{k!(n-k)!}.\frac{(k+1)!(n-k-1)!}{n!}=\frac{1}{2}$ or $\frac{k+1}{n-k}=\frac{1}{2}$

$2k+2=n-k$

$n-3k=2$ …(1)

|||ly $\frac{{}^{n}C_{k+1}}{{}^{n}C_{k+2}}=\frac{2}{3}$

$$\frac{n!}{(k+1)!(n-k-1)!}.\frac{(k+2)!(n-k-2)!}{n!}=\frac{2}{3}$$

$$\frac{k+2}{n-k-1}=\frac{2}{3}$$

$3k+6=2n-2k-2$

$2n-5k=8$ …(2)

From (1) and (2) $n=14$ and $k=4$

$\therefore$ $n+k=18$ Ans.]

Q. 3 (C) Sol Number of terms in $(1+x)^{2009}=2010$

…..(1)

+ addition terms in $(1+x^2)^{2008}=x^{2010}+x^{2012}+.........+x^{4016}=1004$

……(2)

+ addition terms in $(1+x^2)^{2007}=x^{2010}+x^{2013}+.........+x^{4014}+......+x^{6021}=1338$ ……(3)

- (common to 2 and 3) $=x^{2010}+x^{2016}+.........+x^{4014})=335$

Hence total $=2010+1004+1338-335$

$=4352-335=4017$ **Ans.**

Alternatively :

$$n(A\cup B\cup C)=n(A)+n(B)+n(C)-\left(n(A\cap B)+n(B\cap C)+n(C\cap A)+n(A\cap B\cap C)\right)$$

$$(2010+2009+2008)-\underbrace{1005}_{A\cap B}+\underbrace{670}_{B\cap C}+\underbrace{670}_{C\cap A}+335=4017 \quad \text{Ans.]}$$

Q. 4 (C) Sol Let $z=a+ib$ $\Rightarrow$ $\bar{z}=a-ib$

Hence we have

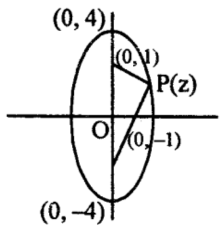
$\therefore$ $|z|^{2008}=|\bar{z}|=|z|$

$$|z|\left[|z|^{2007}-1\right]=0$$

$|z|=0$ or $|z|=1$; if $|z|=0 \Rightarrow z=0 \Rightarrow (0, 0)$

if $|z|=1$ $z^{2009}=z\bar{z}=|z|^2=1 \Rightarrow$ 2009 value of z $\Rightarrow$ Total = 2010 Ans.]

Q. 5 (B) Sol



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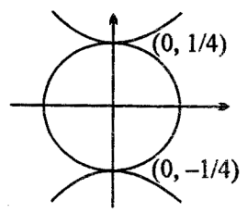
If $|z+i|+|z-i|=8$,

$PF_1+PF_2=8$

$\therefore \quad |z|_{max}=4 \quad \Rightarrow \quad (B)$

Q. 6 (D) Sol



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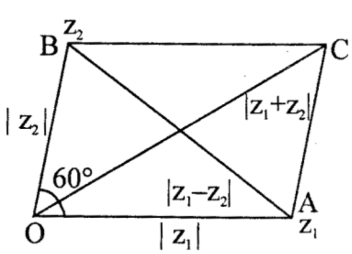
$$\left(1+i\sqrt{3}\right)^n=\left[2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\right]^n=2^n\left(\cos\frac{n\pi}{3}+i\sin\frac{n\pi}{3}\right)$$

$f\left(1+i\sqrt{3}\right)^n=$ real part of $z=2^n\cos\frac{n\pi}{3}$

$$\therefore \quad \sum_{n=1}^{6a}\log_2\left|2^n\cos\frac{n\pi}{3}\right|=\sum\left(n+\log_2\left|\cos\frac{n\pi}{3}\right|\right)=\frac{6a(6a+1)}{2}+\underbrace{(-1-1+0-1-1+0)}_{\text{a such term}}$$

$=3a(6a+1)-4a=18a^2-a$ **Ans.]**

Q. 7 (C) Sol Using consine rule



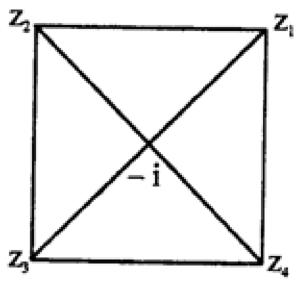
$$|z_1+z_2| = \sqrt{|z_1|^2+|z_2|^2-2|z_1||z_2|\cos 120°}$$

$$\sqrt{4+9+2.3} = \sqrt{19}$$

and $|z_1-z_2| = \sqrt{|z_1|^2+|z_2|^2-2|z_1||z_2|\cos 60°}$

$$= \sqrt{4+9-6} = \sqrt{7}$$

$\therefore$ $\left|\dfrac{z_1+z_2}{z_1+z_2}\right| = \sqrt{\dfrac{19}{7}} = \dfrac{\sqrt{133}}{7} \Rightarrow N = 133$ **Ans.]**



Q. 8 (C) Sol $z^4+4z^3i+6z^2i^2+4zi^3+i^4 = 1+i$

$(z+i)^4 = 1+i \Rightarrow |z+1|^4 = \sqrt{2} \Rightarrow |z+i| = 2^{1/8}$

$|z+i| = 2^{1/8}$

$\text{Area} = \dfrac{d^2}{2} = \dfrac{4|z_1+i|^2}{2}$

$= 2.2^{1/8}.2^{1/8} = 2^{5/4}$ **Ans]**

Q. 9 (A) Sol $W = \dfrac{1}{1-z} = \dfrac{1}{(1-\cos\theta)-i\sin\theta} = \dfrac{1}{2\cos^2(\theta/2)-2i\sin(\theta/2)\cos(\theta/2)}$

$= \dfrac{1}{-2i\sin(\theta/2)\left[\cos(\theta/2)+i\sin(\theta/2)\right]} = \dfrac{\cos(\theta/2)-i\sin(\theta/2)}{-2i\sin(\theta/2)} = \dfrac{1}{2}+\dfrac{1}{2}\cot\dfrac{\theta}{2}i$

Hence $\text{Re}(w) = \dfrac{1}{2}$

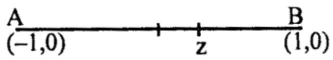
$\therefore$ w moved on the line $2x-1=0$ parallel to y-axis.]

