Total Que 60, Each Subj. 20 (Code 2) Total Marks 240; Each Subj. 80.

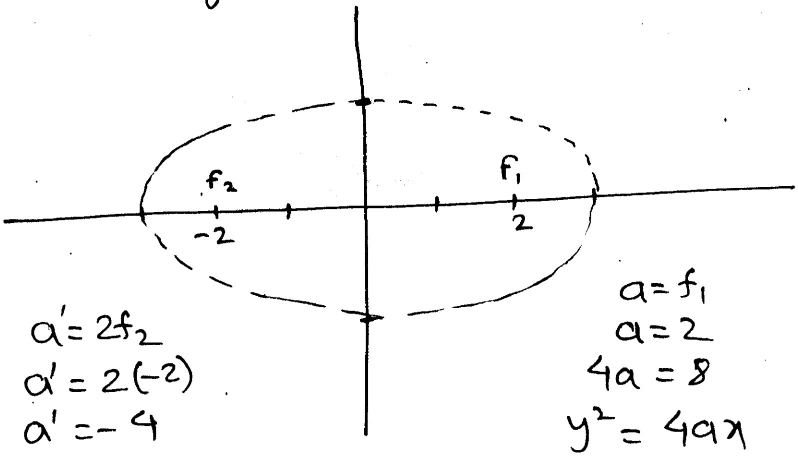
IITJEE Advance Paper 2 Solution of Maths By Suhag Sir and also his Students Anurag Gupta, Lucky Dubey & Manas Deep Vishwakarma, Akshay Swarnakar

Single correct type Q.41 to Q.48

Q.41. Suppose that the foci of the ellipse ----

Sol.

Ans 4.



$a' = 2f_2$

$a' = 2(-2)$

$a' = -4$

$y^2 = -4 \cdot 4 \cdot x$

$y^2 = -16x$.

$y = mx + \frac{a'}{m}$

$y = mx - \frac{4}{m}$

$(2, 0)$

$0 = 2m - \frac{4}{m}$

$\frac{4}{m} = 2m$

$2 = m_2^2$

$a = f_1$

$a = 2$

$4a = 8$

$y^2 = 4ax$

$y^2 = 8x$.

$y = mx + \frac{a}{m}$

$y = mx + \frac{2}{m}$

$(-4, 0)$

$0 = m(-4) + \frac{2}{m}$

$4m = \frac{2}{m}$

$m_1^2 = \frac{1}{2}$

finding

$\frac{1}{m_1^2} + m_2^2$

$2 + 2$

Ans. (4)

Q.42. let m and n be two positive ----

(2) $\lim_{\alpha \to 0} \left(\frac{e^{\cos \alpha^n} - e}{\alpha^m} \right) = -\frac{e}{2}$

S. By L'Hospital $\frac{e^{\cos \alpha^n}(-\sin \alpha^n)\, n\alpha^{n-1}}{m\alpha^{m-1}} \Rightarrow -\frac{en}{m} \frac{\alpha^{2n-1}}{\alpha^{m-1}} = -\frac{e}{2}$

$\Rightarrow 2n = m \Rightarrow \boxed{\frac{m}{n} = 2}$

P.T.O

43. If $\alpha = \int_0^1 \left(e^{9x+3\tan^{-1}x}\right)\left(\frac{12+9x^2}{1+x^2}\right)dx$

(9)

$9x + 3\tan^{-1}x = t$

$dx\left(9+\frac{3}{1+x^2}\right) = dt \Rightarrow dt = \left(\frac{12+9x^2}{1+x^2}\right)dx$

$\int_0^1 e^t dt$

$\Rightarrow e^{(9x+3\tan^{-1}x)} \Rightarrow e^{9+3\pi/4} - 1$

$\left(\log_e\left|1+e^{9+3\pi/4}-1\right| - \frac{3\pi}{4}\right)$

$\Rightarrow \log_e e^{9+3\pi/4} - \frac{3\pi}{4}$

$\Rightarrow 9$

Q.44. Let $f: R \to R$ be a continuous odd func. ----

Soln: $\lim_{x\to 1} \frac{F(x)}{G(x)} = \frac{1}{14}$

(7)

