Q.3 $\lim_{x\to 0^+} x\left[\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]\cdots\left[\frac{15}{x}\right]\right]$.

Sol$^n$: Since $\star - 1 < [\star] \le \star$.

$$\frac{1}{x}-1+\frac{2}{x}-1+\frac{3}{x}-1\cdots\frac{15}{x}-1 \le \left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]\cdots\left[\frac{15}{x}\right]\right) \le \frac{1}{x}+\frac{2}{x}\cdots\frac{15}{x}.$$

$$\frac{15.16}{2x}-15 \le \left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]\cdots\left[\frac{15}{x}\right] \le \frac{15.16}{2x}.$$

$$\frac{120}{x}-15 \le \left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]\cdots\left[\frac{15}{x}\right] \le \frac{120}{x}.$$

$\star$

~~$\lim_{x\to 0^+}$~~ $\lim_{x\to 0^+} x\star$.

By Sandwich theo.

$$\lim_{x\to 0^+}\left(\frac{120}{x}-15\right) \le \lim_{x\to 0^+}\star \le 120.$$

$$\lim_{x\to 0^+} 120-15x \le L \le \lim_{x\to 0^+} 120.$$

$$120 \le L \le 120.$$

Q.21 Two sets A & B are as under :

$A = \{(a,b) \in R \times R : |a-5| < 1 \dots$

Sol$^n$ : $A = \{(a,b) \in R \times R : |a-5| < 1 \text{ & } |b-5| < 1\}$

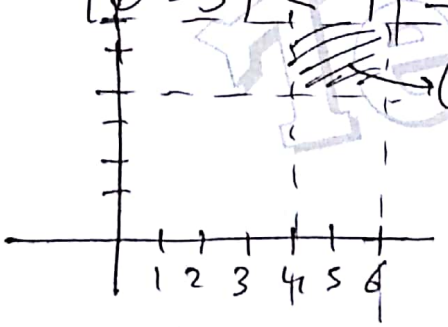
$|a-5| < 1 \Rightarrow -1 < a-5 < 1$

$4 < a < 6.$

& $|b-5| < 1 \rightarrow -1 < b-5 < 1$

$4 < b < 6.$

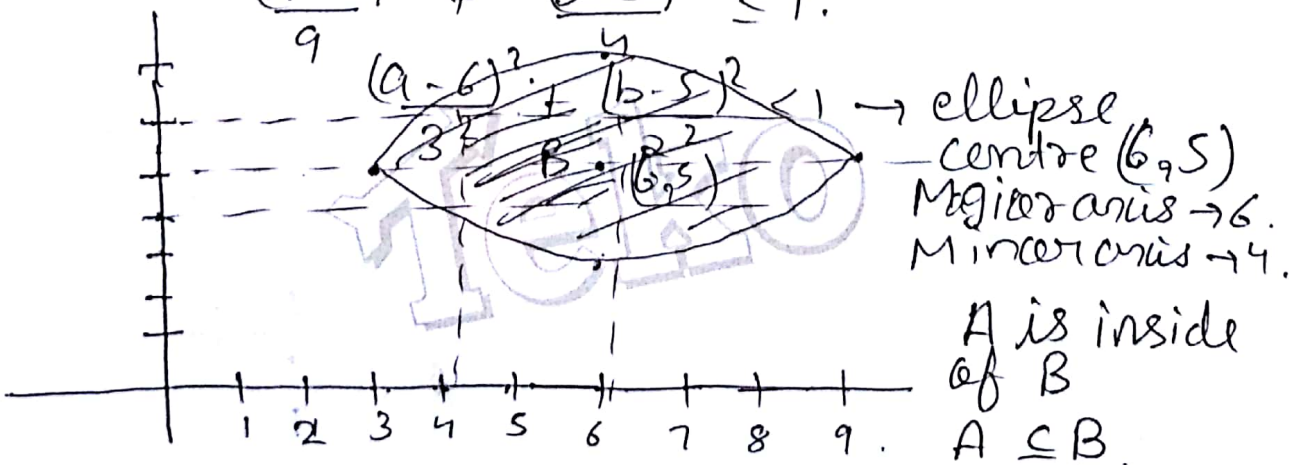
A ⇒




$B = \{(a,b) \in R \times R : 4(a-6)^2 + 9(b-5)^2 \le 36\}.$

$4(a-6)^2 + 9(b-5)^2 \le 36.$

$\frac{(a-6)^2}{9} + \frac{(b-5)^2}{4} \le 1.$



Q.26 Let A be the sum - - - -

(3) $1^2 + 2\cdot 2^2 + 3^2 + 2\cdot 4^2 + 5^2 + 2\cdot 6^2 - -$

If $B - 2A = 100\lambda$. $d = ?$

$S = 1^2 + 2\cdot 2^2 + 3^2 + 2\cdot 4^2 - - -$

$= 1^2 + 3^2 + 5^2 + 2\cdot 2^2 + 2\cdot 4^2 + 2\cdot 6^2 - - -$

A → First 20 terms

$A = \underbrace{1^2 + 3^2 + 5^2 - -}_{10 \text{ terms}} + \underbrace{2\cdot 2^2 + 2\cdot 4^2 + 2\cdot 6^2 - -}_{10 \text{ terms}}$