Q. 10 (D) Sol Given $\left|z-|z+1|\right|^2 = \left|z+|z-1|\right|^2$

$\therefore$ $(z-|z+1|)(\bar{z}-|z+1|) = (z+|z-1|)(\bar{z}+|z-1|)$

$z\bar{z}-z|z+1|-\bar{z}|z+1|+|z+1|^2 = z\bar{z}+z|z-1|+\bar{z}|z-1|+|z-1|^2$

$|z+1|^2-|z-1|^2 = (z+\bar{z})\left[|z-1||z+1|\right]$



$(z+1)(\bar{z}+1)-(z-1)(\bar{z}-1)=(z+\bar{z})[|z-1|+|z+1|]$

$(z\bar{z}+z+\bar{z}+1)-(z\bar{z}-z-\bar{z}+1)=(z+\bar{z})[|z-1|+|z+1|]$

$2(z+\bar{z})=(z+\bar{z})[|z+1|+|z-1|]$

$(z+\bar{z})[|z+1|+|z-1|-2]=0$

$\Rightarrow$ either $z+\bar{z}=$ $\Rightarrow$ z ix purely imaginary

$\Rightarrow$ z lies on $y-$axis $\Rightarrow$ $x=0$

or $|z+1|+|z-1|=2$

$\Rightarrow$ z lie on the segment joining $(-1, 0)$ and $(1, 0)$ $\Rightarrow$ (D)]

Q. 11 (B) Sol $(z+1)^4=16z^4$

$|z+1|=2|z|$

$|z+1|^2=4|z|^2$

$(z+1)(\bar{z}+1)=4z\bar{z}$

$3z\bar{z}-z-\bar{z}-1=0$ or $z\bar{z}-\frac{1}{3}z-\frac{1}{3}\bar{z}-\frac{1}{3}=0$

Center = - coefficient of $\bar{z}=\left(\frac{1}{3}, 0\right)$

Radius $=\sqrt{\alpha\bar{\alpha}-r}=\sqrt{\frac{1}{9}+\frac{1}{3}}=\sqrt{\frac{4}{9}}=\frac{2}{3}$

Hence centre $\left(\frac{1}{3}, 0\right)$ & radius $=\frac{2}{3}$ $\Rightarrow$ (B)]

Q. 12 (B) Sol $\frac{\sqrt{2}+1}{2}=(1-x^2+x^4-x^6+x^8..........)+(x-x^3+x^5-x^7.......)$

$\frac{\sqrt{2}+1}{2}=\frac{1}{1+x^2}+\frac{x}{1+x^2}=\frac{1+x}{1+x^2}$

or $(\sqrt{2}+1)x^2+(\sqrt{2}+1)=2+2x$

$(\sqrt{2}+1)x^2-2x+(\sqrt{2}-1)=0$ (divide by $\sqrt{2}+1$)

$x^2-2(\sqrt{2}-1)x+(\sqrt{2}-1)^2=0$

$\left[x-(\sqrt{2}-1)\right]^2=0$ $\Rightarrow$ $x=\sqrt{2}-1$ Ans.]

Q. 13 (C) Sol $x^2-px+20=0$

$x^2 - 20x + p = 0$

If $p \neq 20$ then

$x^2 - px + 20 = x^2 - 20x + p \quad \Rightarrow \quad (20-p)x + (20-p) = 0 \quad \Rightarrow x = -1$ and $p = -21$

Hence there are 3 values of *x* i.e. $\{10+4\sqrt{5}, 10-4\sqrt{5}, -1\}$]

Q. 14 (A) Sol $D_1 = 4b_1^2 - 4a_1c_1 < 0$

i.e. $a_1c_1 > b_1^2$ .....(1)

$D_2 = 4b_2^2 - 4a_2c_2 < 0$

hence $a_2c_2 > b_2^2$ ....(2)

multiplying (1) and (2)

$a_1a_2c_1c_2 > b_1^2b_2^2$

Now consider for $f(x)$

$D = b_1^2b_2^2 - 4a_1a_2c_1c_2$

$< b_1^2b_2^2 - 4b_1^2b_2^2$

$= -3b_1^2b_2^2$

$\therefore \quad D < 0 \Rightarrow g(x) > 0 \quad \forall x \in R \quad \Rightarrow \quad (A)$]

Q. 15 (C) Sol Product will be divisible by 3 if atleast one digit is 0, 3, 6, 9

Hence total 4 digit numbers = 9.10^3

Number of 4 digit numbers without

0, 3, 6 or 9 = 6^4 = 1296

$\therefore$ Number of numbers $= 9000 - 1296 = 7704$ Ans.]

Q. 16 (B) Sol Sum of single digit number $1+3+5+7+9 = 25 = S$

Sum of two digit number $4S(1+10) = 4(S+10S) = 44S$

Sum of three digit number $12S(1+10+10^2) + (12)(111)S = 1332S$

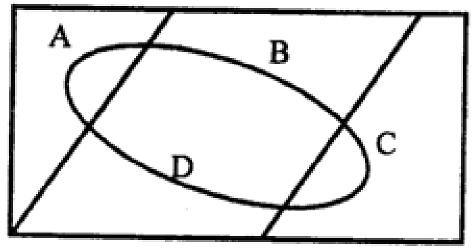
Sum of four digit number $24S(1+10+10^2+10^3) = 24(1111)S = 26664S$

Total $= 28041S$]

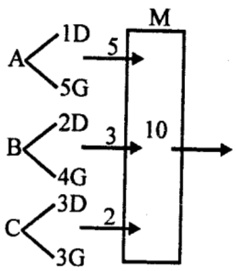
Q. 17 (B) Sol $n = 3$; P (success) $= P(HT \text{ or } TH) = \frac{1}{2} \Rightarrow p = q = \frac{1}{2}$ and $r = 2$

$P(r=2) = {}^3C_2\left(\frac{1}{2}\right)^2 . \frac{1}{2} = \frac{3}{8}$ Ans.]