By L Hospital

$\lim_{x\to 1} \frac{f(x)}{x|f(f(x))|} = \frac{1}{14}$

$\frac{f(1)}{1|f(f(1))|} = \frac{1}{14}$

$\frac{1/2}{|f(\frac{1}{2})|} = \frac{1}{14}$ $\quad |f(\frac{1}{2})| = \frac{1}{7}$

Odd fn. $f(\frac{1}{2}) = 7$

P.T.O

Q.45. Suppose that $\vec{p}, \vec{q}, \vec{r}$ are three non coplaner vectors.....

Sol. Suhag Short Trick let $\vec{p} = \hat{i}$, $\vec{q} = \hat{j}$, $\vec{r} = \hat{k}$

Ans (9)

$$\vec{s} = 4\hat{i} + 3\hat{j} + 5\hat{k}$$

also

$$\vec{s} = x(-i+j+k) + y(i-j+k) + z(-i-j+k)$$

$$\vec{s} = i(-x+y-z) + j(x-y-z) + k(x+y+z)$$

$-x+y-z=4$
$x-y-z=3$
$x+y+z=5$

add (1 & 2) → $z = \frac{-7}{2}$; add (2 & 3) → $x = \frac{8}{2}$; add 1 & 3 → $y = \frac{9}{2}$

Ans $2x+y+z = \frac{16}{2} + \left(\frac{-7}{2}\right) + \frac{9}{2} = \frac{16-7+9}{2} = 9$

Q.46. for any integer k, let $a_k = \cos\left(\frac{k\pi}{7}\right) + i\sin\left(\frac{k\pi}{7}\right)$ ----

Sol. $a_k = e^{k\pi/7}$, $a_{k+1} = e^{(k+1)\pi/7}$, $a_{4k-1} = e^{(4k-1)\pi/7}$, $e_{4k-2} = e^{(4k-2)\pi/7}$

(4)

finding $\dfrac{\sum_{k=1}^{12} |a_{k+1} - a_k|}{\sum_{k=1}^{3} |a_{4k-1} - a_{4k-2}|} = \dfrac{\sum_{k=1}^{12} |e^{(k+1)\pi/7} - e^{k\pi/7}|}{\sum_{k=1}^{3} |e^{(4k-1)\pi/7} - e^{(4k-2)\pi/7}|}$

$$= \frac{\sum_{k=1}^{12} 2\sin\frac{\pi}{14}}{\sum_{k=1}^{3} 2\sin\frac{\pi}{14}}$$

$$= \frac{12}{3} = 4$$

P.T.O.

Q.47. Suppose that all the terms of an arithmetic progr....

Solⁿ :

$$\frac{S_7}{S_{11}} = \frac{\frac{7}{2}[2a+6d]}{\frac{11}{2}[2a+10d]} = \frac{6}{11} \Rightarrow \frac{7[a+3d]}{[a+5d]} = \frac{6}{1} \Rightarrow 7a+21d = 6a+30d$$

$$\Rightarrow \boxed{a=9d}$$

$140 > a_7 > 130 \Rightarrow 140 > a+6d > 130 \Rightarrow 140 > 15d > 130$

$8.66 > d > 9.33$ } Now it is given that A.P. is of only natural nos ∴ d should be only an integer.

∴ Integer b/w 8.66 and 9.33 is 9.

∴ The common difference of AP is $\boxed{9}$

Q.48. The coeff. of x^9 in the ----

Soln. $(1+x)(1+x^2)(1+x^3)\ldots(1+x^{100})$

(8) Here we will get the coeff. of x^9 by following ways

$1.x^9 + x.x^8 + x^2.x^7 + x^3.x^6 + x^4.x^5 + x^1.x^2.x^6$
$+ x^1.x^3.x^5 + x^2.x^3.x^5$.

