Analisys of Paper I

JEE Advance 2014.

Maths by SUHAG KARIYA.

→ Paper I is easy in compair to all JEE (Advn) held in past first of all there is no negative marking, students must apply guess (Tukka) at the end of paper, becaus no negative.

→ Diffential & Integral Calculus → 9 question.
→ 2 Dimensional Geometry → 2 ques
→ 3D & Vectors → 4 questn
→ Permutation Com Com → 2 qu
→ Determinants & Matrices → 2 qu
→ AP & GP → 1 qu

20 qu.

Wait for solution of Paper 2 soon.

Thanks
SUHAG.

SOLUTION OF JEE ADVANCED 2014, PAPER 1, WITH IN 45 MIN., PAGE 2 OF 9

Code 'O' Maths.

More than One May Correct Type

Q.41. For every Pair of Continuous Function $f, g : [0,1] \to$ such that $\max\{f(x) : x \in [0,1]\} = \max\{g(x) : x \in [0,1]\}$, the Correct Statement(s) is (are)

A) $(f(c))^2 + 3f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0,1]$

B) $(f(c))^2 + f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0,1]$

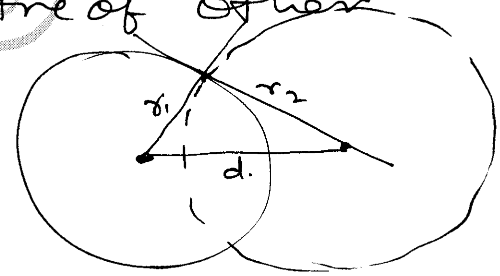
C) $(f(c))^2 + 3f(c) = (g(c))^2 + g(c)$ for some $c \in [0,1]$

D) $(f(c))^2 = (g(c))^2$ for some $c \in [0,1]$

Sol. (A&D) They are continous function in [0,1] & has same Max value it means the cut each other in between [0,1] atleast once let at $c \in [0,1]$ so option A & option D are correct.

Q.42. (B&C) A circle S passes through the point (0,1) and is orthogonal to the circle $(x-1)^2 + y^2 = 16$ & $x^2 + y^2 = 1$. Then. A) radius of S is 8 B) radius of S is 7 C) Centre of S is (−7,1) D) Centre of S is (−8,1)

Sol. (B&C) If two circles are orthogonal means they have relation that tangent at cutting point of each other pass through centre of other so $r_1^2 + r_2^2 = d^2$



As per options when we check of radius (7) & Centre (−7,1) is satisfies reation for both the circles So correct answer are (B)&(C).

P.T.O.

Q.43. From a point $P(\lambda,\lambda,\lambda)$, perpendicular PQ, PR are drawn respectively on the lines $y=x,\ z=1$ & $y=-x,\ z=-1$. If P is such that $\angle QPR$ is a right angle, then the possible value(s) of λ is are
A) $\sqrt{2}$ B) 1 C) -1 D) $-\sqrt{2}$

Sol. (B, C) Lines are

$$\frac{x-0}{1}=\frac{y-0}{1}=\frac{z-1}{0}=t \qquad \frac{x-0}{1}=\frac{-y-0}{1}=\frac{z+1}{0}=K$$

Q | R

$P(\lambda,\lambda,\lambda)$

$Q \to x=t,\ y=t,\ z=1$

$R \to x=K,\ y=-K,\ z=-1$

DR's of $PQ \to \lambda-t,\ \lambda-t,\ \lambda-1$

DR's of PR $\lambda-K,\ \lambda+K,\ \lambda+1$

$\angle QPR$ is 90° so $QP \perp PR$

$$(\lambda-t)(\lambda-K)+(\lambda-t)(\lambda+K)+(\lambda-1)(\lambda+1)=0$$

$$\lambda^2-\lambda K-t\lambda+tK+\lambda^2+\lambda K-t\lambda-tK+\lambda^2-1=0$$

$$3\lambda^2-2t\lambda-1=0$$ as per the given condition $t=\lambda$

that why $\lambda^2=1$

$\lambda=\pm1$

Q.44. Let $\vec{x}$, $\vec{y}$ and $\vec{z}$ be three vectors each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\pi/3$. If $\vec{a}$ is a non zero vector perpendicular to $\vec{x}$ and $\vec{y}\times\vec{z}$ and $\vec{b}$ is a non zero vector perpendicular to $\vec{y}$ & $\vec{z}\times\vec{x}$, then