Q. 18 (A) Sol



$P(A)=\frac{5}{10}, P(B)=\frac{3}{10}; P(C)=\frac{2}{10}$



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$P(D)=P(A).P(D/A)+P(B).P(D/B)+P(C).P(D/C)$

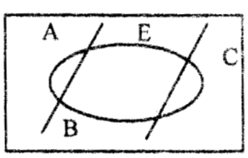
$=\frac{5}{10}.\frac{1}{6}+\frac{3}{10}.\frac{2}{6}+\frac{2}{10}.\frac{3}{6}$

$=\frac{5+6+6}{60}=\frac{17}{60}$

Q. 19 (A) Sol **A: exactly one child**
B: exactly two children
C: exactly 3 children

$P(A)=\frac{1}{4}; P(B)=\frac{1}{2}; P(C)=\frac{1}{4}$

E: couple has exactly 4 grandchildren



$P(E)=P(A).P(E/A)+P(B).P(E/B)+P(C).P(E/C)$

$$=\frac{1}{4}.0+\frac{1}{2}\left[\underbrace{\left(\frac{1}{2}\right)^2}_{2/2}+\underbrace{\frac{1}{4}.\frac{1}{4}.2}_{(1,3)}\right]+\frac{1}{4}\left[3\underbrace{\left(\frac{1}{4}.\frac{1}{4}.\frac{1}{2}\right)}_{1\quad 1\quad 2}\right]$$

$=\frac{1}{8}+\frac{1}{16}+\frac{3}{128}=\frac{27}{128}$ Ans.

|||ly 2/2 denotes each child having two children

$2.\frac{1}{4}.\frac{1}{4}$ denotes each child having 1 and 3 or 3 and 1 children

$=\frac{16}{128}+\frac{8}{128}+\frac{3}{128}=\frac{27}{128}$ **Ans.]**

Q. 20 (A) Sol $\tan\alpha+\tan\beta=-p$

$\tan\alpha\tan\beta=q$

$\tan(\alpha+\beta)=\frac{-p}{1-q}=\frac{p}{q-1}$

$\frac{1}{1+\tan^2(\alpha+\beta)}\left[\tan^2(\alpha+\beta)+p\tan(\alpha+\beta)+q\right]$

$\frac{1}{1+\frac{p^2}{(q-1)^2}}\left[\frac{p^2}{(q-1)^2}+\frac{p^2}{(q-1)}+q\right]$

$\frac{1}{(q-1)^2+p^2}\left[p^2+p^2(q-1)+q(q-1)^2\right]$

$\frac{1}{p^2+(q-1)^2}\left[p^2q+q(q-1)^2\right]$

$q\left[\frac{p^2+(q-1)^2}{p^2+(q-1)^2}\right]=q$]

Q. 21 (D) Sol a, 2a, b, $(a-b-6)$ in A.P.

$a+a-b-6=2a+b$

$b=-3$

$2a-a=b-2a \quad\Rightarrow\quad 3a=b;\ a=-1$

Hence the series is

$-1, -2, -3, -4, -5, \ldots\ldots\ldots\ldots$

$\therefore\quad s_{100}=-[1+2+3+\ldots\ldots..+100]=-5050$ Ans.]

Q. 22 (A) Sol $T_r=\frac{1^2+2^2+3^2+\ldots\ldots+r^2}{r(r+1)}$

$\therefore\quad S=\sum_{r=1}^{60}T_r=\sum_{r=1}^{60}\frac{r(r+1)(2r+1)}{6r(r+1)}=\frac{1}{6}\sum_{r=1}^{60}(2r+1)=\frac{1}{6}\underbrace{[3+5+7+\ldots\ldots.+121]}_{\text{A.P. with a=3, d=2, n=60}}$

$=\frac{60}{2.6}\left[6+(60-1)2\right]=5[6+59\times2]=5[6+118]=620$ Ans.]

Question Type = B.Comprehension or Paragraph

Q. 23 () Sol C

B
C

[Sol. $\therefore$ $\left(1+x+x^2\right)^{2n}=\sum_{r=0}^{4n} a_r x^r$ .....(1)

Replacing *x* by $\frac{1}{x}$ in equation (1) then

$$\left(1+\frac{1}{x}+\frac{1}{x^2}\right)^{2n}=\sum_{r=0}^{4n} a_r\left(\frac{1}{x}\right)^r \quad \text{or} \quad \left(1+x+x^2\right)^{2n}=\sum_{r=0}^{4n} a_r x^{4n-r} \quad ....(2)$$

From equation (1) and (2), we get

$$\sum_{r=0}^{4n} a_r x^r=\sum_{r=0}^{4n} a_r x^{4n-r}$$

Comparing coefficient of x^{4n-r} on both sides, then we get

$a_r=a_{4n-r}$ ....(3)

(10) Put $x=1$ and $x=-1$ in equation (1), then

$9^n=a_0+a_1+a_2+a_3+a_4+....+a_{2n}+....+a_{4n}$

and $1=a_0-a_1+a_2-a_3+a_4-....+a_{2n}+....+a_{4n}$

adding and subtracting, then we get

$$\frac{9^n+1}{2}=a_0+a_2+a_4+.........+a_{2n}+........+a_{4n-2}+a_{4n} \quad ....(4)$$

and $\frac{9^n-1}{2}=a_1+a_3+a_5+.........+a_{2n-1}+........+a_{4n-1}$ ....(5)

Now, $\therefore$ $a_r=a_{4n-r}$

Put $r=0,2,4,6,.........,a_{2n-2},a_{2n}$

$\therefore$ $a_0=a_{4n}$

$a_2=a_{4n-2}$

$a_4=a_{4n-2}$

$\vdots \qquad \vdots$

$a_{2n-2}=a_{2n+2}$

$\therefore$ $a_0+a_2+a_4+.....+a_{2n-2}=a_{2n-2}+......+a_{2n-4}+a_{4n-2}+a_{4n}$