Thus the total cases will be 8

and answer is (8)

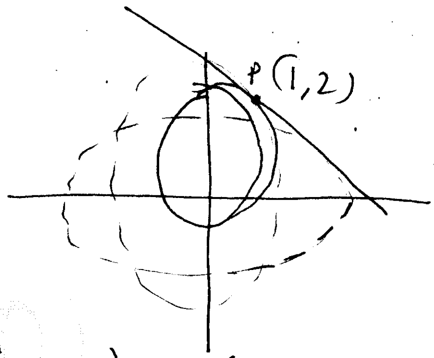
P.T.O.

Q.49 Let E_1 and E_2 be two ellipse whose centres - - -

Sol. $S: x^2 + (y-1)^2 = 2$

Given distance b/w two pt. from P is $\frac{2\sqrt{2}}{3}$

(A, B)



By Polar coordinate line

By $(x_1 + r\cos\theta, y_1 + r\sin\theta)$

$r = \pm\frac{2\sqrt{2}}{3}, \theta = 135°$

$\left(1 + \frac{2\sqrt{2}}{3}\times\frac{1}{\sqrt{2}}, 2 - \frac{2\sqrt{2}}{3}\times\frac{1}{\sqrt{2}}\right) \equiv \left(\frac{5}{3}, \frac{4}{3}\right)$

$\left(1 - \frac{2\sqrt{2}}{3}\times\frac{1}{\sqrt{2}}, 2 + \frac{2\sqrt{2}}{3}\times\frac{1}{\sqrt{2}}\right) \equiv \left(\frac{1}{3}, \frac{8}{3}\right)$

Let the ellipse by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Tangent is $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1 \Rightarrow$ Putting $\left(\frac{5}{3}, \frac{4}{3}\right)$

$\frac{5x}{3a^2} + \frac{4y}{3b^2} = 1$ & $x + y = 3$ [Same]

$a_1 = \sqrt{5}, b_1 = 2$

||ly

$\frac{1}{3a^2}x + \frac{8}{3b^2}y = 1$ & $x + y = 3$ [Same]

$a^2 = 1, b^2 = 8$

$a_2 = 1, b_2 = 2\sqrt{2}$

$e_1^2 = 1 - \frac{4}{5}, \quad e_2^2 = 1 - \left(\frac{1}{2\sqrt{2}}\right)^2$

$e_1 = \frac{1}{\sqrt{5}}, \quad e_2 = \frac{\sqrt{7}}{2\sqrt{2}}$

(A) $e_1^2 + e_2^2 = \frac{1}{5} + \frac{7}{8} = \frac{43}{40}$

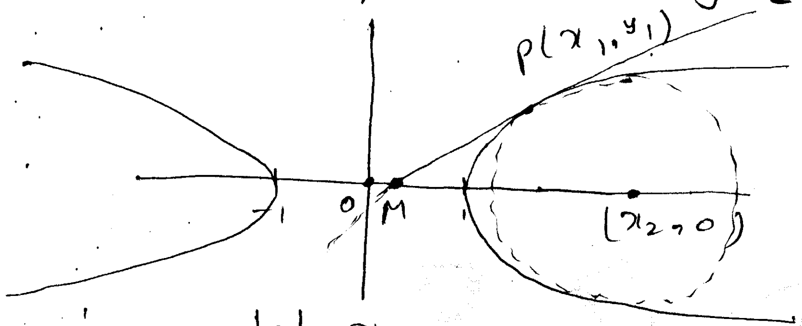
(B) $e_1 e_2 = \frac{\sqrt{7}}{2\sqrt{2}} \times \frac{1}{\sqrt{5}} = \frac{\sqrt{7}}{2\sqrt{10}}$

(C) $|e_1^2 - e_2^2| = \left|\frac{1}{5} - \frac{7}{8}\right| = \frac{27}{40}$ ✗

P.T.O

Q.50. Consider the hyperbola $H: x^2 - y^2 = 1$ and a Circle S....

