ANSWERSHEET (TOPIC = DIFFERENTIAL CALCULUS) COLLECTION #2

Question Type = A.Single Correct Type

Q. 1 (A) Sol least value is 14 which occurs when $x \in [2, 8]$]

Q. 2 (B) Sol $f'(3^+) = \lim_{h\to 0}\frac{f(3+h)-f(3)}{h} = \lim_{h\to 0}\frac{(2-e^h)-1}{h} = -\lim_{h\to 0}\left(\frac{e^h-1}{h}\right) = -1$

$$f'(3^-) = \lim_{h\to 0}\frac{f(3-h)-f(3)}{-h} = \lim_{h\to 0}\frac{\sqrt{10-(3-h)^2}-1}{-h} = -\lim_{h\to 0}\frac{\sqrt{1+(6h-h^2)}-1}{-h}$$

$$= \lim_{h\to 0}\frac{(6h-h^2)}{-h\left(\sqrt{1+6h-h^2}+1\right)} = \lim_{h\to 0}\frac{h(h-6)}{h\left(\sqrt{1+6h-h^2}+1\right)} = \frac{-6}{2} = -3$$

Hence $f'(3^+) \neq f'(3^-)$ $\Rightarrow$ (B)]

Q. 3 (C) Sol $\sum_{k=0}^{2009} g(k) = g(0)+g(1)+g(2)+.........+g(2009) = ?$

Now $\left.\begin{array}{l} f(k)=\frac{k}{2009} \\ f(2009-k)=\frac{2009-k}{2009}\end{array}\right] \Rightarrow f(k)+f(2009-k)=1$ ……(1)

again $g(k) = \frac{f^4(k)}{(1-f(k))^4+f^4(k)}$ …….(2)

Again $g(2009-k) = \frac{f^4(2009-k)}{\left(1-f[2009-k]^4+f^4(2009-k)\right)} = \frac{[1-f(k)]^4}{(f(k))^4+(1-f(k))^4}$ ……..(3)

(2) + (3) gives

$\therefore$ $g(k)+g(2009-k) = \frac{f^4(k)+(1-f(k))^4}{(f(k))^4+(1-f(k))^4} = 1$

$\therefore$ $g(0)+g(2009)=1$

$g(1)+g(2008)=1$

$g(2)+g(2007)=1$

: : :

. . .

$g(1004)+g(1005)=1$

$\sum_{k=0}^{2009} g(k) = 1005$ $\Rightarrow$ [C]]

Q. 4 (C) Sol $g(x) = f(-x + f(f(x)))$; $f(0) = 0$; $f'(0) = 2$

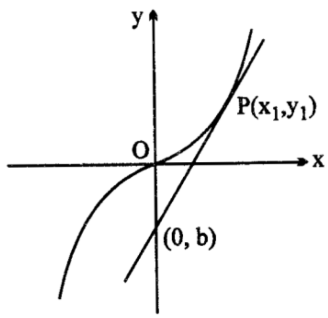
$g'(x) = f(-x + f(f(x)))[-1 + f'(f(x)).f'(x)]$

$g'(0) = f'(f(0)).[-1 + f'(0).f'(0)]$

$= f'(0)[-1 + (2)(2)]$

$= (2)(3) = 6$ Ans.]

Q. 5 (C) Sol $f(x) = \dfrac{41x^3}{3}$



$f'(x) = 41x^2$

$f'(x)|_{x_1, y_1} = 41x_1^2$

$\therefore \quad 41x_1^2 = 2009 = 7^2.41$

$x_1^2 = 49 \quad \Rightarrow \quad x_1 = 7;\ y_1 = \dfrac{41.7^3}{3}(x_1 \neq -7, \text{ think!})$

now $\dfrac{y_1 - b}{x_1 - 0} = 2009 \quad \Rightarrow \quad y_1 - b = 7.2009 = 7^3.41$

$b = \dfrac{41.7^3}{3} - 7^3.41 = \dfrac{41.7^3}{3}(-2) = -\dfrac{82.7^3}{3}$ **Ans.]**