$= \sum (2n-1)^2 + 2\cdot\sum (2n)^2$

$= \sum 4n^2 + 1 - 4n + 8\sum n^2$

$\rightarrow \sum 4n^2 + \sum 1 - 4\sum n + 8\sum n^2 \rightarrow 12\sum n^2 - 4\sum n + \sum 1$

$\rightarrow \frac{12}{2}\frac{(n)(n+1)(2n+1)}{6} - \frac{4}{2}\frac{(n)(n+1)}{2} + n$

$\rightarrow 2n(n+1)(2n+1) - 2n(n+1) + n$

$\rightarrow 2n(n+1)(2n+1-1) + n$

$\rightarrow 4n^2(n+1) + n$ ⇒

For A $n = 10$ & For B $n = 20$.

$A = 4410$ $\quad B = 33620$

$B - 2A = 100\lambda$

$24800 = 100\lambda$

$\lambda = 248$

Q.27 Let $y = y(x)$ be the soln ...

Ans (4) soln: $\sin x \frac{dy}{dx} + y\cos x = 4x$.

$$\frac{d}{dx}(y\sin x) = 4x.$$

$$\int y\sin x = 2x^2 + C.$$

at $x = \frac{\pi}{2}$ $y = 0$.

$$0 = 2\left(\frac{\pi}{2}\right)^2 + C. \rightarrow C = -\frac{\pi^2}{2}$$

$$y\sin x = 2x^2 - \frac{\pi^2}{2} \quad \text{now } x = \frac{\pi}{6}.$$

$$y\cdot\frac{1}{2} = 2\left(\frac{\pi}{6}\right)^2 - \frac{\pi^2}{2}$$

$$y = \frac{\pi^2}{9} - \frac{2\pi^2}{2} \rightarrow -8\pi^2/9.$$

Q.28 The length of the projection ...

(1) $(5,-1,4)$ & $(4,-1,3)$ — $x+y+z=7$.

dr's $\rightarrow (1,0,1)$ dr's of plane $(1,1,1)$

dr's of line

$$\sin\theta = \frac{1.1+0.1+1.1}{\sqrt{1^2+0^2+1^2}\sqrt{1^2+1^2+1^2}} = \frac{2}{\sqrt{6}}.$$

$$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{4}{6}} = \left(\frac{1}{\sqrt{3}}\right).$$

new projection = (Length of segment) $\times \cos\theta$.

$$= \sqrt{1^2+1^2}\cdot\frac{1}{\sqrt{3}} \rightarrow \sqrt{2}\times\frac{1}{\sqrt{3}} = \sqrt{\frac{2}{3}}.$$

Q.29. Let $S = \{x \in R, x \neq 0 \ \& \ 2|\sqrt{x}-3| + \sqrt{x}(\sqrt{x}-6) + 6 = 0\}$.

Sol$^n$: Let $\sqrt{x} = y$. eq$^n$ $2|y-3| + y(y-6) + 6 = 0$.

Case-I: $y \geq 3$. $2(y-3) + y(y-6) + 6 = 0$.

Ans (4) $2y - 6 + y^2 - 6y + 6 = 0 \rightarrow y^2 - 4y = 0$.

$y = 0$ or $y = 4$.
X ✓

Case-II $0 \leq y \leq 3$.

$2.(3-y) + y(y-6) + 6 = 0$

$6 - 2y + y^2 - 6y + 6 = 0$.

$y^2 - 8y + 12 = 0 \rightarrow y = 6, y = 2$.
X ✓

$y^2 - 6y - 2y + 12 = 0$

So $y \in \{2, 4\}$ or $x \in \{4, 16\}$.