(A, B, D)



Let $x_1 = \sec\theta$
$y_1 = \tan\theta$.
eqn of Tangent
$x\sec\theta - y\tan\theta = 1$.
at x axis $y = 0$
$x = \frac{1}{\sec\theta} = \frac{1}{x_1}$. So $M\left(\frac{1}{x_1}, 0\right)$
& $N(x_2, 0)$

$$l = \frac{x_1 + \frac{1}{x_1} + x_2}{3}$$

now eqn of circle
$(x - x_2)^2 + y^2 = r^2$
diff.
$x - x_2 + y\,dy/dx = 0$
Slope of Tangent $(x_1, y_1) = \frac{x_2 - x_1}{y_1} = \frac{x_1}{y_1}$.

$\boxed{x_2 = 2x_1}$

$$l = \frac{x_1 + \frac{1}{x_1} + 2x_1}{3} \qquad l = \frac{3x_1 + \frac{1}{x_1}}{3}$$

$$l = x_1 + \frac{1}{3x_1}$$

$$\frac{dl}{dx_1} = 1 - \frac{1}{3x_1^2} \quad \text{--- (A)}$$

$$M = \frac{y_1}{3} = \frac{1}{3}\sqrt{x^2 - 1}$$

P.T.O.

$$\frac{dM}{dx_1} = \frac{1}{3\sqrt{x^2-1}}, \quad \frac{dM}{dy_1} = \frac{1}{3}$$

Q.51. The Options(s) with the values of a and L ----

Q.51 (A) & (C)

$$I = \int_0^{4\pi} e^t(\sin^6 at + \cos^4 at)\,dt$$

$$= \int_0^{\pi} e^t(\sin^6 at + \cos^4 at)\,dt + \int_{\pi}^{2\pi} ---\,dt + \int_{2\pi}^{3\pi}(\text{——})\,dt + \int_{3\pi}^{4\pi}(\text{——})\,dt$$

$$= \int_0^{\pi} e^t(\sin^6 at + \cos^4 at) + \int_0^{\pi} e^{t+\pi}(\text{——})\,dt + \int_0^{\pi} e^{t+2\pi}(\text{——})\,dt + \int_0^{\pi} e^{t+3\pi}(\text{——})\,dt.$$

$$= (1+e^{\pi}+e^{2\pi}+e^{3\pi})\int_0^{\pi} e^t(\sin^6 at + \cos^4 at)$$

now

$$L = \frac{(1+e^{\pi}+e^{2\pi}+e^{3\pi})\int_0^{\pi} e^t(\sin^6 at + \cos^4 at)}{\int_0^{\pi} e^t(\sin^6 at + \cos^4 at)}$$

$$= 1+e^{\pi}+e^{2\pi}+e^{3\pi}$$

$$= \frac{e^{4\pi}-1}{e^{\pi}-1}$$

P.T.O.

Q.52 let $f, g : [-1, 2] \to R$ be continous function which...

(B) and (C)

$h(x) = f(x) - 3g(x)$

$h(-1) = f(-1) - 3g(-1) = 3 - 0 = 3$

$h(0) = f(0) - 3g(0) = 3$

$h(2) = f(2) - 3g(2) = 3$

$h(-1) = h(0) = h(2) = 3$

hence $h'(x) = 0$ for at least one value of $x \in (-1, 0)$

$h'(x) = 0$ for atleast one value of $x \in (0, 2)$

Let t_1, t_2 be the point where $h'(x) = 0$

hence $h''(x)$ will be zero for atleast one value of $x \in (t_1, t_2)$

so correct option are (B, C)

P.T.O.

Q53 (A) & (B)