Q. 6 (D) Sol $f(x) = \displaystyle\int \frac{x^{2009}}{(1+x^2)^{1006}}dx$

Put $1 + x^2 = t \quad \Rightarrow \quad 2xdx = dt$

$I = \dfrac{1}{2}\displaystyle\int \frac{(t-1)^{1004}dt}{t^{1006}} = \frac{1}{2}\int \left(1 - \frac{1}{t}\right)^{1004}.\frac{1}{t^2}dt$

put $1 - \dfrac{1}{t} = y \quad \Rightarrow \quad \dfrac{1}{t^2}dt = dy$

$\therefore \quad I = \dfrac{1}{2}\displaystyle\int y^{1004}dy = \frac{1}{2}\frac{y^{1005}}{1005} + C = \frac{1}{2010}\left(\frac{t-1}{t}\right)^{1005} + C$

Q. 7 (B) Sol $S = \frac{1+2^{2008}+3^{2008}+.........+n^{2008}}{n^{2009}}$

$Tr = \frac{1}{n}\frac{r^{2008}}{n^{2008}} = \frac{1}{n}.\left(\frac{r}{n}\right)^{2008}$

$S = \int x^{2008}dx = \frac{1}{2009}$]

Question Type = B.Comprehension or Paragraph

Q. 8 () Sol **Q. 1** A
Q. 2 B
Q. 3 D
[Sol.
(1) $\tan^{-1} y = \tan^{-1} x + C$

$x = 0;\ y = 1 \quad \Rightarrow \quad C = \frac{\pi}{4}$

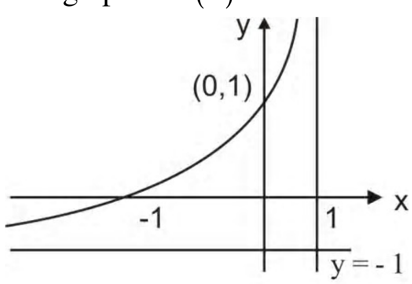
$\Rightarrow \quad \tan^{-1} y = \tan^{-1} x + \frac{\pi}{4} \quad \Rightarrow$

note : even $\frac{-\pi}{4} < \tan^{-1} x + \frac{\pi}{4} < \frac{\pi}{2}$; $\frac{-\pi}{2} < \tan^{-1} x < \frac{\pi}{4}$; $-\infty < x < 1 \Rightarrow (A)$

$x < 1 \quad \Rightarrow \quad (A)$

(3) $\because \quad y = \tan\left(\tan^{-1}+\frac{\pi}{4}\right) = \frac{x+1}{1-x} \quad \Rightarrow$ (D) is correct

(2) The graph of $f(x)$ is as shown.



Hence range is $(-1, \infty) \quad \Rightarrow$ **(B)**

Q. 9 () Sol **Q. 1** A
Q. 2 D
Q. 3 A
[Sol. Since minimum value is zero hence touches the x-axis and mouth opening upwards i.e., $a > 0$ given $f(x-4) = f(2-x)$

$x \rightarrow x+3$

$f(x-1) = f(-1-x)$

$f(-1+x) = f(-1-x)$

Hence *f* is symmetric about the line $x = -1$

$\therefore \quad f(x) = a(x+1)^2$

Now given $f(x) \ge x \ \forall \ x$

$f(1) \ge 1 \qquad \ldots(1)$

and $\quad f(x) \le \left(\dfrac{x+1}{2}\right)^2$ in $(0, 2)$

$f(1) \le 1$

From (1) and (2)

$f(1) = 1$

now $\quad f(x) = a(x+1)^2$

$f(1) = 4a = 1 \quad \Rightarrow \quad a = \dfrac{1}{4}$

$\therefore \quad f(x) = \dfrac{(x+1)^2}{4}$ now proceed]

Q. 10 () Sol **Q. 1** A

Q. 2 D

Q. 3 C

[Sol. $f(0) = 2$

$$f(x) = (e^x + e^{-x})\cos x - 2x - \left[x\int_0^x f'(t)dt - \int_0^x tf'(t)\,dt\right]$$

$$f(x) = (e^x + e^{-x})\cos x - 2x - \left[x\left(f(x) - f(0)\right) - \left\{t.f(t)\Big|_0^x - \int_0^x f(t)\,dt\right\}\right]$$

$$f(x) = (e^x + e^{-x})\cos x - 2x - xf(x) + 2x + \left[xf(x) - \int_0^x f(t)\,dt\right]$$

$$f(x) = (e^x + e^{-x})\cos x - \int_0^x f(t)\,dt \qquad \ldots(1)$$

differentiating equation (1)

$$f'(x) + f(x) + \cos x\left(e^x - e^{-x}\right) - \left(e^x + e^{-x}\right)\sin x$$