(A, B, C)

A) $\vec{b}=(\vec{b}\cdot\vec{z})(\vec{z}-\vec{x})$ B) $\vec{a}=(\vec{a}\cdot\vec{y})(\vec{y}-\vec{z})$

C) $\vec{a}\cdot\vec{b}=-(\vec{a}\cdot\vec{y})(\vec{b}\cdot\vec{z})$ D) $\vec{a}=(\vec{a}\cdot\vec{y})(\vec{z}-\vec{y})$

Sol. Very simple, Class room question

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Q.45 let $f: \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to R$ be given by $f(x) = \left(\log(\sec x + \tan x)\right)^3$ then $f(x)$ is A) an odd function B) is a one one function C) $f(x)$ is onto function D) $f(x)$ is an even function.

Sol. (A & B & C)

$f'(x) = 3\left(\ln(\sec x + \tan x)\right)^2 \cdot \frac{1}{\sec + \tan}(\sec\tan + \sec^2)$

$= \sec x \cdot 3\left(\ln(\sec x + \tan x)\right)^2 \longrightarrow +ve$ for $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

so $f(x)$ is increasing. So it is one one function (B)

$f(x) = \left(\ln(\sec x + \tan x)\right)^3$

$f(-x) = \left(\ln(\sec x - \tan x)\right)^3 = \left(\ln\left(\frac{1}{\sec x + \tan x}\right)\right)^3 = -f(x)$

so $f(x)$ is odd function (A) } $f(x)$ is onto function (C)

Q.48. Let M be a 2x2 symmetric matrix with integer entries. Then M is a invertible if

A) the first column of M is the transpose of second row of M.

B) the second row of M is transpose of the first column of M

C) M is a diagonal matrix with non zero entries in main diagonal

D) The product of entries in main diagonal of M is not the square of an integer.

Sol. 46 & 47 on Page 5

Sol. Ans 'C & D'

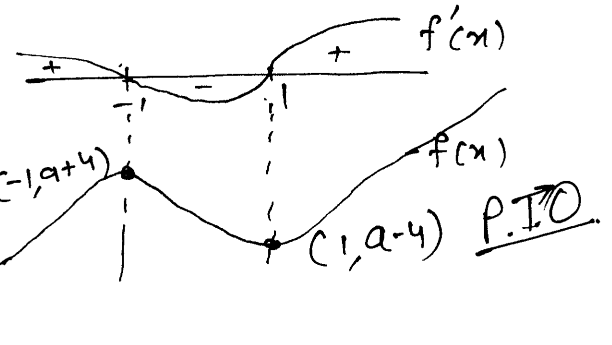
Q.49. let $a \in R$ and let $f: R \to R$ be given by $f(x) = x^5 - 5x + a$ then $f(x)$ has A) three real roots if $a > 4$ B) only one real root if $a > 4$ C) three real root if $a < -4$ D) three real roots if $-4 < a < 4$.

Sol. $f'(x) = 5x^4 - 5 = 5(x-1)(x+1)(x^2+1)$

(B) (D)

$f(-1) = -1 + 5 + a = a + 4$

$f(1) = 1 - 5 + a = a - 4$



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Q.46. let $f:[a,b]\to[1,\infty)$ be a continous fuction and let $g:R\to R$ be defined as

$$g(x)=\begin{cases}0 & ; x<a\\ \int_a^x f(t)dt & ; a\le x\le b\\ \int_a^b f(t)dt & x>b\end{cases}$$ then g(x) is

A) g(x) is Continuous but not differentiable at a

B) g(x) is differentiable on R

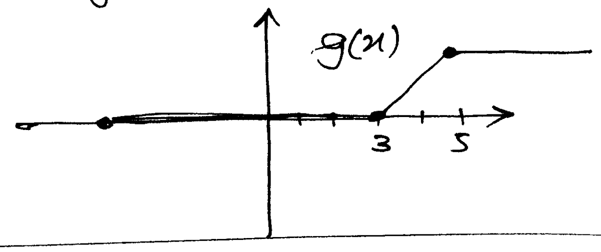
C) g(x) is cont. but not diff. at b.