Let $f(x) = 7\tan^8 x + 7\tan^6 x - 3\tan^4 x - 3\tan^2 x$

$f(x) = 7\tan^6 x[\sec^2 x] - 3\tan^2 x \sec^2 x$

$= (7\tan^6 x - 3\tan^2 x)\sec^2 x$

now

$$\int_0^{\pi/4} f(x)\,dx = \int_0^{\pi/4} (7\tan^6 x - 3\tan^2 x)\sec^2 x\,dx$$

Let $\tan x = t$, $\sec^2 x\,dx = dt$

$$= \int_0^1 (7t^6 - 3t^2)\,dt$$

$= (t^7 - t^3)\Big|_0^1 = 0$ option B is correct

now

$$\int_0^{\pi/4} x f(x)\,dx = x\int_0^{\pi/4} f(x)\,dx - \int_0^{\pi/4} 1\cdot\int f(x)\,dx$$

$$= 0 - \int_0^{\pi/4} \int (7\tan^6 x - 3\tan^2 x)\sec^2 x\,dx$$

$$= -\int_0^{\pi/4} (\tan^7 x - \tan^3 x)\,dx$$

$$= -\int_0^{\pi/4} \tan^3 x[\tan^2 x - 1][\tan^2 x + 1]\,dx$$

$$= -\int_0^{\pi/4} \tan^3 x[\tan^2 x - 1](\sec^2 x\,dx)$$

Let $\tan x = t$, $\sec^2 x\,dx = dt$

$$= -\int_0^1 t^3(t^2 - 1)\,dt$$

$$= -\int_0^1 (t^5 - t^3)\,dt$$

$$= -\left(\frac{t^6}{6} - \frac{t^4}{4}\right)_0^1$$

$= \frac{1}{12}$ (A is correct)

P.T.O

Q.54. let $f'(x) = \dfrac{192x^3}{2+\sin^4 \pi x}$ for all $x \in R$, ...

$0 = f(\tfrac{1}{2})$

Sol.

(D)

$f(x) = f(x) - f(\tfrac{1}{2})$ → because add both side $f(x)$

$= \int_{1/2}^{x} f'(x)\,dx$

now $0 \le \sin^4 \pi x \le 1$

$\Rightarrow \int_{1}^{2} 64x^3 \le \int_{1}^{x} f'(x) \le \int_{1/2}^{x} 96x^3\,dx$

$\Rightarrow (16x^4)_{1/2}^{x} \le f(x) \le [24x^4]_{1/2}^{x}$

$\Rightarrow 16x^4 - 1 \le f(x) \le 24x^4 - \dfrac{3}{2}$

$\int_{1}^{2} (16x^4 - 1)\,dx \le \int_{1}^{2} f(x)\,dx \le \int_{1/2}^{1} \left(24x^3 - \dfrac{3}{2}\right)dx$

$2.6 \le \int_{1}^{2} f(x)\,dx \le 3.9$

only D is true

P.T.O

Q.55 Let S be the set of all non-zero ---

(A, D) Ans.

$\alpha x^2 - x + \alpha = 0$

$D > 0$

$1 - 4\alpha^2 > 0$

$4\alpha^2 < 1$.

$\alpha \in (-\frac{1}{2}, \frac{1}{2}) - \{0\}$

now $|x_1 - x_2| < 1$

$(x_1 - x_2)^2 < 1$

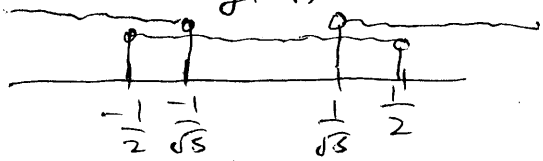
$(x_1 + x_2)^2 - 4x_1x_2 < 1$.

$\frac{1}{\alpha^2} - 4 < 1$.

$\alpha^2 > \frac{1}{5}$

$\alpha \in (-\infty, -\frac{1}{\sqrt{5}}) \cup (\frac{1}{\sqrt{5}}, \infty)$

so common region.



so $\alpha \in (-\frac{1}{2}, -\frac{1}{\sqrt{5}}) \cup (\frac{1}{\sqrt{5}}, \frac{1}{2})$

Q.56. If $\alpha = 3\sin^{-1}\left(\frac{6}{11}\right)$ & $\beta = 3\cos^{-1}\left(\frac{4}{9}\right)$ ---

Ans (BCD)

$\frac{6}{11} > \frac{1}{2}$

$\sin^{-1}\frac{6}{11} > \sin^{-1}\frac{1}{2}$

$3\sin^{-1}\frac{6}{11} > 3\cdot(30^\circ)$

$3\sin^{-1}\frac{6}{11} > 90^\circ$

$\alpha > 90^\circ$

$\cos\alpha = -ve$ (D) ✓

$\frac{4}{9} < \frac{1}{2}$ $\cos^{-1}$ is decreasing.