Hence $\dfrac{dy}{dx} + y = e^x(\cos x - \sin x) - e^{-x}(\cos x + \sin x)$ **Ans. (i)**

(ii) $\quad f'(0) + f(0) = 0 - 2.0 = 0$ **Ans (ii)**

(iii) I.F. of DE (1) is e^x

$$y.e^{x}=\int e^{2x}(\cos x-\sin x)dx-\int(\cos x+\sin x)dx$$

$$y.e^{x}=\int e^{2x}(\cos x-\sin x)dx-(\sin x-\cos x)+C$$

Let $I=\int e^{2x}(\cos x-\sin x)dx=e^{2x}(A\cos x+B\sin x)$

Solving $A=3/5$ and $B=-1/5$ and $C=2/5$

$\therefore \quad y=e^{x}\left(\frac{3}{5}\cos x-\frac{1}{5}\sin x\right)-(\sin x-\cos x)e^{-x}+\frac{2}{5}e^{-x}$ **Ans. (iii)]**

Question Type = C.Assertion Reason Type

Q. 11 (B) Sol $f(x)=\log_{1/4}\left(x-\frac{1}{4}\right)+\frac{1}{2}\log_{4}\left(x^{2}-\frac{x}{2}+\frac{1}{16}\right) \quad \left(x>\frac{1}{4}\right)$

$=\log_{1/4}\left(x-\frac{1}{4}\right)+1+\log_{4}\left(x-\frac{1}{4}\right)$

$=-\log_{4}\left(x-\frac{1}{4}\right)+\log_{4}\left(x-\frac{1}{4}\right)+1$

$=1 \quad \Rightarrow \quad$ f is contant

Hence f is many one as well into. Also range is a singleton $\Rightarrow$ f is constant but a constant function can be anything $\Rightarrow$ not the correct explanation]

Q. 12 (B) Sol Domain is $\{-1, 1\}$ and range is $\left\{-\frac{\pi}{2},\frac{\pi}{2}\right\}$ and domain having two elements $\not\Rightarrow$ range must have two elements]

Q. 13 (A) Sol $f(x)=\frac{1}{2\{-x\}}-\{x\},\ x\notin 1$

Using $\{x\}+\{-x\}=1$ if $x\notin I$

$\{x\}=1-\{-x\}$

$\therefore \quad f(x)=\frac{1}{2\{-x\}}-(1-\{-x\})=\{x\}+\frac{1}{2\{-x\}}-1$

$f(x)/_{\min.}=2.\frac{1}{\sqrt{2}}-1=\sqrt{2}-1$]

Q. 14 (D) Sol $\frac{x}{4e}+\frac{e^{3}}{x}\ge 2\sqrt{\frac{x}{4e}.\frac{e^{3}x}{\quad}}=e$

Hence range is $[0, \infty)$ $\Rightarrow$ $S-1$ is false]

Q. 15 (C) Sol .

Q. 16 (B) Sol Line touches the curve at $(0, b)$ and $\left.\frac{dy}{dx}\right]_{x=0}$ also exists but even if $\frac{dy}{dx}$ fails to exist. tangent line can be drawn.]

Q. 17 (D) Sol $\lim\limits_{x\to\pi/2}\frac{\sin\left(\cot^2 x\right)}{\cot^2 x}\cdot\frac{\cot^2 x}{(\pi-2x)^2}$; put $x=\frac{\pi}{2}-h$

$\lim\limits_{x\to 0}\frac{\tan^2 h}{4h^2}=\frac{1}{4}$]

Q. 18 (B) Sol Range of f is $\left\{\frac{\pi}{2}\right\}$ and domain of f is $\{0\}$. Hence if domain of f is singleton then angle has to be a singleton.

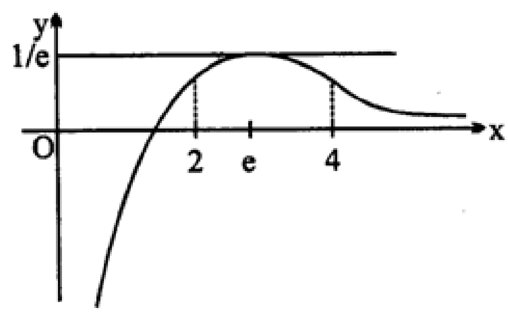
If $S-2$ and $S-1$ are reverse then the answer will be B.]