Now from equation (4)

$$\frac{9^n+1}{2}=2\left(a_0+a_2+a_4+.....+a_{2n-2}\right)+a_{2n}$$

$$\Rightarrow \quad \frac{9^n+1-2a_{2n}}{4}=a_0+a_2+a_4+.....+a_{2n-2}$$

$$\therefore \quad \sum_{r=0}^{n-1} a_{2r}=\frac{9^n+1-2a_{2n}}{4}$$ **Ans.**

(13) $\therefore$ $a_r=a_{4n-r}$

Put $r=1,\ 3,5,\ 7,\ ....,\ 2n-3,\ 2n-1$

$a_1 = a_{4n-1}$
$a_3 = a_{4n-3}$
$a_5 = a_{4n-5}$
: :
$a_{2n-3} = a_{2n+3}$
$a_{2n-1} = a_{2n+1}$

$\therefore \quad a_1 + a_3 + a_5 + ... + a_{2n-1} = a_{2n+1} + a_{2n+3} + ... + a_{4n-3} + a_{4n-1}$

Now from equation (5)

$$\frac{9^n - 1}{2} = 2\left(a_1 + a_3 + a_5 + ... + a_{2n-1}\right)$$

$$\therefore \quad \sum_{r=1}^{n} a_{2r-1} = \left(\frac{9^n - 1}{4}\right) = \left(\frac{3^{2n} - 1}{4}\right) \qquad \textbf{Ans.}$$

(14) $a_2 =$ coefficient of x^2 in $\left(1 + x + x^2\right)^{2n}$

$=$ coefficient of x^2 in $\left\{1 + {}^{2n}C_1\left(x + x^2\right) + {}^{2n}C_2\left(x + x^2\right)^2 + ...\right\}$

$= {}^{2n}C_1 + {}^{2n}C_2$

$= {}^{2n+1}C_2$ **Ans.]**

Q. 24 () Sol **Q. 1** C
Q. 2 B
Q. 3 D

[Sol. $R = (1 + 2x)^n$

put $x = 1$ to get sum of all the coefficients

$\therefore \quad 3^n = 6561 = 3^8 \Rightarrow \quad n = 8$

(i) for $x = \frac{1}{\sqrt{2}}; R = \left(\sqrt{2} + 1\right)^8$

consider $\frac{\left(\sqrt{2}+1\right)^8 + \left(\sqrt{2}-1\right)^8}{1 + f + f'} = 2\left[{}^8C_0\left(\sqrt{2}\right)^8 + ...\right] =$ even integer

since I is integer $\Rightarrow$ $f + f'$ must be an integer
but $0 < f + f' < 2 \Rightarrow f + f' = 1 \Rightarrow f' = 1 - f$
now $n + R - Rf$

$$n + R(1 - f) = 8 + \left(\sqrt{2}+1\right)^n . \left(\sqrt{2}-1\right)^n = 8 + 1 = 9 \qquad \text{Ans.}$$

(ii) T_{r+1} in $(1 + 2x)^8 = {}^8C_r(2x)^r = {}^8C_r$ when $x = \frac{1}{2}$

now $T_{r+1} \ge T_r$

$\frac{T_{r+1}}{T_r} \gtrless 1 \quad \Rightarrow \quad \frac{^8C_r}{^8C_{r-1}} \gtrless 1$

$T_{r+1} \gtrless T_r \quad \frac{8!}{r!(8-r)!}.\frac{(r-1)!;(9-r)!}{8!} \ge 1$

$(9-r) \ge r \quad \Rightarrow \quad 9 \ge 2r$

for $r = 1, 2, 3, 4$ this is true

i.e. $T_5 > T_4$

but for $r = 5$ $T_6 < T_5$

$\Rightarrow$ T_5 is the greatest term $\Rightarrow$ (B)

(iii) again $T_{k+1} = ^8C_k.2^k.x^k$; $T_k = ^8C_{k-1}.2^{k-1}.x^{k-1}$

$T_{k-1} = ^8C_{k-2}.2^{k-2}.x^{k-2}.x^{k-2}$

we want to find the term having the greatest coefficient

$\therefore$ $2^{k-1}.{}^8C_{k-1} > 2^k.{}^8C_k$ ...(1)

and $2^{k-1}.{}^8C_{k-1} > 2^{k-2}.{}^8C_{k-2}$ ...(2)

from (1)

$\frac{8!.2^{k-1}}{(k-1)!(9-k)!} > \frac{2^k.8!}{k!(8-k)!} \Rightarrow \frac{1}{(9-k)} > \frac{2}{k} \Rightarrow k > 18-3k \Rightarrow k > 6$

Again $2^{k-1}.{}^8C_{k-1} > 2^{k-2}.{}^8C_{k-2}$

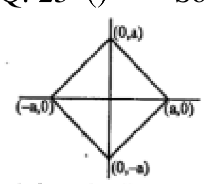
$\frac{8!.2^{k-1}}{(k-1)!(9-k)!} > \frac{2^{k-2}.8!}{(k-2)!(10-k)!} \Rightarrow \frac{2}{k-1} > \frac{1}{10-k}$

$\Rightarrow$ $20-2k > k-1$ $\Rightarrow 21 > 3k$ $\Rightarrow$ $k < 7$

$\Rightarrow$ $6 < k < 7$ $\Rightarrow$ T_6 and T_7 term has the greatest coefficient

$\Rightarrow$ $k = 6$ or 7 $\Rightarrow$ sum $= 6+7 = 13$ Ans.]

Q. 25 () Sol **Q. 1**



D

Q. 2 B

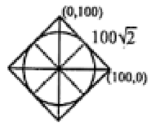
Q. 3 C

[Sol.

(i) $|x|+|y| = a$

Figure is a square Ans.

(ii) Area of the circle $= \frac{\pi d^2}{4}$ (where $d =$ diameter of circle = side of the square)

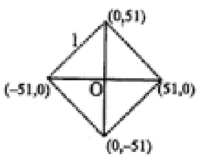


$= \frac{\pi(100)^2 .2}{4}$

$= 5000\pi$ Ans.