D) g(x) is cont. & diff at either a or b but not both

Sol. let consider f(x) = 1. now get g(x) as.
& a = 3, b = 5

$$g(x)=\begin{cases}0 & ; x<3\\ x-3 & ; 3\le x\le 5\\ 2 & ; 5<x\end{cases}$$

(A & C)



Q.47. let $f:(0,\infty)\to R$ be given by $f(x)=\int_{1/x}^{x} e^{-\left(t+\frac{1}{t}\right)}\frac{dt}{t}$ then

A) f(x) is montonically ↑ on $[1,\infty)$

B) f(x) ———— ↓ on (0, 1)

C) $f(x)+f\left(\frac{1}{x}\right)=0$ for all $x\in(0,\infty)$

D) $f(2^x)$ is an odd fution of x on R

Sol. Apply newton lebnitz $f'(x)=\frac{e^{-(x+\frac{1}{x})}}{x}\cdot 1-\frac{e^{-(\frac{1}{x}+x)}}{(1/x)}\left(-\frac{1}{x^2}\right)$

$f'(x)=2\left(\frac{e^{-(x+\frac{1}{x})}}{x}\right)=+ve$

f(x) ↑

(A)(C)(D)

→ Simply

→ $f(2^x)+f(2^{-x})=0$

Sol. of 48 & 49 on Page 4, Sol of Q50 on Page 6

P.T.O.

Q.50. (A,B) Let M and N be two 3×3 matrices such that MN = NM. Further, if $M \neq N^2$ and $M^2 = N^4$, then

A) determinant of $(M^2 + MN^2)$ is 0
B) there is a 3×3 non zero matrix U such that $(M^2 + MN^2)U$ is the zero matrix.
C) determinant of $(M^2 + MN^2) \geq 1$
D) for a 3×3 matrix U, if $(M^2 + MN^2)U$ equals the zero matrix then U is the zero matrix.

Sol. $M^2 - N^4 = 0$

$(M+N^2)(M-N^2) = 0 \quad \because MN = NM$

as $M - N^2 \neq 0 \Rightarrow |M+N^2| = 0$

Integer Type

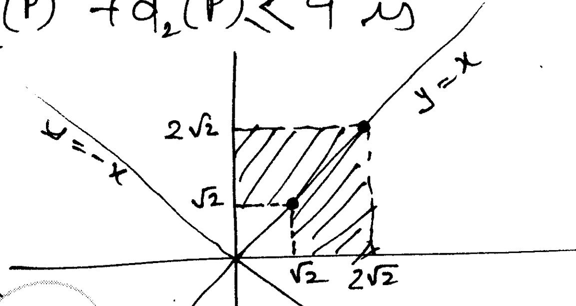
Q.51. For a point P in the plane, let $d_1(P)$ & $d_2(P)$ be the distance of the point P from the lines $x - y = 0$ & $x + y = 0$ respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane & satisfying $2 \leq d_1(P) + d_2(P) \leq 4$ is

Sol. R = big square − small square

$= (2\sqrt{2} \times 2\sqrt{2}) - (\sqrt{2})(\sqrt{2})$

$= 8 - 2$

$= 6$



Q.52. Let $\vec{a}, \vec{b}, \vec{c}$ be three noncoplanar unit vectors such that the angle between every pair of them is $\frac{\pi}{3}$. If $\vec{a}\times\vec{b} + \vec{b}\times\vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$, where p, q, r scalars, then the value of $\frac{p^2 + 2q^2 + r^2}{q^2}$ is

Ans '4'

P.T.O.

Q.53. The largest value of the non-negative integer 'a' for which

$$\lim_{x\to 1}\left\{\frac{-ax+\sin(x-1)+a}{x+\sin(x-1)-1}\right\}^{\frac{1-x}{1-\sqrt{x}}} = \frac{1}{4}$$

Sol.