Q. 19 (A) Sol $y=|\ln x|$ not differentiable at $x=1$

$y=|\cos|x||$ is not differentiable at $x=\frac{\pi}{2},\frac{3\pi}{2}$

$y=\cos^{-1}(\operatorname{sgn} x)=\cos^{-1}(1)=0$ differentiable $\forall\ x\in(0, 2\pi)$]

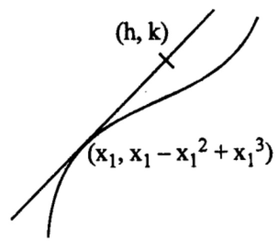
Q. 20 (A) Sol $f'(x)=\frac{1-\ln x}{x^2}$; note that $f(2)=f(4)$



f is increasing $x\in(0, e)$ and f is decreasing (e, ∞)]

Q. 21 (B) Sol $f'(x)=1-2x+3x^2>0$

$\Rightarrow\quad -\frac{a}{b}>0\qquad\Rightarrow\qquad ab<0$



$$\frac{x_1^3 - x_1^2 + x_1 - k}{x_1 - 1/3} = 3x_1^2 - 2x_1 + 1$$

$$g(x_1) = 2x_1^3 - 2x_1^2 + \frac{2}{3}x_1 + k - \frac{1}{3}$$

$$g'(x_1) = 6x_1^2 - 4x_1 + \frac{2}{3} = \frac{2}{3}(3x_1 - 1)^2 \quad]$$

Q. 22 (A) Sol .

Q. 23 (C) Sol .

Q. 24 (A) Sol Let $f(x) = 0$ has two roots say $x = r_1$ and $x = r_2$ where $r_1, r_2 \in [a, b]$

$\Rightarrow \quad f(r_1) = f(r_2)$

Hence $\exists$ there must exist some $c \in (r_1, r_2)$ where $f'(c) = 0$

but $f'(x) = x^6 - x^5 + x^4 - x^3 + x^2 - x + 1$

for $x \ge 1$, $f'(x) = (x^6 - x^5) + (x^4 - x^3) + (x^2 - x) + 1 > 0$

for $x \le 1$, $f'(x) = (1 - x) + (x^2 - x^3) + (x^4 - x^5) + x^6 > 0$

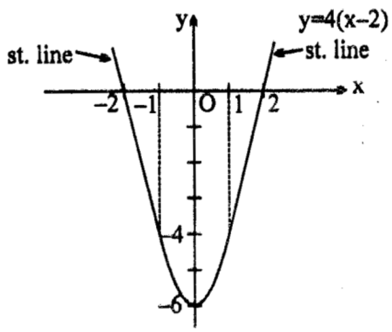
hence $f'(x) > 0$ for all x

$\therefore$ Rolles theorem fails $\Rightarrow$ $f(x) = 0$ can not have two or more roots.]

Q. 25 (D) Sol $f(x) = x^2 - |x^2 - 1| + 2||x| - 1| + 2|x| - 7$

$f(-x) = f(x) \Rightarrow$ Area $x < 0 =$ area $x > 0$

Case - I : for $0 < x < 1$



$y = x^2 - (1-x^2) + 2(1-x) + 2x - 7 = 2(x^2-3)$

If $- < x < 0$

$f(x) = 2(x^2-3)$

now $f'(0^+) = f'(0^-) = 0$

for $x > 1$

$f(x) = x^2 - (x^2-1) + 2(x-1) + 2x - 7$

$f(x) = 4(x-2)$

note $\lim_{x\to 1} f(x) = -4 = f(1)$

$\Rightarrow$ f is continuous. Also $f'(1^-) = f'(1^+) = 4$

$\Rightarrow$ f is derivable at $x = 1$]

