

Q.60. Let X & Y be two events, ----

Sol. $P(X|Y) = \frac{P(X \cap Y)}{P(Y)}$ $P(Y|X) = \frac{P(X \cap Y)}{P(X)}$

$$\frac{1}{2} = \frac{1/6}{P(Y)}$$

$$P(Y) = \frac{1}{3}$$

$$\frac{1}{3} = \frac{1/6}{P(X)}$$

$$P(X) = \frac{1}{2}$$

✓(A) $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y) = \frac{1}{3} + \frac{1}{2} - \frac{1}{6} = \frac{5}{6} - \frac{1}{6} = \frac{2}{3}$ correct

✓(B) $P(X \cap Y) = \frac{1}{6}$ } $P(X) \cdot P(Y) = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$ correct

(C) Wrong.

(D) $P(X^c \cap Y) = P(X^c) \cdot P(Y) = (1 - \frac{1}{2}) \cdot \frac{1}{3} = \frac{1}{6}$ wrong.

59. Let $f: (-1, 1) \rightarrow \mathbb{R}$ be such that

(A, B) $f(\cos 2\theta) = \frac{2\theta}{2 - \sec^2 \theta}$

Soln. $\cos 4\theta = \frac{1}{2} \rightarrow 2\cos^2 2\theta - 1 = \frac{1}{2}$

$$2\cos^2 2\theta = \frac{4}{3}$$

$$\cos^2 2\theta = \frac{2}{3} \rightarrow (2\cos^2 2\theta - 1)^2 = \frac{2}{3}$$

$$2\cos^2 2\theta - 1 = \pm \sqrt{\frac{2}{3}}$$

$$\cos^2 2\theta = \frac{1}{2} \pm \frac{1}{\sqrt{6}} = \frac{\sqrt{6} \pm 2}{2\sqrt{6}}$$

$$\sec^2 2\theta = \frac{2\sqrt{6}}{\sqrt{6} \pm 2}$$

$$f\left(\frac{1}{2}\right) = \frac{2}{\frac{2 - 2\sqrt{6}}{\sqrt{6} \pm 2}} = \frac{2\sqrt{6} \pm 4}{\pm 4} = \pm \frac{\sqrt{6} + 1}{2} = 1 \pm \sqrt{\frac{3}{2}}$$