$$\log_e \frac{1}{4} = \lim_{x\to 1}\left(\frac{1-x}{1-\sqrt{x}}\right)\left(\log_e\left(\frac{a(1-x)+\sin(x-1)}{(x-1)+\sin(x-1)}\right)\right)$$

$$= \lim_{x\to 1}\left(\frac{2(1-\sqrt{x})}{(1-\sqrt{x})}\right)\log_e\left(\frac{-a+\frac{\sin(x-1)}{x-1}}{1+\frac{\sin(x-1)}{x-1}}\right)$$

$$= \lim_{x\to 1}(2)\log_e\left(\frac{-a+1}{1+1}\right)$$

$$\log_e\frac{1}{4} = \log_e\left(\frac{1-a}{2}\right)^2$$

$$\pm\sqrt{\frac{1}{4}} = \frac{1-a}{2} \quad\rightarrow\quad \pm\frac{1}{2} = \frac{1-a}{2}$$

$$\pm 1 = 1-a$$

$$a = 1\mp 1 < \begin{matrix}0\\2\end{matrix}$$

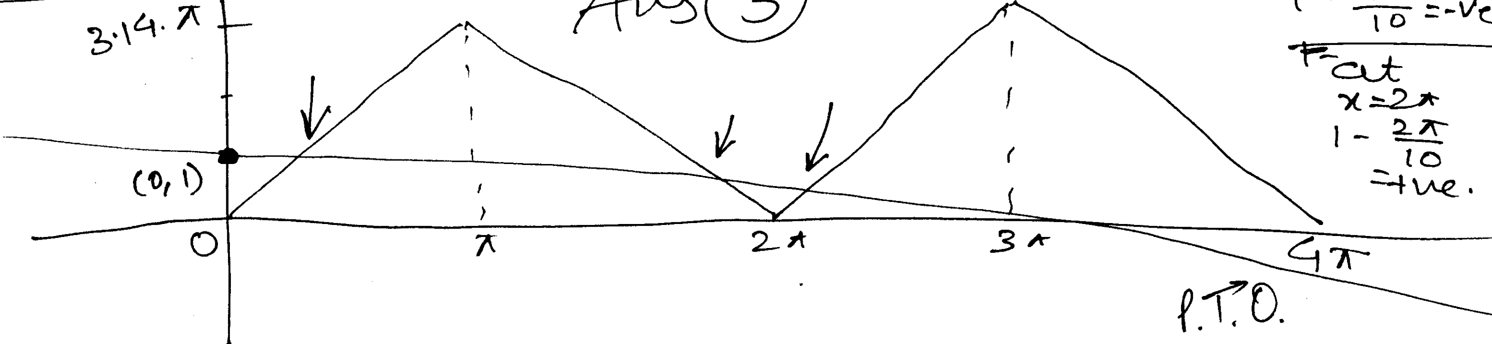
Ans "2"

Q.54. let $f:[0,4\pi)\to[0,\pi]$ be defined by $f(x)=\cos^{-1}(\cos x)$ the number of points $x\in[0,4\pi]$ satisfying the eq $f(x)=\frac{10-x}{10} = 1-\frac{x}{10} = -\frac{1}{10}x+1$

at $x=4\pi$: $1-\frac{4\pi}{10}$ = −ve

at $x=2\pi$: $1-\frac{2\pi}{10}$ = +ve.

Sol. Ans (3)



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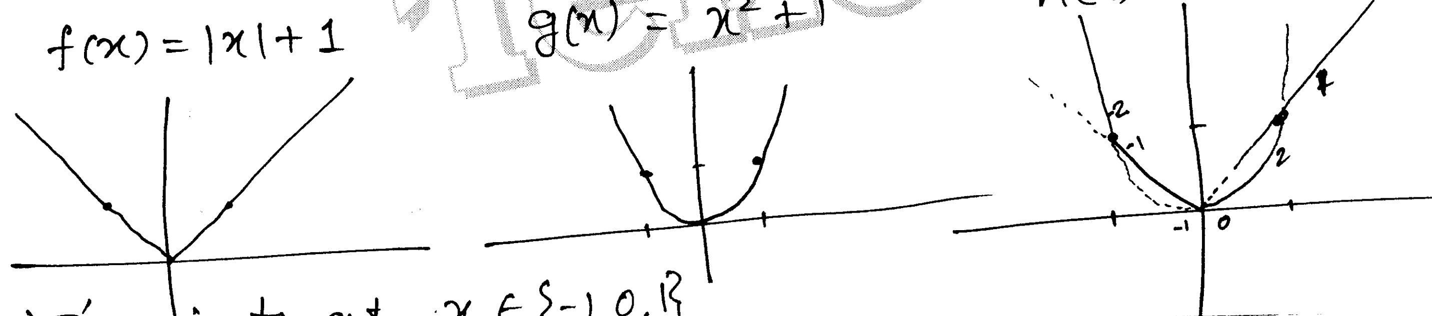
Q.55. let $f: R \to R$ and $g: R \to R$ be respectively given by $f(x) = |x| + 1$ and $g(x) = x^2 + 1$. Define $h: R \to R$ by