Q. 26 (D) Sol Let $b > 0$, then $f(1) = b > 0$ and

$f(5) = 2a + 3b - 6 = 2(a+2b) - b - 6 = 4 - b - 6 = -(2+b) < 0$

Hence by IVT, $\exists$ some $c \in (1, 5)$ s.t. $\Rightarrow$ $f(c) = 0$

If $b = 0$ then $a = 2$

$f(x) = 2\sqrt{x-1} - \sqrt{2x^2-3x+1} = 0$

$\Rightarrow$ $4(x-1) = 2x^2 - 3x + 1 = (2x-1)(x-1)$

$(x-1)(2x-5) = 0 \Rightarrow x = \frac{5}{2}$

Hence $f(x) = 0$ if $x = \frac{5}{2}$ which lies in $(1, 5)$

If $b < 0$, $f(1) = b < 0$ and

$f(2) = a + b\sqrt{3} - \sqrt{3}$

$= (a+2b) + (\sqrt{3}-2)b - \sqrt{3}$

$= (2-\sqrt{3}) - (2-\sqrt{3})b$

$= (2-\sqrt{3})(1-b) > 0 \quad (\text{as } b < 0)$

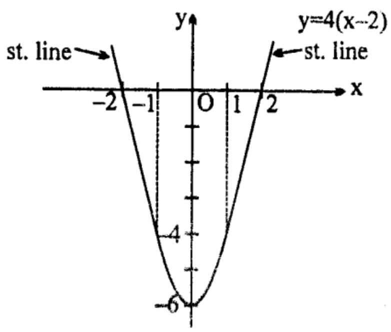
Hence $f(1)$ as $f(2)$ have opposite signs

$\exists$ some $c \in (1, 2) \subset (1, 5)$ for which $f(c) = 0$

$\Rightarrow$ Statement -1 is valid for all $b \in R$ $\Rightarrow$ statement -1 is false.

Statement -2 is obviously true $\Rightarrow$ (D)]

Q. 27 (D) Sol $f(x) = x^2 - |x^2 - 1| + 2||x| - 1| + 2|x| - 7$



$f(-x) = f(x) \Rightarrow$ Area for $x < 0 =$ area of $x > 0$

Case-I : for $0 < x < 1$

$y = x^2 - (1 - x^2) + 2(1 - x) + 2x - 7 = 2(x^2 - 3)$

For $x > 1$

$f(x) = x^2 - (x^2 - 1) + 2(x - 1) + 2x - 7$

$f(x) = 4(x - 2)$

note $\lim_{x \to 1} f(x) = -4 = f(1)$

$\Rightarrow$ f is continuous $\forall\, x \in R$. Also $f'(1^{-1}) = f'(1^{+}) = 4$

$\Rightarrow$ f is derivable at $x = 1$

Area bounded by the $y = f(x)$ and +ve x-axis is

Area $= \left| 2\int_0^1 (x^2 - 3)\,dx \right| + 2 = \left| 2\left(\frac{1}{2} - 3\right) \right| + 2 = \frac{16}{3} + 2 = \frac{22}{3}$

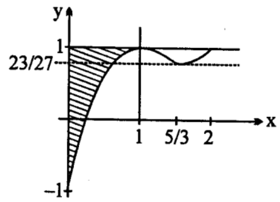
$\therefore$ Area bounded by the $f(x)$ and x-axis $= 2\left(\frac{22}{3}\right) = \frac{44}{3}$ Ans.]

Question Type = D.More than one may corect type

Q. 28 () Sol A, B, D

[Hint. A=1; $A = 1$; $B = 1$; $C =$ aperiodic; $D = 2\pi$]

Q. 29 () Sol



]

B, C, D

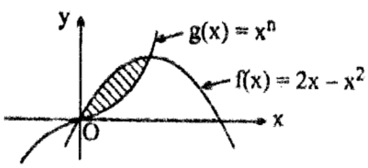
[Sol. The graph of $y = f(x) = (x-1)^2(x-2)+1$

$f(1) = f(2) = 1$ and $f(0) = -1$

Verify alternatives

Q. 30 () Sol **Q. 1** B, C, D

[Sol. Solving $f(x) = 2x - x^2$ and $g(x) = x^n$



We have $2x - x^2 = x^n \Rightarrow x = 0$ and $x = 1$

$$A = \int_0^1 \left(2x - x^2 - x^n\right)dx - x^2 - \frac{x^3}{3} - \frac{x^{n+1}}{n+1}\Bigg]_0^1$$

$$= 1 - \frac{1}{3} - \frac{1}{n+1} = \frac{2}{3} - \frac{1}{n+1}$$

hence, $\frac{2}{3} - \frac{1}{n+1} = \frac{1}{2} \Rightarrow \quad \frac{2}{3} - \frac{1}{2} = \frac{1}{n+1}$

$\Rightarrow \quad \frac{4-3}{6} = \frac{1}{n+1} \Rightarrow \quad n+1 = 6 \Rightarrow n = 5$

Hence n is a divisor of 15, 20, 30 $\Rightarrow$ B, C, D]