Two elements

Q-30 Let $a_1, a_2$ -- A.P. then m is equal to --

(4) $\sum_{k=0}^{12} a_{k+1} = 416. \rightarrow \frac{13}{2}(2a_1 + (13-1)(d)) = 416.$

$a_9 + a_{43} = 66.$ $a_1 + 24d = 32$ —(i)

$a_1 + 8d + a_1 + 42d = 66 \rightarrow a_1 + 25d = 33$ —(ii)

$2a + 50d = 66$ (ii) − (i) $d = 1$ & $a_1 = 8$

Now $\sum_{k=1}^{17} a_k^2 = \sum_{n=1}^{17} (7+n)^2 \rightarrow \sum_{n=1}^{17} 49 + n^2 + 14n. \rightarrow 49n + \frac{n(n+1)(2n+1)}{6} + 14\frac{(n)(n+1)}{2}$

where $n = 17$

$140m = 4760$ $m = 34$.

Q.16 (1) The sum of coefficients of all odd a —
$(x+\sqrt{x^3-1})^5 + (x-\sqrt{x^3-1})^5$ is --

$(x+y)^n + (x-y)^n = 2({}^nC_0 y^0 x^n + {}^nC_2 y^2 x^{n-2} \cdots)$

$\Rightarrow 2\cdot({}^5C_0 x^5 + {}^5C_2 (x^3-1)\cdot x^3 + {}^5C_4 (x^3-1)^2 x)^2$

$\Rightarrow 2(x^5 + 10(x^6 - x^3) + 5(x^6 + 1 - 2x^3)x)$

$\Rightarrow 2(11x^5 - 10x^3 + 5x^7 + 5x - 10x^4)$
odd.

$= 2(1 - 10 + 5 + 5) \Rightarrow 2$

Q.15. If $\sum_{i=1}^{9}(x_i - 5) = 9$ ..... $x_1, x_2 \ldots x_9$ is ..

Ans (4)

Soln: $\sum_{i=1}^{9}(x_i - 5) \Rightarrow \sum_{i=1}^{9} x_i - \sum_{i=1}^{9} 5 = 9.$

$\sum_{i=1}^{9} - 45 = 9. \rightarrow \sum_{i=1}^{9} x_i = 54. \quad -(i).$

$\hookrightarrow \bar{x} = \frac{\sum_{i=1}^{9} x_i}{9} = \frac{54}{9} = 6$

$\text{var}(x) = \sum_{i=1}^{9}(x_i - 6)^2$ .....

$\text{var}(x) = \sum_{i=1}^{9}(x_i^2 + 36 - 12x_i) / 9 \quad -(ii)$

but $\sum_{i=1}^{9}(x_i - 5)^2 = 45 \rightarrow \sum_{i=1}^{9} x_i^2 + 25 - 10x_i = 45.$

$\sum_{i=1}^{9} x_i^2 + \sum_{i=1}^{9} 25 - 10\sum_{i=1}^{9} x_i = 45.$

$\sum_{i=1}^{9} x_i^2 + 25 \times 9 - 10 \times 54 = 45.$

$\sum_{i=1}^{9} x_i^2 = 360 \quad -(iii)$

so $\text{var}(x) = \left(\sum_{i=1}^{9} x_i^2 + \sum_{i=1}^{9} 36 - 12\sum_{i=1}^{9} x_i\right)/9.$

$= (360 + 36 \times 9 - 12 \times 54)/9. = 4.$

S.D $= \sqrt{\text{var}} x = \sqrt{4} = 2$ Ans. (4 Option)

Q.24 The boolean expression.
$\sim(p \vee q) \vee (\sim p \wedge q)$ is eq$^n$ to ---
(2)

Soln:
$(\sim p \wedge \sim q) \vee (\sim p \wedge q)$
Let $\sim p = A$ $\sim q = B$ & $q = C$.
$(A \wedge B) \vee (A \wedge C)$
which $A \wedge (B \vee C)$
$\Rightarrow \sim p \wedge (\sim q \vee q) \rightarrow (T)$.
$\Rightarrow \sim p \wedge (T)$
$\Rightarrow \sim p \wedge T$.
$\Rightarrow \sim p$.

Q.25 Let m be the slope of line.
$L \Rightarrow (y-3) = m(x-2)$
(4) $y = mx + 3 - 2m$.
at B $x = 0$. $y = 3 - 2m$.
$B(0, 3-2m)$
& at A $y = 0$.
$x = \frac{2m-3}{m}$.
so $R\left(\frac{2m-3}{m}, 3-2m\right)$
$\downarrow$ $\downarrow$
$h$ , $k$.

$k = 3 - 2m$ $\Big\}$ $h = \frac{2m-3}{m}$.
$m = \frac{3-k}{2}$.
$h = 2 - \frac{3}{m}$
$h = 2 - \frac{3}{\frac{3-k}{2}}$
$h = 2 - \frac{6}{3-k}$
$h = \frac{6-2k-6}{3-k}$
$h = \frac{-2k}{3-k}$
$3h - hk = -2k$
$3h + 2k = hk$