$\cos^{-1}\frac{4}{9} > \cos^{-1}\frac{1}{2}$

$\cos^{-1}\frac{4}{9} > 60^\circ$

$3\cos^{-1}\frac{4}{9} > 3 \times 60^\circ$

$\beta > 180^\circ$

$\cos\beta = -ve$ (A) ✗, $\sin\beta = -ve$ (B) ✓

$\alpha + \beta > 270^\circ$

$\cos(\alpha + \beta) = +ve$ (C) ✓

P.T.O.

Paragraph-2 (cont.)

Let $F: R \to R$ be a thrice differentiable - - -

Q.57 The correct statement(s) is(are)

(A)(B)(C)

$f(x) = xF(x)$

$f'(x) = F(x) + xF'(x)$

for $x = 1$

$f'(1) = F(1) + 1F'(1)$

$f'(1) = 0 + xF'(1)$

Given $F'(x) < 0$ for $x \in (\frac{1}{2}, 3)$

so $f'(1) < 0$ A is correct

$f(x) = xF(x)$

$f(2) = 2F(2)$

now $F(1) = 0$ & function is decreasing

so $F(2) < 0$

$f(2) < 0$ B is correct.

$f'(x) = F(x) + xF'(x)$

now $F(x) < 0$ for $x \in (1, 3)$ &

$F'(x) < 0$

so $f'(x) < 0$ for $x \in (1, 3)$

so $f'(x) \neq 0$ for $x \in (1, 3)$

so C is correct

Q.58 (C&D)

$f(x) = xF(x)$

$f'(x) = F(x) + xF'(x) \Rightarrow x^2 f'(x) - x^2 F(x)$

$\int_1^3 x^3 F''(x) = 40 \quad [x^3 F'(x)]_1^3 - 3\int_1^3 x^2 F'(x) = 40$

$27F'(3) - F'(1) - 3(-12) = 40$

$27F'(3) - F'(1) = 4$

$9f'(3) - 9F(3) - [f'(1) - F(1)] = 4$

$9f'(3) - f'(1) + 32 = 0$

$f'(x) = F(x) + xF'(x)$

$xf'(x) = xF(x) + x^2F'(x)$

$f(x) = xf'(x) - x^2F'(x)$

$\int_1^3 f(x)dx = \int_1^3 xf'(x) - \int_1^3 x^2F'(x) = (xf(x))_1^3 - \int_1^3 f(x)dx + 12.$

$\int_1^3 f(x)dx = -12$

P.T.O

Paragraph-2 (two)

Q.59 → Let n_1 & n_2 be the number ----

(A) & (B)

Total prabablity of selecting a red ball one of the two ----
$$= \frac{1}{2}\left(\frac{n_1}{n_1+n_2}\right) + \frac{1}{2}\left(\frac{n_3}{n_3+n_4}\right)$$

prapablity of selecting a red ball from II
$$= \frac{1}{2}\left(\frac{n_3}{n_3+n_4}\right)$$

For conditional prabablity
$$= \frac{\frac{1}{2}\left(\frac{n_3}{n_3+n_4}\right)}{\frac{1}{2}\left(\frac{n_1}{n_1+n_2}\right) + \frac{1}{2}\left(\frac{n_3}{n_3+n_4}\right)}$$

putting n_1, n_2, n_3, n_4 from options

A & B are correct.

Q.60 → A ball is drawn ----

(C & D)

From option A.

$n_1 = 4 \quad n_2 = 6$

~~If red ball is transferred then~~

$$P(R) = \frac{4}{4+6} = \frac{4}{10} = \frac{2}{5} \times \text{ from (C) } \frac{10}{30} = \frac{1}{3}$$

none of the other option satisfy this

only C & D satisfy.

P.T.O