Q. 31 () Sol **Q. 1** A, B, D

[Sol. $\frac{dy}{dx} + y = f(x)$

$I.F. = e^x$

$ye^x = \int e^x f(x)dx + C$

now if $0 \le x \le 2$ then $ye^x = \int e^x e^{-x} dx + C \quad \Rightarrow \quad ye^x = x + C$

$x = 0, \quad y(0) = 1, \qquad C = 1$

$\therefore \quad ye^x = x + 1 \qquad ....(1)$

$y = \frac{x+1}{e^x}$; $y(1) = \frac{2}{e}$ Ans. $\Rightarrow$ (A) is correct

$y' = \frac{e^x - (x+1)e^x}{e^{2x}}$;

$y'(1) = \frac{e - 2e}{e^2} = \frac{-e}{e^2} = -\frac{1}{e}$ Ans. $\Rightarrow$ (B) is correct

if $x > 2$

$ye^x = \int e^{x-2} dx$

$ye^x = e^{x-2} + C$

$y = e^{-2} + Ce^{-x}$

as y is continuous

$\therefore$ $\lim_{x\to 2} \frac{x+1}{e^x} = \lim_{x\to 2}\left(e^{-2} + Ce^{-x}\right)$

$3e^{-2} = e^{-2} + Ce^{-2}$ $\Rightarrow$ $C = 2$

$\therefore$ for $x > 2$

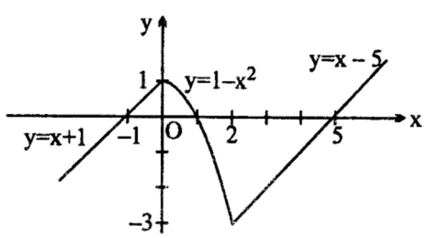
$y = e^{-2} + 2e^{-x}$ hence $y(3) = 2e^{-3} + e^{-2} = e^{-2}\left(2e^{-1} + 1\right)$

$y' = -2e^{-x}$

$y'(3) = -2e^{-3}$ Ans. $\Rightarrow$ (D) is correct]

Question Type = E.Match the Columns

Q. 32 () Sol **Q. 1** (A) P, S, (B) Q, R; (C) Q, R (D) P.S.



[Sol. Let $g(x) = \begin{cases} \frac{\pi}{2} - 2\tan^{-1} f(x) & f(x) \in (-1, 1) \\ -\frac{\pi}{2} - 2\tan^{-1} f(x) & f(x) \in (-\infty, -1) \\ \frac{3\pi}{2} - 2\tan^{-1} f(x) & f(x) \in (1, \infty) \end{cases}$

(A) $\frac{d(x)}{d(x)} = -\frac{2}{1+f^2(x)} = -\frac{1}{13}$ $\Rightarrow$ $f(x) \pm 5$

$\Rightarrow$ $x = -6, 10$ $\Rightarrow$ $x = -6, 10$ $\Rightarrow$ P,S

(B) refer to graph of $y = f(x)$ $\Rightarrow$ **Q,R**

(C) $-k \in (-3, 1) \Rightarrow k \in (-1, 3) \Rightarrow$ **Q, R**

(D) $g'(x) = \frac{-2f'(x)}{1+f^2(x)} < 0 \Rightarrow f'(x) > 0 \Rightarrow x = -6, 10 \Rightarrow$ **P,S**]

Q. 33 () Sol (A) Q; (B) S; (C) P; (D) R

[Sol. $f(x) = \frac{\ln x}{8} - ax + x^2$; $f'(x) = \frac{1}{8x} - a + 2x$ ....(1) $\Rightarrow f'(x) = \frac{16x^2 - 8ax + 1}{8x}$

If $a = 1$, $f'(x) = \frac{(4x-1)^2}{8x} = 0 \Rightarrow x = \frac{1}{4}$

Hence $x = 1/4$ is the point of inflection and $a = 1 \Rightarrow (C) \Rightarrow (P)$

now $f'(x) = 0$ gives $\frac{16x^2 - 8ax + 1}{8x} = 0$ or $16x^2 - 8ax + 1 = 0$

$x = \frac{8a \pm \sqrt{64a^2 - 64}}{32} \Rightarrow x = \frac{a + \sqrt{a^2 - 1}}{4} (a > 1)$ or $x = \frac{a - \sqrt{a^2 - 1}}{4} (a > 1)$

and $f''(x) = 2 - \frac{1}{8x^2}$

$f''\left(\frac{a + \sqrt{a^2 - 1}}{4}\right) = 2 - \frac{16}{8\left(a + \sqrt{a^2 - 1}\right)^2} = 2 - \frac{2}{\left(a + \sqrt{a^2 - 1}\right)^2} \quad (a > 1)$