(iii) $x + y < 51$ $x \ge 0,\ y \ge 0$

or $x + y \le 50$



give one each to *x* and *y*

$x + y \le 48$

$x + y + z = 48 \Rightarrow$ number of solutions $= {}^{50}C_2$

$\frac{50\times 49}{2}$

Number of solutions in all the four quadrants $= 100.49 = 4900$

Number of solutions except $(0, 0)$ on *x* and *y* axis from $(-51, 0)$ to $(51, 0)$ and $(0, 51)$ to $(0, -51)$ are 200

Total solutions $= 4900 + 200 + 1 = 5101$ Ans.]

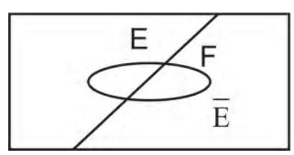
Q. 26 () Sol **Q. 1** D

Q. 2 A

Q. 3 B

[Sol. $P(E) = P$

$P(F) = P(E \cap F) + P(\overline{E} \cap F)$



$P(F) = P(E)P(F/E) + P(\overline{E})P(F/E)$

$= p.1 + (1-p).\frac{1}{5} = \frac{4p}{5} + \frac{1}{5}$

(i) if $p = 0.75$

$p(F) = \frac{1}{5}(4p+1) = \frac{1}{5}(4) = 0.8$

$\therefore$ $P(E/F) = \frac{P(E\cap F)}{P(F)} = \frac{0.75}{0.80} = \frac{15}{16}$ Ans.

(ii) now $P(E/F) = \frac{5p}{(4p+1)} \ge p$

Equality holds for $p = 0$ or $p = 1$

For all others value of $p \in (0, 1)$, LHS > RHS, hence **(A)**

(iii) If each question has n alternatives than

$$P(F) = p + (1-p)\frac{1}{n} = p\left(1 - \frac{1}{n}\right) + \frac{1}{n} = \frac{(n-1)p+1}{n}$$

$\therefore$ $P(E/F) = \frac{np}{(n-1)p+1}$ which increases as n increases for a fixed p $\Rightarrow$ **(B)]**

Q. 27 () Sol **Q. 1** B

Q. 2 A

Q. 3 C

[Sol. Urn – I $<^{5R}_{1B}$

Urn – I $<^{2R}_{4B}$

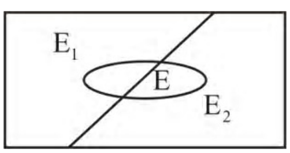
A : first two draws resulted in a blue ball.

B_1 : urn-I is used $P(B_1) = \frac{1}{2}$

B_2 : urn-II is used $P(B_2) = \frac{1}{2}$

$$P(A/B_1) = \frac{1}{6}.\frac{1}{6} = \frac{1}{36}$$

$$P(A/B_2) = \frac{4}{6}.\frac{4}{6} = \frac{16}{36} = \frac{4}{9}$$ **Ans.(i)**



$$\underbrace{P(B_1/A)}_{E_1} = \frac{\frac{1}{2}.\frac{1}{36}}{\frac{1}{2}.\frac{1}{36} + \frac{1}{2}.\frac{16}{36}} = \frac{1}{17}$$

$$\underbrace{P(B_2/A)}_{E_2} = \frac{\frac{1}{2}.\frac{16}{36}}{\frac{1}{2}.\frac{16}{36} + \frac{1}{2}.\frac{16}{36}} = \frac{16}{17}$$

$\Rightarrow$ **Ans. (ii)**

E: third ball drawn is red

$$P(E) = P(E \cap E_1) + P(E \cap E_2)$$

$$= \frac{1}{17}.\frac{5}{6} + \frac{16}{17}.\frac{2}{6} = \frac{5}{102} + \frac{32}{102} = \frac{37}{102}$$ **Ans. (iii)**

Q. 28 () Sol **Q. 1** [C]
Q. 2 [A]
Q. 3 [B]
[Sol.
(1) A: 3 balls drawn found to be one each of different colours.

B_1: $1(W)+1(G)+4(R)$ are drawn; $P(B_1)=\frac{1}{10}$

B_2: $1(W)+4(G)+1(R)$ are drawn; $P(B_2)=\frac{1}{10}$

B_3: $4(W)+1(G)+1(R)$ are drawn; $P(B_3)=\frac{1}{10}$

B_4: They are drawn in groups of 1, 2, 3 (WGR) - (6 cases); $P(B_4)=\frac{6}{10}$

B_5: $2(W)+2(G)+1(R)$; $P(B_5)=\frac{1}{10}$ **Ans.**

$P(A/B_1)=\frac{{}^4C_1}{{}^6C_3}=\frac{4}{20}$ W G R R R R

$P(A/B_2)=\frac{{}^4C_1}{{}^6C_3}=\frac{4}{20}$ W G G G G R

$P(A/B_3)=\frac{{}^4C_1}{{}^6C_3}=\frac{4}{20}$ W W W W G R

$P(A/B_4)=6.\frac{{}^1C_1.{}^2C_1.{}^3C_1}{{}^6C_3}=\frac{36}{20}$ W G G R R R,

$P(A/B_5)=\frac{{}^2C_1.{}^2C_1.{}^2C_1}{{}^6C_3}=\frac{8}{20}$ W W G G R R

$$\sum_{r=1}^{5}P(B_i).P(A/B_i)=\frac{1}{10}.\frac{4}{20}+\frac{1}{10}.\frac{4}{20}+\frac{1}{10}.\frac{4}{20}+\frac{1}{10}.\frac{36}{20}+\frac{1}{10}.\frac{8}{20}=\frac{56}{200}$$

(2) $P(B_1/A)=\frac{\frac{1}{10}.\frac{4}{20}}{\frac{56}{200}}=\frac{4}{56}=\frac{1}{14}$ **Ans.**

(3) $P(B_5/A)=\frac{\frac{1}{10}.\frac{8}{20}}{\frac{56}{200}}=\frac{8}{56}=\frac{2}{14}$

Hence P (bag had equals number of W and G balls/A)

