Q.46. If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}+\vec{b}| = \sqrt{29}$

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Given

$$\vec{a} \times (2i+3j+4k) = (2i+3j+4k) \times b$$

$$\vec{a} \times (2i+3j+4k) + \vec{b} \times (2i+3j+4k) = 0$$

$$(\vec{a}+\vec{b}) \times (2i+3j+4k) = 0$$

Clearly $\vec{a}+\vec{b} \parallel (2i+3j+4k)$

$\therefore$ Angle b/w $\vec{a}+\vec{b}$ and $2i+3j+4k$ is equal to angle b/w $2i+3j+4k$ & $(-7i+2j+3k)$

$$\cos\theta = \frac{(-14+6+12)}{\sqrt{29}\sqrt{62}}$$

$$\therefore (\vec{a}+\vec{b}).(-7i+2j+3k) = |\vec{a}+\vec{b}|.|-7i+2j+3k| \cos\theta$$

$$= \frac{\sqrt{29}\times\sqrt{62}\times 4}{\sqrt{29}\quad\sqrt{62}}$$

$\therefore$ Ans = 4

(47) $\int_{-\pi/2}^{\pi/2}\left(x^2+\ln\left(\frac{\pi+x}{\pi-x}\right)\right)\cos x\,dx$ ($\ln\left(\frac{\pi+x}{\pi-x}\right)\cos x$: odd = 0; $x^2\cos x$: even)

$$= 2\int_0^{\pi/2} x^2.\cos x\,dx.$$

Taking by parts twice

(47) → (B) Ans = $\frac{\pi^2}{2} - 4$