$$h(x) = \begin{cases} \text{Max } \{f(x), g(x)\} & \text{if } x \le 0 \\ \text{min } \{f(x), g(x)\} & \text{if } x > 0 \end{cases}$$

The number of points at which $h(x)$ is not differentiable is

Sol. $f(x) = |x| + 1$ $\quad g(x) = x^2 + 1$ $\quad h(x)$ will be.



Ans '3' points at $x \in \{-1, 0, 1\}$

Q.56. let $n_1 < n_2 < n_3 < n_4 < n_5$ be positive integers such that $n_1 + n_2 + n_3 + n_4 + n_5 = 20$. Then the number of such distinct arrangements $(n_1, n_2, n_3, n_4, n_5)$ is

Sol.

1, 2, 3, 4, 10 → 1, 2, 4, 5, 8 $\qquad$ 2, 3, 4, 5, 6

1, 2, 3, 5, 9 → 1, 3, 5, 4, 7

1, 2, 3, 6, 8 → 1, 2, 4, 6, 7

Ans (7)

Q.57. let $n \ge 2$ be an integer. Take n distinct points on a circle and join each pair of points by a line segment. Colour the line segment joining every pair of adjacent point by blue and the rest by red. If the number of red and blue line segments are equal, then the value of n is

Sol. (5) $\quad$ number of diagonals = number of sides

$${}^nC_2 - n = n \implies {}^nC_2 = 2n$$

$n = 5$

P.T.O.

SOLUTION OF JEE ADVANCED 2014, PAPER 1, WITH IN 45 MIN., PAGE 9 OF 9

Q.58. The slope of the tangent to the curve $(y-x^5)^2 = x(1+x^2)^2$ at the point $(1,3)$ is

Sol. diff.

$$2(y-x^5)\left(\frac{dy}{dx}-5x^4\right) = 1(1+x^2)^2 + x.2(1+x^2)(0+2x)$$

$x=1$ & $y=3$

Ans (8)

$$2(3-1)(m-5) = (1+1)^2 + 1.2.(1+1)(0+2.1)$$

$$4(m-5) = 4+4(2)$$

$$4(m-5) = 12$$

$$m-5 = 4$$

$$m = 8$$

Q.59. Let a, b, c be positive integers such that $\frac{b}{a}$ is an integer. If a, b, c are in geometric progression and the arithmetic mean of a, b, c is b+2, then the value of $\frac{a^2+a-14}{a+1}$

Sol. let a, b, c

Ans (4)

$$\frac{b}{r}, b, br$$

$$\frac{\frac{b}{r}+b+br}{3} = b+2$$

Ans (4)

$$b\left(\frac{1}{r}+r-2\right) = 6$$

let $r=2$, $b=12$

a, b, c → 6, 12, 24

$a=6$

So $\frac{a^2+a-14}{a+1} = \frac{36+6-14}{6+1}$

$$= \frac{42-14}{7}$$

$$= 6-2 = 4$$

Q.60. The value of $\int_0^1 4x^3\left\{\frac{d^2}{dx^2}(1-x^2)^5\right\}dx$. Ans (2)

(I = $4x^3$, II = $\frac{d^2}{dx^2}(1-x^2)^5$)

$$= \left[4x^3\left((1-x^2)^5\right)'\right]_0^1 - \int_0^1 12x^2\left((1-x^2)^5\right)'$$

$$= (4.0-0) - \left[12x^2(1-x^2)^5 - \int 24x(1-x^2)^5\right]_0^1$$

$= 0$, 0

$$= +12\int_0^1 2x(1-x^2)^5 dx \qquad 1-x^2 = t, \quad 2x = -dt$$

$$= -12\int_1^0 t^5 = 2\left[t^6\right]_0^1 = 2$$

P.T.O.