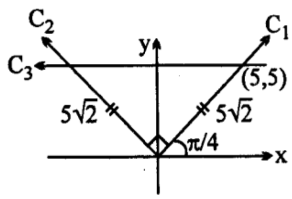
$=P(B_1/A)=P(B_5/A)=\frac{1}{14}+\frac{2}{14}=\frac{3}{14}$ **Ans.**]

Question Type = C.Assertion Reason Type

Q. 29 (C) Sol Statement-2 is False

Take eg. $z = 2 + 3i$

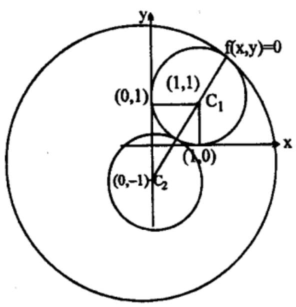
$\bar{z} = 2 - 3i$



$-z = -2 - 3i$

$-\bar{z} = -2 + 3i$ then figure is rectangle]

Q. 30 (D) Sol Area $= \frac{5\sqrt{2}.5\sqrt{2}}{2} = 25$

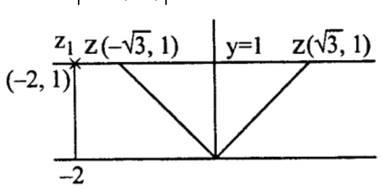


Hence S-1 is false and S-2 is true.]

Q. 31 (B) Sol z_1, z_2 and '0' are on the same side then only $S-2$ is the reason of $S-1$]

Q. 32 (C) Sol For $\lambda = |\text{Im}(z_1),|$ then number of values of $z = 1$

For $\lambda > |\text{Im}(z_1)|$, then number values of $z = 2$



]

Q. 33 (A) Sol **Let** $z = \cos\theta + i\sin\theta$ where $\cos\theta, \sin\theta \in Q$

$z^{2n} - 1 = -1 + \cos 2n\theta + i\sin 2n\theta$

$= -2\sin^2 n\theta + 2i\sin n\theta\cos n\theta$

$= -2\sin n\theta(\sin n\theta - i\cos n\theta)$

$|z^{2n} - 1| = 2|\sin n\theta|$

Now $P(n): \sin n\theta, \cos n\theta \in Q \quad \forall\, n \in N$ can be provided by induction if $\sin\theta, \cos\theta \in Q$]

Q. 34 (A) Sol **Let** $A = \begin{bmatrix} a & p \\ b & q \\ c & e \end{bmatrix}$ $A^T = \begin{bmatrix} a & b & c \\ p & q & r \end{bmatrix}$

$$AA^T = \begin{bmatrix} a^2+p^2 & ab+pq & ac+pr \\ ab+pq & b^2+q^2 & bc+qr \\ ac+pr & bc+qr & c^2+r^2 \end{bmatrix}$$

$$\left|AA^T\right| = \begin{vmatrix} a & p & 0 \\ b & q & 0 \\ c & r & 0 \end{vmatrix} \begin{vmatrix} a & b & c \\ p & q & r \\ 0 & 0 & 0 \end{vmatrix} = 0 \quad \Rightarrow \quad AA^T \text{ is singular.}$$]

Q. 35 (A) Sol **Given** $AB + A + B = 0$

$AB + A + B + I = I$

$A(B+I) + (B+I) = I$

$(A+I)(B+I) = I$

$\Rightarrow$ $(A+I)$ and $(B+I)$ are inverse of each other $\Rightarrow$ $(A+I)(B+I) = (B+I)(A+I)$

$\Rightarrow$ $AB = BA$]

Q. 36 (B) Sol Let $x_1, x_2, x_3 \in R$ be the roots of $f(x) = 0$

$\therefore$ $f(x) = (x - x_1)(x - x_2)(x - x_3)$

$f(i) = (i - x_1)(i - x_2)(i - x_3)$

$|f(i)| = |x_1 - i||x_2 - i||x_3 - i| = 1$

$\therefore$ $\sqrt{x_1^2+1}\sqrt{x_2^2+1}\sqrt{x_3^2+1} = 1$

This is possible only if $x_1 = x_2 = x_3 = 0$

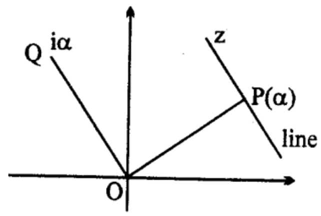
$\Rightarrow$ $f(x) = x^3$ $\Rightarrow$ $a = 0 = b = c$ $\Rightarrow$ $a + b + c = 0$*

Q. 37 (D) Sol $ix^2 + (1+i)x + i = 0$ $\Rightarrow$ $\alpha\beta = 1 \Rightarrow$ $\text{Im}(\alpha\beta) = 0$.]

Question Type = D.More than one may corect type

Q. 38 () Sol B, D

[Sol. Required line is passing through $P(\alpha)$ and parallel to the vector $\overrightarrow{OQ}$



Hence $z = \alpha + i\lambda a,\ \lambda \in R$

$\dfrac{z-\alpha}{\alpha} =$ purely imaginary

$\Rightarrow \quad \text{Re}\left(\dfrac{z-\alpha}{\alpha}\right) = 0 \Rightarrow \quad (\mathbf{B})$

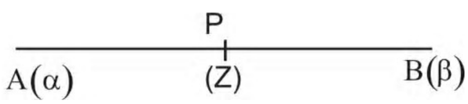
$\Rightarrow \quad \text{Re}\left((z-\alpha)\bar{\alpha}\right) = 0 \quad \Rightarrow \quad \text{Re}\left(z\bar{\alpha} - |\bar{\alpha}|\right) = 0$

Also $\quad \dfrac{z-\alpha}{\alpha} + \dfrac{\bar{z}-\bar{\alpha}}{\bar{\alpha}} = 0$

$\bar{\alpha}(z-\alpha) + \alpha(\bar{z}-\bar{\alpha}) = 0$

$\bar{\alpha}z + \alpha\bar{z} - 2|\alpha|^2 = 0 \quad \Rightarrow \quad (\mathbf{D})$]

Q. 39 () Sol [A, B, C, D]



[Sol. $AP + PB = AB$

$|z-\alpha| + |\beta - z| = |\beta - \alpha| \quad \Rightarrow \quad$ A is true

Now $z = \alpha + t(\beta - \alpha)$

$= (1-t)\alpha + t\beta$ where $t \in (0, 1) \quad \Rightarrow \quad$ B is true

Again $\dfrac{z-\alpha}{\beta-\alpha}$ is real $\Rightarrow \quad \dfrac{z-\alpha}{\beta-\alpha} = \dfrac{\bar{z}-\bar{\alpha}}{\bar{\beta}-\bar{\alpha}}$

$\Rightarrow \quad \begin{vmatrix} z-\alpha & \bar{z}-\bar{\alpha} \\ \beta-\alpha & \bar{\beta}-\bar{\alpha} \end{vmatrix} = 0 \quad$ Ans.