Hence for $a > 1$ and $x = \frac{a + \sqrt{a^2 - 1}}{4}$, f( ) has a local minima

$\therefore$ (**B**) $\Rightarrow$ (S)

|||ly for $a > 1$ and $x = \frac{a - \sqrt{a^2 - 1}}{4}$

we have local mixima

$\therefore$ (**A**) $\Rightarrow$ (**Q**)

finally for $0 \le a < 1$

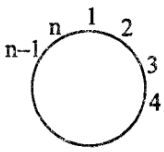
$f'(x) = \frac{16x^2 - 8ax + 1}{8x}$

$\Delta = 64a^2 - 64 < 0$

Hence $f'(x) > 0 \Rightarrow$ f is monotonic $\Rightarrow$ (**D**) $\Rightarrow$ (**R**)

Q. 34 () Sol (A) S; (B) Q; (C) P; (D) R

[Sol.



(A) **1^{st} vertex** $^{n}C_1$ way

2 and n can not be taken. Remaining vertices are

$\underbrace{3, 4, 5...............(n-1)}_{(n-3)\text{vertices}}$

$\underbrace{OOOO}_{\text{four to be taken}}$

$\underbrace{|x||x||x|.....|x|}_{(n-7)\text{not to be taken}}$ $\Rightarrow$ number of gaps $(n-6)$ out of which 4 can be selected in $^{n-6}C_4$ ways.

Hence required number of ways $\frac{^{n-6}C_4.n}{5} = 36$

which is satisfied by $n = 12$ Ans. $\Rightarrow$ (S)

(B) $x^3 + ax^2 + bx + c \equiv (x^2+1)(x+k) = x^3 + kx^2 + x + k$

$\Rightarrow$ $b = 1$ and $a = c$

Now 'a' can be taken in 10 ways and as $a = c$ hence 'c' can be only in one way

Also $b = 1$. Hence total 10 Ans. $\Rightarrow$ (Q)

Alternatively: $-i - a + bi + c = 0 + 0i$

$\therefore$ $c - a + (b-1)i = 0 + 0i$ $\Rightarrow$ $a = c$ and $b = 1$]

(C) $z^6(1+i) = \bar{z}(i-1)$ ....(1)

$\therefore$ $|z|^6|1+i| = |\bar{z}||-1+i|$ $\Rightarrow$ $|z|^6 = |z|$ $\Rightarrow$ $|z| = 0$ or $|z| = 1$

if $|z| = 0$ $\Rightarrow$ $z = 0$

if $|z| = 1$ then $z\bar{z} = 1$ $\Rightarrow$ $\bar{z} = \frac{1}{z}$

hence equation (1) becomes

$z^6(1+i) = \frac{1}{z}(-1+i)$

$z^7 = \frac{-1+i}{1+i} = \frac{(-1+i)(1-i)}{2} = i$

$z = \cos\frac{2m\pi + \frac{\pi}{2}}{7} + i\sin\frac{2m\pi + \frac{\pi}{2}}{7}$

Where $m = 0, 1, 2, ..., 6$ are the other solutions

Total solutions = 8 Ans. $\Rightarrow$ (**P**)

(D) $2^{f(x)+g(x)} = x$

Put $x=4$ $\quad 2^{f(4)+g(4)}=4=2^2$

$f(4)+g(4)=2$

$g(4)=2-f(4)$

$\therefore \quad 0 \le 2-f(4) < -1$

$-2 \le f(4) < -1$

$1 < f(4) \le 2 \quad \Rightarrow \quad f(4)=2$ (as $f(x)$ is a non negative integer)

again put

$2^{f(1000)+g(1000)}=1000$

$f(1000)+g(1000)=\log_2(1000)$

$g(1000)=\log_2(1000)-f(1000)$

$\therefore \quad 0 \le \log_2 1000 - f(1000) < 1$

$-\log_2 1000 \le -f(1000) < 1-(\log_2 1000)$

$(\log_2 1000)-1 \le f(1000) \le \log_2 1000$

$\Rightarrow \quad f(1000)=9$ as f is an integer

Hence $f(4)+f(1000)=11$ Ans. $\Rightarrow$ (**R**)]