Again $\begin{vmatrix} z & \bar{z} & 1 \\ \alpha & \bar{\alpha} & 1 \\ \beta & \bar{\beta} & 1 \end{vmatrix} = 0$ if and only if $\begin{vmatrix} z-\alpha & \bar{z}-\bar{\alpha} & 0 \\ \alpha & \bar{\alpha} & 1 \\ \beta-\alpha & \bar{\beta}-\bar{\alpha} & 0 \end{vmatrix} = 0$

$\Rightarrow \quad \begin{vmatrix} (z-\alpha) & \bar{z}-\bar{\alpha} \\ \beta-\bar{\alpha} & \bar{\beta}-\bar{\alpha} \end{vmatrix} = 0 \quad$ **Ans.**

Q. 40 () Sol **Q. 1** A, B, C, D

[Sol. $\frac{z^n - 1}{z - 1} = (z - \alpha_1)(z - \alpha_2)............(z - \alpha_{n-1})$

put $z = i$

$$\prod_{r=1}^{n-1}(i - \alpha_r) = \frac{i^n - 1}{i - 1} = \begin{bmatrix} 0 & \text{if } n = 4k \\ 1 & \text{if } n = 4k+1 \\ 1+i & \text{if } n = 4k+2 \\ i & \text{if } n = 4k+3 \end{bmatrix}$$

Q. 41 () Sol **Q. 1** B, C, D

[Hint. $PQ = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow$ **B, C, D**]

Q. 42 () Sol A, B

[Sol. $\therefore \quad \frac{t_{2p}}{t_p} = \frac{t_{4p}}{t_{2p}} = r \text{ (say)}$

If we start from t_p, then t_{2p} is the $(p+1)^{th}$ term and if we start from t_{2p}, then t_{4p} is the $(2p+1)^{th}$ term

$\therefore \quad t_{2p} = t_p + pd \quad \ldots (1)$

and $\quad t_{4p} = t_{2p} + 2pd \ (d = c.d)$

$\Rightarrow \quad t_{4p} = t_{2p} + 2(t_{2p} - t_p)$ (from equation (1))

$\Rightarrow \quad t_{4p} = 3t_{2p} - 2t_p \Rightarrow \quad \frac{t_{4p}}{t_{2p}} = 3 - \frac{2t_p}{t_{2p}} \quad \Rightarrow \quad r = 3 - \frac{2}{r}$

$\Rightarrow \quad (r-1)(r-2) = 0 \quad \Rightarrow \quad r = 1, 2$ **Ans.**

Alternative solution : $R = \frac{A+(2p-1)D}{A+(p-1)D} = \frac{A(4p-1)D}{A+(2p-1)D}$

$R = \frac{(\ )-(\ )}{(\ )-(\ )}; \ R = \frac{2PD}{PD} = 2$

Also if $PD = 0 \quad \Rightarrow \quad D = 0 \Rightarrow T_p = T_{2p} = T_{4p} \Rightarrow R = 1$]

Q. 43 () Sol B, C

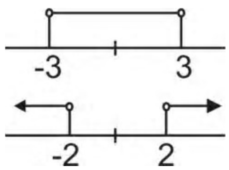
[Sol. $(\log_2 x)^4 - \left(\log_2\left(\frac{x}{2}\right)^3\right)^2 + 9\left[\log_2 32 - \log_2 x^2\right] < 4(\log_2 x)^2$

$(\log_2 x)^4 - (3\log_2 x - 3)^2 + 45 - 15\log_2 x < 4(\log_2 x)^2$

Let $\log_2 x = t$

$t^4 - (3t-3)^2 + 45 - 18t < 4t^2 \Rightarrow \quad t^4 - (9t^2 + 9 - 18t) - 18t + 45 < 4t^2$

$\Rightarrow \quad t^4 - 13t^2 + 36 < 0 \quad \Rightarrow \quad (t^2-4)(t^2-9) < 0$



$\Rightarrow \quad 4 < t^2 < 9$

$t^2 < 9 \Rightarrow \quad -3 < t < 3$

and $t^2 > 4 \Rightarrow \quad t > 2$ or $t < -2$

hence, $t \in (-3, -2) \cup (2, 3)$

$x \in \left(\frac{1}{8}, \frac{1}{4}\right) \cup (4, 8) \quad \Rightarrow \quad B, C$

Q. 44 () Sol **Q. 1** B, D

[Sol. $x^2 - 2x + 4 = -3\cos(ax+b)$

$(x-1)^2 + 3 = -3\cos(ax+b)$

for above equation to have atleast one solution

let $f(x) = (x-1)^2 + 3$ and $f(x) = -3\cos(ax+b)$

if $x = 1$ then L.H.S. = 3

and R.H.S. $= -3\cos(a+b)$

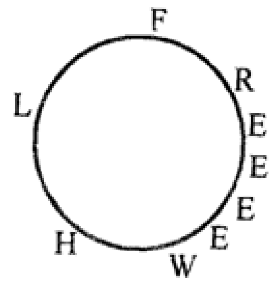
hence, $\cos(a+b) = -1$

$\therefore \quad a + b = \pi, 2\pi, 5\pi$

but $0 \le a + b \le 10 \Rightarrow \quad a + b = \pi$ or $3\pi \quad \Rightarrow \quad B, D]$

Q. 45 () Sol [A, B, C, D]

[Sol. $S = 111111 = 3.7.11.13.37 \quad \Rightarrow \quad [ABCD]$



Q. 46 () Sol **Q. 1** B, C, D

[Sol. **(A)** False is should be $^9P_5 - 1$

(B) $x.4! = 8!$

$\therefore \quad x = \frac{8!}{4!} = {}^8C_4$

(C) Vowels E E E E select 4 places in $^9C_4$ ways arrange consonant alphabetically only us one ways.

$\therefore {}^9C_4 = 126 = \frac{1}{2}.256 = \frac{1}{2}.{}^{10}C_5$

(D) True

$\therefore$ correct answer are $(B), (C)$ and (D)

Q. 47 () Sol B, C, D

[Sol. Let number of blue marbles is b and number of green marbles is g

Hence $\frac{bg}{^{b+g}C_2} = \frac{1}{2}$

$(b+g)(g+b-1) = 4bg$

$(b+g)^2 - (b+g) = 4bg$

$b^2 + g^2 + 2bg - b - g = 4bg$

$g^2 - 2bg - g + b^2 - b = 0$

$D = (2b+1)^2 - 4(b^2 - b)$

$= 8b + 1$ must a perfect square. Hence3 possible values of b are $3, 6, 10 \Rightarrow [B, C, D]$]

Q. 48 () Sol B, C, D

[Sol. Let the H.P. be $\frac{1}{A} + \frac{1}{A+D} + \frac{1}{A+2D} + ...$

Corresponding A.P. $A + (A+D) + (A+2D) + ...$

T_p of $AP = \frac{1}{q(p+q)} = A + (p-1)D$ ....(1)

T_q of $AP = \frac{1}{p(p+q)} = A + (q-1)D$ ....(2)

T_{p+q} of $AP = A+(P+q-1)D$

Now solving equation (1) and (2), we get

$$A = D = \frac{1}{pq(p+q)}$$

$\therefore \quad T_{p+q}$ of $AP = A+(p+q-1)D = (p+q)D = \frac{1}{pq}$

And T_{pq} of $AP = A+(pq-1)D = pqD = \frac{1}{p+q}$

$\Rightarrow \quad T_{p+q}$ of $HP = pq$ and T_{pq} of $HP = p+q$

also $\because p > 2,\ q > 2$

$\therefore \quad pq > p+q \quad$ i.e. $\quad T_{p+q} > T_{pq}$]

Question Type = E.Match the Columns

Q. 49 () Sol (A) Q, R; (B) P, S; (C) Q, S; (D), P, R

[Sol. (A) $z = \frac{1 \pm \sqrt{-3}i}{2} = \frac{1+\sqrt{-3}i}{2}$ or $\frac{1-\sqrt{-3}i}{2}$

amp $z = \frac{\pi}{3}$ or $\text{amp}z = -\frac{\pi}{3} \Rightarrow$ **Q,R**

(B) $z = \frac{-1 \pm \sqrt{3}i}{2} = \frac{-1+\sqrt{3}i}{2}$ or $\frac{-1-\sqrt{3}i}{2}$

amp $z = \frac{2\pi}{3}$ or $-\frac{2\pi}{3} \Rightarrow$ **P,S**

(C) $2z^2 = -1-i\sqrt{3} \Rightarrow z^2 = \frac{-1-\sqrt{3}i}{2} = \cos\left(-\frac{2\pi}{3}\right) + i\sin\left(-\frac{2\pi}{3}\right)$

$$z = \cos\left(\frac{2m\pi-(2\pi/3)}{2}\right) + i\sin\left(\frac{2m\pi-(2\pi/3)}{2}\right)$$

$m = 0,\ z = \cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)$

$m = 1,\ z = \cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right) \Rightarrow \text{amp}\,z = -\frac{\pi}{3}$ or $\frac{2\pi}{3} \Rightarrow$ **Q,S**

(D) $2z^2 + 1 - i\sqrt{3} = 0$

$$z^2 = \frac{-1+i\sqrt{3}}{2} = \cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)$$

$$z = \cos\left(\frac{2m\pi+(2\pi/3)}{2}\right) + i\sin\left(\frac{2m\pi+(2\pi/3)}{2}\right)$$

$m = 0, \quad z = \cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)$

$m = 1,\quad \cos\left(\frac{4\pi}{3}\right)+i\sin\left(\frac{4\pi}{3}\right)$ or $\cos\left(-\frac{2\pi}{3}\right)+i\sin\left(-\frac{2\pi}{3}\right)\Rightarrow$ **P, R]**

Q. 50 () Sol (A) Q: (B) R; (C) S

[Sol: (A) No pair $= {}^6C_4.2^4 = 15.16 = 240$ Ans. $\Rightarrow$ (Q)

(B) atleast one pair = exactly one + both pair

$= {}^6C_1.{}^5C_2.2^2 + {}^6C_2$

$= 240 + 15 = 255$ Ans. $\Rightarrow$ (R)

(C) fewer than 2 pairs = no pair + exactly one pair

$= {}^6C_4.2^4 + {}^6C_1.{}^5C_2.2^2$

$= 240 + 240 = 480$ Ans $\Rightarrow$ (S)]

Q. 51 () Sol (A)-R (B)-S (C)-P (D)-Q

[Sol: (A) $fog : f\left[g(x)\right]$

$= \ln\left[g(x)\right] = \ln\left(x^2-1\right)$

$\therefore\quad x^2 - 1 > 0 \quad\Rightarrow\quad (-\infty, -1)\cup(1, \infty) \quad\Rightarrow\quad$ R

(B) $gof : g\left[f(x)\right] = \ln^2 x - 1 \quad\Rightarrow\quad (0, \infty) \quad\Rightarrow\quad$ S

(C) $fof : f\left[f(x)\right] = \ln\left[\ln(x)\right] \quad\Rightarrow \ln x > 0$

$x > 1$

$\therefore\quad (1, \infty) \quad\Rightarrow\quad$ P

(D) $gog : g\left[g(x)\right] = g^2(x) - 1$

$\left(x^2-1\right)^2 - 1 \Rightarrow x \in (-\infty, \infty) \quad\Rightarrow\quad$ Q]